A Text Book of Physics

C. R. DASGUPTA

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For H. S. courses of West Bengal & Tripura, +2 Courses of Central and Delhi Board and other states

A Text Book of PHYSICS

PART II

[Light, Magnetism, Electrostatics, Current electricity and Modern Physics]

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&
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Preface to the Sixth Edition

I have the pleasure in presenting to the readers the revised sixth edition of the 2nd part of 'A Text Book of Physics'. I took this opportunity to revise thoroughly the entire text of the book and had added some important and valuable information in various chapters. The number of worked out sums has also been increased. The exercises have been made up-to-date in the light of recent I.I.T., J.E.E. and H.S. examinations.

I am confident that this revised volume will be able to serve the H.S. students better than before.

Department of Physics
City College, Calcutta.
7th June, 1988

C. R. Dasgupta

Preface to the First Edition

The educational pattern of the state has again undergone a drastic change. The old XI-class H. S. Course has been scrapped and a 10-year secondary course followed by a 2-year H. S. course has been introduced in its place. Students, who passed the 'Madhyamik' examination in 1976, got admitted in the 1st year of the so-called+2 course and will appear in the final examination in 1978. The present volume is the second part of the book entitled 'A Text Book of Physics' written on the syllabus of the general stream for Physics. The first part, it may be mentioned, was published in July, 1976.

It goes without saying that the present volume has been written in the same style and manner as the first part which had, in the meantime, been recommended widely to the students of schools and colleges imparting H. S. education. This volume contains the subject-matter of the second paper viz. light, magnetism, electrostatics, current electricity and modern physics.

Some errors due to 'Printer's devil' may creep in and the author will be grateful to the readers if such errors are brought to his notice.

Department of Physics
City College, Calcutta.
March, 1977.

C. R. Dasgupta

Syllabus for H. S. Course of W. B. Higher Secondary Council

PAPER II

1. Optics

Recapitulation of basic topics on rectilinear propagation of light. Photometry—basic concepts, Lummer-Brodhun Photometer.

Reflection at plane surfaces—law of reflection, Periscope.

Reflection at curved surfaces—basic definitions and mathematical relations between u, v, f and r.

Refraction of light—laws of refraction, total reflection, critical angle, formation of image, refraction through a prism, minimum deviation, relation between δ and μ.

Image formation by thin convex and concave lenses ; basic definitions : deduction of formula connecting u, v and f. Spherical and chromatic aberrations (qualitative discussion).

Dispersion of light, formation of a spectrum, different types of spectra (line and continuous), emission and absorption spectra—discussions on a very limited level. Atmospheric refraction.

Optical instruments : Camera, Compound microscope, Astronomical telescope, Binoculars.

The human eye—defects of vision ; remedy as example of combination of lenses.

2. Magnetism

Recapitulation of basic concepts with demonstration. Magnetic field, magnetic lines of force, Coulomb's law, unit pole, magnetic intensity.

Molecular theory of magnetism. Fero-para-and dia-magnetic substances. Permeability, Susceptibility.

Terrestrial magnetism—Declination, Dip and Horizontal intensity (methods of measurement not required).

3. Electrostatics

Basic electrostatic phenomena. Electroscopes—simple experiments. Conductors and insulators. Electrostatic induction. Distribution of charge on conductors. Surface charge density.

Coulomb's law of electrostatics. Electric field. Potential and its measure ; qualitative idea of relation between intensity and potential. Capacitance, Capacitance of a sphere, Simple capacitors. Factors determining capacitance, Capacitors in series and parallel.

Electrostatic machines—Electrophorous and Van de Graaff generator.

4. Current Electricity

Voltaic cells—Primary and secondary cells. Ohm's law ; Resistivity. Temperature coefficient of resistances. Resistances in series and parallel.

Wheatstone bridge (measurement of resistances of resistors with this bridge). Post office Box. Metre bridge. Potentiometer—measurement of potential difference and current.

Heating effect of current—Joule's law. Determination of '*J*' by electrical method ; Thermo-electricity. Peltier and Thomson's effect (general outline).

Chemical effect of current—Electrolysis, Faraday's laws, Practical applications.

Electromagnetism :

Magnetic effect of current—action of magnet on current, current on magnet and current on current. Demonstrations. Simple galvanometers, (tangent, D' Arsonval). Voltemeters, Ammeters and their uses.

Electromagnetic induction : Laws of Faraday and Lenz. Self and mutual inductance. Induction coil.

Ideas about alternating current (qualitative discussion). Principles of motors and dynamos.

5. Modern Physics

Cathode rays and their properties. X-rays—discovery and properties. Nature of X-rays ; Practical applications.

Thermionic emission—descriptive ; vacuum tubes, diodes—principle of rectification. Use. Triodes—principle of amplification, Elementary principles of radio.

Photo electric emission. Photo tube and its uses. Particle nature of radiation. Elementary ideas of Quantum theory (No details on the determination of Plancks constant).

Electronic structure of atom—brief introduction to the Bohr model of atom : Semi-conductor ; diodes and triodes, transistors and their uses (descriptive).

The nucleus of the atom and its structure : Atomic number and mass number ; isotopes.

Radioactivity—its discovery, alpha, beta and gamma rays and their properties. Radioactive decay (no mathematics—graphical illustration only), Radio-isotopes, Artificial transmutation of elements with illustrations. Mention of mass-energy equivalence. Nuclear fission and its uses. Nuclear fusion—its importance.

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OPTICS

1

PROPAGATION OF LIGHT AND PHOTOMETRY

1.1. Nature of light :

We become acquainted with our surroundings mainly through our sense of vision. Whenever we open our eyes, we see various objects around us. But can we see things simply by opening our eyes ? In pitch-darkness, we cannot see anything even if we keep our eyes wide open. To see things some external stimulus is necessary for our eyes. In other words, when light from objects around us falls on our eyes, the objects become visible to us. Hence, light may be regarded as an external stimulus which causes the sensation of sight.

Like heat, electricity etc, *light is a form of energy.* When a metallic ball is heated, it emits heat energy. The chemical potential energy of coal is, in this case, converted into heat energy. If the heating is continued, the ball will become red-hot and will emit light. In this case, a part of chemical energy is transformed into light energy. From all these examples, we conclude that light is a form of energy. Light, itself, is invisible but it makes other things visible to us. We cannot see light but we see lighted objects. Like other forms of energy, *light is invisible to us.*

Light is propagated from one place to another in the form of waves which are transverse in nature and very small in length. Light waves have an extremely high velocity of about 1,86,000 miles per second in vacuum.

1.2. Rectilinear motion of light :

One of the most obvious facts about the behaviour of light is that it travels from one place to another in *a straight line.* When motor head-lights are switched on in a dark road the streaks of light are found to travel in straight lines. You are familiar with the appearance of shafts of sunlight coming through a rift in the clouds or with the passage of a ray of light into a dark room through a hole in the window. In all these cases, you must have seen that light travels in straight line.

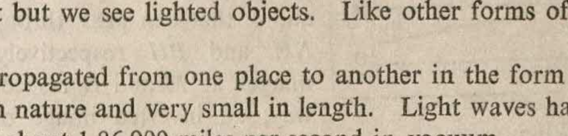


Fig. 1.1

We can also demonstrate this fact by a simple experiment. Three card-board screens having small holes in their centres stand in a row (Fig. 1.1). These are arranged so that the holes are in a straight line along with a candle flame placed on the left of the card-board *A*. Light can be received by an eye placed

on the right of the card-board *C*, provided the eye is in the same line with the holes. If, however, one of the boards is displaced so that the holes are no longer in a straight line, the light is cut off. This conclusively proves that light travels in a straight line, for if light could have moved in a zig-zag way, it could have easily passed through the hole of the displaced board and reached the eye.

1.3. Pin-hole camera :

The principle of action of this camera illustrates the rectilinear propagation of light.

Fig. 1.2 shows a pin-hole camera. It is a rectangular box. Its front surface is made of card-board with a very fine hole at *H*. The opposite back surface *X* is made of ground glass. The inside of the box is painted black in order to prevent reflection of light by the sides of the box. An inverted image of an object held in front of the pin-hole is formed on the ground glass plate.

Suppose that a candle is kept in front of a pin-hole camera [Fig. 1.2]. From the point *A* of the flame, say, rays of light travel in all directions but the only narrow pencil that can pass through the hole is *AH* which produces the image *B* of the point *A* of the flame. Similarly, the only narrow pencils starting from *N* and *P* that can pass through the hole are *NH* and *PH* respectively which form images at *M* and *S* respectively. In this way, an inverted image of the flame is produced on the ground glass screen.

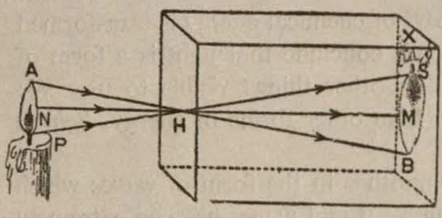


Fig. 1.2

If the screen at the back be replaced by a photographic plate, then a photograph may be obtained on giving a comparatively long exposure.

Some discussions on the pin-hole camera :

(i) If the size of the pin-hole be increased, we get no well defined inverted image on the screen. This is due to the fact that a large hole is an aggregation of a number of small pin-holes, each of which will cast its corresponding image on the screen. The superposition of these images will produce a general blurring effect. The image will be sharp if the hole is very small.

(ii) The image does not depend upon the shape of the hole as long as the hole is very small. The image will be of the shape of the object, if the hole is a pin-hole—no matter whether the hole is circular, oval or triangular. For this reason, horizontal sun-rays coming through a *triangular* pin-hole on a window cast circular image on the wall behind.

(iii) If the object be moved away from the pin-hole, keeping the screen fixed in position, the size of the image diminishes.

(iv) If the screen be moved away from the pin-hole, keeping the object fixed in position, the size of the image increases.

If a straight line *NHM* be drawn through the pin-hole *H* perpendicular to the object and the image (Fig. 1.2), then the sizes of the object and its image bear the following relation with their distances from the pin-hole ;

$$\frac{\text{Size of the object}}{\text{,, ,, ,, image}}=\frac{\text{Distance of the object from the hole } (NH)}{\text{,, ,, ,, image ,, ,, ,, } (HM)}$$

Example 1. *In a pin-hole camera, the screen is 6 inches away from the hole. How far a man will stand from the camera in order to produce an image on the screen half the size of the man ?*

Ans. We know, $\frac{\text{Size of the object}}{\text{,, ,, ,, image}}=\frac{\text{Distance of the object from the hole}}{\text{,, ,, ,, image ,, ,, ,,}}$

or, $2=\frac{\text{Distance of the object}}{6}$

$\therefore$ the distance of the object from the hole $=6\times2$ inches $=1$ ft.

Hence the man should stand 1 ft. away from the camera.

Example 2. *An image 1·5 inches high of a building is formed in a pin-hole camera whose screen is 2·6 inches away from the hole. If the camera is placed 91 ft. away from the building, what is the height of the building ?*

Ans. We know, $\frac{\text{height of the object}}{\text{height of the image}}=\frac{\text{object distance}}{\text{image distance}}$

or, $\frac{\text{height of the object}}{\frac{1\cdot5}{12}}=\frac{91}{\frac{2\cdot6}{12}}$

$\therefore$ height of the object $=\frac{91\times1\cdot5}{2\cdot6}$ ft. $=52\cdot5$ ft.

Example 3. *The distance between the screen and the hole of a pin-hole camera is 20 cm. Find out the distance of an object from the hole when the image on the screen will be ⅓rd the size of the object.* *[H.S. Exam. 1983]*

Ans. We have, $\frac{\text{size of the object}}{\text{,, ,, ,, image}}=\frac{\text{Distance of the object from the pinhole}}{\text{,, ,, ,, image ,, ,, ,,}}$

In this case, $3=\frac{\text{Distance of the object from the pin-hole}}{20}$

$\therefore$ Required distance $=60$ cm.

Example 4. *A pin-hole camera takes the picture of the sun. The screen is 100 cm. from the hole and the sun subtends an angle ½° at the hole. Find the diameter of the image formed on the screen.*

Ans. The sun subtends an angle of $\frac{1}{2}^\circ=\frac{1}{2}\times\frac{2\times3\cdot14}{360}$ rad. $=\frac{3\cdot14}{360}$ rad. at the hole.

So, $\frac{\text{diameter of the sun}}{\text{distance of the sun from the hole}}=\frac{3\cdot14}{360}$

Again, from the principle of pin-hole camera, we can write,

$$\frac{\text{diameter of the image}}{\text{distance of the screen from the hole}}=\frac{\text{diameter of the sun}}{\text{distance of the sun from the hole}}$$

or, $\frac{\text{diameter of the image}}{100}=\frac{3\cdot14}{360}$

$\therefore$ Diameter of the image $=\frac{3\cdot14}{360}\times100=0\cdot872$ cm.

1.4. Formation of shadows :

We know that opaque objects produce shadows. Formation of shadows affords us further illustration of the fact that light travels in straight lines. If light could have travelled in a zig-zag way, shadows would not have been formed. Depending upon the relative sizes of the obstacle and the source of light, the shadows may be of different shape and nature. Several cases of formation of shadow are considered below :

(i) A point source and an extended obstacle :

S is a point source and *AB* is a spherical obstacle (opaque). *M* is a screen held behind the obstacle (fig. 1.3). Rays of light from the point source *S* travel in all directions. Of these rays, those which pass just by the side of the obstacle, like *SA*, *SB* etc and those above or below them, can easily reach the screen but no light is found to reach the screen, which lies within the conical space *SAB* because they are obstructed by the opaque obstacle *AB*. As a result, the portion *A'B'* of the screen will be dark and will be circular in shape.

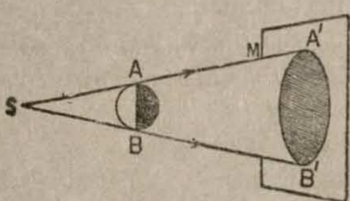


Fig. 1.3

Thus *A'B'* is the shadow of the obstacle *AB*. It has the shape of the contour of the obstacle as seen from the source. If the screen be moved away, the shadow increases in size but it becomes less dark.

(ii) A source of finite size and an obstacle larger in size than the source :

S_1S_2 is a source of light of finite size, *AB* the opaque obstacle having size greater than that of the source and *M* a screen (fig. 1.4). The extended source S_1S_2 may be looked upon as an assemblage of a number of point sources. S_1 and S_2 are two such extreme point sources.

Now, the cone of rays starting from the source S_1 and bounded by the lines like S_1A, S_1B, etc is obstructed by the obstacle *AB* and is unable to reach the screen. They produce a shadow on the screen extending from *E* to *F*. Similarly, the cone of rays starting from the lowest point S_2 and bounded by the lines like S_2A, S_2B etc, is not capable of reaching the screen. The shadow they produce extends from *G* to *H*. The shadow formed due to other intermediate points on the source will occupy the intervening position, between *G* and *F*. From the fig. 1.4, it is clear that the portion *EH* is *completely dark* because no light reaches this portion either from S_1 or from S_2 and hence from no other intermediate points of the source. The portions *GE* and *HF* are *semi-dark* because in *GE* light reaches from the upper part of the source and in *HF* from the lower part.

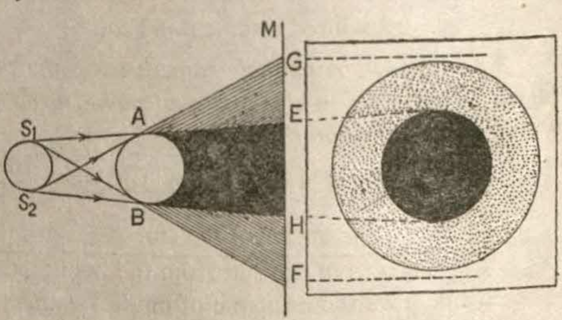


Fig. 1.4

Thus in the case of a source of finite size, the shadow consists of dark region surrounded by semi-dark region. The dark region is called *Umbra* and the semi-dark region is called *Penumbra.* The right hand portion of the fig. 1.4 shows the actual appearance of the shadow. The central dark circular portion is the umbra and the outer semi-dark portion is the penumbra.

From the figure it is clear that when the screen is moved away from the obstacle, the size of the shadow increases but its character remains unchanged.

(iii) A source of finite size larger than the obstacle :

S_1S_2 is a source of finite size larger than the obstacle AB. M is a screen placed behind the obstacle (Fig. 1.5). As before the extended source may be regarded as an aggregation of a number of point sources. Suppose S_1 and S_2 are two such extreme point sources.

The cone of rays starting from S_1 and bounded by the lines like S_1A, S_1B etc is obstructed by the obstacle and is unable to reach the screen. A shadow extending from K to D is thereby formed on the screen.

Similarly, the cone of rays starting from S_2 and bounded by the lines like S_2A, S_2B etc is obstructed by the obstacle and they produce a shadow on the screen extending from G to C. The shadow formed due to other intermediate points on the source will occupy the intervening positions between C and D.

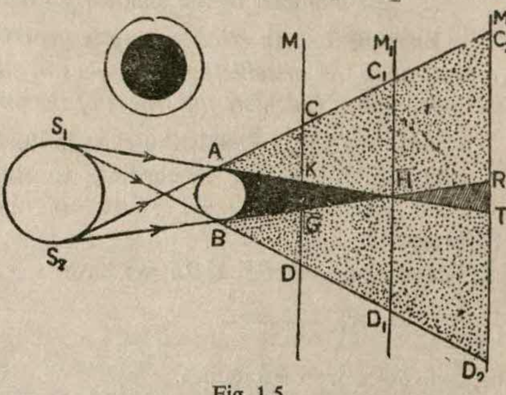


Fig. 1.5

It is clear from the figure that KG is the umbral portion of the shadow and KC and GD constitute the surrounding penumbral portions. In this case, the umbral cone is found to be converging and the penumbral cone diverging. As a result, the character of the shadow will change with the displacement of the screen. As the screen is shifted away from the obstacle, the umbral cone is gradually reduced until at the position M_1 it becomes merely a point (H). If the screen be moved further away as at M_2 say, there is no umbra at the central region; instead there is an opposite diverging cone HRT within which light from the peripheral part of the source can enter. An eye placed anywhere in this diverging cone and looking towards the obstacle would see it as a dark object with light above and below as shown in left-hand top corner of Fig. 1.5. If the screen be gradually moved further away, the penumbral portion becomes fainter and fainter until it becomes difficult to distinguish between light and shade.

In this connection it may be mentioned that shadows of the leaves of trees on the ground exhibit umbra and faint penumbra. Here, the sun is the source of light, the leaves are the opaque obstacles and the ground is the screen. Since the ground and the leaves are near to each other and the sun is far away, we see both umbra and penumbra in the shadow. Similarly, when an aeroplane flies at a very

low altitude, its shadow on the ground is distinguishable. But as the aeroplane moves higher up (*i.e.* the distance between the object and the screen increasing), the shadow becomes gradually indistinguishable.

Example 1. *A spherical opaque body of diameter 4 inches is kept at a distance of 1 ft. from a point source. A screen is placed 1 ft. away from the centre of the spherical body. What is the diameter of the shadow formed on the screen ?*

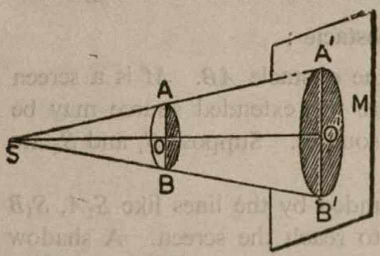


Fig. 1.6

Ans. Let S be the source, AB, the obstacle and M the screen. $A'B'$ is the shadow formed on the screen [Fig. 1.6].

According to the question, $SO=1$ ft ; and $OO'=1$ ft. Hence $SO'=2$ ft ; Also $AB=4$ inches.

$$\text{Now,}\quad \frac{AB}{A'B'}=\frac{SO}{SO'} \quad \text{or,} \quad \frac{4}{A'B'}=\frac{1\times 12}{2\times 12}$$

$\therefore$ $A'B'=8$ inches,

$\therefore$ the diameter of the shadow$=8$ inches,

Example 2. *An electric lamp is kept in a glass bulb of diameter 4 inches in a dark room. A metallic ball, 2 inches in diameter, is placed 6 inches away from the glass bulb. Calculate the length of the umbral cone of the ball.*

Ans. In Fig. 1.7, B is the glass bulb, D the metallic ball and CE the length of the umbral cone. According to the problem, $AB=2''$; $CD=1''$; $AC=6''$ and $CE=x$ (say).

From similar Δ^s ABE, CDE, we have,

$$\frac{AB}{AE}=\frac{CD}{x} \text{ or, } \frac{2}{6+x}=\frac{1}{x}$$

or, $2x=6+x$ or, $x=6$ inches.

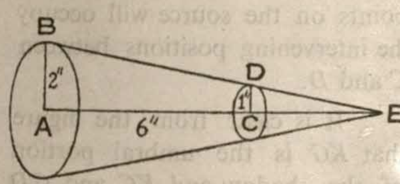


Fig.1.7

So, the length of the umbral cone is 6 inches.

Example 3. *An opaque circular disc of 2·6 cm. diameter is held normally on the path of rays from the sun. A screen is placed behind the disc such that the diameter of the umbra on the screen is zero. Calculate the distance between the disc and the screen if the diameter of the sun be $1{\cdot}30\times 10^6$ km and the distance between the sun and the disc be $1{\cdot}50\times 10^8$ km.*

Ans. AB=diameter of the sun$=1{\cdot}3\times 10^6$ km ; CD=the diameter of the disc$=2{\cdot}6$ cm; R=the point on the screen where the umbra ends ; EF is the screen [Fig. 1.8]. From the figure, we get, $\dfrac{AB}{CD}=\dfrac{PR}{QR}$

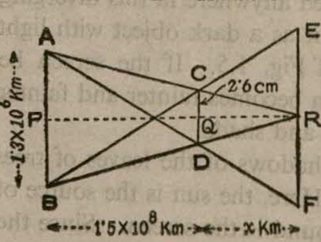


Fig. 1.8

$$\text{or,} \quad \frac{1{\cdot}3\times 10^6}{2{\cdot}6\times 10^{-5}}=\frac{1{\cdot}5\times 10^8+x}{x}=\frac{1{\cdot}5\times 10^8}{x}+1$$

$$\text{or,} \quad \frac{10^{11}}{2}=\frac{1{\cdot}5\times 10^8}{x} \quad \text{[Neglecting 1]}$$

$\therefore \quad x=\frac{3}{10^3}$ km$=3$ metre

Example 4. *If the sun subtends an angle of 32 minutes of arc at a point on the earth, calculate the least height of an aeroplane of wing span 100 ft. such that its shadow on the ground may consist of penumbra only.*

Ans. At the least height from the given point, the umbral region of the shadow will reduce to a point and the whole of the shadow will be occupied by penumbra. In Fig. 1.7, E represents the given point on the surface of the earth, DC is the position of the aeroplane and CE is the height of the aeroplane from the ground. Now, if the sun subtends an angle of 32 minutes of arc at E, the wings of the plane also subtend the same angle at E.

$$\text{Now, } 32'=\frac{32^\circ}{60}=\frac{32}{60}\times\frac{2\times3{\cdot}14}{360} \text{ radian.}$$

According to circular measure, the angle subtended at E by the wings of the plane$=\dfrac{\text{length of the wings}}{CE}=\dfrac{100}{CE}$

$$\therefore \quad \frac{100}{CE}=\frac{32}{60}\times\frac{2\times3{\cdot}14}{360} \quad \text{or,} \quad CE=\frac{100\times60\times360}{32\times2\times3{\cdot}14}=10{,}748 \text{ ft. (nearly)}$$

1.5. Eclipses :

The eclipses of the sun and the moon afford further illustration of the formation of shadows. A solar eclipse is caused by the passage of the moon between the sun and the earth at new moon and a lunar eclipse is caused by the passage of the earth between the sun and the moon at full moon. In the first case, the moon may be considered as the obstacle and the earth as the screen and *vice versa* in the second case.

[N.B. The distances of the sun and the moon from the earth are 9×10^6 miles and 21×10^4 miles respectively. The sun's diameter is about 109 times greater than that of the earth. The umbral cone of the earth's shadow is long enough to be about $8{\cdot}6\times10^5$ miles and it far extends the moon.

These distances are so large that it is almost impossible to draw diagrams up to scale in a small compass. For these reasons, the diagrams illustrating the eclipses have not been drawn according to the scale ; they qualitatively illustrate the occurrence of different types of eclipses.]

Solar eclipse : Solar eclipse may be of three different types :—(i) *Total* (ii) *Partial* and (iii) *Annular.*

During rotation in their respective orbits, when the moon (M) comes between the earth (E) and the sun (S) in new moon as shown in fig. 1.9, the shadow of the moon is cast behind it. The portion CD of the shadow is umbra and the portion CG or DF is penumbra. When the umbra of the moon's shadow touches any part of the earth as in CD, the sun becomes totally invisible to

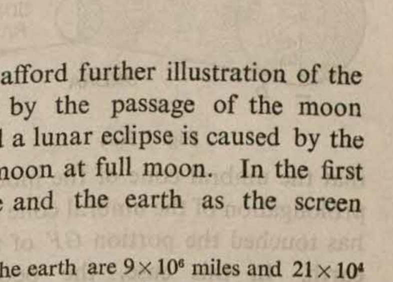


Fig. 1.9

the people of that part while to the people of penumbral regions of the earth as in *DF* or *CG*, the sun becomes partially visible. The people of the region *CG* can observe only the upper part of the sun and the people of the region *DF* the lower part. Consequently, it is a **total eclipse** for the people of the region *CD* and a **partial eclipse** for the people in the region *GC* or *DF*.

Since the moon is much smaller than the earth, its shadow is also smaller. So, the umbral portion of the moon's shadow covers a small region of the earth. For this reason total eclipse of the sun becomes visible over a limited space on the surface of the earth. Further, since the moon's shadow is not very wide, it cannot extend over the whole of the illuminated hemisphere of the earth. So, solar eclipse is not visible from all places on the illuminated hemisphere. Look at the fig. 1.9(*a*). In the figure, *ab* is the penumbral region of the moon's shadow. It has not covered the whole of the illuminated hemisphere of the earth.

During total solar eclipse, it becomes as dark as night and the moon appears on the sky. The moon looks like a copper-coloured disc from the earth because the surface of the moon turned towards the earth cannot receive the direct sunlight but receives the feeble light reflected by the earth. The feeble reflected light makes it look like a copper-coloured disc.

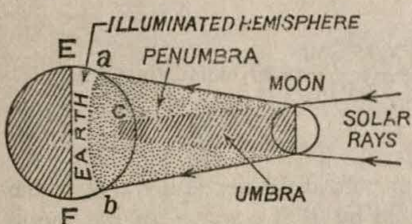


Fig. 1.9(*a*)

Since the sun is much bigger than the moon and since the distance between them varies, it may so happen that the umbral cone of the moon's shadow does not touch the earth ; instead a prolongation of the umbral cone touches the earth. In fig. 1.10, such prolongation has touched the portion *GF* of the earth. In this case, the people occupying the region *GF* will observe a luminous bright outer ring of the sun's disc, the central portion being cut off from view by the moon. This is known as the **annular eclipse** of the sun.

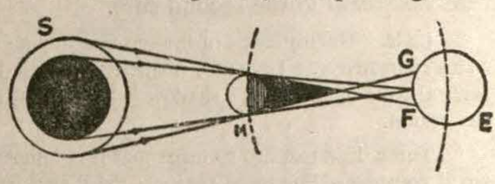


Fig. 1.10

Lunar eclipse : We know that the moon has no light of its own. The moon looks bright because it reflects the sun-light falling upon it. At the time of full moon the earth comes between the sun and the moon.

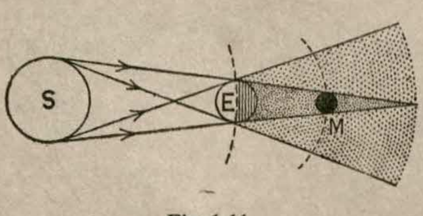


Fig. 1.11

During rotation in their respective orbits, when the earth (*E*) comes between the sun (*S*) and the moon (*M*) in full moon, the shadow of the earth falls on the moon. When the umbral region of the earth's shadow fully covers the moon (fig. 1.11), it becomes invisible from the earth. Total lunar eclipse

then takes place. If a part of the moon falls inside the penumbra and the rest inside umbra, it causes a partial eclipse of the moon.

Before entering the umbral region of the shadow, the moon enters the penumbral region where the sunlight is not strong. For this reason, the lunar disc appears dull before the commencement of the eclipse. For the same reason, the moon looks dull for sometime after the eclipse is over ; for the moon again enters into the penumbra after coming out of the umbra.

Since the size of the earth is much bigger than the size of the moon the length of the earth's shadow cone is always greater than the distance between the earth and the moon. Thus, *the lunar eclipse can never be annular.*

Example : *A man on the earth observes annular eclipse of sun on a new moon. How high should the man ascend from the surface of the earth so that he may just see the total solar eclipse ? The diameters of the sun and the moon are $8{\cdot}6\times10^5$ miles and 2×10^3 miles respectively. The distances of the sun and the moon from the man on the earth are respectively 93×10^6 and $2{\cdot}4\times10^5$ miles.*

Ans. To see the annular eclipse of the sun, the umbral cone of the moon, when prolonged, ends in a point and then produces a diverging cone and falls on the earth. In fig. 1.12, the positions of the sun (S_1S_2) the moon (M_1M_2) and the earth E have been shown. To see the total solar eclipse the man should ascend a height of x miles. From the figure,

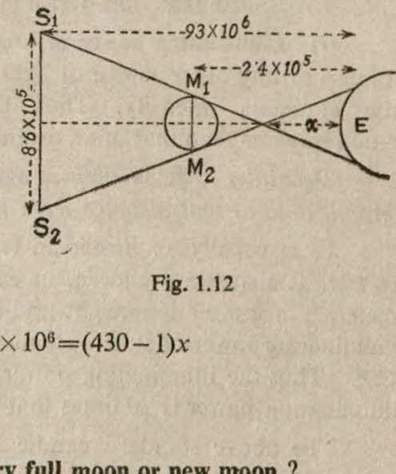


Fig. 1.12

we can write $\dfrac{S_1S_2}{M_1M_2}=\dfrac{93\times10^6-x}{2{\cdot}4\times10^5-x}$

or $\dfrac{8{\cdot}6\times10^5}{2\times10^3}=\dfrac{93\times10^6-x}{2{\cdot}4\times10^5-x}$

or $4{\cdot}3\times10^2=\dfrac{93\times10^6-x}{2{\cdot}4\times10^5-x}$ or $103{\cdot}2\times10^6-93\times10^6=(430-1)x$

$\therefore \quad x=\dfrac{10{\cdot}2\times10^6}{429}=2{\cdot}4\times10^4$ miles.

Why eclipses are not found to occur at every full moon or new moon ?

We do not see eclipses occurring at every full moon and new moon. The reason is that the plane of the moon's orbit is inclined at an angle of about 5° to the plane in which the earth and the sun lie. If the earth, the sun and the moon were in the same plane, then at every new moon there would have been a solar eclipse and at every full moon, a lunar eclipse. Because of the inclination mentioned above, at every full moon, the moon cannot enter the shadow cone of the earth. Sometimes the moon passes above and sometimes below the shadow cone and similarly at every new moon the sun's view is not obstructed by the moon.

To understand when eclipses will occur, the following facts are to be considered. The plane in which the earth and the sun lie, called the *plane of ecliptic*, intersects the plane of the moon's orbit in a straight line. This line is

called the *nodal line.* If full moon occurs on or near this nodal line there will be a lunar eclipse and if a new moon occurs on or near the nodal line, there will be a solar eclipse.

In order that there should be a lunar eclipse two conditions are therefore, to be satisfied :—(i) *the moon must be full* and (ii) *the moon must be on or near the nodal line.*

Similarly for a solar eclipse, the conditions are (i) *the moon must be new* and (ii) *the moon must be on or near the nodal line.*

1.6. Photometry :

The branch of Physics which deals with the subject of measurement of light is called *photometry* and the instruments used for this purpose are, called *photometers.*

Some quantities are to be defined before we take up a discussion of the subject.

1.7. Some important terms in connection with photometry :

(i) **Illuminating power or luminous intensity of a source :** The strength of a lamp or any other source of light is specified by the term illuminating power or the luminous intensity. Thus the luminous intensity of an electric lamp is much brighter than that of an ordinary oil lamp.

Definition : It is defined as *the amount of light falling normally per second on unit area at unit distance from the source.*

It is usually expressed in terms of the luminous intensity of a standard candle. The standard chosen, in earlier days, was a sperm candle $\frac{7}{8}$ inch in diameter, $\frac{1}{6}$ of a pound in weight and burning at the rate 120 grains per hour. The illuminating power of such a standard candle is called *candle power*, abbreviated as *C.P.* Thus the illuminating power of an electric lamp is 20 *C.P.* means that its illuminating power is 20 times that of a standard candle.

The above standard candle has been found to be inconvenient in many ways. Officially, **Hercourt Pentane lamp** is now taken as a standard lamp. In this lamp, a mixture of air and pentane vapour is burned which gives ten times light than the previous standard candle. So, international candle power now means $\frac{1}{10}$ of the illuminating power of a Hercourt Pentane lamp.

In Germany **Hefner lamp** is used as a standard. Amyl acetate is the fuel in this lamp. Its illuminating power is 0·9 times the international candle power. The standard used in France is, however, a **Carcel lamp** whose illuminating power is 9·6 times the international candle power.

Since standard light used in different countries is different and its illuminating power is also different compared to the old standard candle, inconvenience was felt in international work and the necessity of an international standard, acceptable to all was uppermost in the minds of the scientists. Finally in 1948, they unanimously adopted an international standard. It is $\frac{1}{60}$th of the luminosity coming

out of a hole of 1 sq. cm. area in a black-body kept at the temperature of solidifying platinum. The unit is called **Candela.** Its symbol is *cd.*

(ii) **Luminous flux or flux of light** : Consider a point source of light, which emits light energy uniformly in all directions. Also consider a closed surface round the point source. Then, the rate at which light energy crosses the closed surface, is called the flux or the luminous flux.

This unit of luminous flux is **lumen.** It is defined as *the luminous flux emitted through unit solid angle from a uniform point source of illuminating power 1 candela.* Its symbol is *lm.*

Since the solid angle subtended by a closed surface at a point inside it is 4π,

$$\textbf{1 candela} = 4\pi \textbf{ lumens.}$$

(iii) **Intensity of illumination or Illumination** : We all know that the illumination of an open space where sunlight can fall directly, is greater than that of a shaded place. In other words, the same source can illuminate different surfaces differently. By intensity of illumination, we ordinarily mean how far a surface is illuminated by a source. Its exact definition is as follows :

The illumination at a point is defined as the amount of light energy falling normally per second on unit area round the point.

If Q is the amount of light energy falling normally per second on area A round a point, then the illumination of the point

$$I = Q \div A$$

In the F.P.S. system, the unit of illumination is **foot-candle.** If 1 lumen of light falls normally per second on 1 sq. ft. area the illumination of the area is 1 foot candle. It is also known as lumen/sq. ft.

In the M.K.S. system, the unit of illumination is **lux.** If 1 lumen of light falls normally per second on 1 sq. metre area, the illumination of the area is 1 lux. It is also known as **metre-candle** or lumen/sq. metre. The symbol of lux is *lx.*

If, however, 1 lumen of light falls normally per second on 1 sq. cm. area, the illumination in C.G.S. system is 1 **phot.** It is also, called lumen/sq. cm.

The following relations may be obtained from the above definitions :

1 lux = 1 metre-candle = 1 lumen/sq. metre.

1 phot = 1 centimetre-candle = 1 lumen/sq. cm. = 10^4 lux.

1 foot-candle = 1 lumen/sq. ft. = 10·764 lux.

It may be pointed out here that when the sun shines brightly the illumination of the surface of the earth may reach 60,000 ft. candles. In the full moon, the illumination of the moon-lit surface of the earth is only about 1 foot-candle. It may be recalled that the moon has no light of its own ; it reflects the sunlight to the earth.

(iv) **Brightness or Luminance** : If the source of light has a finite size, its illuminating power is expressed by the brightness or luminance. It is defined as *the amount of light energy emitted normally per second in a particular direction from unit area of the source.* In other words, the brightness or luminance is the illuminating power per unit area of the source.

If P be the illuminating power of a source of area A, its brightness, $B = P/A$.

1.8. Inverse square law in connection with illumination :

The law states that *the illumination of a surface by a point source is inversely proportional to the square of its distance from the source.* It may be established in the following way.

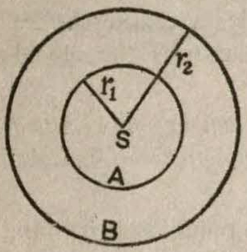


Fig. 1.13

S is a point source, emitting light uniformly in all directions (Fig. 1.13). Suppose the source is emitting Q amount of light energy per second. Consider two spheres A and B of radii r_1 and r_2 respectively having the point S as their common centre.

Now, Q amount of light from the source S will cross the surface of the sphere A in one second. If I_1 be the illumination at any point inside the sphere A,

then, $I_1 = \dfrac{Q}{4\pi r_1^2}$ [$4\pi r_1^2$=surface area of the sphere A]

If, now, the sphere A is replaced by the sphere B, the same amount of light will cross its surface in the same time. Hence I_2, the illumination at any point on its inside surface, is given by $I_2 = \dfrac{Q}{4\pi r_2^2}$ [$4\pi r_2^2$=surface area of the sphere B]

$$\therefore \quad \frac{I_1}{I_2} = \frac{Q}{4\pi r_1^2} \bigg/ \frac{Q}{4\pi r_2^2} = \frac{r_2^2}{r_1^2} \quad \therefore \quad I \propto \frac{1}{r^2};$$

This is the inverse square law.

1.9. Relation between the illuminating power and illumination :

Consider two concentric spheres A and B (Fig. 1.13). Suppose, the radius of the sphere A is 1 (unity) and that of the sphere B is r. A source S kept at their centre, is emitting Q amount of light energy uniformly in all directions per second. Suppose the illuminating power of the source S is L and the illumination at an inside point of the sphere B is I.

Now, from the definitions of the illuminating power of a source and the illumination at a point, we may say that the illuminating power of a source is its illumination at unit distance away. Since same quantity of light (Q) is passing across the spheres A and B, we have, for the sphere A,

$$L = \frac{Q}{4\pi(1)^2} \; [4\pi(1)^2\text{=surface area of the sphere } A] \quad \text{or, } Q = 4\pi L.$$

Now, for the sphere B, $I = \dfrac{Q}{4\pi r^2} = \dfrac{4\pi L}{4\pi r^2} = \dfrac{L}{r^2}$

[$4\pi r^2$=surface area of the sphere B]

$$\text{So, illumination} = \frac{\text{Illuminating power}}{\text{square of the distance}}$$

Example : *Two lamps hang directly above a table. The illuminating power of one is 40 c.p. and that of the other is 63 c.p. Their vertical heights are respectively*

2 ft. and 3 ft. Compare the illuminations of the table when the lamps are put on alternately.

Ans. We know, $I=\frac{L}{r^2}$. For the first lamp, $I_1=\frac{40}{2\times 2}=10$ ft.-candles.

For the second lamp $I_2=\frac{63}{3\times 3}=$ 7 ft.-candles. $\therefore\ \frac{I_1}{I_2}=\frac{10}{7}$

1.10. Principle of Photometry :

Consider two sources of light S_1 and S_2 kept at distances r_1 and r_2 respectively from a screen M (Fig. 1.14). Suppose L_1 and L_2 are the illuminating powers of the sources. If they produce same illumination on the screen, then, the principle of photometry may be established in the following way ;

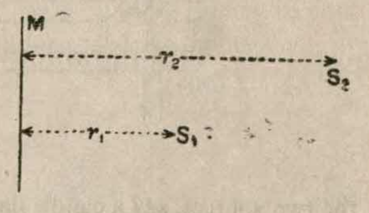


Fig. 1.14

The illumination produced on the screen by the first source$=L_1/r_1^2$. Similarly, the illumination produced by the second source$=L_2/r_2^2$.

Since the illuminations are equal, we have, $L_1/r_1^2=L_2/r_2^2$

or, $\frac{L_1}{L_2}=\frac{r_1^2}{r_2^2}$

Hence *if two sources produce equal illumination on a screen, their illuminating powers are proportional to the squares of their distances from the screen.* This is the basic principle of photometry. This principle is applied in different photometers for the comparison of illuminating powers.

1.11. Comparison of illuminating powers by photometer :

(A) Rumford's photometer : Rumford's photometer consists of an upright metallic rod R placed at a short distance in front of a white screen of paper or glass (S). Two lamps—one a candle C and the other an electric lamp L—whose illuminating powers are to be compared, are placed in such a way that the shadows (A and B) of the rod which they form on the screen lie just side by side [Fig. 1.15].

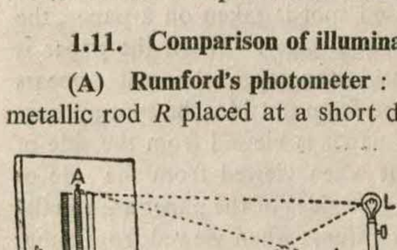


Fig. 1.15

The shadow formed by the lamp C receives light only from the lamp L and vice versa. Hence, if the distance r_1 and r_2 are now adjusted until both the shadows are equally illuminated, the two lamps will be producing equal illumination on the screen. It follows that $\frac{L_1}{L_2}=\frac{r_1^2}{r_2^2}$.

Measuring the distances r_1 and r_2 by a scale, the illuminating powers of the lamps may be compared.

(B) Bunsen's grease-spot photometer :

The photometer consists of an optical bench MN provided with several uprights carrying sources of light, (C and L) and a paper screen P having a greased spot at its centre as shown in the fig. 1.16. Place the paper screen in between

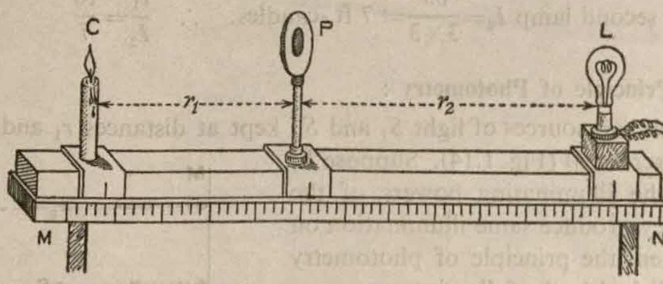


Fig. 1.16

the two sources, say a candle flame and an electric bulb. Adjust the position of the screen such that the greased and ungreased portions of the screen appear equally bright from whatever side the screen is observed. When this is done, the intensitities of illumination produced by the sources on the screen are equal. Mearure the distances of the sources from the screen. Let r_1 and r_2 be the distances of the sources from the screen, L_1 and L_2 are the illuminating powers of the candle flame and the electric bulb respectively. Then

$$\frac{L_1}{L_2}=\frac{r_1^2}{r_2^2}$$

Action of the greased spot : When a greased spot is taken on a paper, the spot becomes more transparent than the rest of the paper. When the paper is held near a window and it is observed from inside the room, the spot appears brighter than the rest of the paper. If it is viewed from outside, the spot appears darker. We, therefore, conclude that when the paper is viewed from the side of greater illumination, the spot appears dark, but when viewed from the side of lesser illumination, it appears brighter. If, then, two sides of the paper are equally illuminated, the spot ought to be of the same brightness when viewed from either sides. The following calculations also show the above fact.

Let I_1 and I_2 be the quantities of light falling per unit area on the screen from the candle flame and the electric bulb respectively. Let α be the fraction of light diffusively reflected from the ungreased portion of the paper. Then $(1-\alpha)$ is the fraction of light transmitted through the ungreased portion. If we place our eye on the screen from the side of the candle flame, the amount of light reaching the eye from unit area of the ungreased portion is proportional to $I_1\alpha+I_2(1-\alpha)$. Similarly, the amount of light reaching the eye from unit area of the greased portion is proportional to $I_1\beta+I_2(1-\beta)$, where β=fraction of light diffusively reflected by the greased spot and $(1-\beta)$=the fraction transmitted through the greased spot. When greased and ungreased portions appear equally bright, the above two quantities are equal.

$\therefore \quad I_1\alpha+I_2(1-\alpha)=I_1\beta+I_2(1-\beta)$ or, $I_1(\alpha-\beta)=I_2(\alpha-\beta) \quad \therefore \quad I_1=I_2$

Therefore, $\dfrac{L_1}{r_1^2}=\dfrac{L_2}{r_2^2}$ or $\dfrac{L_1}{L_2}=\dfrac{r_1^2}{r_2^2}$

This explains why we consider intensities of illumination at the screen produced by the two sources equal when the greased and the ungreased portions appear equally illuminated.

(C) Lummer Brodhun photometer : The essential parts of a Lummer-Brodhun photometer are shown in fig. 1.17. A white diffusing screen A is illuminated normally on the two sides by light sources C_1 and C_2. The sources can be moved along an optical bench. The illuminating powers of these two sources are to be compared. D and E are total reflection prisms which reflect the light coming from A to two right angled prisms F and G. These two prisms F and G are so shaped and cemented that they come in contact only at the central circular area, while a thin film of air exists between the two in their outer region. The central part transmits light coming from C_1 into the telescope T and the outer parts reflect light coming from C_2 into the telescope T. The rest of the rays fall upon the blackened surface, where they are absorbed. The field of view inside the telescope appears to be divided into two concentric circular strips, the central strip (S_1) illuminated by C_1 and the outer annular space (S_2) by C_2 [Fig. 1.18]. If C_1 produces greater illumination than C_2, the central part will be brighter than the outer part and if C_2 produces greater illumination, the outer part will be brighter than the central part [Fig. 1.18 (a) and (b)]. The distances of C_1 and C_2 from A are adjusted till the whole field appears uniformly illuminated. The central spot will then become indistinguishable from the outer [Fig. 1.18(c)]. In this circumstances, photometric balance is said to have been attained.

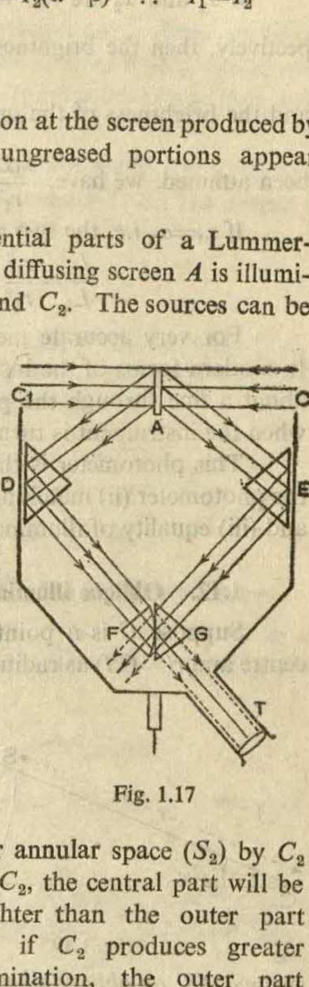


Fig. 1.17

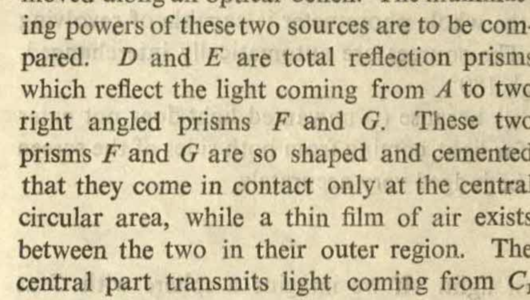


Fig. 1.18

Theory : After the photometric balance is attained, let the distance of the source C_1 from the screen A be r_1 and that of the source C_2 be r_2. Then, the intensity of illumination produced by the sources on the screen may be written as $I_1=\dfrac{L_1 \cos\theta}{r_1^2}$ and $I_2=\dfrac{L_2 \cos\theta}{r_2^2}$ respectively. Since $\theta=0$, $I_1=\dfrac{L_1}{r_1^2}$ and $I_2=\dfrac{L_2}{r_2^2}$.

If α_1 and α_2 are the reflecting powers of the two surfaces of the screen respectively, then the brightness of the surface turned towards the source $C_1 = \frac{\alpha_1 L_1}{r_1^2}$ and the brightness of the opposite surface$=\frac{\alpha_2 L_2}{r_2^2}$. Since photometric balance has been attained, we have, $\frac{\alpha_1 L_1}{r_1^2} = \frac{\alpha_2 L_2}{r_2^2}$

If $\alpha_1 = \alpha_2$ *i.e.* the two surfaces have the same reflecting power then,

$$\frac{L_1}{L_2} = \frac{r_1^2}{r_2^2}$$

For very accurate measurements, the sources C_1 and C_2 are interchanged. In modern forms of the instrument, the whole photometer is capable of revolving about a line through the point A. The sources are automatically interchanged, when the instrument is turned through 180°.

This photometer is the best of all because (i) unwanted light does not enter the photometer (ii) matching is done for light coming from both sides of the screen and (iii) equality of illumination can be judged very accurately.

1.12. Oblique illumination :

Suppose C is a point source of light. If we imagine a sphere with C as centre and $r(=CS)$ as radius, then SA is a small area on the surface of that sphere.

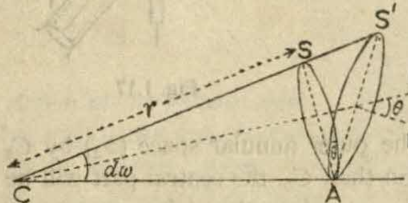


Fig. 1.19

Consider another small surface AS', making an angle θ with the surface AS [Fig. 1.19]. Both the surfaces are the sections of the conical surfaces subtending the same solid angle $d\omega$ at C. It is clear that the rays of light are falling normally on the surface AS but obliquely on AS' *i.e.* the surface AS is having a *direct* illumination while the surface AS' *an oblique illumination.*

If L be the luminous intensity of the source of light, then the amount of light incident on both the surfaces$=L.d\omega$. Let the illuminations on the surfaces AS and AS' be I_0 and I respectively.

Then, $I_0 = \frac{L.\, d\omega}{\text{Area of the surface } AS}$ and $I = \frac{L.\, d\omega}{\text{Area of the surface } AS'}$

But we know, the area of the surface $AS = \cos\theta \times$ area of the surface AS'

$$\therefore \quad \frac{I}{I_0} = \frac{\text{Area of the surface } AS}{\text{,, ,, ,, ,, } AS'} = \cos\theta.$$

or $\quad I = I_0 \cos\theta$.

Hence, the illumination is proportional to the cosine of the angle of incidence. This is known as *Lambert's cosine law.*

When $\theta = 0$ (*i.e.* normal incidence), $I = I_0$.

Example 1. *Two lamps balance on a shadow photometer at distances 60 cm. and 42 cm. from the screen. The stronger lamp is then covered with a shade which transmits 80% of the incident light. How far must this lamp be displaced in order to restore balance ?*

Ans. Suppose, the illuminating powers of the sources are L_1 and L_2 respectively. L_1 is evidently greater than L_2 because its distance from the screen is greater than that of the other.

In the first case, $\dfrac{L_1}{L_2}=\left(\dfrac{60}{42}\right)^2=\left(\dfrac{10}{7}\right)^2$

When the stronger lamp is covered, its illuminating power will be $\frac{4}{5}L_1$, because it, now, gives 80% light. Suppose, in order to restore balance, the lamp is placed at a distance x cm. from the screen. We get,

$$\frac{\frac{4}{5}L_1}{L_2}=\left(\frac{x}{42}\right)^2 \text{ or, } \frac{4}{5}\cdot\left(\frac{10}{7}\right)^2=\left(\frac{x}{42}\right)^2$$

Taking square root, we get, $\dfrac{2}{2{\cdot}24}\times\dfrac{10}{7}=\dfrac{x}{42}$ or, $x=53{\cdot}57$ cm.

Hence the lamp is to be moved $(60-53{\cdot}57)=6{\cdot}43$ cm. towards the screen.

Example 2. *Correct exposure for a photographic print is 10 seconds at a distance of 1 ft. from a 20 c.p. lamp. For how many seconds, will you expose the print at a distance of 2 ft. from a 16 c.p. lamp ?*

Ans. In the first case, the illumination $I_1=\dfrac{20}{(1)^2}=20$ ft. candles.

Hence, the amount of light necessary for correct exposure$=20\times10$ units.

The illumination in the second case, $I_2=\dfrac{16}{(2)^2}=4$ ft. candles.

If 't' be the time of exposure, the amount of light necessary$=4\times t$ units

Since the same photographic print is taken, the amounts of light in the two cases will be equal.

$$\therefore\quad 4\times t=20\times10 \text{ or } t=50 \text{ sec.}$$

Example 3. *On a photometer bench, there is a lamp with dirty chimney. In order to balance it on a screen, another lamp is placed at a distance of 10 cm. from the screen on the other side. When the chimney is cleaned, the lamp on the other side is shifted through 2 cm. to restore balance. Find the percentage of light absorbed by the dirty chimney.*

Ans. Let L_1 be the illuminating power of the lamp with dirty chimney and L_2 that when the chimney is cleaned. Let its distance from the screen be r.

In the first case, $\dfrac{L_1}{r^2}=\dfrac{1}{(10)^2}=\dfrac{1}{100}$. In the second case, $\dfrac{L_2}{r^2}=\dfrac{1}{(10-2)^2}=\dfrac{1}{64}$

$$\therefore\quad \frac{L_1}{L_2}=\frac{64}{100} \text{ or } \frac{L_2-L_1}{L_2}=\frac{100-64}{100}=\frac{36}{100}$$

So, the percentage of light absorbed$=\dfrac{L_2-L_1}{L_2}\times100=\dfrac{36}{100}\times100=36\%$

Example 4. *Two lamps A and B produce equal illumination on a screen when placed 4 meters and 1 metre respectively on the opposite sides. If now a lamp of 9 c.p. is placed by the side of A, it is found that the pair will have to be moved to a distance of 5 metres to restore the balance, B remaining at a distance of 1 metre. Calculate the c.p. of the lamps A and B.*

Ans. If L_A and L_B be the candle powers of the lamps A and B respectively, then in the first position, $\frac{L_A}{L_B}=\left(\frac{4}{1}\right)^2$ or $L_A=16.L_B$.

In the second position, $\frac{L_A+9}{L_B}=\left(\frac{5}{1}\right)^2=25$ or $L_A+9=25L_B$ or $16L_B+9=25L_B$

$\therefore$ $L_B=1$ c.p. and $L_A=16$ c.p.

Example 5. *Two lamps are placed 100 cm. apart. A screen is placed between them so as to balance the lamps. Now the positions of the lamps are interchanged. To restore balance, the screen has to be shifted through 30 cm. Find the ratio of the illuminating powers of the lamps.*

Ans. Let the illuminating powers be L_1 and $L_2(L_1>L_2)$. If they are at distances r_1 and r_2 from the screen, then $\frac{L_1}{L_2}=\frac{r_1^2}{r_2^2}$ ..(i) It is easily seen that $r_1>r_2$.

When the positions of the lamps are interchanged, the screen is to be shifted towards the lamp of illuminating power L_2. If the displacement is x, the distance of the screen from $L_1=x+r_2$ and the distance from $L_2=r_1-x$.

So, $\frac{L_1}{L_2}=\frac{(r_2+x)^2}{(r_1-x)^2}$ .. (ii)

From eqns (i) and (ii), we get $\frac{r_1}{r_2}=\frac{r_2+x}{r_1-x}$

or, $r_1^2-r_1.x=r_2^2+r_2.x$ or $(r_1+r_2)(r_1-r_2)=x(r_1+r_2)$ $\therefore$ $r_1-r_2=x=30$ cm.
Again, $r_1+r_2=100$ cm ; so $r_1=65$ cm and $r_2=35$ cm

$$\therefore \frac{L_1}{L_2}=\left(\frac{65}{35}\right)^2=\frac{169}{49}$$

Example 6. *A table can turn about a horizontal axis passing through the centre of the table. A lamp of 100 c.p. is suspended at a vertical height of 4 ft. from the centre. How will the intensity of illumination at the centre diminish if the table turns through 30° from the horizontal position ?*

Ans. If I_0 be the intensity of illumination at the centre when the table was horizontal, $I_0=\frac{100}{(4)^2}=\frac{25}{4}$ ft. candle. When the table turns through 30°, the intensity of illumination at the centre is given by $I=I_0 \cos\theta=\frac{25}{4}.\cos 30°$ $=\frac{25}{4}\times\frac{\sqrt{3}}{2}$ ft. candle.

$\therefore$ Percentage diminution of intensity of illumination

$$=\frac{I_0-I}{I_0}\times 100=\left(1-\frac{I}{I_0}\right)\times 100=\left(1-\frac{\sqrt{3}}{2}\right)\times 100=13{\cdot}4\,\%$$

Example 7. *At what height should a lamp be hung above the centre of a round table so as to obtain a maximum illumination on its edges ? Find the height in terms of radius of the table.*

[*Jt. Entrance 1984*]

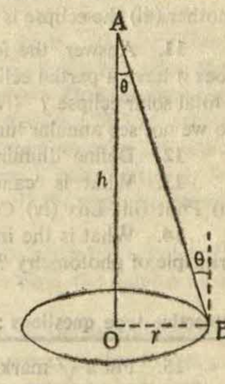


Fig. 1.20

Ans. From fig 1.20, we have the intensity of illumination at a point B on the edge, $I=\frac{L}{(AB)^2}.\cos\theta$, where L is the illuminating power of the lamp.

Now, $(AB)^2=(r^2+h^2)$ and $\cos\theta=\frac{h}{\sqrt{r^2+h^2}}$;

$$\therefore\ I=\frac{L}{(r^2+h^2)}\times\frac{h}{\sqrt{r^2+h^2}}=\frac{Lh}{(r^2+h^2)^{\frac{3}{2}}}\ ;$$

For maximum illumination, $\frac{dI}{dh}=0$ *i.e.* $\frac{d}{dh}\left[\frac{Lh}{(r^2+h^2)^{\frac{3}{2}}}\right]=0$

$$i.e.\quad \frac{L}{(r^2+h^2)^{\frac{3}{2}}}-\frac{3.L.h.2h}{2(r^2+h^2)^{\frac{5}{2}}}=0\quad i.e.\quad r^2+h^2=3h^2\quad\therefore\quad h=\frac{r}{\sqrt{2}}.$$

Exercises

Essay type questions :

1. Explain with illustration and experimentation that light travels in straight line.

2. Describe the construction and explain the action of a pin-hole camera.

3. How are shadows formed ? Draw diagrams and explain how umbra and penumbra are formed when rays of light are intercepted by a spherical extended obstacle.

4. What is the difference between illuminating power and illumination ? Which quantity is determined by a photometer ? Describe any one photometer you know.

5. Describe a Lummer-Brodhun photometer and explain the method by which the intensity of two sources can be compared with it. [*H. S. Exam. 1978*]

Short answer type questions :

6. Answer the following questions in connection with a pin-hole camera :—(i) What happens when the hole is enlarged ? (ii) What happens when the ground glass is moved further away from the hole ? (iii) What happens when the object is moved away from the hole ? (iv) What happens when the shape of the hole is changed ?

7. Horizontal sunlight coming through a triangular pin-hole in a window falls on the opposite wall. Explain why we see a circular patch of light on the wall.

8. What is the distinction between umbra and penumbra ? A bird flying close to the ground casts a shadow on the ground ; but when it flies up in the air there is no detectable shadow. Explain.

9. Draw diagrams to show the formation of umbra and penumbra for a spherical obstacle in the following cases :—(i) When the source is a point source (ii) When the source is a sphere smaller in size than the obstacle (iii) When the source is a sphere larger in size than the obstacle.

10. Draw two neat diagrams to illustrate the occurrence of solar and lunar eclipses. From the diagram of the solar eclipse you have drawn, answer why (i) the eclipse is not visible from all places of illuminated hemisphere of the earth (ii) the eclipse is partial at one place and total at another (iii) the eclipse is not seen at every new moon ?

11. Answer the following questions :—(i) When does lunar eclipse occur ? (ii) When does it have a partial eclipse ? (iii) Why does the moon look like copper-coloured disc during a total solar eclipse ? (iv) Why does lunar eclipse not take place on every full moon ? (v) Why do we not see annular lunar eclipse ?

12. Define 'illuminating power' and 'intensity of illumination.' [*H. S. Exam. 1978*]

13. What is 'candle power' of a lamp ? Explain the following terms :—(i) Lumen (ii) Phot (iii) Lux (iv) Candela.

14. What is the inverse square law in connection with illumination ? What is the basic principle of photometry ?

Objective type questions :

15. Put a $\surd$ mark on the correct answer in the following cases :—

(*a*) In which case do we see an annular eclipse ? [Ans. Solar, lunar]

(*b*) What are the relative positions of sun, moon and earth at the time of solar eclipse ?

[Ans. Moon is between the sun and the earth ; the earth is between the sun and the moon.]

(*c*) What does a photometer measure ?

[Ans. Luminous intensity ; illumination ; Lumen.]

(*d*) Why does moon appear copper-coloured at the time of total solar eclipse ?

[Ans. Sunlight falls on the moon ; light reflected by the earth falls on the moon ; the moon has no light of its own.]

Numerical Problems :

16. A dark room 10 ft. square with white walls has a small hole on the centre of one wall. An image of a tree 11 inches high is formed on the opposite wall, the tree being 55 ft. high and situated at certain distance outside the hole. How far is the tree from the hole? [Ans. 600 ft.]

17. The distance of the pin-hole to the plate, in a pin-hole camera, is 8 inches. How far from a tree 200 ft. high must the camera be placed to get the whole image of the tree on the plate if it is 6 inches high ? [Ans. 266·6 ft.]

18. A candle flame 2 cm. high is at a distance of 15 cm. from the pin-hole of a pin-hole camera. Find the size of the image when the screen of the camera is placed 25 cm. from the hole. [Ans. 3·33 cm.]

19. There is a pin-hole on the window of a room 10 ft. wide. The image of a tree outside is formed on the opposite wall. If the height of the image is 4 ft. and the distance of the tree from the window 30 ft., what is the height of the tree ? [Ans. 12 ft.]

20. A 6 cm high image of a tower is formed inside a pin-hole camera when placed at some distance from the tower. From another point 10 metre further from the previous position in the same straight line, the image is 4 cm in height. What is the height of the tower ? Camera box is 20 cm long. [*H.S. Exam 1985*] [Ans. 6 m]

21. A pin-hole camera takes a picture of the sun. The screen is 100 cm. from the hole and the sun subtends an angle of $\frac{1}{2}°$ at the hole. Find the diameter of the image formed on the screen. What would happen if a convex lens of focal length 100 cm. is kept at the position of the hole ? [Ans. 8·72 mm. ; brighter image]

22. A circular uniform source of light, 4 inches in diameter is placed at a distance of 3 ft. from a spherical opaque body 2 inches in diameter. Find the shortest distance from the latter at which a screen may be placed so as to have no umbra in the shadow cast upon it.

[Ans. 3 ft.]

23. Find the least height from the surface of the earth at which, when a bird flies, it will cast a shadow on the earth without umbra ? The wings of the bird are 2 ft. wide, the diameter of sun $=9\times10^5$ miles and the distance of the sun from the earth $=9\times10^7$ miles. [Ans. 200 ft.]

24. The diameter of the sun being taken as 9×10^5 miles and its distance from the earth 9×10^7 miles and the diameter of the moon 2100 miles, find the distance of the earth from the moon at the time of a solar eclipse when the eclipse is total only at a single point on the earth.

[Ans. 21×10^4 miles]

25. The diameter of the sun is 9×10^5 miles, its distance from the earth 9×10^7 miles, the diameter of the moon 2100 miles and its distance from the earth 2,09,000 miles. Find the diameter and the area on the earth within which the solar eclipse is total. [Ans. 10 miles ; 78·5 sq. miles]

26. What is the luminous flux in space from a point source of luminous intensity 250 basic S.I. units ? A small piece of paper is held at a distance of 5 metres from it perpendicularly to the rays of light. What is the illumination on the surface of the paper ? If the paper be turned through an angle of 60°, what would be the illumination ?

[*Jt. Entrance 1983*] [Ans. 3140 lumen ; 10 candle per m^2; 5 candle/m^2]

27. Illuminating powers of two sources 12 ft. apart are as 16 : 25. Find the point on the line joining them, where the illumination due to the sources is the same.

[Ans. 5·3 ft. from the weaker source]

28. A photographic print is found to be satisfactory when the exposure was for 15 sec. at a distance of 2 ft. from a 16 *c.p.* lamp. At what distance must the same paper be held from a 32 *c.p.* lamp in order that an exposure of 20 sec. will give the same result ? [Ans. 3·23 ft.]

29. An illumination of 6ft. candle is needed at a distance of 4 ft. from a candle. What should be the candle power of the lamp ? [Ans. 96 c.p.]

30. A standard candle and a gas-lamp are placed 6 ft. apart. The illuminating power of the gas-lamp is 4 c.p. Find, at what point on the line joining them the illumination on a screen due to the sources is the same. [Ans. 2 ft from the standard candle]

31. Two lamps of illuminating powers 32 and 16 c.p. are placed 1 metre apart. Find at what point on the line joining them, the illumination on a screen due to the sources is the same.

[Ans. 41·4 cm. from the second lamp or 58·6 cm. from the first lamp.]

32. A 300 c.p. lamp is suspended 5 ft. above the centre of a rectangular table 8 ft. $\times$ 6 ft. Calculate the illumination at (i) the centre and (ii) a corner of the table.

[Ans. (i) 12 ft. candle (ii) 4·24 ft. candle]

33. A small screen is held 12 ft. from a source of light in such a position that the light is incident on it normally. It is then removed to 15 ft. and turned round so that the light is incident on its surface at an angle of 60°. Compare the illuminations in the two cases. [Ans. 25 : 8]

34. Two lamps *A* and *B* producea balance on a photometer when placed 4 metres and 1 metre respectively from a screen. If now a lamp 9 c.p. is placed by the side of *A*, it is found that the pair will have to be moved to a distance of 5 metres to restore the balance, *B* remaining at a distance of 1 metre. Calculate the c.p. of the lamps *A* and *B*. [Ans. *A*—16 c.p. *B*—1 c.p.]

Harder Problems :

35. Two lamps *A* and *B*, illuminate a photometer screen equally when their distances from the screen are 60 cm. and 30 cm. respectively. A plane mirror, with its reflecting face at right angles to the axis of the photometer bench is placed 7·5 cm. from *B* on the side remote from the screen. To restore balance, *A* is then moved 9 cm. nearer the screen. Compare the luminous intensity of *B* with that of its image formed by the mirror. [Ans. 1156/999]

[Hints : $\dfrac{L_A}{(60)^2}=\dfrac{L_B}{(30)^2}$ .. (i) and $\dfrac{L_A}{(51)^2}=\dfrac{L_B}{(30)^2}+\dfrac{L_M}{(45)^2}$ .. (ii)

where L_M = luminous intensity of the image.]

36. Two lamps are placed 150 cm. apart. A screen is placed between them so as to balance the lamps. Now the positions of the lamps are interchanged. To restore balance the screen has to be shifted through 45 cm. Find the ratio of the illuminating power of the lamps.

[Ans. 169:49]

37. Two lamps A and B produce a balance on a photometer when placed 6 metres and 2 metres respectively from a screen. If now a lamp, 12 candle power, is placed by the side of A, it is found that the pair will have to be moved to a distance of 8 metres to restore the balance, B remaining at a distance of 2 metres as before. Calculate the candle powers of the lamps A and B. [Ans. $A=15{\cdot}4$; $B=1{\cdot}7$]

38. The shadow which causes an eclipse is geometrically speaking a solid cone with its axis on the line joining the centre of the sun and the shadow-casting body. If the sun is 93,000,000 miles from the earth and subtends an angle 32′ at the earth, find how far the earth's shadow extends from the centre of the earth. Find also the diameter of the earth's shadow at the point where the moon crosses it. The distance of the moon is 60 times the earth's radius.

[*Jt. Entrance 1981*] [Ans. $8{\cdot}7\times10^5$ miles ; 5788 miles].

39. An electric lamp is kept in a glass sphere of radius 20 cm. and is suspended from a height of 300 cm over a table. An opaque ball of radius 10 cm. is held under the lamp at a height of 100 cm. Find (*a*) the size of the umbra and penumbra formed by the ball (*b*) At what height should the ball be kept in order that its umbra vanishes ? [Ans. (*a*) 5 cm. ; 25 cm ; (*b*) 150 cm.]

40. Two lamps A and B are placed 3 ft. apart and 4 ft above a horizontal platform. The illumination at a point on the platform vertically below B is 10 ft. candles, when only the lamp B is used. When both the lamps are switched on, the illumination at the same point is 14 ft candles. Calculate the luminous intensities of the lamps. [Ans. $A=125$ c.p. ; $B=160$ c.p.]

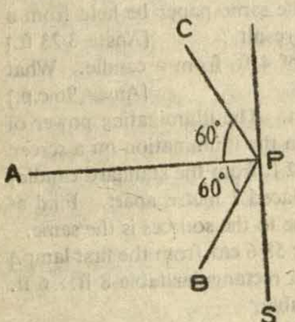


Fig. 1.21

41. A screen S is illuminated by two point sources A and B. Another source C sends a parallel beam of light towards the point P on the screen [Fig 1.21]. Line AP is normal to the screen and the lines AP, BP and CP are in one plane. The distances AP, BP and CP are 3 metres, 1·5 metres and 1·5 metres respectively. The radiant powers of sources A and B are 90 watts and 180 watts respectively. The beam from C is of intensity 20 watts/m^2. Calculate the intensity at P on the screen.

[*I.I.T. 1982*] [Ans. 14 watts/m^2]

42. A lamp of illuminating power L is hanging vertically at a height h above the centre of a circular table. If the diameter of the circular top of the table be d, prove that the intensity of illumination at the centre is $\left(1+\frac{d^2}{4h^2}\right)^{3/2}$ times the intensity at any point on the circumference of the table.

43. A lamp is hanging along the axis of a circular table of radius r. At what height should the lamp be placed above the table so that the intensity of illumination at the edge of the table is $\frac{1}{8}$ of that at the centre ? [*I.I.T. 1978*] [Ans. $r/\sqrt{3}$]

44. Two sources of light, each 2 candle power, are placed on the same side of a Bunsen photometer. One is at a distance of 1 ft. and the other at 2 ft from the spot. Where must a third source of 5 candle power be placed in order that the appearance of each side of the photometer may be the same ? [Ans. 1·414 ft. on the other side]

45. A lamp produces a certain intensity of illumination at a screen when situated at a distance of 85 cm. from it. On placing a sheet of glass between the lamp and the screen, the lamp must be moved 5 cm nearer to the screen to produce the same illumination as before. What percentage of light is stopped by the glass ? [Ans. 11·42%]

2

REFLECTION OF LIGHT AT PLANE AND CURVED SURFACES

2.1. Reflection of light :

In the preceding chapter, it has been pointed out that light travels in straight lines in a homogeneous medium. But when light moves from one medium to another, a part of the light returns to the first medium in a straight line from the surface of separation of the two media. This phenomenon is known as **reflection of light.** Reflection of light produces a very important effect. The common objects around us are not self luminous but we are able to see them because they reflect light from the sun or other sources in all directions. Mirrors and polished surfaces reflect light strongly. We shall, in the present chapter, discuss the laws and other associate matters governing the reflection of light at plane and curved surfaces.

2.2. Regular reflection :

If the reflecting surface is smooth and polished, the reflected rays move in a particular direction, depending upon the direction of the incident rays. In Fig. 2.1. a beam of parallel rays is incident on a smooth reflecting surface. The reflected beam is also constituted of parallel rays. Such reflection is known as *regular reflection.*

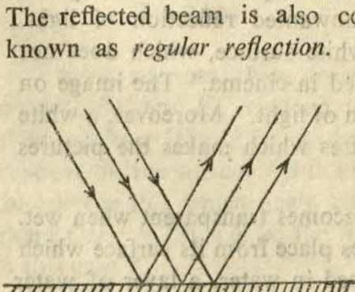


Fig. 2.1

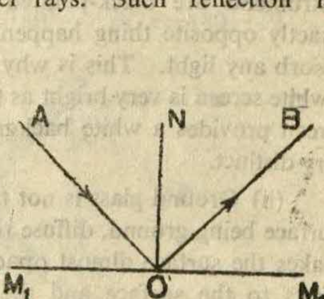


Fig. 2.2

In fig. 2.2, a single ray AO has undergone regular reflection at the reflecting surface M_1M_2. Here, AO, called the *incident ray* is the direction in which the light falls on the reflecting surface. OB, the *reflected ray*, gives the direction in which the ray is reflected. The point O where the light is incident on the reflecting surface, is called the *point of incidence*.

A perpendicular ON drawn to the reflecting surface at the point of incidence, is called the *normal.*

The angle made by the incident ray with the normal (*i.e.*, $\angle AON$) is called the *angle of incidence* and the angle made by the reflected ray with the same normal (*i.e.*, $\angle BON$) is called the *angle of reflection.*

2.3. Diffuse reflection :

When is light is incident on a surface which is not smooth but rough, the reflected light travels in a large number of directions, most of which bear no simple relation to the direction of the incident light. In fig. 2.2 (a) a beam of parallel rays is incident on a rough surface. Each ray of the beam undergoes regular reflection but since the surface is uneven, the noımals drawn at different points on the surface will be directed in different directions. Hence the reflected rays will be scattered in all directions and will not form a parallel reflected beam although the incident beam is parallel. This kind of reflection is known as *diffuse reflection.*

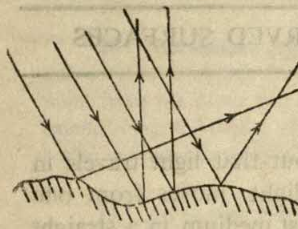


Fig. 2.2(a)

Ground glass, white paper, cinema screen, the walls of a room etc. produce diffuse reflection of light because all these surfaces are rough and uneven. As a result, these surfaces look equally bright whatever be the direction at which they are looked. In the case of a plane mirror, however, since the reflection is regular, the portion of the mirror which reflects light, appears dazzling.

A few important points : (i) When light falls on a black surface, no light is reflected by or transmitted through the surface. Black surface completely absorbs the light. For this reason, the inside of camera, telescopes and other optical instruments are black-painted in order to stop unwanted reflection of light. Exactly opposite thing happens in the case of a white surface, which does not absorb any light. This is why a white screen is used in cinema. The image on a white screen is very bright as there is no absorption of light. Moreover, a white screen provides a white background to black pictures which makes the pictures very distinct.

(ii) Ground glass is not transparent ; but it becomes transparent when wet. Surface being ground, diffuse reflection of light takes place from its surface which makes the surface almost opaque. But when dipped in water, a layer of water sticks to the surface and the surface becomes almost smooth. In that case, emergence and regular reflection of light take place and the surface behaves like a transparent one.

(iii) The eastern and western sky become red a little before the sunrise and sunset respectively. This is known as *dawn* and *twilight* respectively. This happens due to diffuse reflection of sun light by the dust particles and water particles suspended in air.

2.4. Laws of reflection :

(1) The incident ray, the reflected ray and the normal at the point of incidence lie in the same plane.

(2) The angle of incidence is equal to the angle of reflection.

Normal incidence : When a ray of light is incident on a reflecting surface normally, the angle of incidence is zero. According to the laws of reflection, the angle of reflection is, therefore, zero *i.e.* the ray after reflection retraces its path.

A ray AB is incident normally ($\angle i = 90^\circ$) on a plane mirror MN [Fig. 2.3 (a)]. After reflection, it retraces the path and travels along BA.

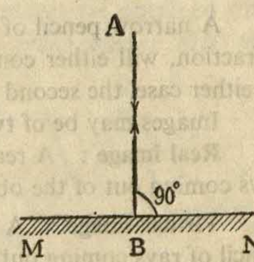


Fig. 2.3 (a)

2.5. Deviation of a ray due to reflection :

When a ray is reflected by a reflecting surface, it suffers some deviation. Consider a ray AO incident on the reflecting surface XY at O, at an angle of incidence $i = \angle AON$. After reflection, the ray travels along OB [Fig. 2.3(b)] Angle of reflection $\angle BON = \angle i$ according to the laws of reflection.

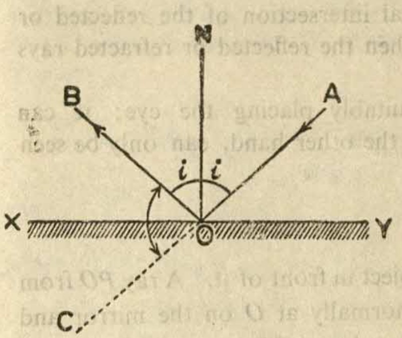


Fig. 2.3(b)

In absence of the reflecting surface, the ray would have travelled straight along AOC but due to reflection the ray suffers a deviation which is measured by the angle $\angle BOC$, known as the angle of deviation. From the figure, it is clear that $\angle BOC = 180^\circ - \angle BOA = 180^\circ - 2i$.

Now, suppose, there are two reflecting surfaces PB and PC inclined to each other at an angle α [Fig. 2.3 (c)], and a ray AB, incident on the surface PB is reflected on to the surface PC, which again reflects the ray along CD. What will be the total deviation in this case ?

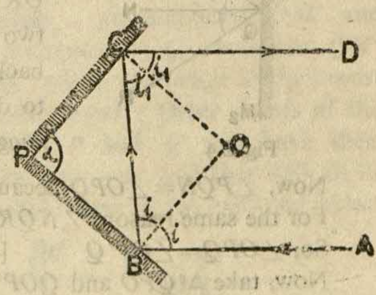


Fig. 2.3(c)

If the angle of incidence at B is i and that at C is i_1, then from the preceding discussion, we can say that the deviation due to reflection at $B = 180^\circ - 2i$ and that at $C = 180^\circ - 2i_1$.

Therefore, total deviation $= 180^\circ - 2i + 180^\circ - 2i_1 = 360^\circ - 2(i_1 + i)$

Now, from the ΔCBO, $i + i_1 = 180^\circ - \angle BOC$

Again from the quadrilateral $PCBO$, $\alpha = 180^\circ - \angle BOC$

[$\because$ $\angle PCO$ and $\angle PBO$ each equal to 90°]

$\therefore$ $\alpha = i + i_1$; hence the total deviation after two reflections $= 360^\circ - 2\alpha$.

If the reflectors are inclined at an angle $\alpha = 90^\circ$, the total deviation $= 360^\circ - 2 \times 90^\circ = 180^\circ$ which means that the incident and the emergent rays will be parallel to each other but their directions are opposite.

2.6. Image :

A narrow pencil of rays coming out of a point source, after reflection or refraction, will either converge to a point or appear to diverge from a point. In either case, the second point is called the geometrical image of the point source.

Images may be of two types : (i) real image and (ii) virtual image.

Real image : A real image of an object is formed when a narrow pencil of rays coming out of the object converges to a point after reflection or refraction.

Virtual image : A virtual image of an object is formed when a narrow pencil of rays coming out of the object appear to diverge from a point after reflection or refraction.

Distinction between real and virtual images :

(i) A real image is formed by the actual intersection of the reflected or refracted rays but a virtual image is formed when the reflected or refracted rays appear to diverge from a point.

(ii) A real image can be seen by suitably placing the eye; it can also be cast on a screen. A virtual image, on the other hand, can only be seen but cannot be cast on a screen.

2.7. Image formed by a plane mirror :

M_1M_2 is a plane mirror and P is a point object in front of it. A ray PO from the object falls normally at O on the mirror and is reflected back along OP. Another ray PQ incident obliquely on the mirror is reflected along QR so that $\angle PQN = \angle RQN$ (Fig. 2.4). The two reflected rays OP and QR, when produced backwards, meet at P' *i.e.* the reflected rays appear to diverge from P' which, therefore, is the *virtual image* of P.

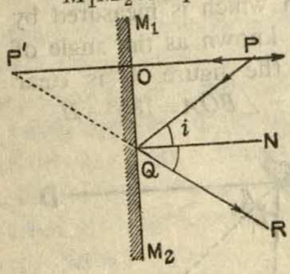


Fig. 2.4

Now, $\angle PQN = \angle OPQ$ because QN and OP are parallel.

For the same reason, $\angle NQR = \angle OP'Q$

So, $\angle OPQ = \angle OP'Q$ $[\because \angle PQN = \angle NQR]$

Now, take $\Delta^s QPO$ and QOP'. We have, $\angle OPQ = \angle OP'Q$

and $\angle QOP = \angle QOP'$ $[\because$ each is $90°]$ and QO is common to both.

$\therefore$ The triangles are equal in all respects. Hence, $OP = OP'$.

i.e., the image P' is as far back from the mirror as the object P is in front of it and the line PP' intersects the section of the mirror perpendicularly.

So, the image formed by a plane mirror has the following properties :

(1) The distance of the object (OP) from the mirror = the distance of the image (OP') from the mirror. They are also at equal distances from any point on the mirror ($QP = QP'$)

(2) The line joining the object and the image intersects the mirror perpendicularly.

(3) The size of the object = the size of the image.

It must not be thought, however, that only virtual images are obtained with a plane mirror. If a *convergent* beam is incident on a plane mirror MN, the reflected rays pass through a point P in front of the mirror [Fig. 2.5]. Here P is a *real image* because it can be received on a screen. The point Q where the convergent beam would have met had there been no plane mirror, acts, in this case, as a *virtual object.*

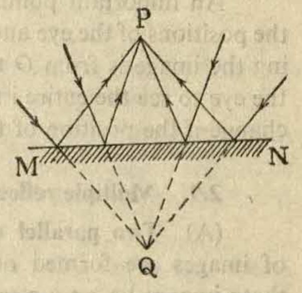


Fig. 2.5

Comparing the figs. 2.4 and 2.5, we see that a real object (divergent beam) gives rise to a virtual image while a virtual object (convergent beam) gives rise to a real image. In each case, however, the image and the object are at equal distances from the mirror.

2.8. Image of an extended object :

PQ is an extended object in front of a plane mirror MM'. An extended object may be regarded as a combination of a large number of point objects, each of which will be imaged by the mirror. The combination of these images will give the final image of the extended object.

From the point P of the object PQ, drop a perpendicular PM and produce it backward to P' such that $PM=P'M$. P' will be the image of the top-most point P of the object (Fig. 2.6). Similarly, from the point Q, drop a perpendicular QM' and produce it backward to Q' such that $QM'=Q'M'$. Q' will be the image of the lowest point Q of the object. Other points of the object between P and Q will have their images in the same way, and they will all lie between P' and Q'. So, $P'Q'$ will be the extended image of the extended object PQ [Fig. 2.6].

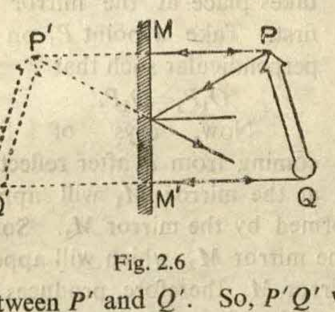


Fig. 2.6

Fig. 2.7 shows how an eye can see the image by the reflected rays. Rays of light PO and PO', coming from the point P, after reflection at the mirror MM', appear to the eye to diverge from P'. Similarly, rays QS and QS', coming from the lowest point Q of the object, after reflection, appear to come from Q'. In this way, rays from other points of the object, will after reflection, appear to

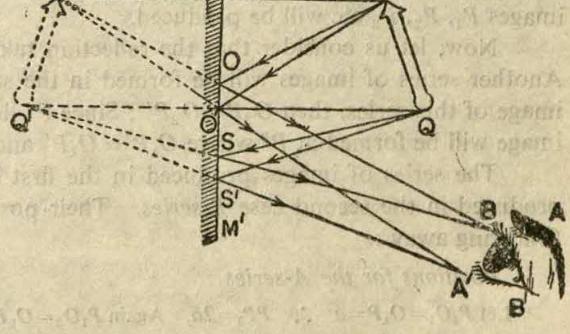


Fig. 2.7

come from points lying between P' and Q' and the eye will see the whole image $P'Q'$.

An important point to be noted in this connection is that depending upon the positions of the eye and the object, the effective portion of the mirror in producing the image is from O to S'. So, a mirror of length OS' will be sufficient for the eye to see the entire image. The effective portion of the mirror will, of course, change if the position of the eye or of the object is changed.

2.9. Multiple reflections at two mirrors :

(A) **Two parallel mirrors** : You may have noticed that a large number of images are formed of an object placed between two parallel mirrors. All these images lie on a straight line which passes through the object perpendicular to the mirrors.

M_1 and M_2 are two plane mirrors kept parallel to each other and P is a point object placed between them. A perpendicular is drawn from P on M_1 M_2 and it is produced on both sides of the mirrors. Let the perpendicular intersect the mirrors M_1 and M_2 at O_1 and O_2 respectively (Fig. 2.8).

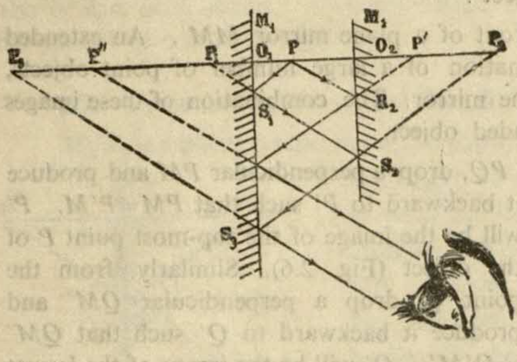


Fig. 2.8

Suppose the reflection takes place at the mirror M_1 first. Take a point P_1 on the perpendicular such that

$$O_1P_1=O_1P.$$

Now, rays of light coming from P after reflection at the mirror M_1 will appear to come from P_1 which is the image of P formed by the mirror M_1. Some of the rays, like P_1P_2, will again be reflected at the mirror M_2, which will appear to come from P_2 such that $O_2P_1=O_2P_2$. The mirror M_2, therefore, produces an image P_2 of the object P_1. Since P_2 is in the front of the mirror M_1, the mirror M_1 will again form an image P_3 of it, so that $O_1P_2=O_1P_3$. In this way, a number of images P_1, P_2, P_3 etc will be produced.

Now, let us consider that the reflection takes place at the mirror M_2 first. Another series of images will be formed in the same manner. If P' be the first image of this series, then $O_2P=O_2P'$; Since P' lies in front of the mirror M_1, an image will be formed at P'' where $O_1P'=O_1P''$ and so on.

The series of images produced in the first case is called *A-series* and that produced in the second case *B-series*. Their positions may be found out in the following away :

Positions for the A-series :

Let $P_1O_1=O_1P=a$ $\therefore$ $PP_1=2a$, Again $P_1O_2=O_2P_2$;

Now, $PP_2=PO_2+O_2P_2=PO_2+P_1O_2=PO_2+(P_1P+PO_2)=2PO_2+P_1P$

$$=2b+2a=2c \text{ (say) where } PO_2=b.$$

Also, $O_2P_2=O_1P_3$

Now, $PP_3=PO_1+O_1P_3=PO_1+O_1P_2=PO_1+(PO_1+PP_2)=2PO_1+PP_2=2a+2c$

In this way, if P_4 be the image of P_3, then $PP_4=4c$ and $PP_5=4c+2a$

Hence, for $2n^{th}$ image, $PP_{2n}=2n.c$ and for $(2n+1)^{th}$ image, $PP_{2n+1}=2nc+2a$

Positions for the B-series :

The first image being P', $O_2P=O_2P'$; proceeding in the same way, it may be shown that if P'_{2n} be the $2n_{th}$ image in the 'B-series, then $PP'_{2n}=2nc$ and $PP'_{2n+1}=2nc+2b$.

Theoretically, therefore, we will get an infinite number of images of an object placed between two parallel mirrors. The more remote the images, the fainter they become, since some of the light energy is absorbed by the mirror at each successive reflections.

Example : *Two parallel plane mirrors facing each other are a cm. apart. A small luminous object is placed between them b cm. from one of the mirrors. Find the distance from the object of an image produced by four reflections.*

[*H. S. Exam. 1984*]

Ans. P is the object between the plane mirrors M_1 and M_2 placed b cm. from the mirror M_1 [Fig 2.9]. The distance of the first image P_1 from the mirror $O_1M_1=b$. So, the distance of P_1 from O_2M_2, the second mirror$=O_1P_1+O_1O_2$ $=(b+a)$; now the distance of the image (2nd) P_2 formed by the mirror M_2 from it$=O_2P_2=O_2P_1=(a+b)$. Again distance of this image from the mirror M_1 $=O_2P_2+O_1O_2=a+b+a=2a+b$. So, the distance of the image P_3 (3rd) formed by the mirror M_1 from it$=(2a+b)$ and its distance from $M_2=2a+b+a=3a+b$

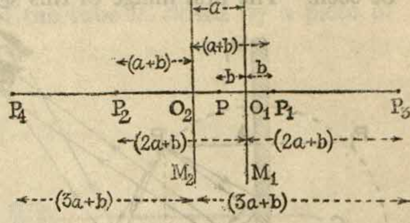


Fig. 2.9

Thus, the distance of the fourth image P_4 from $M_2=3a+b$. *i.e.* $O_2P_4=3a+b$.

Now, the distance of P_4 from the object $P=PP_4=O_2P_4+O_2P=(3a+b)$ $+(a-b)=4a$.

Practical application of parallel mirrors :

Simple periscope : A simple periscope consists of two plane parallel mirrors M_1 and M_2 facing each other at an angle of 45° to the line joining them (Fig. 2.10).

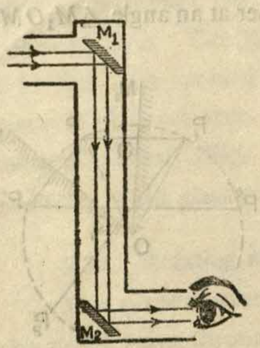


Fig. 2.10

They are mounted in a wooden frame or in a metallic tube and are capable of turning about a vertical axis. Keeping the instrument vertical and looking through the lower mirror M_2, distant objects will be visible. The periscope is used to see over the heads of a crowd or over the top of any obstacle. Rays of light coming from distant object will be reflected along the axis of the tube by the mirror M_1 and will fall on the mirror M_2 which, again, reflects the rays in a horizontal direction. The rays, then, reach the eye of the observer.

Many people watch football matches in Calcutta with the help of simple periscopes over the heads of a crowd in the maidan. Soldiers use this periscope to watch the movements of the enemy soldiers from a trench. Periscopes used in submarines are more elaborate than the simple type described here. Prisms are used instead of mirrors in such periscopes (See prism periscope).

(B) Two mirrors at right angles to each other :

M_1 and M_2 are two plane mirrors inclined to each other at right angles *i.e.*, $\angle M_1OM_2$ is 90°. P is a point source of light (Fig. 2.11). Dropping a perpendicular PAP_1 on M_1O, if P_1A is made equal to PA, then P_1 is the image of P formed by the mirror M_1O. Again, since P_1 lies in front of the mirror M_2O, the mirror will produce an image of P_1. To get the position of the image, produce the line M_2O and draw a perpendicular P_1CP_2 on it, such that $P_1C=P_2C$. P_2 will, then, be the image of P_1. Fig. 2.11 shows how the rays of light will be reflected in order that an eye may see these images. Since P_2 lies at the back of both the mirrors, no further images will be produced.

Again, looking through the mirror M_2O first, another set of images will be seen. The first image of this series is at P' such $BP=BP'$. The mirror M_1O will again produce an image of P' since P' lies in front of it. To get the position of this image, produce the line M_1O and draw a perpendicular $P'D$ and extend it to P'', making $P''D=P'D$. P'' will be the image of P'. No further images will be formed ; for P'' lies at the back of both the mirrors. With the help of simple geometry, it can be proved that P_2 coincides with P''. Geometrically, the object P and all the images (P_1, P' and P_2 or P'') lie on a circle whose centre is the intersection (O) of the mirrors and the radius is OP.

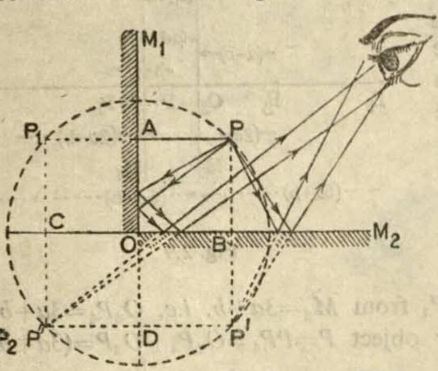


Fig. 2.11

(C) Two mirrors inclined at any angle :

M_1 and M_2 are two plane mirrors inclined to each other at an angle $\angle M_1OM_2$ (Fig. 2.12). P is a point source of light placed between them. Draw a perpendicular PQ from P on M_1O and produce it to P_1 such that $PQ=P_1Q$. P_1 is the image of P. The mirror M_2O will now produce an image of P_1 and its position P_2 will be obtained by drawing a perpendicular $P_1Q_1P_2$ on M_2O so that $P_1Q_1=P_2Q_1$. In this way, images will be formed until the final image falls behind the mirrors.

Again, considering the reflection by the mirror M_2O, the point source P will have another series of images like P', P'' etc.

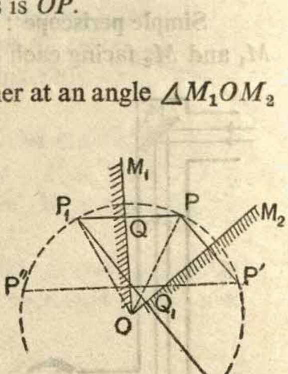


Fig. 2.12

Now, take $\Delta^s POQ$ and P_1OQ. $PQ=P_1Q$ and $\angle OQP=\angle OQP_1$ (each being 90°).

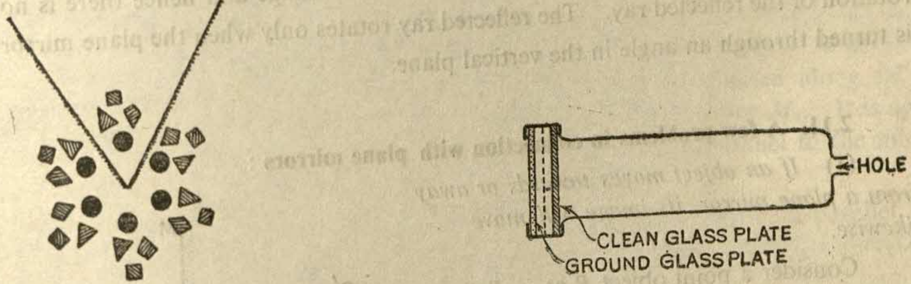
QO is common to both the triangles.

Hence, the triangles are equal in all respects. We, therefore, get $PO=P_1O$. In the same way, it may be proved that $P_1O=P_2O=P'O=P''O=$etc.

Geometrically, the object P and all the images lie on a circle whose centre is O and radius is OP. If $\angle M_1OM_2=\theta$, it may be proved that the number of images produced $n=\left(\frac{360}{\theta}-1\right)$; For example, if the mirrors make an angle 60° between themselves the number of images of an object produced by them is given by, $n=\left(\frac{360}{60}-1\right)=5$.

Applications : the Kaleidoscope : It is an interesting toy for the children. Its basic principle is the formation of a number of images by inclined mirrors.

It consists of three strips of plane mirror, placed at an angle of 60° to one another inside a cylindrical tube. One end of the tube is closed by a piece of



(i) (ii)

(i) Fig. 2.13

hard cardboard with a hole at its centre. At the other end of the tube, there is a ground-glass plate to admit light. On the plate are placed small pieces of coloured glass. The pieces are then covered by a transparent glass plate [Fig. 2.13 (ii)]. On looking through the hole in the cardboard, five images of each piece of glass are seen. All these images together with the objects form a symmetrical pattern [Fig. 2.13 (i)]. As the tube is slowly rotated, the respective positions of the glass pieces change and a variety of patterns is obtained.

2.10. Rotating mirror :

If a plane mirror is turned through an angle θ, a fixed ray of light reflected therefrom, turns through 2θ. This is the principle of a rotating mirror.

Consider a plane mirror MM on which a ray of light AO is incident (Fig. 2.14). The ray is reflected along OB. ON is the normal to the reflecting surface at O. According to the laws of reflection, $\angle AON = \angle BON$. Suppose, each of them is α. Then, $\angle AOB = 2\alpha$.

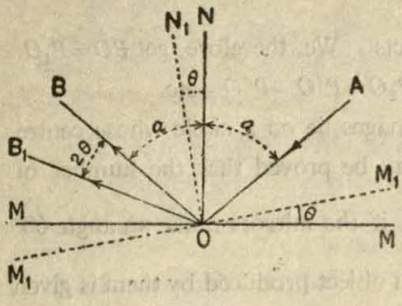


Fig. 2.14

Suppose now that the mirror is rotated through an angle θ to the position M_1M_1. The normal ON will also rotate through the same angle. Let the new position of the normal be ON_1.

Suppose, that the ray AO is now reflected along OB_1. So, the reflected ray rotates through an angle $= \angle BOB_1$.

According to the laws of reflection, $\angle AON_1 = \angle B_1ON_1$

But $\angle AON_1 = \alpha + \theta$; So, $\angle AOB_1 = 2(\alpha + \theta)$.

$\therefore \quad \angle BOB_1 = \angle AOB_1 - \angle AOB = 2(\alpha+\theta) - 2\alpha = 2\theta$.

Hence, it may be said that the reflected ray turns through 2θ.

It is to be noted that if the plane mirror MM is rotated in a horizontal plane about the axis ON, the angle of incidence does not change and hence there is no rotation of the reflected ray. The reflected ray rotates only when the plane mirror is turned through an angle in the vertical plane.

2.11. A few problems in connection with plane mirrors :

(1) *If an object moves towards or away from a plane mirror, its image will move likewise.*

Consider a point object P at a distance D from a plane mirror M (Fig. 2.15). Its image P' will be formed at the back of the mirror at the same distance D from it. If the object now moves through X towards the mirror, its distance from the mirror $= D - X$. Hence, the distance of its image from the mirror is also $D - X$. The displacement of the image is, therefore $= D - (D - X) = X$.

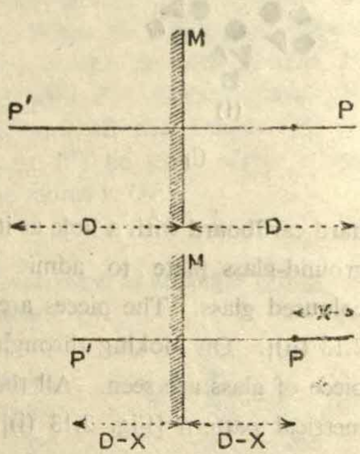


Fig. 2.15

(2) *If a plane mirror moves towards or away from an object, its image will likewise move through double the distance.*

Ans. Suppose that the object point P is at a distance D from the plane mirror M, which will produce an image P' at the same distance behind

it [Fig. 2.16 (i)]. If the mirror now moves through a distance X toward the object, then, the present object distance $=D-X$ [Fig. 2.16 (ii)].

So, the image P will now be at a distance $(D-X)$ behind the mirror.

Initial distance between the object and its image$=2D$.

Present distance between the object and its image$=2(D-X)$.

Since the object is fixed, the displacement of the image$=2D-2(D-X)=2X$.

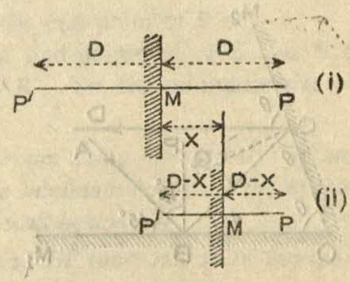


Fig. 2.16

So, if the mirror moves through a distance X towards the object, the image moves through a distance $2X$ likewise.

(3) *Two plane mirrors are inclined to each other. A ray of light coming parallel to the first mirror falls on the second and after reflection, is incident on the first. Being reflected by the first mirror the ray emerges parallel to the second mirror. Find the angle between the mirrors.*

Ans. Suppose two mirrors M_1 and M_2 are inclined to each other at an angle $\angle M_1OM_2$. A ray of light AB, parallel to the mirror M_1 is incident at B on the mirror M_2 and is reflected along BC so that it falls on the mirror M_1. It is again reflected along CD, parallel to the mirror M_2 (Fig. 2.17).

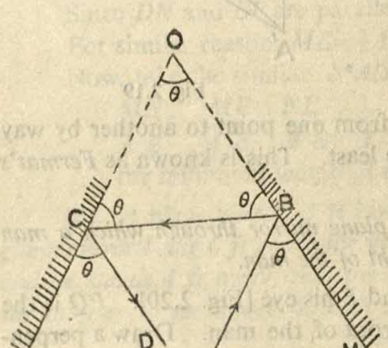


Fig. 2.17

Since AB and M_1O are parallel and OM_2 intersects them, $\angle ABM_2=\angle M_1OM_2$ $=\theta$ (say).

Again, since CD and M_2O are parallel and M_1O cuts them $\angle M_1CD=$ $\angle M_1OM_2=\theta$.

Also, since AB is the incident ray and BC its reflected ray, $\angle ABM_2=\angle CBO$ $=\theta$; for the same reason, $\angle M_1CD=$ $\angle BCO=\theta$.

So, the three angles of the ΔOBC are equal to one another. Hence $\angle M_1OM_2=60°$.

(4) *A ray of light is incident on a horizontal plane mirror at an angle of 45°. After reflection, some of the light, is incident on another plane mirror. If the ray reflected by the second plane mirror travels horizontally, find the angle between the mirrors.*

Ans. Suppose OM_1 is the horizontal plane mirror and OM_2 the second mirror [Fig. 2.18]. Let the angle between the mirrors be θ. According to the laws of reflection, $\angle ABM_1=\angle OBC=45°$

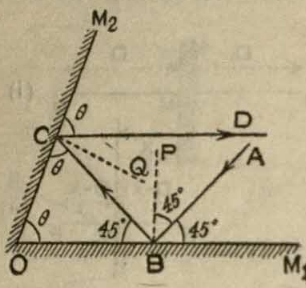


Fig. 2.18

Again, since CD and OM_1 are parallel, $\angle DCM_2=\theta$.

According to the laws of reflection, $\angle DCM_2=\angle BCO=\theta$.

Now, in ΔCOB, $\angle OCB+\angle CBO+\angle COB=180°$

or $\theta+45°+\theta=180°$; $\therefore\ \theta=67\frac{1}{2}°$

So, the angle between the plane mirrors$=67\frac{1}{2}°$.

(5) *MN is a plane mirror. AB and BC are the incident and reflected rays. D is any point on the mirror. Prove that* $AB+BC<AD+DC$.

Ans. In fig. 2.19, AB is the incident ray, and BC, the reflected ray. A' is the image of A. Hence CBA' is one single line.

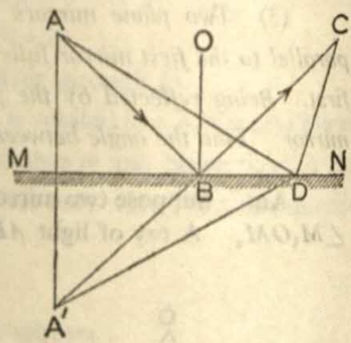


Fig. 2.19

$\Delta^s AMB$ and MBA' are equal in all respects because $AM=A'M$, MB is common to both and $\angle AMB=\angle A'MB=90°$.

$\therefore\ AB=A'B$

For the same reason, $AD=A'D$.

Now, $AB+BC=A'B+BC=A'C$.

But we know, sum of the two sides of a triangle is greater than the third side *i.e.*, $A'D+DC>A'C$

or $AD+DC>AB+BC$ $[\because A'D=AD]$

or $AB+BC<AD+DC$

Thus, the path followed by a ray of light from one point to another by way of reflection at a plane reflecting surface is the least. This is known as *Fermat's principle of least path*.

(6) *Prove that the minimum length of a plane mirror through which a man can see the full image of his body is half the height of the man.*

Ans. Let AB be the height of the man and E his eye [Fig. 2.20]. PQ is the mirror in front of the man. Draw a perpendicular from A on PQ and produce it equally to A' so that $AP=A'P$. A' is the image of A. Join A' with E and suppose it cuts the mirror at M_1. Rays of light, starting from A, after being reflected by the mirror, will appear to the eye E, as if they come from the point A'. In other words, if the mirror extends up to M_1, the image A_1 will be visible. Similarly, to make the lowest point B visible to the eye, the mirror should extend up to M_2. So, the minimum length of the plane mirror is M_1M_2.

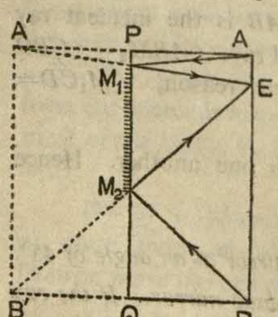


Fig. 2.20

Since, P is the mid-point of the arm AA' of the $\Delta AA'E$, and since PM_1 is parallel to AE, the point M_1 is the mid point of $A'E$.

For similar reason, M_2 may be proved to be the mid-point of $B'E$.

Now, in the $\Delta EA'B'$, M_1 and M_2 are the mid-points of $A'E$ and $B'E$ respectively. So, M_1M_2 is parallel and half of $A'B'$. So, the minimum size of the mirror is half the height of the man.

(7) *A plane mirror is fixed on the wall of a room and a man stands in front of the mirror at the middle of the room. Find the minimum height of the mirror through which the man can see the full image of the wall behind him.*

Ans. Suppose AB and CD are the two walls and EF the position of the man at the middle of the room, E being the eye of the person (Fig. 2.21). Produce AC to A' to make $AC=A'C$. Similarly, produce BD to B' to make $BD=DB'$. Evidently, $A'B'$ is the image of the wall AB.

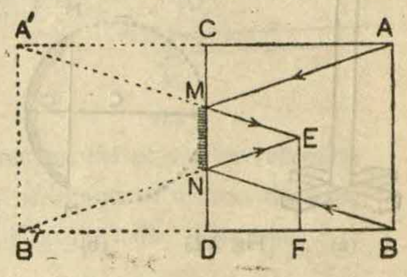


Fig. 2.21

Join E with A' and B'. Suppose, the lines intersect the wall CD at M and N respectively. MN will be the required length of the mirror for, the rays of light, starting from A and B after being reflected by the mirror MN, will appear to come from A' and B' respectively.

Now $FD=FB$ and $B'D=DB$. $\therefore$ $DF=\frac{1}{3}B'F$

Since DN and EF are parallel and $DF=\frac{1}{3}B'F$, $NE=\frac{1}{3}B'E$

For similar reason, $ME=\frac{1}{3}A'E$.

Now, take the similar $\Delta^s A'EB'$ and MNE. We have,

$$\frac{MN}{A'B'}=\frac{ME}{A'E}=\frac{NE}{B'E}=\frac{1}{3} \quad \therefore \quad MN=\frac{1}{3}A'B'=\frac{1}{3}AB$$

$\therefore$ the minimum length of the mirror $=\frac{1}{3}\times$ height of the wall.

(8) *A plane mirror, 3 ft high, is fixed against a vertical wall, the top of the plane mirror being 6 ft above the ground. The eye of a boy is 5 ft above the ground and he stands 4 ft away from the wall. (i) What length of the mirror is effective for the boy to see image through the mirror ? (ii) What portion of his body will be visible through the mirror ?*

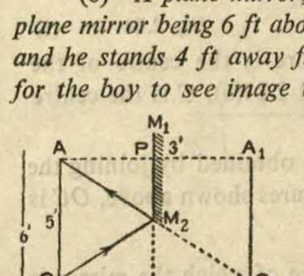


Fig. 2.22

Ans. Let $M_1M_2(=3$ ft) be the plane mirror and BB_1 the floor of the room. AB is the height ($=5$ ft) of the eye of the boy [Fig. 2.22]. According to the problem $M_1Q=6$ ft and $M_1M_2=3$ ft.

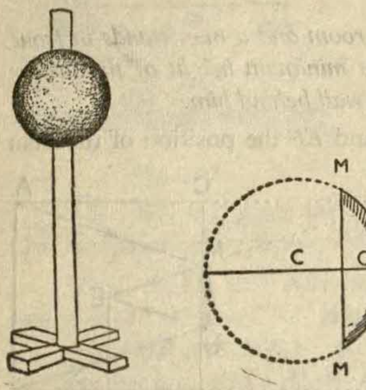
If AP is produced to A_1 such that $AP=A_1P$, A_1 represents the image of the boy's eye. It is clear from the figure that the portion PM_2 of the mirror is effective in producing the image.

Now, $PM_2=M_1M_2-M_1P=M_1M_2-(M_1Q-AB)=3-(6-5)=3-1=2$ ft.

Joining A to M_2 and producing the line AM_2 to cut the line of the image A_1B_1 at C_1, it is clear that the boy will see the portion AC or A_1C_1 of his body through the mirror. Now, the $\Delta^s APM_2$ and AA_1C_1 are similar. So, $\frac{PM_2}{A_1C_1}=\frac{AP}{AA_1}$ or, $\frac{2}{A_1C_1}=\frac{4}{8}$.

$\therefore\ A_1C_1=4$ ft.

So, the boy will see 4 ft length of his body.



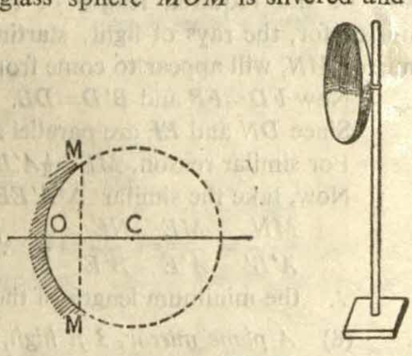
(a) Fig. 2.23 (b)

2.12. Spherical mirrors :

A spherical mirror is a portion of a glass sphere silvered on one surface to reflect light regularly. There are in general, two types of spherical mirrors :—(1) *Convex* and (2) *Concave*.

If the outside surface of a portion of glass sphere MOM is silvered and light is reflected by that surface, the mirror is called a convex one [Fig. 2.23 (b)]. Fig. 2.23 (a) shows the actual shape of a convex mirror.

If the inside surface of a portion of a glass sphere MOM is silvered and light is reflected by that surface, the mirror is called a concave one [Fig. 2.24 (*a*)]. Fig. 2.24 (*b*) shows the actual shape of a concave mirror.



(*a*) Fig. 2.24 (*b*)

2.13. Some important definitions in connection with a spherical mirror :

(i) **Pole :** The pole of a mirror is the centre of the reflecting surface or the face. In figures 2.23(b) and 2.24(a), O is the pole of the mirror.

(ii) **Centre of curvature :** The centre of curvature is the centre of the sphere of which the mirror is a part. In figs. 2.23(*b*) and 2.24(*a*), C is the centre of curvature of the mirror MOM.

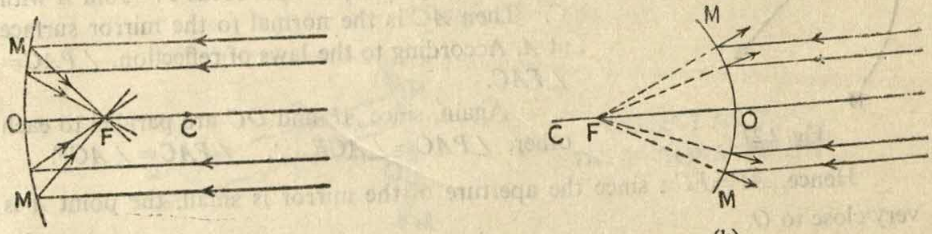
(iii) **Principal axis :** The principal axis is the line obtained by joining the pole and the centre of curvature of the mirror. In the figures shown above, OC is the principal axis of the mirror MOM.

(iv) **Radius of curvature :** The radius of the sphere of which the mirror is a part is called the radius of curvature of the mirror. In figs. 2.23(*b*) and 2.24(*a*), CO is one of the radii of curvature of the mirror MOM.

(v) **Aperture :** The aperture of a spherical mirror is the angle subtended by the mirror itself at its centre of curvature. In figs. 2.23(*b*) and 2.24(*a*) the angle subtended by the arc MOM at the centre of curvature C of the mirror is the

aperture of the mirror. In our subsequent discussion of the spherical mirrors, we shall always consider mirrors of small aperture not exceeding 10°.

(vi) **Principal focus :** Suppose MOM is a convex or a concave mirror. A parallel beam of rays, parallel to the axis of the mirror falls on the mirror



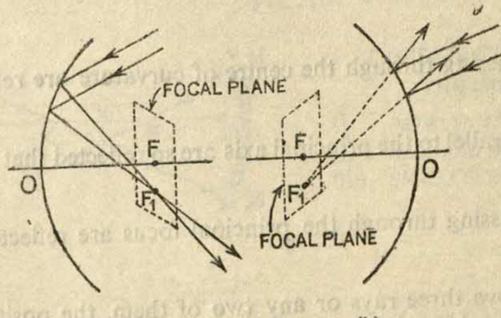
(a) Fig. 2.25 (b)

[Fig. 2.25(*a*) and (*b*)]. In the case of a concave mirror, the rays after reflection meet at a point F on the axis and in the case of a convex mirror, the rays after reflection, appear to diverge from a point F on the axis. This point F is called the principal focus of the mirror.

(vii) **Focal length :** The focal length of a spherical mirror is the distance between the pole of the mirror and the principal focus.

(viii) **Focal plane and secondary focus :** If a plane be imagined to be drawn through the principal focus of a mirror, perpendicular to the axis, the plane is called the focal plane of the mirror [Fig. 2.26].

If a beam of parallel rays be incident on a mirror slightly inclined to the axis, the rays after reflection converge to a point F_1 lying on the focal plane in



(a) Fig. 2.26 (b)

the case of a concave mirror [Fig. 2.26(*a*)] and appear to diverge from a point F_1 lying on the focal plane in the case of a convex mirror [Fig. 2.26(*b*)]. F_1 is called the secondary focus of the mirror.

It is to be borne in mind that the principal focus of a spherical mirror is a fixed point but the secondary focus is not a fixed point.

2.14. Relation between the focal length and the radius of curvature of a mirror :

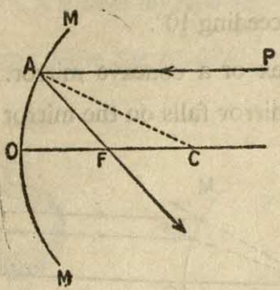


Fig. 2.27

In Fig. 2.27 *PA* is an incident ray parallel and close to the principal axis *OC* of a concave mirror *MOM*. The reflected ray *AF* passes through the principal focus *F*. Join *A* with *C*. Then *AC* is the normal to the mirror surface at *A*. According to the laws of reflection, $\angle PAC = \angle FAC$.

Again, since *AP* and *OC* are parallel to each other, $\angle PAC = \angle ACF \quad \therefore \quad \angle FAC = \angle ACF$.

Hence, $AF = FC$; since the aperture of the mirror is small, the point *A* is very close to *O*.

$$\therefore \quad AF = OF \text{ and } OF = FC \text{ i.e., } OF = \frac{OC}{2}.$$

If f be the focal length and r the radius of curvature, $f = \frac{r}{2}$.

In the same way, it can be proved that for a convex mirror $f = \frac{r}{2}$.

2.15. Image formation by spherical mirrors :

When an object is held in front of a spherical mirror, an image is formed whose position, size and nature can be found out by geometrical ray drawing, for if the focus and the centre of curvature of a mirror are located we can determine the direction in which some specific rays are reflected by the mirror. For example,

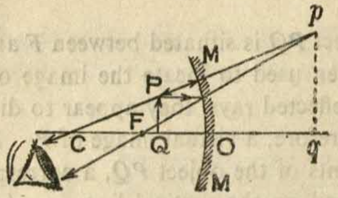
(i) Rays passing through the centre of curvature are reflected back along their own paths.

(ii) Rays parallel to the principal axis are so reflected that they pass through the principal focus.

(iii) Rays passing through the principal focus are reflected parallel to the principal axis.

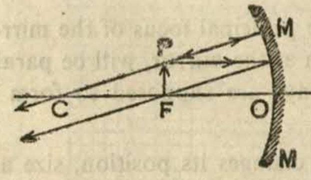
Using the above three rays or any two of them, the position, size, etc., of the image may be found out.

Figs. 2.28-2.33 show how the image formed by a concave mirror for different positions of an object placed on the principal axis can be obtained with the help of the above-mentioned rays. In all these diagrams, the object is represented by a vertical line *PQ* and the image by another vertical line *pq*.



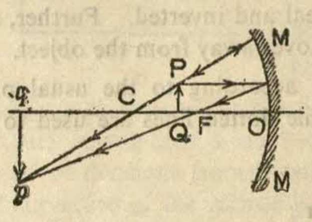
OBJECT BETWEEN F AND O;
THE IMAGE IS
(i) BEHIND THE MIRROR
(ii) VIRTUAL AND ERECT
(iii) LARGER THAN OBJECT

Fig. 2.28



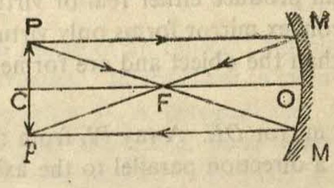
OBJECT AT F
THE IMAGE IS AT INFINITY

Fig. 2.29



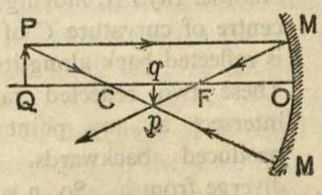
OBJECT BETWEEN F AND C
THE IMAGE IS
(i) BEYOND C
(ii) REAL AND INVERTED
(iii) LARGER THAN OBJECT

Fig. 2.30



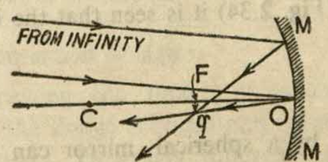
OBJECT AT C
THE IMAGE IS
(i) AT C
(ii) REAL AND INVERTED
(iii) SAME SIZE AS THE OBJECT

Fig. 2.31



OBJECT BEYOND C
THE IMAGE IS
(i) BETWEEN C AND F
(ii) REAL AND INVERTED
(iii) SMALLER THAN OBJECT

Fig. 2.32



OBJECT AT INFINITY
THE IMAGE IS
(i) AT F
(ii) REAL AND INVERTED
(iii) SMALLER THAN OBJECT

Fig. 2.33

Starting with the fig. 2.28, where the object PQ is situated between F and O, it will be seen that rays no. 1 and 2 have been used to locate the image of the point P of the object. On drawing the two reflected rays, they appear to diverge from the point p behind the mirror. p is, therefore, a virtual image of P. If the same construction is carried out for other points of the object PQ, a corresponding set of virtual image points will be obtained on the vertical line pq. In this case, the image formed is larger than the object, virtual, erect and behind the mirror.

In fig. 2.29, the object is shifted to the principal focus of the mirror. Rays from any point of the object, after reflection at the mirror, will be parallel to one another. Under these circumstances the rays are supposed to form an image at infinity.

Figs. 2.30—2.33 show how the image changes its position, size and nature as the object PQ is gradually moved away from the mirror to infinity. It is to be noted that in all these cases, the image is real and inverted. Further, the image becomes smaller and smaller as the object moves away from the object.

In the diagrams, full lines are used, according to the usual practice to represent real rays, objects and images, while dotted lines are used to represent virtual rays and images.

Formation of images by a convex mirror :

Unlike the concave mirror, which can produce either real or virtual images according to the position of the object, a convex mirror forms only virtual images. The images are always erect and smaller than the object and are formed between the focus and pole of the mirror.

PQ is an object in front of a convex mirror OL. A ray PL from the point P on the object is incident on the mirror in a direction parallel to the axis. It will be reflected along LM such that the ray appears to diverge from the focus F. Another ray PN, moving towards the centre of curvature C of the mirror, is reflected back along its own path. These two reflected rays do not intersect at any point but when produced backwards, appear to diverge from p. So, p is the virtual image of P. In the same way, the images of other points on the object may be found out. From the diagram (Fig. 2.34) it is seen that the image pq is virtual, erect and smaller than the object.

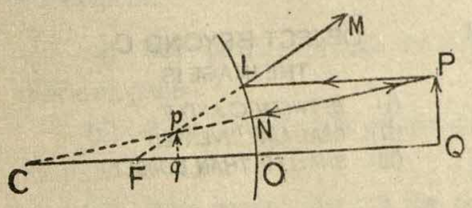


Fig. 2.34

Graphical construction of image :

The position of an image formed by a spherical mirror can be located accurately by graphical construction. The procedure is as follows :

Draw a horizontal line OC on a graph paper. The line will represent the

principal axis of a spherical mirror [Fig. 2.35]. Draw another line MOM' perpendicular to CO. This line represents the plane through the pole of the mirror.

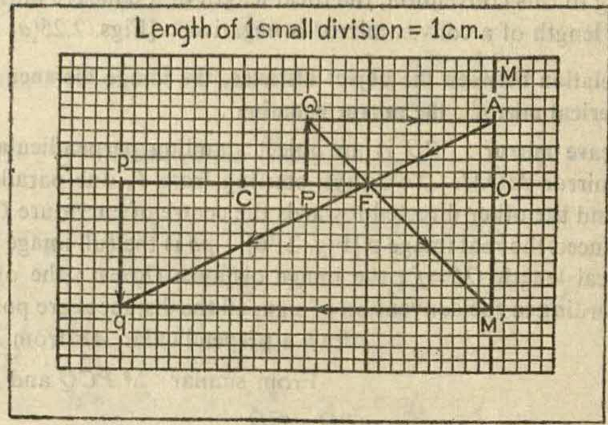


Fig. 2.35

Suppose, the mirror is concave with a focal length of 10 cm. Now, if the scale is selected in such a way that 1 small division represents 1 cm, then a point F at a distance of 10 divisions from O will locate the focus of the mirror. Since, the radius of curvature of the mirror is double the focal length, a point C at a distance of 20 small divisions from O will give the centre of curvature of the mirror.

Now, an image of an object 5 cm high and situated 15 cm from the mirror is to be constructed. In this case, a point P is to be taken at a distance of 15 small divisions from O. A perpendicular PQ, 5 small divisions in height is to be drawn on the line OC. PQ, then, represents the object. Two incident rays are now taken—one parallel to the axis and incident on the mirror at A and the other passing through the focus and incident at M'. The first ray, after reflection at the mirror passes through the focus F and the other goes parallel to the axis. The reflected rays intersect at q which is the real image of Q. Drawing a perpendicular pq on the axis, the whole image is obtained. It is found that the image is situated 30 small divisions from O. So, the image distance, according to the scale $=30$ cm. Also, the height of the image pq is equal to 10 small divisions. So, the height of the image, according to the scale is 10 cm. The magnification$=\frac{10}{5}=2$.

Same results will be obtained if calculations are made with the help of mirror equation. Needless to say that the same graphical method may be employed in the case of a convex mirror for locating the position and height of the image.

2.16. Convention of sign :

While drawing the images of an object placed at different positions, we have seen that the image is sometimes formed in front of and sometimes, at the back of the mirror. To specify different image distances and object distances, suitable sign convention need be adopted. The usual convention is as follows :

Taking the pole of the reflecting surface as the origin, distances measured in

the same direction as the incident light will be counted negative and those measured in the opposite direction will be counted positive.

According to this convention, the focal length of a concave mirror is positive while the focal length of a convex mirror is negative. [Figs. 2.25(*a*) and (*b*)].

2.17. Relation between the object distance, the image distance and the focal length of a spherical mirror ; the mirror equation :

(i) **Concave mirror** : PQ is an object standing perpendicular to the axis of a concave mirror MOM. Two rays, starting from P, one parallel to the axis of the mirror and the other directed towards the centre of curvature C, after reflection have produced the real image p [Fig. 2.36]. pq is the full image of the object.

Here, focal length $OF=f$; the image distance $Oq=v$; the object distance $OQ=u$. According to the convention of sign, all the distances are positive. Now, draw a perpendicular AB from A on the axis.

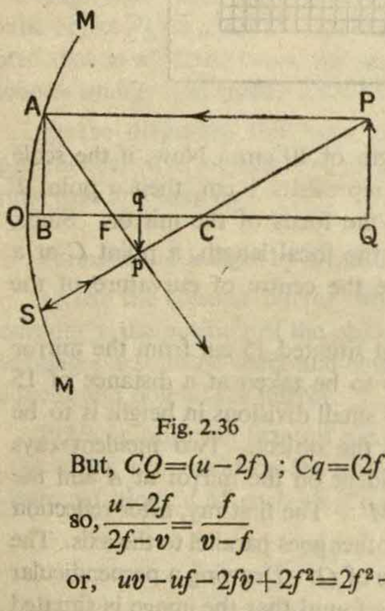


Fig. 2.36

From similar Δ^s PCQ and pCq, we have,

$$\frac{PQ}{pq}=\frac{CQ}{Cq}$$

Again, from similar Δ^s ABF and pFq we have, $\dfrac{AB}{pq}=\dfrac{BF}{Fq}$

But as $AB=PQ$, we get, $\dfrac{PQ}{pq}=\dfrac{AB}{pq}$

$$\therefore \quad \frac{CQ}{Cq}=\frac{BF}{Fq}=\frac{OF}{Fq}$$

[Since the aperture of the mirror is very small]

But, $CQ=(u-2f)$; $Cq=(2f-v)$; $OF=f$; $Fq=(v-f)$

so, $\dfrac{u-2f}{2f-v}=\dfrac{f}{v-f}$

or, $uv-uf-2fv+2f^2=2f^2-fv$ or, $uv=uf+vf$

or, $\dfrac{1}{f}=\dfrac{1}{v}+\dfrac{1}{u}$ [Dividing by $u.v.f.$]

(ii) **Convex mirror** : Consider an object PQ standing perpendicular to the axis of a convex mirror MOM. Two rays, starting from P, one parallel to the axis of the mirror and the other directed towards the centre of curvature C, after reflection have produced the virtual image p [Fig. 2.37]. pq is the full image of the object. Here, focal length $OF=-f$; the image distance $Oq=-v$ and the object distance $OQ=u$.

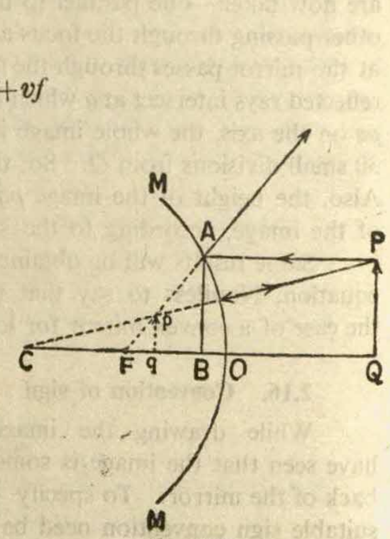


Fig. 2.37

A perpendicular AB is drawn from A on the axis OC. Now, from similar $\Delta^s PCQ$ and pCq, we have, $\frac{PQ}{pq}=\frac{CQ}{Cq}$

Also from similar $\Delta^s ABF$ and pFq we get $\frac{AB}{pq}=\frac{BF}{Fq}$

Since $AB=PQ$, we can write, $\frac{AB}{pq}=\frac{PQ}{pq}$

$\therefore \quad \frac{CQ}{Cq}=\frac{BF}{Fq}=\frac{OF}{Fq}$ (since the aperture is very small)

But, $CQ=u+(-2f)$; $Cq=-2f-(-v)$; $OF=-f$; $Fq=-f-(-v)$.

$\therefore \quad \frac{u+(-2f)}{-2f-(-v)}=\frac{-f}{-f-(-v)}$; or, $\frac{u-2f}{v-2f}=\frac{f}{f-v}$

or, $uf-uv-2f^2+2f.v=f.v-2f^2$ or, $uv=f.v+uf$

or, $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$ [Dividing both sides by $u.v.f.$]

The above equation is called the mirror equation.

2.18. Pair of conjugate focii :

If the distances of two points from the pole of a mirror are u and v along the principal axis, and if they are applicable to the mirror equation *viz*, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, the points are called the pair of conjugate focii. From the above equation, it is clear that the equation remains unchanged if u and v are interchanged. In other words, the positions of an object and its real image are interchangeable. Referring to the figs. 2.30—2.33, it is seen that if the object PQ is placed in the position of the image pq, the image will be formed in the previous position of the object.

So, conjugate focii are any pair of points such that an object placed at one of them, gives rise to a real image at the other.

2.19. Linear magnification :

Linear magnification is defined as the ratio of height of the image to the height of the object. Thus,

$$\text{Linear magnification } (m)=\frac{\text{Height of the image}}{\text{Height of the object}}$$

From figs. 2.36 and 2.37, it may be said that $m=\frac{pq}{PQ}$

From, Δ^s PCQ and pCq, we have, $\frac{pq}{PQ}=\frac{Cq}{CQ}=\frac{OC-Oq}{OQ-OC}=\frac{r-v}{u-r}$.

From the general formula of spherical mirrors, we get,

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}=\frac{2}{r} \quad \text{or,} \quad \frac{1}{v}-\frac{1}{r}=\frac{1}{r}-\frac{1}{u}$$

$$\text{or,} \quad \frac{r-v}{v.r}=\frac{u-r}{u.r} \quad \text{or,} \quad \frac{r-v}{u-r}=\frac{v}{u}$$

$$\therefore \quad m=\frac{pq}{PQ}=\frac{r-v}{u-r}=\frac{v}{u}=\frac{\text{image distance}}{\text{object distance}}$$

Relation between m and v :

Since $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, multiplying by v, we get $1+\frac{v}{u}=\frac{v}{f}$ or $1+m=\frac{v}{f}$

When the image and the object have the same size, then $m=1$. From the above equation it follows that $v=2f$. Now $2f=r=$ radius of curvature of the mirror. Thus, $v=r$. Hence $u=r$, since $m=v/u=1$.

So, when an object is placed at the **centre of curvature** of a concave mirror, an image of equal size is formed at the same place (Fig. 2.31).

Example 1 : *An object is placed at distance (a) 20 cm. and (b) 4 cm. in front of a concave mirror of focal length 12 cm. Determine the position, size and nature of the image in each case.*

Ans. (*a*) Here, $u=+20$ cm. ; $f=+12$ cm. ; $v=$?

We have, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ or, $\frac{1}{v}+\frac{1}{20}=\frac{1}{12}$

$$\therefore \quad \frac{1}{v}=\frac{1}{12}-\frac{1}{20}=+\frac{2}{60} \qquad \therefore \quad v=30 \text{ cm.}$$

Since, the image distance is $+ve$, the image is real and is formed at a distance of 30 cm. from the mirror. Magnification $=\frac{v}{u}=\frac{30}{20}=1{\cdot}5$

(*b*) Here, $u=+4$ cm. ; $f=+12$ cm. ; $v=$?

Now, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ or, $\frac{1}{v}+\frac{1}{4}=\frac{1}{12}$ $\therefore$ $\frac{1}{v}=\frac{1}{12}-\frac{1}{4}=-\frac{1}{6}$ $\therefore$ $v=-6$ cm.

Since v is—ve, the image is virtual and is formed at the back of the mirror at a distance of 6 cm. Magnification $=\frac{v}{u}=\frac{6}{4}=1{\cdot}5$

Example 2 : *A convex mirror of focal length 18 cm. forms a virtual image on its axis at a distance of 6 cm. from it. Find the position of the object.*

Ans. Here, $v=-6$ cm. (negative because image is virtual) ; $f=-18$ cm.

Now, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$; Here, $-\frac{1}{6}+\frac{1}{u}=-\frac{1}{18}$

$$\text{or,} \quad \frac{1}{u}=\frac{1}{6}-\frac{1}{18}=\frac{2}{18} \qquad \therefore \quad u=9 \text{ cm.}$$

So, the object is at a distance of 9 cm. from the mirror.

Example 3 : *The sun subtends an angle of $\frac{1}{2}°$ at the centre of curvature of a concave mirror whose radius of curvature is 36 ft. What will be the size of the image formed by the mirror ?*

Ans. $(\frac{1}{2})° = \frac{1}{2} \times \frac{\pi}{180} = \frac{\pi}{360}$ radian. According to circular measure,

radian$=\frac{\text{arc}}{\text{radius}} = \frac{D}{u} = \frac{\pi}{360}$ $\therefore$ $u = \frac{360 \times D}{\pi}$ ft. where D=diameter of the sun and u=the distance of the sun from the mirror.

As the sun is far away from the mirror, the image will be formed at the focus of the mirror. Hence, $v = f = \frac{r}{2} = \frac{36}{2} = 18$ ft. So, $m = \frac{v}{u} = \frac{18 \times \pi}{360 \times D}$

Now, the diameter (*i.e.* size) of the image=magnification×diameter of the sun$=\frac{18 \times \pi}{360 \times D} \times D = \frac{18 \times \pi}{360}$ ft.$=\frac{18 \times \pi \times 12}{360}$ inches$=1{\cdot}88$ inches.

Example 4 : *A concave mirror has focal length f. A point object is placed at a distance xf away from the focus. Prove that the image will be formed at a distance f/x away from the focus and the magnification is 1/x.*

Ans. In this case, $u = f + xf$; From the equation $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we have

$\frac{1}{v} + \frac{1}{f+xf} = \frac{1}{f}$ $\therefore$ $\frac{1}{v} = \frac{1}{f} - \frac{1}{fx+f} = \frac{xf}{f(f+xf)} = \frac{x}{f+xf}$ $\therefore$ $v = \frac{f}{x} + f$. This shows that the image is formed f/x away from the focus.

Further, magnification$=\frac{v}{u} = \frac{f+xf}{x(f+fx)} = \frac{1}{x}$.

Example 5 : *A convergent beam of rays is so incident on a convex mirror that in absence of the mirror, it would have converged at a point 10 cm. away from the pole of the mirror. Find the point where the rays reflected by the mirror converge. Focal length of the mirror is 15 cm.*

Ans. The point P where the beam would have converged in absence of the mirror, will, in the present case act as a virtual object point [Fig. 2.38]. Hence, we have $OP = u = -10$ cm and $f = -15$ cm (convex).

From $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ we get $\frac{1}{v} - \frac{1}{10} = -\frac{1}{15}$

or $\frac{1}{v} = \frac{1}{10} - \frac{1}{15} = \frac{1}{30}$ $\therefore$ $v = 30$ cm

So, the image Q is formed at a point 30 cm away from the mirror. Since v is $+ve$, the image is real.

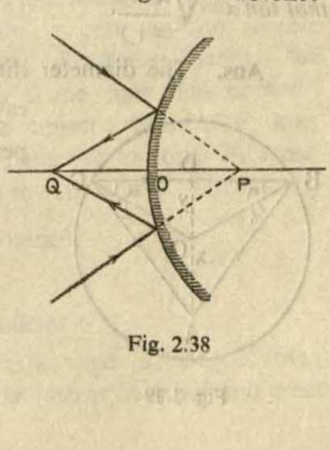


Fig. 2.38

Example 6 : *A concave mirror of radius of curvature 20 cm. and a convex mirror of radius of curvature 30 cm. are placed, facing each other, on the same axis, 40 cm. apart. An object 5 cm. high is placed vertically on the axis at a distance of 15 cm. from the concave mirror. Rays of light are first reflected by the concave mirror and then by the convex mirror. Find the position and size of the final image.*

Ans. The image formed by the first reflection at concave mirror will serve the purpose of object to the convex mirror. The final image will be formed by the convex mirror. Now, for the concave mirror, $u=15$ cm. ; $r=20$ cm. ; $v=$?

From $\frac{1}{v}+\frac{1}{u}=\frac{2}{r}$, we have $\frac{1}{v}+\frac{1}{15}=\frac{2}{20}$ $\therefore$ $\frac{1}{v}=\frac{1}{10}-\frac{1}{15}=\frac{1}{30}$

or $v=+30$ cm. This shows that the image is 30 cm. away from the concave mirror or $(40-30)=10$ cm. in front of the convex mirror. So, for the convex mirror, $u=10$ cm ; $r=-30$ cm. $v=$?

From $\frac{1}{v}+\frac{1}{u}=\frac{2}{r}$, we have, $\frac{1}{v}+\frac{1}{10}=-\frac{2}{30}=-\frac{1}{15}$

$$\therefore \quad \frac{1}{v}=-\frac{1}{15}-\frac{1}{10}=-\frac{1}{6} \quad \text{or,} \quad v=-6 \text{ cm.}$$

So, the final image is formed 6 cm. behind the convex mirror and it is virtual.

Again, magnification by the concave mirror, $m_1=\frac{v}{u}=\frac{30}{15}=2$

and magnification by the convex mirror, $m_2=\frac{v}{u}=\frac{6}{10}=\frac{3}{5}$

$\therefore$ Final magnification$=m_1\times m_2=2\times\frac{3}{5}=\frac{6}{5}$

So, the final size of the image=final magnification×size of the object $=\frac{6}{5}\times5=6$ cm.

Example 7 : *A luminous point A is inside a circle. A ray emerges from A and after two reflections at the circle, returns to A. If α be the angle of incidence, x the distance of A from the centre of the circle and y the distance of the centre from the point where the ray in its course crosses the diameter through A, prove that* $\tan\alpha=\sqrt{\frac{x-y}{x+y}}$.

Ans. The diameter through the point A has intersected the course of the ray at D [Fig 2.39]. It is easy to see that AD is perpendicular to BC. So, $AD.\cot 2\alpha=BD=OD.\cot\alpha$.

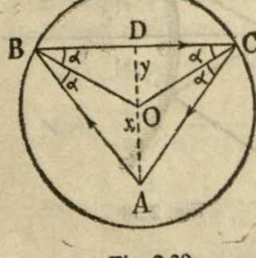


Fig. 2.39

$$\therefore \quad (x+y)\cot 2\alpha=y.\cot\alpha$$

$$\text{or, } (x+y)\tan\alpha=y.\tan 2\alpha=\frac{2y.\tan\alpha}{1-\tan^2\alpha}$$

$$\therefore \quad (x+y)(1-\tan^2\alpha)=2.y$$

$$\text{or, } \tan^2\alpha=1-\frac{2y}{x+y}=\frac{x-y}{x+y} \quad \therefore \quad \tan\alpha=\sqrt{\frac{x-y}{x+y}}.$$

2.20. Determination of focal length of a concave mirror :

(i) **By pins :** We know that when an object is placed at the centre of curvature of a concave mirror, its image is also formed at the centre of curvature. The image becomes inverted. This fact is used in determining the focal length of a concave mirror.

The concave mirror *O* [Fig. 2.40] is supported vertically in a suitable holder and a shining brass pin *P* fixed in another holder is adjusted so that the tip of the pin is at the same level as the centre of the mirror. The pin is then moved along the table in front of the mirror until a real and inverted image of the pin is seen somewhere in front of the mirror. The pin is then moved to and fro until there is no parallax between the top of the pin *P* and the tip of the inverted image *p*. Measure the distance between the pin *P* and the centre of the mirror. It is equal to the radius of curvature of the mirror, half of which gives the focal length.

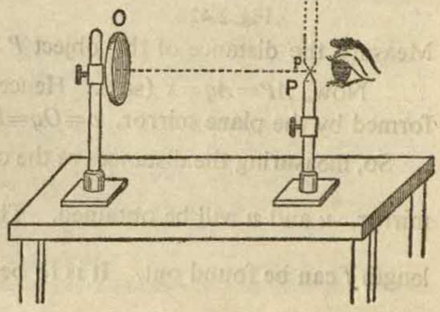


Fig. 2.40

(ii) **By U-V method :** In this method, an optical bench is required. The bench is made of brass or wood and a scale is attached to it. Some suitable uprights are provided which can move along the bench. These uprights hold the mirror, paper screen, pin, etc.

Place a concave mirror *O* and a candle *P* on the bench at a distance apart. Adjust the candle flame and the centre of the mirror in the same level (Fig. 2.41). Now place a paper-screen *Q* at some distance away from the candle as shown in the figure. Keeping the mirror and the candle fixed, move the screen to and fro until a sharp and inverted image of the candle-flame is cast on the screen. Now, determine from the bench scale, the distance between the mirror and the flame and that between the screen and the mirror. The first one is the object distance *u* and the second one, the image distance *v*. Keeping the flame at various distances from the mirror, the observations are to be repeated several times. Then using the formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, the focal length *f* can be determined.

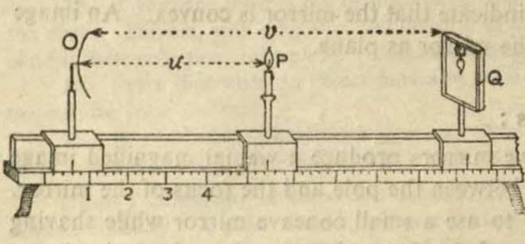


Fig. 2.41

2.21. Determination of focal length of a convex mirror :

Place a convex mirror *MO* vertically on a table and a long shining pin *P* in front of it [Fig. 2.42]. Looking through the mirror, a small and erect

image p will be visible. Now, covering the lower half of the convex mirror, place a plane mirror A vertically between the object P and the mirror. The plane mirror will form an image of P at its back. Now, move the plane mirror to and fro until the image p formed by the convex mirror and the image q formed by the plane mirror are in the same vertical line without any parallax.

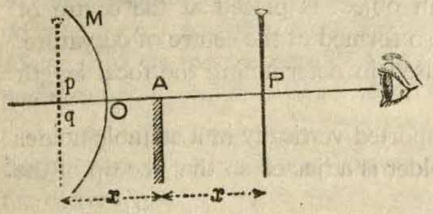


Fig. 2.42

Measure the distance of the object P from the convex mirror. It is u.

Now, $AP=Aq=x$ (say). Hence $Pq=2AP=2x$ because q is the image of P formed by the plane mirror, $v=Oq=Pq-OP=2x-u$.

So, measuring the distances of the object P from the convex mirror and the plane mirror, u and v will be obtained. Then using the formula, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, the focal length f can be found out. It is to be remembered that v is $-ve$ in this case.

2.22. Identification of mirrors :

How can you indentify in a simple way whether a mirror is plane, concave or convex ? We know, a plane mirror produces an image which is of the same size as the object. But when an object is placed very close to a concave mirror, a magnified virtual image is produced while in the similar circumstances, a convex mirror will produce a diminished image of the object. So, the simple way of identifying a mirror is to hold a finger very near the mirror and to look for its image through the mirror. A magnified image will indicate that the mirror is concave but a diminished image will indicate that the mirror is convex. An image of same size will, however, identify the mirror as plane.

2.23. Uses of spherical mirrors :

(i) **Concave mirror** : Concave mirrors produce a virtual magnified image (Fig. 2.26) when the object is placed between the pole and the focus of the mirror. For this reason, many people prefer to use a small concave mirror while shaving because it produces a magnified image of the face. So, concave spherical mirrors are widely used as shaving mirrors.

Reflecting telescopes also utilise concave mirrors. The largest telescope in the world at Mount Palomar has a huge concave mirror. Dentists also use concave mirrors while examining the teeth of a patient.

(ii) **Convex mirror** : As convex mirrors always produce a diminished virtual image and as the image is always situated between the pole and the focus of the mirror, cars and other vehicles use convex mirrors as view finders. A small convex mirror is usually fitted in front of and slightly to the right of the driver of the car. The driver, while driving the car, can see through this mirror, all the vehicles and other things coming from behind the car.

Exercises

Essay type :

1. Show that a beam of rays from a point source after reflection at a plane mirror appears to diverge from another point. What is that point called ? Where is it situated ? What is its nature ?

2. Explain, with diagram, how a plane mirror produces an image. Prove that the image is as far back of the mirror as the object is in front of it.

3. Explain, with a neatly drawn diagram how two plane mirrors inclined at right angles to each other form images of an object placed between them. [*H. S. Exam. 1979*]

4. Prove that a ray reflected from a plane mirror turns through twice the angle through which the mirror turns. [*H. S. Exam. 1970, '74, '81, '84*]

5. Two plane mirrors are at right angles to each other. A ray of light is reflected successively by the mirrors. Show that the incident ray and the final emergent ray are parallel to each other.

6. Prove, with the help of a diagram, that for a person of given height standing upright, the minimum length of a vertical plane mirror in which he can see his feet and the top of his head at the same time, is independent of the distance between his eyes and the top of his head.

7. A man stands at the centre of a room. A plane mirror hangs in the wall in front of him. Show that the minimum height of the mirror through which the man can see the full image of the wall behind him is one third the height of the man.

8. Explain, with the help of a diagram, the action of a periscope. For what purpose is it used ?

9. Prove, in the case of a concave mirror, that $\frac{1}{v}+\frac{1}{u}=\frac{2}{r}=\frac{1}{f}$ [*H. S. Exam. 1978*]

10. How will you proceed to determine the focal length of a convex mirror ?

Short answer type :

11. What is reflection of light ? What are the laws of reflection ?

12. A ray of light is incident on a plane mirror at an angle of 60°. What will be its deviation after reflection ? Explain with a diagram.

13. Two plane mirrors are inclined at an angle of 60°. An object lies on the bisector of the angle. Explain, with diagrams, the position of the image when (i) two reflections take place and (ii) three reflections take place. What are the total number of images produced ?

14. Prove that when an object moves in front of a plane mirror, its image moves equally through the same distance.

15. At night, it is difficult to see through a closed glass window from a well-lighted room ; but it is relatively easy when the room lights are switched off. Why ?

[**Hints** : Internally reflected and externally transmitted light will interfere and this will obscure visibility. A glass window, however transparent, always reflects a part of incident light. The reflected light interferes with the rays of light transmitted from outside and therefore disturbs vision. When room lights are switched off, there is no reflection and hence no interference with the transmitted rays of light from outside objects.]

16. Why are cinema screens made white and rough ?

17. Define the following terms in connection with a concave mirror : (a) focus (b) focal length (c) radius of curvature (d) centre of curvature (e) pole (f) conjugate pair of focii.

18. What restriction was made in deducing the mirror equation ? What is the harm if the restriction is not adhered to ?

19. How would you identify whether a mirror is plane, concave or convex ? [*H. S. Exam. 1979*]

20. State the nature of the mirror to be used and the position where an object is to be placed so as to obtain an image (i) real and magnified (ii) virtual and magnified (iii) real and diminished (iv) vertual and diminished (v) real and of equal size (vi) virtual and of equal size.

Objective type :

21. Answer the following questions by writing 'Yes' or 'No'.

(i) Can a virtual image be photographed ?——

(ii) Can a convex mirror form a magnified real image ?——

(iii) Are convex mirrors used as 'view-finder' in motor cars ?——

(iv) Can diffuse reflection of light form images ?——

(v) Is the principle of parallel mirrors applied in a periscope ?——

Numerical Problems :

22. An object P lies between two parallel plane mirrors M_1 and M_2. Its distance from the mirror M_1 is 4 cm. and the second image seen through the mirror M_2 is at a distance of 22 cm. from M_1. Find the distance between the mirrors. [Ans. 9 cm.]

23. A point source of light is situated between two parallel mirrors A and B. The distance between the mirrors is 3 inches and the object is 2 inches away from one mirror. How far is the third image behind A from the third image behind B ? [Ans. 18 inches]

24. (*a*) Find the angle between two plane mirrors in order that a ray incident on the first mirror in a direction parallel to the second may after reflection in the second mirror retrace its path. [Ans. 45°]

(*b*) Two perpendicular mirrors form the sides of a vessel filled with water. A ray of light is incident from above, normal to the water surface. Show that the emerging ray is parallel to the incident ray. Assume that there are two reflections at the mirror surfaces.

25. A man stands at the centre of a room. A plane mirror hangs in the wall in front of him. What should be the minimum height of the mirror through which the man can see the full image of the wall behind. The height of the wall is 4·2 metres.

[*H. S. Exam. 1978*] [Ans. 1·4 metres]

26. How many images of himself can an observer see in a room whose ceiling and two adjacent walls are mirrors ? [Ans. 6]

27. If a man runs at the rate of 5 ft./sec towards a mirror, at what rate will he approach his image formed by the mirror ? [Ans. 10 ft/sec]

28. Two plane mirrors are inclined at an angle of 35°. A ray of light is incident on one mirror at 60° and undergoes two successive reflections at the mirrors. Show by accurate drawing that the angle of deviation produced is 70°.

29. Two plane mirrors inclined to each other form images of an object placed between them. Prove that the object and its images will lie on the circumference of a circle.

30. A ray is so incident on a plane mirror that the incident ray and its reflected ray subtend an angle of 20° between them. If the mirror is now rotated through 15°, what are the possible angles between the incident ray and the new reflected ray ? [Ans. 50° ; 10°]

31. Two plane mirrors are inclined to each other at a fixed angle. If a ray travelling in a plane perpendicular to both mirrors is reflected first from one and then from the other, show that the angle through which it is deflected does not depend on the angle at which it strikes the first mirror.

32. Two mirrors A and B are inclined to each other at 60°. A ray of light is incident parallel to A upon B. Obtain the magnitude of deviation of the ray after the second reflection.

[*H. S. Exam. 1982*] [Ans. 240°]

33. A concave mirror forms an erect image three times magnified at a distance 24 cm on its axis. What is the focal length of the mirror ? [Ans. 9 cm.]

34. The magnification of an image formed by a convex mirror is $1/p$. Calculate the object distance if the focal length of the mirror be f. [*H. S. Exam. 1983*] [Ans $(p-1)f$]

35. A converging beam of rays, coming from an object, would have met at a point 30 cm. behind a mirror but after reflection, actually met at a point 15 cm. in front of the mirror. Was the mirror concave or convex ? What was its focal length ? [Ans. Concave ; 30 cm.]

36. An arrow, 15 cm. long, is lying *along* the axis of a concave mirror. The pointed end of the arrow is 30 cm. away from the mirror whose radius of curvature is 20 cm. Find the magnification of the image of the arrow. [Ans. 0·143]

37. A concave mirror forms a virtual image three times the size of an object when the distance between the object and the mirror is 10 cm. Find the position and size of the image when an object 1″ high is placed at a distance of 40 cm. from the same concave mirror.

[Ans. 24 cm. ; 0·6″]

[Ans. 4·16 cm. 25 cm.]

38. A coin 2·54 cm in diameter held 254 cm. from the eye just covers the full moon. What is the diameter of the image of the moon formed by a concave mirror of radius of curvature 127 cm. ? [Ans. 0·635 cm]

39. A concave mirror forms a doubly magnified virtual image of an object placed 10 cm. away from the mirror. If the object is kept at a distance of 30 cm. from the mirror, where would the image be formed ? What will be the size and the nature of the image ?

[*H. S. Exam. 1979*] [Ans. 60 cm ; double, real]

40. If an object be kept at a distance x from a concave mirror of radius of curvature r, show that the image distance is given by $v = r.x/(2x-r)$.

Harder Problems :

41. The floor of a room $ABCD$ is rectangular in shape. The sides AB and CD are 12 ft long and the other sides are 16 ft long. The walls AB and CD have two plane mirrors facing each other. A man stands at the middle of the side CD facing the wall AB. The first image of side AB seen through the mirror fixed on AB just fills the mirror. Find the width of the mirror.

[Ans. 4 ft]

42. A plane mirror is rotated through an angle θ about two different axes. For a given incident ray, it is found that the reflected ray in one case turns through an angle 2θ but in other case, the reflected ray remains as it is. How is this possible ? [*Jt. Entrance 1976*]

43. A ray of light, after being reflected, is allowed to fall on a scale. When the reflecting mirror (plane) is slightly turned, the spot of light is displaced through 25 cm on the scale. If the distance of the scale from the mirror is 100 cm, find the angle of rotation of the mirror.

[Ans. 7° (nearly)]

44. A small linear object of length l is placed *along* the axis of a spherical mirror at a distance u from the mirror. Show that the length of the image is $l\left(\dfrac{f}{u-f}\right)^2$ where f=focal length of the mirror and $l \ll u$.

45. A convex mirror of focal length f produced an image whose size is $1/r$th the size of an object. Show that the object distance from the mirror$=(r-1)f$.

46. A concave mirror produced on a screen an image twice the length of the object. Now both the object and the screen are shifted so that the length of the image becomes three times the length of the object. If the displacement of the screen be 25 cm, find the displacement of the object and the focal length of the mirror. [Ans. displacement$=4{\cdot}2$ cm. $f=25$ cm]

47. The interior of a hollow sphere is polished to serve as a reflecting mirror. At a point 12 cm from the centre of the sphere a point source of light is placed. If the radius of curvature of the sphere be 24 cm, locate the image formed by two successive reflections, the first taking place at (a) far wall and (b) the near wall. [Ans. (*a*) 30 cm from the near wall (*b*) 12 cm from the far wall]

48. An object is kept at a distance of 40 cm from a convex mirror. At a distance of 25 cm. from the object a plane mirror is so placed that it covers one half of the convex mirror. As a result, the two images produced by the mirrors are found to coincide. What is the radius of curvature of the convex mirror ? [Ans. $-26{\cdot}67$ cm]

49. An object is placed in front of a convex mirror at a distance of 50 cm. A plane mirror is introduced covering the lower half of the convex mirror. If the distance between the object and the plane mirror is 30 cm., it is found that there is no parallax between the images formed by the two mirrors. What is the radius of curvature of the convex mirror ?

[*I.I.T. 1973*] [Ans. 25 cm.]

50. Two mirrors—one concave and the other convex—have the same focal length 15 cm. and are on the same axis. The distance between their poles is 30 cm. An object is placed at a distance of 45 cm from the concave mirror. Find the position of the final image assuming that the first reflection takes place at the concave and the second at the convex.

[Ans. 5 cm. at the back of the convex mirror]

51. A plane mirror is placed in front of a concave mirror of focal length 10 cm, at a distance of 22·5 cm from the pole of the concave mirror. Where should an object be placed in between the mirrors so that the image formed by the concave mirror may coincide with that formed by the plane mirror ? [Ans. 15 cm]

52. A concave mirror and a convex mirror are placed co-axially at a distance $2R$, from each other. Both the mirrors have radius of curvature R. A small circle of radius a is drawn on the convex mirror near its pole. Show that the radii of the three successive images are $a/3$, $a/11$ and $a/41$.

[Hints : For the Ist image, $\frac{1}{v_1}+\frac{1}{2R}=\frac{2}{R}$ $\therefore v_1=\frac{2}{3}R$; $\text{mag}^n=\frac{2}{3}\cdot\frac{R}{2R}$

$\therefore$ Radius of 1st image$=\text{mag}^n\times$radius of the object$=\frac{2}{3}\times\frac{R}{2R}\times a=a/3$

Object distance for the 2nd image$=2R-\frac{2}{3}R=\frac{4}{3}R$

$$\therefore\ \frac{1}{v_2}+\frac{3}{4R}=-\frac{2}{R}\quad\therefore\quad v=-\frac{4}{11}.R$$

$$\text{Magnification}=\frac{4R}{11}\times\frac{3}{4R}=3/11$$

$\therefore$ 2nd radius$=$Ist radius$\times\text{mag}^n=\frac{a}{3}\times\frac{3}{11}=\frac{a}{11}$ and so on for the 3rd image.]

3

REFRACTION OF LIGHT AT A PLANE SURFACE

3.1. Refraction of light :

The action of mirrors, plane or spherical, depends on the fact that light striking a surface is turned back into space from which it comes. This happens always to some extent, but if the surface is that of a transparent material, some of the light passes through into the second substance. If the light is incident obliquely, it will change its direction sharply on going through the boundary. This change of direction of the rays is called **refraction.**

Suppose, a ray of light AB, travelling through air, is incident obliquely on a glass block (Fig. 3.1). The ray now enters into the block but the direction it takes up in the glass is different from the direction AB because the ray will suffer refraction at B. Suppose, the ray travels along BC in the glass. Here, AB is called the **incident ray,** BC the **refracted ray** and B, **the point of incidence.** If a perpendicular be drawn through B, on PQ, the line of separation of the two media, it is called **the normal** (NBN'). The angle made by the incident ray AB with the normal BN is called **the angle of incidence** ($\angle ABN$) and the angle made by the refracted ray BC with the same normal is called **the angle of refraction.** ($\angle CBN'$).

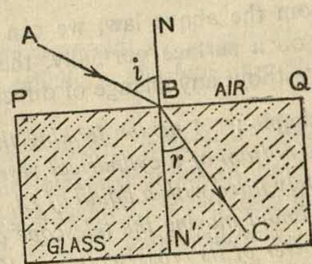


Fig. 3.1

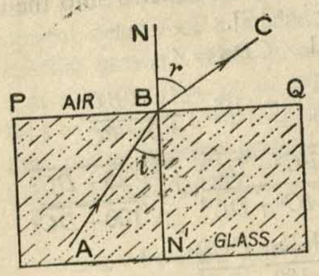


Fig. 3.2

It is important to remember that when a ray passes from one medium to a more optically dense medium (*e.g.* from air to glass) the refracted ray bends towards the normal *i.e.* the angle of incidence is greater than the angle of refraction [Fig. 3.1].

Conversely, a ray passing from one medium to a less optically dense medium (e.g. from glass to air), the refracted ray bends away from the normal *i.e.* the angle of incidence is smaller than the angle of refraction [Fig. 3.2.]

3.2. Laws of refraction :

The laws of refraction of light are as follows :

(*i*) *The incident ray, the refracted ray and the normal to the surface of separation of the two media at the point of incidence lie in one plane and the incident and the refracted rays are on the opposite sides of the normal.*

(*ii*) *The ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant which depends upon the nature of the media and the colour of the light used.*

If i be the angle of incidence and r the angle of refraction then according to the above law, $\dfrac{\sin i}{\sin r}=\mu$, a constant.

The constant μ is called the **refractive index** of the second medium (*i.e.* the medium in which the ray is refracted) with respect to the first medium (*i.e.* the medium through which the light was travelling initially). For example, when a ray of light, travelling through air, is refracted into glass, the ratio of the sines of the above two angles is 1·51. In other words, the refractive index of glass with respect to air is 1·51.

The second law of refraction is sometimes referred to as **Snell's law** as it was discovered by Dr. Edward Snell in 1621.

From the above law, we can write : when $i=0$, $r=0$ *i.e.*, when a ray is incident on a surface normally, the refracted ray goes straight into the second medium without any change of direction.

Example : *A person looking through a telescope just sees the point A on the rim at the bottom of a cylindrical vessel when the vessel is empty. When the vessel is completely filled with a liquid of r.i. 1·5, he observes a mark at the centre B of the bottom, without moving the telescope or the vessel. What is the height of the vessel if the diameter of its cross-section is 10 cm ?* [*I.I.T. 1977*]

Ans. When the vessel is empty, light rays from A travel straight towards the telescope T along ADT. When the vessel is full of liquid, the ray BD is refracted into the telescope along DT [Fig. 3.3].

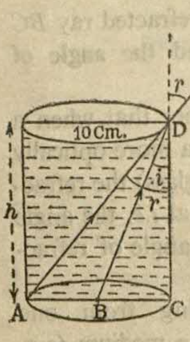


Fig. 3.3

Here $\angle BDC=\angle i$ and $\angle CDA=\angle r$,

$$\text{so, } \mu=\frac{\sin r}{\sin i}=\frac{AC}{AD}\Big/\frac{BC}{BD} \quad \text{or} \quad 1{\cdot}5=\frac{AC\times BD}{AD\times BC}$$

$$\text{or, } \quad 1{\cdot}5=\frac{10\sqrt{BC^2+DC^2}}{5\sqrt{AC^2+DC^2}}=\frac{2\sqrt{25+DC^2}}{\sqrt{100+DC^2}}$$

$$\text{Squaring, } 2{\cdot}25=\frac{4(25+DC^2)}{100+DC^2}$$

$$\text{or} \quad 225+2{\cdot}25\times DC^2=100+4DC^2$$

$$\text{or} \quad 1{\cdot}75\times DC^2=125 \quad \text{or} \quad DC^2=\frac{500}{7} \quad \therefore \; DC=\sqrt{\frac{500}{7}}=8{\cdot}45 \text{ cm.}$$

3.3. Relative and absolute refractive indices :

Definition : *When a ray of light, travelling through a medium 'a' is refracted into another medium 'b', the ratio of the sine of the angle of incidence to the sine of the angle of refraction is called the refractive index (abbreviated as r.i.) of the*

medium 'b' with respect to the medium 'a'. It is written as ${}_a\mu_b$ *i.e.* ${}_a\mu_b = \dfrac{\sin i}{\sin r}$, where i and r are the angles of incidence and refraction respectively. This refractive index is called **the relative refractive index.**

Since the path of a ray of light is reversible, if a ray travelling through the medium 'b' is incident at an angle r on the surface of separation, then the ray will be refracted into the medium 'a' with an angle of refraction i. In this case.

$${}_b\mu_a = \frac{\sin r}{\sin i}$$

Therefore, ${}_a\mu_b \times {}_b\mu_a = \dfrac{\sin i}{\sin r} \times \dfrac{\sin r}{\sin i} = 1$ or, ${}_a\mu_b = \dfrac{1}{{}_b\mu_a}$

For example, the *r.i.* of glass with respect to air is $\frac{3}{2}$; the *r.i.* of air with respect to glass is therefore, $\frac{2}{3}$.

Definition : *If a ray of light coming from vacuum or air, is refracted into a medium, the consequent refractive index will be called the absolute refractive index of the medium.*

Refractive index of a medium in general, means refractive index of the medium with respect to air. For example, the refractive index of glass is 1·5—this statement means that when a ray, travelling through air is refracted into glass, the ratio of the sines of the angles of incidence and refraction will be 1·5.

It has been mentioned earlier that *r.i.* of a medium depends on the colour of the light used. Thus, the refractive index of glass for red light is different from the *r.i.* of glass for blue or violet light. A medium is said to be optically denser than another if the *r.i.* of the first medium is greater than that of the second. This optical density of a medium is however, not related to the physical density or specific gravity. For example, the physical density of turpentine is less than that of water (sp. gr. of turpentine=0·87) but its optical density is greater than that of water (*r.i.* of turpentine=1·47). It is, therefore, important to remember that higher specific gravity of a substance does not necessarily endow it with higher optical density.

3.4. Relation between the refractive index and the velocity of light :

The refractive index has an important significance. From wave theory of light, it may be proved that μ of a medium is expressed by

$$\mu = \frac{\text{Velocity of light in vacuum}}{\text{,, \quad ,, \quad ,, \quad in the medium}}$$

Consider two media 'a' and 'b'. If ${}_a\mu_b$ be the *r.i.* of the medium 'b' with respect to the medium 'a' then,

$${}_a\mu_b = \frac{\text{Velocity of light in the medium '}a\text{'}}{\text{,, \quad ,, \quad ,, \quad ,, \quad ,, \quad ,, \quad '}b\text{'}}$$

If the medium 'b' is denser than the medium 'a', ${}_a\mu_b > 1$, and in that case the velocity of light in 'a' is greater than the velocity of light in 'b'. From this, we get a very important result *viz.* the *velocity of light in rarer medium is greater than the velocity of light in denser medium.*

Further, from the above relation, we may have an explanation as to why a ray of light changes its direction while crossing the boundary between two different media.

(i) If the velocity of light in both the media is the same ${}_a\mu_b = 1$ or $\sin i = \sin r$, which gives $i = r$; in other words, there will be no change of direction of the ray while crossing the surface of separation or there will be no refraction.

(ii) If the medium 'b' is denser, the velocity of light in it will be less and ${}_a\mu_b > 1$, or $\sin i > \sin r$, which gives $i > r$; in other words, the refracted ray will move towards the normal after refraction.

(iii) If the medium 'a' is rarer, the velocity of light in it will be greater and ${}_a\mu_b < 1$ or, $\sin i < \sin r$ which give $i < r$; in other words, the refracted ray will move away from the normal after refraction.

All these results have been experimentally verified. Hence it may be said that the light is refracted while going from one medium to another because the velocity of light in different media is different.

Example : *A light wave of frequency 5×10^{14} Hz enters a medium of refractive index 1·5. Find the velocity and the wavelength of light wave in the medium. Velocity of light in vacuum$=3 \times 10^8$ m/s.* [*I.I.T. 1983*]

Ans. We know, $\mu = \dfrac{\text{Velocity of light in vacuum}}{\text{,, ,, ,, ,, medium}}$

$$\text{or, } 1{\cdot}5 = \frac{3 \times 10^8}{V} \text{ or } V = \frac{3 \times 10^8}{1{\cdot}5} = 2 \times 10^8 \text{ metre/s.}$$

Again, $\lambda = \dfrac{V}{n}$ where n is the frequency of the wave.

$$\therefore \quad \lambda = \frac{2 \times 10^8}{5 \times 10^{14}} = 0{\cdot}4 \times 10^{-6} \text{ metre.}$$

Table of refractive indices of some substances.

Solid	R.I.	Liquid	R.I.
Crown glass	1·5	Water	1·33
Flint glass	1·62	Glycerine	1·47
Diamond	2·6	Turpentine	1·47
Ice	1·31	Alcohol	1·37

3.5. Geometrical construction of refracted ray :

Consider a ray of light incident at an angle of 70° on a glass slab whose *r.i.* is $\frac{3}{2}$. Its refracted ray is to be constructed geometrically.

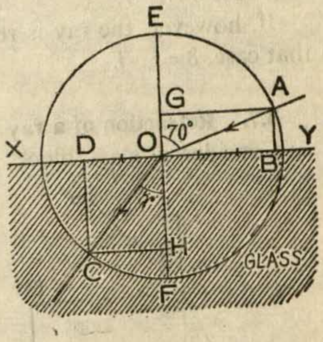


Fig. 3.4

Let XY represent the boundary between air and glass. (Fig 3.4). Take a point O on the middle of the block and draw a perpendicular EOF on the line XY. Draw a line OA inclined to the normal EOF at an angle of 70°. AO represents the incident ray. On the right of O, take three equal lengths along OY, B being the end-point of the last length. Mark off two equal lengths on the same scale on the left of O, D being the end-point of the last length. From the point B, draw a perpendicular on XY interesecting the incident ray at A. With O as centre and OA as radius, draw a circle. Drop a perpendicular through D on XY so that it meets the circle at C. Now join OC. It will be the required refracted ray.

To prove it, draw perpendiculars AG and CH from A and C respectively on EOF. Now, $\sin 70° = \frac{AG}{AO}$ and $\sin r = \frac{CH}{CO}$

$$\therefore \quad \frac{\sin 70°}{\sin r} = \frac{AG}{AO} \Big/ \frac{CH}{CO} = \frac{AG}{CH} \quad [\because AO = CO] \text{ But } AG = OB \text{ and } CH = OD.$$

$$\text{So, } \quad \frac{AG}{CH} = \frac{OB}{OD} = \frac{3}{2} \text{ (according to construction)}$$

$$\therefore \quad \frac{\sin 70°}{\sin r} = \frac{3}{2} = r.i. \text{ of glass. (given)}$$ Hence, OC is the required refracted ray.

3.6. Deviation of a ray due to refraction :

When a ray of a light is refracted from one medium to another, it suffers a certain deviation. The angle between the directions of incidence and refraction measures the deviation undergone by the ray.

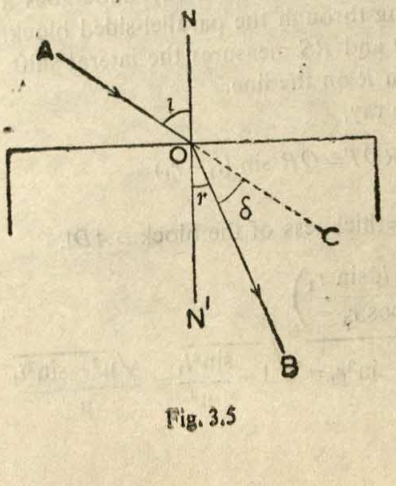


Fig. 3.5

Consider an incident ray AO and its refracted ray OB. The angle of incidence $\angle AON = i$ and the angle of refraction $\angle N'OB = r$. Produce AO to C. The direction of incidence is AOC while that of refraction is OB.

Hence, the deviation of the ray due to refraction $(\delta) = \angle BOC$ [Fig. 3.5].

Now, $\delta = \angle BOC = \angle N'OC - \angle N'OB = \angle NOA - \angle N'OB = i - r$
$[\because \angle NOA = \angle N'OC]$

If the ray is refracted from a rarer to a denser medium, $i > r$ and in that case $\delta = i - r$.

If, however, the ray is refracted from a denser to a rarer medium, $i < r$ and in that case, $\delta = r - i$.

3.7. Refraction of a ray of light through a parallel block :

Consider a glass block $ABCD$ with parallel and plane faces. A ray of light PQ is incident at an angle $\angle i_1$ on the face AB. The ray is refracted as it enters the block. Suppose it is refracted along QR at an angle of refraction $\angle r_1$. The ray now falls on the lower face DC and emerges into air again at an angle of emergence $= \angle i_2$. It may be proved in the following way that the incident ray PQ and the emergent ray RS are parallel to each other.

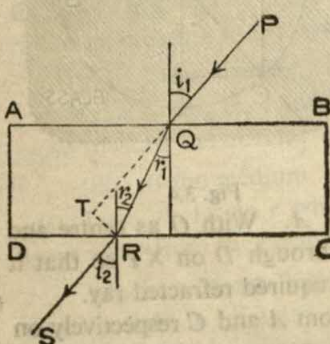


Fig. 3.6

Proof : Let ${}_a\mu_g$ be the *r.i.* of glass with respect to air. Then ${}_a\mu_g = \dfrac{\sin i_1}{\sin r_1}$

Again, if ${}_g\mu_a$ be the *r.i.* of air with respect to glass then considering refraction at the point R, we have ${}_g\mu_a = \dfrac{\sin r_2}{\sin i_2}$

But we know, ${}_a\mu_g = \dfrac{1}{{}_g\mu_a}$ $\therefore$ $\dfrac{\sin i_1}{\sin r_1} = \dfrac{1}{\dfrac{\sin r_2}{\sin i_2}} = \dfrac{\sin i_2}{\sin r_2}$

Now, from the diagram, it is clear that $\angle r_1 = \angle r_2$.

Hence $\sin r_1 = \sin r_2$. or, $\sin i_1 = \sin i_2$ which gives $i_1 = i_2$.

So, the incident ray PQ is parallel to the emergent ray RS.

Lateral shift : It is to be noted that PQ and RS are no doubt parallel but they do not lie in the same straight line. In other words, the ray undergoes a *lateral shift* or *lateral displacement* while passing through the parallel-sided block. The distance between the two parallel rays PQ and RS measures the lateral shift. Produce PQ and draw a perpendicular RT from R on the line.

Here, RT measures the lateral shift of the ray.

Now, $\sin RQT = \dfrac{RT}{QR}$ $\therefore$ $RT = QR \sin RQT = QR \sin (i_1 - r_1)$

Also, $\cos r_1 = \dfrac{t}{QR}$ $\therefore$ $QR = \dfrac{t}{\cos r_1}$ [t=thickness of the block=AD].

$$\therefore \; RT = t.\frac{\sin (i_1 - r_1)}{\cos r_1} = t\left(\sin \; i_1 - \frac{\cos i_1 \sin r_1}{\cos r_1}\right)$$

Again, $\sin r_1 = \dfrac{\sin i_1}{\mu}$ and $\cos r_1 = \sqrt{1 - \sin^2 r_1} = \sqrt{1 - \dfrac{\sin^2 i_1}{\mu^2}} = \dfrac{\sqrt{\mu^2 - \sin^2 i_1}}{\mu}$

$$\therefore \quad RT = t\left(\sin i_1 - \frac{\cos i_1 \sin i_1 \times \mu}{\mu\sqrt{\mu^2 - \sin^2 i_1}}\right) = t \sin i_1\left(1 - \frac{\cos i_1}{\sqrt{\mu^2 - \sin^2 i_1}}\right)$$

Knowing the thickness (t) of the block, the angle of incidence ($\angle i_1$) and the *r.i.* (μ) of the material of the block, the lateral shift can be calculated. On the other hand, it may be said that the lateral shift depends upon (i) the thickness of the block, (ii) the angle of incidence and (iii) the *r.i.* of the material of the block.

Example : *A ray of light is incident on the upper surface of a rectangular glass block at an angle 30°. Show that the emergent ray from the lower surface of the block is parallel to the incident ray. Find also the lateral shift of the ray if the height of the block is 10 cm.*

Ans. See Fig. 3.6. Here, $i_1 = 30°$; Now $\mu = \dfrac{\sin 30°}{\sin r_1}$

$\therefore \sin r_1 = \dfrac{\sin 30°}{\mu} = \dfrac{1}{2 \times 1{\cdot}5} = \dfrac{1}{3}$; considering the lower surface, $\mu = \dfrac{\sin i_2}{\sin r_2}$

$\therefore \sin i_2 = \mu \times \sin r_2 = 1{\cdot}5 \times \frac{1}{3} = 0{\cdot}5 \quad [r_1 = r_2] \quad \therefore i_2 = 30°$

This shows that the emergent ray is parallel to the incident ray.

Again lateral shift $= t.\sin i_1\left(1 - \dfrac{\cos i_1}{\sqrt{\mu^2 - \sin^2 i_1}}\right)$

$$= 10 \times \sin 30°\left(1 - \frac{\cos 30°}{\sqrt{(1{\cdot}5)^2 - (\sin 30°)^2}}\right)$$

$$= 10 \times \tfrac{1}{2}\left(1 - \frac{\sqrt{3}}{2\sqrt{(1{\cdot}5)^2 - (0{\cdot}5)^2}}\right) = 1{\cdot}95 \text{ cm.}$$

3.8. Refraction of light through a number of parallel media of increasing density:

a, b, c, etc. are several parallel-sided media arranged in order of increasing density *i.e.* the medium '*b*' is denser than '*a*', the medium '*c*' is denser than '*b*' and so on. But the first and the last media are the same. If a ray of light be incident on this pile, then after passing through different media, when it will emerge into the medium '*a*', the emergent ray will be found to be parallel to the incident ray.

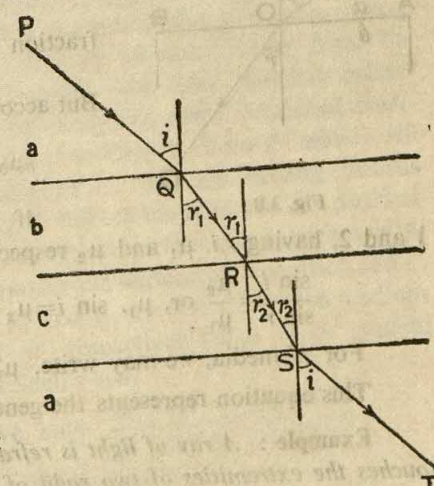


Fig. 3.7

Here, the incident ray PQ is parallel to the emergent ray ST. [Fig 3.7]

Now, considering refraction at Q, we get $\dfrac{\sin i}{\sin r_1} = {}_a\mu_b$

Similarly, considering refractions at R and S, we get

$$\frac{\sin r_1}{\sin r_2} = {}_b\mu_c \text{ and } \frac{\sin r_2}{\sin i} = {}_c\mu_a.$$

Multiplying, ${}_a\mu_b \times {}_b\mu_c \times {}_c\mu_a = \dfrac{\sin i}{\sin r_1} \times \dfrac{\sin r_1}{\sin r_2} \times \dfrac{\sin r_2}{\sin i} = 1. \quad \ldots \quad$ (i)

The above result holds good not only for three media as shown above, but for any number of parallel media provided the first and the last media are the same. For '*n*' media, we can write, ${}_a\mu_b \times {}_b\mu_c \times {}_c\mu_d \times \ldots . \times {}_n\mu_a = 1$

If the medium '*a*' be air, then from the above relation, we have

$${}_{air}\mu_b \times {}_b\mu_c \times {}_c\mu_{air} = 1.$$

$$\therefore \quad {}_b\mu_c = \frac{1}{{}_{air}\mu_b \times {}_c\mu_{air}} = \frac{{}_{air}\mu_c}{{}_{air}\mu_b} = \frac{r.i. \text{ of } 'c'}{r.i. \text{ of } 'b'}.$$

Example 1 : *If the r.i. of water with respect to air be* $\frac{4}{3}$ *and that of glass with respect to air be* $\frac{3}{2}$, *what is the r.i. of glass with respect to water ?*

Ans. We know, ${}_w\mu_g = \dfrac{{}_{air}\mu_g}{{}_{air}\mu_w} = \dfrac{\frac{3}{2}}{\frac{4}{3}} = \dfrac{3}{2} \times \dfrac{3}{4} = \dfrac{9}{8}.$

Example 2 : *R. I. of glycerine with respect with glass is 0·98 and that of glycerine with respect to air is 1·47. Determine the r.i. of glass with respect to air and of air with respect to glass.*

Ans. We know, ${}_{glass}\mu_{gly} = \dfrac{{}_{air}\mu_{gly}}{{}_{air}\mu_{glass}}$; $\therefore \; 0{\cdot}98 = \dfrac{1{\cdot}47}{{}_{air}\mu_{glass}}$

or, ${}_{air}\mu_{glass} = \dfrac{1{\cdot}47}{0{\cdot}98} = 1{\cdot}5$ Again, ${}_{glass}\mu_{air} = \dfrac{1}{{}_{air}\mu_{glass}} = \dfrac{1}{1{\cdot}5} = 0{\cdot}66.$

3.9. General form of Snell's law :

AB is the surface of separation of two media '*a*' and '*b*'. The medium '*b*' is denser than the medium '*a*'. Consider a ray of light *PO* incident on *AB* at *O* and refracted along *OQ*. If $\angle i$ and $\angle r$ be the angles of incidence and refraction we have, $\dfrac{\sin i}{\sin r} = {}_a\mu_b.$

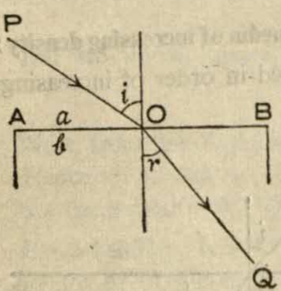


Fig. 3.8

But according to art. 3.8, we can write,

$${}_a\mu_b = \frac{{}_{air}\mu_b}{{}_{air}\mu_a} \quad \therefore \quad \frac{\sin i}{\sin r} = \frac{{}_{air}\mu_b}{{}_{air}\mu_a}$$

If now, the media '*a*' and '*b*' be denoted by 1 and 2, having *r.i.* μ_1 and μ_2 respectively, then,

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} \text{ or, } \mu_1 . \sin i = \mu_2 \sin r.$$

For '*n*' media, we may write, $\mu_1 \sin i_1 = \mu_2 \sin r_2 = \mu_3 \sin r_3 = \ldots = \mu_n \sin r_n$

This equation represents the general form of Snell's law.

Example : *A ray of light is refracted through a sphere in such a way, that it touches the extremities of two radii of the sphere which are inclined at an angle* θ *to each other. If* μ *be the r.i. of the material of the sphere and* δ *the deviation of the ray, show that* $\mu = \dfrac{\cos (\theta - \delta)/2}{\cos \theta/2}.$

Ans. Suppose, the ray PA is incident on the sphere at an angle i and is refracted through the sphere along the path AB. Finally the ray emerges from the sphere in the direction BQ [Fig. 3.9]. From the figure, it is evident that the deviation of the ray caused by the passage through the sphere$=\delta$. Let the angle of refraction at $A=r$. Since $OA=OB$, the angle of incidence at $B=r$. So, the angle of emergence of the ray at $B=i$. Further the $\angle AOB=\theta$ (given). Now, $\delta=\angle CAB+\angle CBA=(i-r)+(i-r)=2(i-r)$.

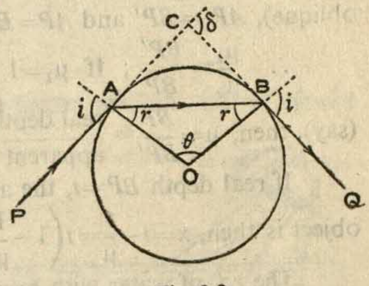


Fig. 3.9

Again from ΔAOB, we get $2r+\theta=180°$ or $r+\frac{\theta}{2}=90°$ .. .. (i)

$\therefore\ \delta=2(i-90°+\theta/2)$ or $\frac{\delta}{2}-\frac{\theta}{2}=i-90°$ or $\frac{1}{2}(\theta-\delta)=90°-i$ .. (ii)

Considering refraction at A, we get $\sin i=\mu \sin r=\mu \sin (90°-\theta/2)=\mu \cos \theta/2$.

From eqn. (ii), we have, $\cos \frac{1}{2}(\theta-\delta)=\cos (90°-i)=\sin i$

$$\therefore\ \mu . \cos \theta/2=\cos \tfrac{1}{2}(\theta-\delta) \text{ or } \mu=\frac{\cos \frac{1}{2}(\theta-\delta)}{\cos \frac{1}{2}\theta}$$

3.10. Formation of image by refraction at a plane surface :

(*i*) *The object is placed in a denser medium and the eye in a rarer medium.*

Consider an object P placed in the medium '*a*' which is denser than the medium '*b*' from which a person is viewing the object. A ray PB from the object is incident normally on the refracting surface AB. It passes straight into the medium '*b*' along BC [Fig. 3.10]. Another ray PA from the object is incident on the refracting surface obliquely and is refracted away from the normal along the path AD. The two refracted rays BC and AD, when produced backwards, appear to diverge from P' which will be the image of P. The eye looking towards the object P, will see the object in the position of P'. The object, in this case, appears to be raised *towards* the surface of separation AB.

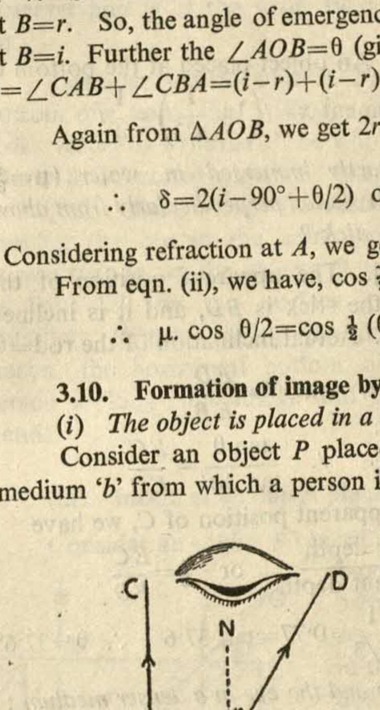


Fig. 3.10

If μ_1 and μ_2 are the *r.i.* of the medium '*b*' and '*a*' respectively then from general form of Snell's law,

$$\text{We have } \frac{\mu_1}{\mu_2}=\frac{\sin i}{\sin r}=\frac{\sin PAN'}{\sin DAN}$$

But, $\angle PAN'=\angle APB$ and $\angle DAN=\angle P'AN=\angle APB$.

$$\text{Hence, } \frac{\mu_1}{\mu_2}=\frac{\sin APB}{\sin AP'B}=\frac{\frac{AB}{AP}}{\frac{AB}{AP'}}=\frac{AP'}{AP}$$

Since the points A and B are very near to each other (for the ray PA is slightly oblique), $AP'=BP'$ and $AP=BP$.

$\therefore\ \dfrac{\mu_1}{\mu_2}=\dfrac{BP'}{BP}$; If $\mu_1=1$ (*i.e.* the medium '*b*' is replaced by air) and $\mu_2=\mu$ (say), then, $\mu=\dfrac{BP}{BP'}=\dfrac{\text{real depth of the object}}{\text{apparent ,, ,, ,, ,,}}$

If real depth $BP=t$, the apparent depth$=t/\mu$. The displacement PP' of the object is then, $x=t-\dfrac{t}{\mu}=t\left(1-\dfrac{1}{\mu}\right)$

The *r.i.* of water with respect to air is $\frac{4}{3}$; An object placed at the bottom of a tank of depth t will have an upward displacement $x=t\left(1-\dfrac{1}{4/3}\right)=\dfrac{1}{4}.t$

Example : *A straight stick of wood, partly immersed in water* ($\mu=\frac{4}{3}$) *appears to be inclined at 30° with the surface when viewed perpendicularly from above the water. What is the actual inclination of the stick ?*

Ans. Let ABC be the stick [Fig 3.11]. The apparent position of the immersed portion of the stick is BD, and it is inclined at 30° with the surface. Actual inclination of the rod$=\theta$.

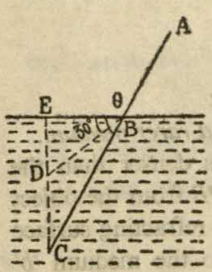


Fig. 3.11

From the figure, we get, $\tan 30°=\dfrac{ED}{EB}$.

Again $\tan\theta=\dfrac{EC}{EB}$. $\therefore\ \dfrac{\tan\theta}{\tan 30°}=\dfrac{EC}{ED}$

But as D is the apparent position of C, we have

$$\mu=\frac{\text{Real depth}}{\text{Apparent depth}} \quad\text{or}\quad \tfrac{4}{3}=\frac{EC}{ED}$$

Hence, $\dfrac{\tan\theta}{\tan 30°}=\frac{4}{3}$ or $\tan\theta=\frac{4}{3}\times\tan 30°=\frac{4}{3}\times\dfrac{1}{\sqrt{3}}=0{\cdot}77=\tan 37{\cdot}6°$ $\therefore\ \theta=37{\cdot}6°$

(*ii*) *The object is placed in a rarer medium and the eye in a denser medium :*

P is a point object in the medium '*b*' which is rarer than the medium '*a*' from which a person is viewing the object. Two rays, PB and PA, starting from the object P after being refracted at the surface AB, enter into the denser medium '*a*'. When these rays reach the eye of the observer, they appear to come from P' which is the image of P. The object, in this case, appears to move *away* from the surface of separation AB (Fig. 3.12).

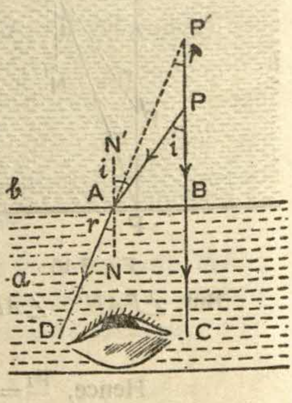


Fig. 3.12

Here, $\dfrac{\mu_2}{\mu_1}=\dfrac{\sin i}{\sin r}=\dfrac{\sin PAN'}{\sin DAN}$

But, $\angle PAN'=\angle APB$ and $\angle DAN=\angle P'AN'$ $=\angle AP'B$,

So, $\dfrac{\mu_2}{\mu_1}=\dfrac{\sin APB}{\sin AP'B}=\dfrac{AB}{AP}\Big/\dfrac{AB}{AP'}=\dfrac{AP'}{AP}$

Since, A and B are very close points, $AP'=BP'$ and $AP=BP$.

$$\therefore \frac{\mu_2}{\mu_1}=\frac{BP'}{BP}=\frac{\text{Apparent height of the object}}{\text{Real } \quad ,, \quad ,, \quad ,, \quad ,,}$$

An important fact : Keeping an object in a denser medium, if it is viewed obliquely from a rarer medium, the image changes position. With increasing obliquity of line of vision, the image moves towards the observer and is at the same time raised higher up.

Consider several objects, P, Q, R at the bottom of a tank [Fig. 3.12(a)]. If the observer looks vertically towards P, he will see the object almost at its actual position ; the image P' will be slightly raised along the vertical line. But keeping the eye in the same position, if the person tries to see the objects Q and R, the rays of light fall on the surface of separation obliquely, thereby raising the images Q' and R' more and more. For this reason, the horizontal bottom of a tank full of water appears concave to a person looking from above and the pool of water appears deepest where he stands.

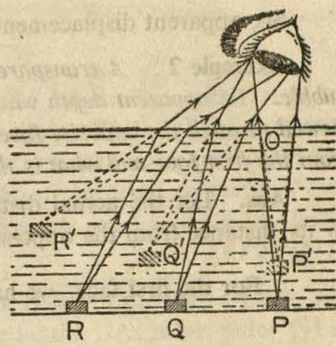


Fig. 3.12(a)

(iii) **Image of an object placed below a parallel-sided glass block :**

Consider an object P placed some distance in air below a parallel sided glass block of thickness t [Fig. 3.13]. The ray PA normal to the surface emerges along AB, while the ray PN close to the normal is refracted along NO in the glass and emerges in air along OQ in a direction parallel to PN (See art 3.7). An observer (not shown) above the block thus sees the object at P', the point of intersection of QO and BA.

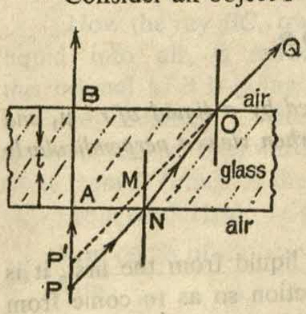


Fig. 3.13

Suppose the normal at N, intersects $P'O$ at M. Then, since MN is parallel to PP' and $P'M$ is parallel to PN, $MNPP'$ is a parallelogram. Thus $PP'=NM$. But PP' is the displacement of the object P. Hence NM is equal to the displacement. Since the apparent position of an object at N is at M (compare fig. 3.8), we can say that *the displacement of P is independent of the position of P below the glass block* and it is given by $PP'=t\left(1-\frac{1}{\mu}\right)$

Example 1 : *The height of a glass slab is 10 cm. There is a dot at the bottom of the block. What will be the apparent displacement of the dot when viewed through the block ? μ of glass=1·5.*

Ans. When the object is in the denser medium and the eye in the rarer, we know, $\mu=\dfrac{\text{real distance of the object}}{\text{apparent ,, ,, ,, ,,}}$ or, $1{\cdot}5=\dfrac{10}{\text{apparent distance of the image}}$

$\therefore$ the apparent distance of the dot$=\dfrac{10}{1{\cdot}5}=6{\cdot}6$ cm.

So, apparent displacement of the dot$=10-6{\cdot}6=3{\cdot}4$ cm.

Example 2 : *A transparent cube of glass, 1·5 cm. edge, contains a small air bubble. Its apparent depth when viewed through one face of the cube is 6 cm. and viewed through the opposite face is 4 cm. What is the actual distance of the bubble from the first face and what is the r.i. of the glass ?*

Ans. Let the actual distance of the bubble from the first face be x cm. ; so its distance from the opposite face$=(15-x)$ cm.

For the first face, we have $\mu=\dfrac{\text{actual distance}}{\text{apparent ,,}}=\dfrac{x}{6}$

and ,, ,, ,, second ,, ,, ,, $\mu=\dfrac{15-x}{4}$

$\therefore \dfrac{x}{6}=\dfrac{15-x}{4}$ or, $x=9$ cm. Also, $\mu=\dfrac{9}{6}=1{\cdot}5$

Example 3 : *A vessel full of water is 12 ft. deep. If r.i. of water with respect to air be $\frac{4}{3}$ find the apparent depth of the vessel.*

Ans. In this case $\mu=\dfrac{\text{Real height}}{\text{Apparent ,,}}$ or $\dfrac{4}{3}=\dfrac{12}{\text{App. height}}$

$\therefore$ the apparent height of the vessel$=\dfrac{12\times3}{4}=9$ ft.

Example 4 : *A vessel has depth 2d and is half-filled by a liquid of r.i. μ_1 and the other half by another liquid of r.i. μ_2. Prove that when viewed perpendicularly the apparent depth of the vessel$=\left(\dfrac{1}{\mu_1}+\dfrac{1}{\mu_2}\right)d$.*

Ans. Suppose, when the ray enters the second liquid from the first, it is refracted in a direction so as to come from the point P_1 [Fig. 3.14]. According to the art 3.10(ii).

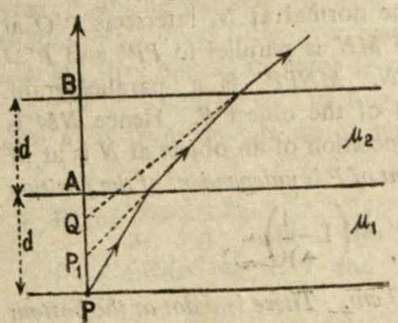


Fig. 3.14

$$\frac{AP}{AP_1}=\frac{\mu_1}{\mu_2} \quad \therefore \quad AP_1=\frac{\mu_2}{\mu_1}.AP=\frac{\mu_2}{\mu_1}d.$$

Now, as the ray of light is refracted into air from the second liquid, suppose, it appears to come from the point Q.

Here, $\dfrac{BP_1}{BQ}=\mu_2$

$$\therefore \quad BQ=\frac{BP_1}{\mu_2}=\frac{AB+AP_1}{\mu_2}=\frac{d}{\mu_2}+\frac{AP_1}{\mu_2}=\frac{d}{\mu_2}+\frac{d}{\mu_1}=d\left(\frac{1}{\mu_1}+\frac{1}{\mu_2}\right)$$

[N.B. If d_1 be the depth of the liquid of *r.i.* μ_1 and d_2 that of the liquid of *r.i.* μ_2 then $BQ=\frac{d_1}{\mu_1}+\frac{d_2}{\mu_2}$. In general, if there be a number of liquids arranged in order of density, then their apparent depth$=\frac{d_1}{\mu_1}+\frac{d_2}{\mu_2}+\ldots+\frac{d_n}{\mu_n}=\Sigma\frac{d}{\mu}$]

Example 5 : *A vertical microscope is focussed on a point at the bottom of an empty tank. Water (r.i.$=\frac{4}{3}$) is then poured into the tank. The height of the water column is 4 cm. Another lighter liquid which does not mix with water and which has a r.i. of $\frac{3}{2}$ is then poured over water. The height of the liquid column is 2 cm. What is the vertical distance through which the microscope must be moved to bring the point object in focus again* ? [*I.I.T. 1971*]

Ans. A is the point at the bottom of the tank [Fig. 3.15], $DA=4$ cm, the depth of water and $ED=2$ cm, the depth of lighter liquid. As μ for water ($\frac{4}{3}$) is less than that for the liquid ($\mu=\frac{3}{2}$), water is the rarer medium and the liquid the denser one. Now a ray AB from A, travelling through the rarer medium water, is refracted into the denser medium bending towards the normal. When produced backward, the refracted ray appears to come from P *i.e.* P is the virtual image of A.

According to art 3.11(*a*). $\frac{\mu_l}{\mu_\omega}=\frac{\text{App. height of the object}}{\text{Real }\ ,,\ \ ,,\ \ ,,\ \ ,,}$

$$\text{or} \quad \frac{3}{2}\Big/\frac{4}{3}=\frac{DP}{DA} \quad \text{or} \quad DP=DA\times\frac{3}{2}\times\frac{3}{4}=4\times\frac{3}{2}\times\frac{3}{4}=\frac{9}{2}\text{ cm.}$$

Now the ray BC, travelling from the liquid into air, is refracted away from the normal as it is going from a denser to a rarer medium. When the refracted ray is produced backward, it appears to come from P' which is the final image of A. From art 3.11(*a*), we get,

$$\mu_l=\frac{\text{Real depth of the object }(EP)}{\text{App. }\ ,,\ \ ,,\ \ ,,\ \ ,,\ (EP_1)} \text{ or } \frac{3}{2}=\frac{EP}{EP'}\,;$$

$$\text{Now } EP=ED+DP=2+\frac{9}{2}=\frac{13}{2}\text{ cm.}$$

$$\therefore \quad EP'=\frac{2}{3}\times EP=\frac{2}{3}\times\frac{13}{2}=4\tfrac{1}{3}\text{ cm.}$$

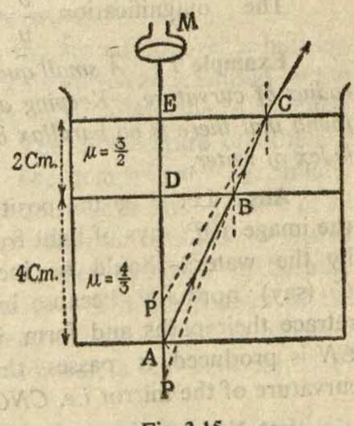


Fig. 3.15

So, $AP'=AE-EP'=6-4\frac{1}{3}=1\frac{2}{3}$ cm. *i.e.* the microscope should be raised through $1\frac{2}{3}$ cm.

Example 6 : *A small object is placed on the principal axis of a concave mirror of focal length 10 cm at a distance of 32 cm. By how much will the position and the size of the image alter when a parallel sided glass slab of thickness*

6 cm. and r.i. 1·5. is introduced between the centre of curvature and the object ? The parallel sides of the slab are perpendicular to the principal axis of the mirror.

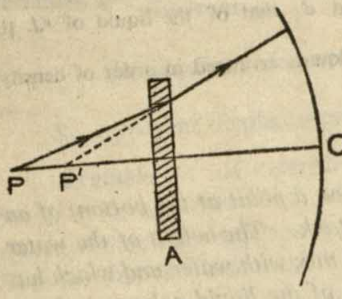


Fig. 3.16

Ans. Let P be the position of the object. Its image given by the mirror O when the block of glass is not placed, may be found out from the equation, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\text{or } \frac{1}{v}+\frac{1}{32}=\frac{1}{10} \text{ or } \frac{1}{v}=\frac{1}{10}-\frac{1}{32}=\frac{11}{160}$$

$$\therefore \quad v=\frac{160}{11}=14{\cdot}5 \text{ cm.}$$

The magnification $m=\frac{v}{u}=\frac{160}{11\times 32}=\frac{5}{11}=0{\cdot}45.$

When the glass slab A is placed, the rays from P appear to come from P' whose displacement from $P=t\left(1-\frac{1}{\mu}\right)=6\left(1-\frac{1}{1{\cdot}5}\right)=2$ cm.

The distance of P' from O, the pole of the mirror is, therefore$=32-2=30$ cm.

Again, from the equation $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, we have,

$$\frac{1}{v}+\frac{1}{30}=\frac{1}{10} \text{ or } \frac{1}{v}=\frac{1}{10}-\frac{1}{30}=\frac{1}{15} \quad \therefore \quad v=15 \text{ cm.}$$

The image is displaced by $=15-14{\cdot}5=0{\cdot}5$ cm.

The magnification$=\frac{v}{u}=\frac{15}{30}=0{\cdot}5.$

Example 7 : *A small quantity of water is kept in a concave mirror of 30 cm. radius of curvature. Keeping a pin at a distance of 22 cm. from the mirror it was found that there is no parallax between the pin and its image. Find the refractive index of water.*

Ans. Let P be the position of the pin [Fig. 3.17]. For the formation of the image at P, rays of light from P, after being refracted by the water, should be incident on the mirror at C (say) normally because in that case, the rays will retrace their paths and form image at P. So, when CN is produced, it passes through O, the centre of curvature of the mirror *i.e.* CNO is a radius of curvature.

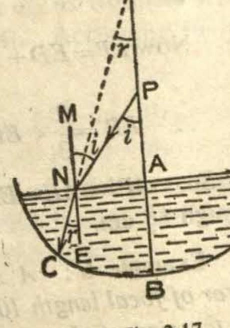


Fig. 3.17

Let NM be drawn perpendicular to the surface of water at N. Here, angle of incidence $i=\angle PNM=\angle NPA$ and angle of refraction $r=\angle CNE=\angle NOA$. If μ be the *r.i.* of water,

$$\mu=\frac{\sin i}{\sin r}=\frac{\sin NPA}{\sin NOA}=\frac{NA/PN}{NA/ON}=\frac{ON}{PN}$$

As the ray PN is very near to the principal axis of the mirror. $ON=OA$ and $PN=PA$. So $\mu=\frac{OA}{PA}$.

Again if the depth AB of water be very small compared to PA or OA, then $OA=OB$ and $PA=PB$. Hence $\mu=\frac{OB}{PB}$. Now OB=radius of curvature of the mirror=30 cm. and PB= distance of the pin=22 cm. So $\mu=\frac{30}{22}=1{\cdot}36$ (nearly).

[N. B. For the sake of understanding, the depth of the liquid has been exaggerated.]

3.11. Atmospheric refraction :

The atmosphere, we all know, gradually loses its density as its height increases above the sea level. So the rays of light proceeding from the sun or the moon cannot travel in straight lines but are refracted more and more towards the normal as they pass through the successive lower layers of the atmosphere. Since an observer on the earth's surface sees any heavenly body in the direction of the rays reaching him, the objects always appear higher up in the sky than they actually are. Due to such atmospheric refraction, the sun or the moon becomes visible sometime before they rise and remains in view for some time after they set.

Twinkling of stars : If you look at the night sky, you will see that some of the heavenly bodies are giving out steady light while the brightness of some other is constantly changing. The first type of heavenly bodies are planets which are situated comparatively nearer to the earth and the second type are stars which are situated far away from the earth. Why do stars twinkle ?

Have you ever tried to see an object through the heated air over a burning oven ? You will see that the object is quivering. This quivering is caused by the continuous change of density and hence of refractive index of air due to irregular heating. The stars twinkle for the same reason. The temperature of different layers of atmosphere does not remain steady. The temperature continually changes causing a change in the refractive index of atmospheric air. Now a good concentration of rays in a particular direction must reach an observer to enable him to see a star brightly. This cannot, however, happen continuously because of the variations of refractive index of the air due to which the courses of the rays alter at every instant. The amount of light reaching the eye of an observer in a given direction being thus not steady every moment, a star appears to be twinkling.

Planets, however do not appear to twinkle, for planets being comparatively nearer to the earth the amount of light received from them on the earth is much stronger. Atmospheric refraction cannot cause any appreciable change to their brightness.

A few interesting phenomena concerning refraction :

(i) Keep a glass rod partly immersed in glycerine kept in a glass vessel. If you, now, try to see the portion of the rod immersed, you will not be able to see

it. Refractive indices of glass and glycerine are exactly equal. They behave as one single homogeneous medium. As a result, rays of light are not reflected or refracted through glass. Moreover, glass and glycerine both being transparent glass becomes invisible through glycerine. In the same way, a rod of flint glass immersed in carbon disulphide becomes invisible because the refractive indices of carbon disulphide and flint glass are equal.

(ii) Ordinarily glass is a transparent substance ; light easily passes through glass. But glass, when powdered, becomes opaque. Rays of light cannot pass easily through glass powders ; they are refracted several times by numerous glass particles. For this reason, powdered glass becomes opaque to light. If water is poured in the powder, it becomes transparent again. Light finds easy path through water layer and is transmitted through it.

3.12. Total internal reflection :

When light passes from one medium to a more optically dense medium, there will always be both reflection and refraction for all angles of incidence, more light being refracted than reflected. In fig. 3.18., a ray of light first passes from air into glass and then from glass into air. In each case, there are reflected and refracted rays. But this is not always the case when light passes from a denser to a rarer medium.

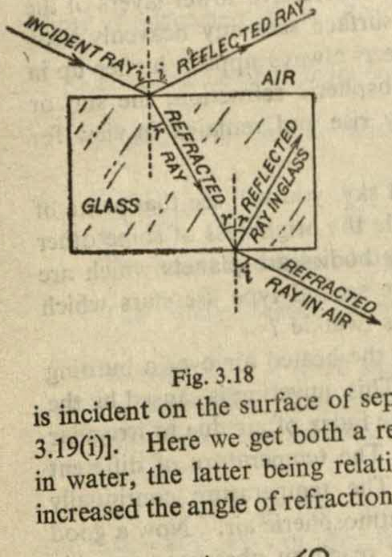


Fig. 3.18

Suppose AB is the surface of separation between air and water. A ray of light P_1O from a point object P_1 in water is incident on the surface of separation at O with a small angle of incidence [Fig. 3.19(i)]. Here we get both a refracted ray (OQ_1) in air and a reflected ray (OR_1) in water, the latter being relatively weak. If the angle of incidence is gradually increased the angle of refraction also increases until, for a certain angle of incidence

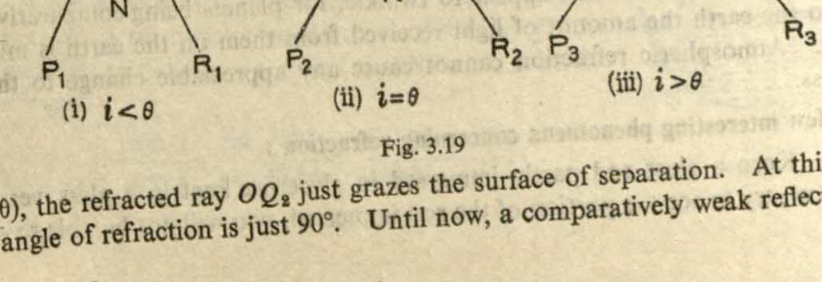


Fig. 3.19

($i=\theta$), the refracted ray OQ_2 just grazes the surface of separation. At this stage the angle of refraction is just 90°. Until now, a comparatively weak reflected ray

OR_2 is obtained inside water and a strong refracted ray goes out into the air. Fig. 3.19(ii)].

Since it is impossible to have an angle of refraction greater than 90°, it follows that for angles of incidence slightly greater than before ($i>\theta$) all the light is internally reflected making the reflected ray suddenly brighter and there is no refracted ray at all. Fig. 3.19(iii) shows such an angle of incidence for which the whole light is internally reflected along OR_3. This condition is described as **total internal reflection.** In this condition the surface of separation between the two media behaves like a shining mirror.

The angle of incidence in the denser medium corresponding to which the angle of refraction is 90°, is called the **critical angle** for the pair of media concerned. In fig. 3.19(ii), $\angle P_2ON$ is the critical angle.

Definition : *While passing from a denser to a rarer medium, if a ray of light is incident at an angle greater than the critical angle for the pair of media concerned, then the ray of light is totally reflected in the denser medium according to the ordinary laws of reflection. This is known as total internal reflection of light.*

Conditions for total internal reflection :

It should be carefully noted that the phenomenon of total internal reflection can occur only when light travels from denser to rarer medium. The phenomenon cannot occur when light travels from rarer to denser medium, for example from air to water, as a refracted ray is then always obtained.

The following conditions are, therefore, necessary for total internal reflection of light : (1) *The ray of light must pass from denser to rarer medium* and (2) *angle of incidence must be greater than the critical angle for the pair of media.*

Relation between critical angle and refractive index :

Suppose $\angle P_2ON=\theta$, the critical angle between water and air [Fig. 3.19(ii)]. The refracted ray OQ_2 grazes the surface of separation and the angle of refraction $=90°$. If ${}_a\mu_w$ be the *r.i.* of water with respect to air, then,

$$\frac{\sin\theta}{\sin 90°}=\frac{1}{{}_a\mu_w} \quad \therefore \quad \sin\theta=\frac{1}{{}_a\mu_w}$$

This is the relation between the refractive index and the critical angle.

Example 1 : *If r.i. of glass with respect to air is 1·52, what will be the critical angle between them* ?

Ans. Let the critical angle be θ. We have $\sin\theta=\frac{1}{{}_a\mu_g}$ Here ${}_a\mu_g=1{\cdot}52$.

Hence, $\sin\theta=\frac{1}{1{\cdot}52}=0{\cdot}6579=\sin 41°$ (nearly) $\therefore$ $\theta=41°$ (nearly).

Example 2: *A ray of light passes from glass to water in such a manner that the refracted ray just grazes the surface of separation. If the r.i. of glass and water with respect to air be 1·5 and 1·33 respectively, find the angle of incidence of the ray.*

Ans. We know, ${}_w\mu_g=\frac{\text{air }\mu_g}{\text{air }\mu_w}=\frac{1{\cdot}5}{1{\cdot}33}=1{\cdot}12$

Since the refracted ray just grazes the surface of separation the critical angle between glass and water will be the angle of incidence.

$$\text{Now, } \sin\theta = \frac{1}{{}_w\mu_g} = \frac{1}{1{\cdot}12} = 0{\cdot}89 \quad \therefore \quad \theta = 62°54' \text{ (nearly).}$$

Example 3 : *A ray of light is travelling from diamond to glass. Calculate the minimum angle of incidence of the ray on the diamond-glass interface such that no light is refracted into glass. μ for glass=1·51 and that for diamond=2·47.* [*I.I.T. 1977*]

Ans. It is clear that the minimum angle of incidence is the critical angle between diamond and glass. If θ be the critical angle, then,

$$\sin\theta = \frac{1}{{}_g\mu_d} = \frac{\mu_g}{\mu_d} = \frac{1{\cdot}51}{2{\cdot}47} = 0{\cdot}6113 = \sin 37°42'$$

∴ the minimum angle of incidence=37°42′.

Example 4 : *The refractive index of a liquid for red light is 1·64 and the difference between the critical angles for red and blue light at the liquid-air interface is 0°·53′. What is the refractive index of the liquid for blue light ? sin 37°30′= 0·609 ;* sin *36°37′=0·596.*

Ans. Suppose μ_r and θ_r are the *r.i.* and the critical angle of the liquid for red light. Then, $\sin\theta_r = \frac{1}{\mu_r} = \frac{1}{1{\cdot}64} = 0{\cdot}609 \quad \therefore \quad \theta_r = 37°30'$

The critical angle θ_b for blue light is evidently less than that for red light. Hence $\theta_b = 37°30' - 0°53' = 36°37'$.

The *r.i.* for blue light, $\mu_b = \frac{1}{\sin\theta_b} = \frac{1}{\sin 36°37'} = \frac{1}{0{\cdot}596} = 1{\cdot}67$

Example 5 : *The base of a cube of glass of r.i.* μ_1 *is in contact with the surface of a liquid of r.i.* μ_2*. Light incident on one vertical face of the cube is reflected internally from the base and emerges again from the opposite vertical face in a direction making an angle θ with its normal. Assuming* $\mu_1 > \mu_2$*, show that the light has just been totally reflected internally if* $\mu_2 = \sqrt{(\mu_1^2 - \sin^2\theta)}$.

Ans. For total internal reflection at glass-liquid interface the angle of incidence should be equal to the critical angle, say, θ_c [Fig. 3.20]. According to the condition of total internal reflection,

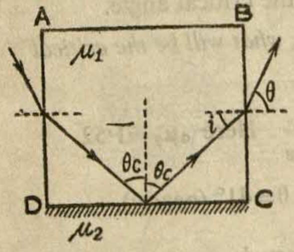


Fig. 3.20

$\sin\theta_c = \frac{\mu_2}{\mu_1}$. Now, if i be the angle of incidence on the face BC, $i = 90° - \theta_c$.

$$\text{Hence, } \mu_1 = \frac{\sin\theta}{\sin i} = \frac{\sin\theta}{\sin(90° - \theta_c)} = \frac{\sin\theta}{\cos\theta_c}$$

$$\text{Again, } \cos\theta_c = \sqrt{1 - \sin^2\theta_c} = (\sqrt{\mu_1^2 - \mu_2^2})/\mu_1$$

$$\therefore \quad \mu_1 = \frac{\sin\theta}{\cos\theta_c} = \frac{\mu_1 \sin\theta}{\sqrt{\mu_1^2 - \mu_2^2}} \quad \text{or,} \quad \mu_1^2 - \mu_2^2 = \sin^2\theta$$

So, $\mu_2 = \sqrt{\mu_1^2 - \sin^2\theta}$.

3.13. Proof of total internal reflection from the laws of refraction :

In the preceding article, we have seen that $\sin\theta = \frac{1}{\mu}$ where θ is the critical angle and μ, the *r.i.* of the denser medium with respect to the rarer. Now, it can be proved in the following way that for an angle of incidence greater than the critical there can be no real value for the angle of refraction which means that there cannot be any refracted ray.

Let us suppose that if possible, refraction takes place for an angle of incidence $\angle i$ greater than the critical angle [Fig. 3.19(iii)] and the angle of refraction $= \angle r$. According to the laws of refraction, $\frac{\sin i}{\sin r} = \frac{1}{\mu}$. or, $\sin r = \mu \sin i$. . . .(i)

Since $i > \theta$, $\sin i > \sin\theta$. or, $\sin i > \frac{1}{\mu}\left[\because \sin\theta = \frac{1}{\mu}\right]$.

So from eqn. (i), we get $\sin r > 1$. But $\sin r$ cannot be greater than 1 for any real value of r. Hence, under the above condition (*i.e.* for angle of incidence greater than critical angle), the ray of light cannot be refracted; it is totally reflected in the denser medium.

3.14. A fish eye-view :

Suppose a man is standing at the bank of a pond. If a ray starting from the man almost grazes the surface of water and is refracted to the eye of a fish, then the angle of refraction in the water will be 49°, because the critical angle between air-water interface is 49° [Fig. 3.21]. No other ray from above the surface of water will be able to reach the eye of the fish at an angle greater than this. So, the fish will see the man along the line *OAB* which makes an angle of 49° with *OP*. If there be a tree on the opposite bank it will appear to the eye of the fish, as if situated along the line *OCD* which also makes an angle of 49° with *OP*. Hence, *all objects above the surface of water will appear to be situated within a cone of angle 98° to the eye of the fish.* We, on the surface of the earth, see the sun to describe an arc of a circle of 180° in air but a fish in water will see it as an arc of circle of 98° for the above reason.

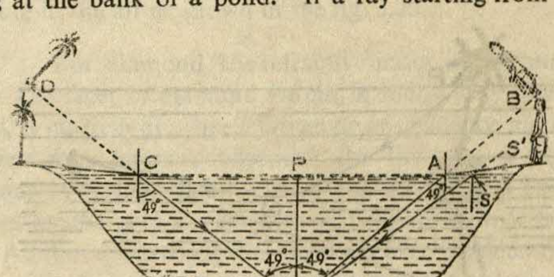


Fig. 3.21

If the fish looks beyond the cone mentioned above, it will see objects lying in the water. For example, a ray starting from an object *S* in the water, when incident on the surface, will have an angle of incidence greater than 49° and as such, it will be totally reflected inside water and will reach the eye of the fish.

The object S will, as a result, appear to be at S' to the fish. For this reason, the water surface will appear to the fish as a shining mirror with a hole at the centre through which all objects lying above the surface will be visible to the fish. The radius of the hole is evidently CP or AP.

If $PC=PA=r$ and $PO=h$, then $\tan \angle POC=\frac{r}{h}$

Again if θ_c be the critical angle (in this case 49°) $\angle POC=\theta_c$

$$\therefore \quad r=h \tan \theta_c=h\times\frac{\sin \theta_c}{\cos \theta_c}=\frac{h}{\mu\sqrt{1-\frac{1}{\mu^2}}}=\frac{h}{\sqrt{\mu^2-1}}. \quad \left[\because \sin \theta_c=\frac{1}{\mu}\right]$$

3.15. Natural illustration of total internal reflection :

An optical illusion takes place in deserts or in cold countries. The traveller in a desert often sees what appears to be a sheet of water a short distance ahead of him. This he is never able to reach. In cold countries, an inverted image of a distant object is sometimes found hanging in the sky. Such optical illusions are called **mirage.** They are brought about by total reflection of light.

(*a*) **Mirage in the desert** : The layer of air in contact with the hot sand bed of a desert becomes very hot and expands, thus becoming less dense than cooler layers above. In the absence of wind, the atmosphere of a desert may be thought of as consisting of layers of air of increasing density upwards as the altitude increases. A ray of light coming from the point P of a distant tree through the cool air layers are refracted away from the normal. The refraction is thus increased as the ray passes down to successively lower layers and so it travels along a curved path as shown in fig. 3.22. Finally, the ray meets a hot surface layer near the ground, say at Q where the angle of incidence becomes greater than the critical angle for the two successive layers at Q. Total reflection then occurs and the ray passes upwards being continually refracted into denser and denser layers, finally reaching the eye of the observer. A *virtual image* of the point P is therefore seen at P'. The hot surface layer of air thus acts as a mirror in which an inverted image of the tree is seen. The inversion of the image produces in the mind of the traveller an impression that the image is formed by a pool of water.

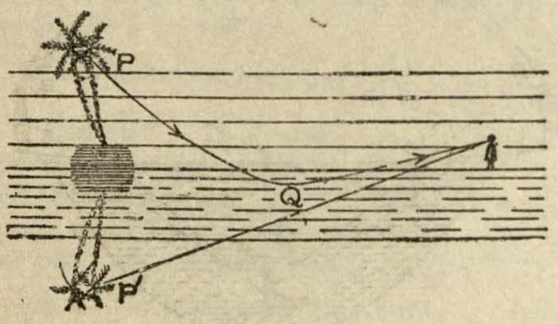


Fig. 3.22

Further, due to continuous change of temperature, the density and hence the refractive index of the layers of air change. This gives the image a quivering appearance, as if the pool of water is disturbed by slow air current. All these

combine to give a complete impression about the existence of a pool of water ahead but on approaching it, the pool disappears and the traveller is very much disappointed.

For the same reason, when a long and straight asphalted road bathed by strong sun-light is viewed, a portion of the road ahead appears shining and wet, as if there has been rain there. It is also a sort of optical illusion.

(*b*) **Mirage in cold countries** : In cold countries, the density and hence *r.i.* of layers of air decreases as the altitude increases. So, a ray of light from a distant object (say, a ship) while proceeding upwards, is refiacted away from the normal at each upper layer. The angle of incidence, gradually increasing in this way, becomes so at a particular layer that it exceeds the critical angle and the ray suffers total reflection. It then begins to travel downwards. The object, therefore, appears inverted and hanging in the air as shown in the fig. 3.23.

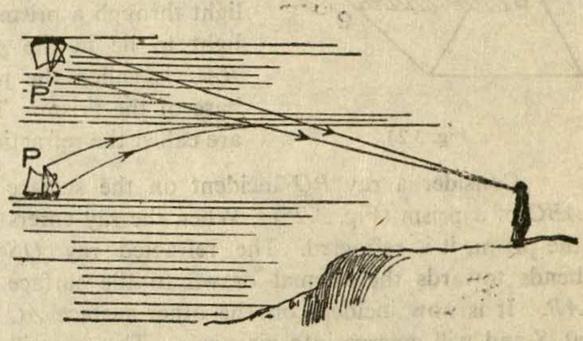


Fig. 3.23

(*c*) **Sparkling of diamond** : For diamond, the refractive index is 2·42 and the critical angle is only 24·4°. The faces of diamond are cut in such a way, that a ray of light entering the crystal, is incident at a face at an angle of incidence more than the critical angle and suffers total internal reflection. In this way, a ray suffers multiple internal reflections at various faces. There are, however, a few faces available where the angle of incidence is less than the critical angle and the rays energe through those faces. All the rays entering into the crystal are condensed as a result of multiple internal reflections and when they finally emerge, they produce the sparkling.

As a matter of fact, the brilliance of all precious gems is due to total internal reflection.

(*d*) **Total internal reflection of radio waves** :

Radio waves are electromagnetic waves. Light waves and radio waves are same in nature. Radio waves have all optical properties. Propagation of short wave length radio waves from one place to another is due to total internal reflection of the waves by ionised layer known as *Appleton layer*.

When a radio wave from a transmitter is sent skyward, it is refracted away from the normal on entering the ionised layer, just like ordinary light waves. At some height, a critical angle is reached and the wave then begins to be refracted downward. After emerging from the ionised layer it returns to the earth, where its presence is detected by a radio receiver.

3.16. Refraction of light by a prism :

Prism : It is a triangular block of a refracting material, say glass, the surfaces of which are all inclined to one another and edges are all parallel. Of the five surfaces of a prism, three are rectangular and two triangular. Fig. 3.24, shows a prism. *EH* is one of the edges of the prism. *ABC* is the principal section of the prism. It is at right angles to the edges of the prism. Whenever we shall discuss refraction of light through a prism, we shall consider the ray of light to lie in the principal section. The angle *BAC* is called the refracting angle and *BC*, the base of the prism. The two surfaces *AB* and *AC* are called the refracting surfaces.

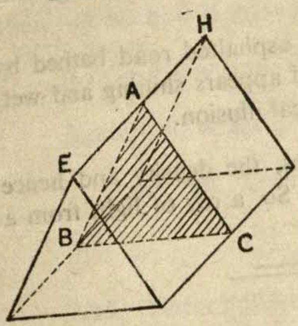


Fig. 3.24

Consider a ray *PQ* incident on the surface *AB* of the principal section *ABC* of a prism (Fig. 3.25). When the ray enters the prism, it is refracted. The refracted ray *QS* bends towards the normal drawn to the surface *AB*. It is now incident on the other surface *AC* at *S* and will emerge into air again. The ray will again be refracted at *S* and the emergent ray *ST* will bend away from the normal drawn to the surface *AC*. So, *PQST* is the complete course of ray through the prism. It is clear from the figure that the ray, in passing through the prism, is turned towards the base *BC* of the prism *i.e.* the ray undergoes some deviation. The angle between the direction of the incident ray *PQ* and the direction of the emergent ray *ST* measures *the angle of deviation.*

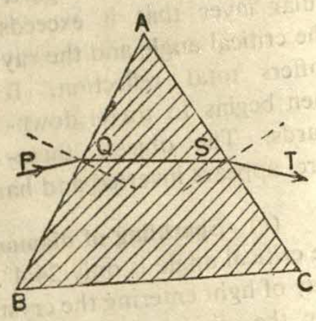


Fig. 3.25

It is to be noted here that the ray will be deviated in the opposite direction *i.e.* towards the vertex *A* if the material of the prism is less dense than the surrounding medium.

Measure of the angle of deviation : Let *PQST* be the complete course of a ray of light through the prism *ABC* (Fig. 3.26). *PQ* and *TS*, when produced, include an angle δ which is the angle of deviation of the ray. The angles of incidence and refraction at the face *AB* are i_1 and r_1. The angles of incidence and emergence at the face *AC* are r_2 and i_2 respectively.

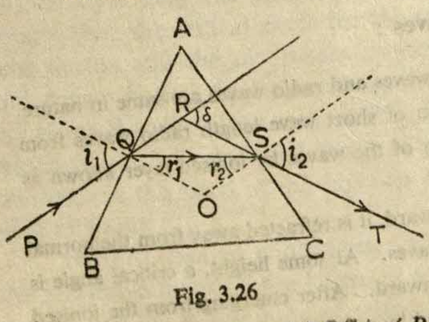


Fig. 3.26

Now, since the arm *QR* of the ΔRQS is produced, the exterior

$$\angle\delta = \angle RQS + \angle RSQ = (i_1 - r_1) + (i_2 - r_2) = i_1 + i_2 - (r_1 + r_2).$$

In the quadrilateral $AQOS$, the sum of the four angles$=4rt\angle s$. i.e. $\angle A+\angle O+\angle AQO+\angle ASO=4rt.\angle s$.

But $\angle AQO+\angle ASO=2rt.\angle s$. [$\because$ QO and SO are normals to AB and AC respectively]

$$\angle A+\angle O=2rt,\ \angle s.$$

Again, in ΔQSO, $\angle O+\angle r_1+\angle r_2=2rt.\angle s$

$$\therefore\quad \angle A=\angle r_1+\angle r_2 \qquad \ldots \quad \ldots \quad \text{(i)}$$

Hence, $$\delta=i_1+i_2-A \qquad \ldots \quad \ldots \quad \text{(ii)}$$

3.17. Angle of minimum deviation :

From the eqn. (ii) of the preceding article, it is clear that for a prism of given refracting angle, the deviation δ depends upon the angle of incidence i_1. If i_1 be changed, the deviation also changes. But it has been seen that for a given angle of incidence the deviation becomes minimum *i.e.* if the angle of incidence is greater or less than the given value, the deviation always increases. If a ray be allowed to be incident on a prism at various angles of incidence and the corresponding deviations be measured, then a graph drawn between i and δ—known as i—δ curve—looks almost like a parabola as shown in fig. 3.27.

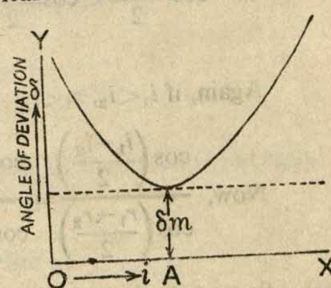


Fig. 3.27

From the figure, it is seen that for a particular value of the angle of incidence (given by OA) the deviation becomes minimum (δ_m). For any other angle of incidence, the deviation is greater. The minimum value of the angle of deviation is known as *the angle of minimum deviation*. If a prism is so placed that a ray suffers minimum deviation through it, the position of the prism is called *the position of minimum deviation.*

3.18. Condition of minimum deviation :

Mathematically it may be proved in the following way that in the minimum deviation position, the angle of incidence (i_1) equals the angle of emergence (i_2).

From fig. 3.26, we have,

$$\frac{\sin i_1}{\sin r_1}=\frac{\sin i_2}{\sin r_2}=\mu \quad [\mu=r.i.\text{ of the prism}] \quad \therefore \quad \frac{\sin i_1+\sin i_2}{\sin r_1+\sin r_2}=\mu$$

$$\text{or,}\quad \frac{2\sin\frac{i_1+i_2}{2}\cos\frac{i_1-i_2}{2}}{2\sin\frac{r_1+r_2}{2}\cos\frac{r_1-r_2}{2}}=\mu \quad \text{or,}\quad \frac{\sin\frac{\delta+A}{2}}{\sin\frac{A}{2}}\times\frac{\cos\frac{i_1-i_2}{2}}{\cos\frac{r_1-r_2}{2}}=\mu$$

[Using eqns (i) & (ii) of art. 3.16]

Since μ is a constant quantity, the product of the two factors on the L.H.S. of the above equation is also constant. Hence, when δ is minimum,

$$\frac{\left(\sin\frac{\delta+A}{2}\right)}{\left(\sin\frac{A}{2}\right)}$$ is also minimum, or $$\frac{\left(\cos\frac{i_1-i_2}{2}\right)}{\left(\cos\frac{r_1-r_2}{2}\right)}$$ is maximum because the product of the two is constant.

If $i_1>i_2$, $r_1>r_2$, Again $(i_1-i_2)>(r_1-r_2)$ because rotation of the incident ray in air is greater than the rotation of the refracted ray in the denser medium.

$$\therefore \quad \cos\frac{i_1-i_2}{2}<\cos\frac{r_1-r_2}{2} \quad \text{or,} \quad \frac{\cos\frac{i_1-i_2}{2}}{\cos\frac{r_1-r_2}{2}}<1$$

Again, if $i_1<i_2$, $r_1<r_2$; so, $(i_2-i_1)>(r_2-r_1)$ $\therefore$ $\cos(i_2-i_1)<\cos(r_2-r_1)$

$$\text{Now,} \quad \frac{\cos\left(\frac{i_1-i_2}{2}\right)}{\cos\left(\frac{r_1-r_2}{2}\right)}=\frac{\cos-\left(\frac{i_2-i_1}{2}\right)}{\cos-\left(\frac{r_2-r_1}{2}\right)}=\frac{\cos\left(\frac{i_2-i_1}{2}\right)}{\cos\left(\frac{r_2-r_1}{2}\right)}<1$$

So, we see that whether $i_1>i_2$ or $i_1<i_2$,

$$\frac{\cos\left(\frac{i_1-i_2}{2}\right)}{\cos\left(\frac{r_1-r_2}{2}\right)}$$ is always less than 1.

But when $i_1=i_2$ or for that reason $r_1=r_2$, the aforesaid quantity is equal to 1 which is its maximum value.

Hence, for the deviation to be minimum $i_1=i_2$ or, $r_1=r_2$.

It follows that *in the minimum deviation position, the light passes symmetrically through the prism*

[Alternative calculus method :

We have seen that $\delta=i_1+i_2-A$ and $A=r_1+r_2$

Differentiating we get, $\frac{d\delta}{di_1}=1+\frac{di_2}{di_1}$ and $\frac{dA}{dr_1}=0=1+\frac{dr_2}{dr_1}$ [A is constant]

When δ becomes minimum $\frac{d\delta}{di_1}=0$ $\quad\therefore\quad \frac{di_2}{di_1}=-1$ Also $\frac{dr_2}{dr_1}=-1$

Considering refractions at Q and S (fig. 3.26) we have,

$$\sin i_1=\mu\sin r_1$$

$$\text{and} \quad \sin i_2=\mu\sin r_2$$

Differentiating the above equations, we get,

$$\cos i_1.di_1=\mu \cos r_1.dr_1$$

$$\text{and} \quad \cos i_2.di_2=\mu \cos r_2.dr_2$$

$$\therefore \quad \frac{\cos i_1}{\cos i_2}\cdot\frac{di_1}{di_2}=\frac{\cos r_1}{\cos r_2}\cdot\frac{dr_1}{dr_2}$$

Since, $\dfrac{di_2}{di_1}=-1$ and $\dfrac{dr_2}{dr_1}=-1$, we have, $\dfrac{\cos i_1}{\cos i_2}=\dfrac{\cos r_1}{\cos r_2}$

Squaring and changing we get, $\dfrac{1-\sin^2 i_1}{1-\sin^2 i_2}=\dfrac{1-\sin^2 r_1}{1-\sin^2 r_2}$

Multiplying both the numerator and the denominator of the R.H.S. of the above equation by μ^2,

$$\frac{1-\sin^2 i_1}{1-\sin^2 i_2}=\frac{\mu^2-\mu^2\sin^2 r_1}{\mu^2-\mu^2\sin^2 r_2}=\frac{\mu^2-\sin^2 i_1}{\mu^2-\sin^2 i_2} \quad \text{(From Snell's law)}$$

$$=\frac{(\mu^2-\sin^2 i_1)-(1-\sin^2 i_1)}{(\mu^2-\sin^2 i_2)-(1-\sin^2 i_2)}=\frac{\mu^2-1}{\mu^2-1}=1$$

$$\therefore \quad \sin^2 i_1=\sin^2 i_2 \text{ or } i_1=i_2$$

So, the minimum deviation occurs when the angle of incidence (i_1) is equal to the angle of emergence (i_2). From this we get $r_1=r_2$].

3.19. Refractive index of the material of the prism :

We have seen, $\delta=i_1+i_2-A$ and $A=r_1+r_2$ .. (i)

When a ray passes through a prism with minimum deviation, we have seen in the preceding article that the angle of emergence (i_2) becomes equal to the angle of incidence (i_1). In other words, when $\delta=\delta_m$ (minimum), $i_1=i_2$ and $r_1=r_2$.

So, $\delta_m=2i_1-A$ or $i_1=\dfrac{\delta_m+A}{2}$ [From eqn. (i)] Further, $A=2r_1$ or $r_1=\dfrac{A}{2}$

Now considering refraction on the refracting surface AB, the angle of incidence$=i_1$ and the angle of refraction$=r_1$. If μ be the *r.i.* of the material of the prism, then, $\mu=\dfrac{\sin i_1}{\sin r_1}=\dfrac{\sin\left(\dfrac{\delta_m+A}{2}\right)}{\sin\dfrac{A}{2}}$

So, knowing δ_m and the refracting angle A of the prism, the *r.i.* of the material of the prism can be found out.

Example 1 : *The refracting angle of a prism is 60° and the angle of minimum deviation of a ray passing through the prism is 30°. What is the r.i. of the material of the prism ?*

Ans. Here, $A=60°$ and $\delta_m=30°$.

We know, $\mu=\dfrac{\sin\left(\dfrac{\delta_m+A}{2}\right)}{\sin\dfrac{A}{2}}=\dfrac{\sin\left(\dfrac{30+60}{2}\right)}{\sin\dfrac{60}{2}}=\dfrac{\sin 45°}{\sin 30°}=\dfrac{1}{\sqrt{2}}\times 2=\sqrt{2}$

Example 2 : *The refracting angle of a prism is 60° and the r.i. of its material is 1·5. What is the angle of minimum deviation ?* sin $48°35' = 0·75$.

Ans. Here $A = 60°$ and $\mu = 1·5$.

We know, $\mu = \dfrac{\sin\left(\dfrac{\delta_m + A}{2}\right)}{\sin\dfrac{A}{2}}$ or, $1·5 = \dfrac{\sin\left(\dfrac{\delta_m + 60}{2}\right)}{\sin\dfrac{60}{2}}$

$$= \frac{\sin\left(\dfrac{\delta_m + 60}{2}\right)}{\sin 30°} = \frac{\sin\left(\dfrac{\delta_m + 60}{2}\right)}{\dfrac{1}{2}} \quad \therefore \quad 0·75 = \sin\left(\frac{\delta_m + 60}{2}\right)$$

or, $\sin 48°35' = \sin\left(\dfrac{\delta_m + 60}{2}\right)$ $\therefore$ $\dfrac{\delta_m + 60}{2} = 48°35'$ or, $\delta_m = 37°10'$.

Example 3 : *One face of a prism of r.i. 1·5 and angle 75° is covered with a liquid of r.i.* $\dfrac{3\sqrt{2}}{4}$. *What should be the angle of incidence of the light on the other face of the prism for which light is just totally reflected at the liquid covered face?* [*Jt. Entrance 1982*]

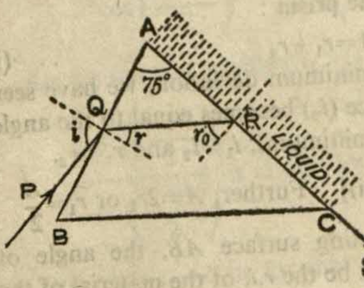


Fig. 3.28

Ans. Let r_0 be the critical angle between prism-liquid interface. Let the ray QR be incident on the face AC at an angle r_0. Under this circumstances,

$$\sin 90° \times \frac{3\sqrt{2}}{4} = \sin r_0 \times 1·5$$

or, $\sin r_0 = \dfrac{3\sqrt{2}}{4} \times \dfrac{10}{15} = \dfrac{1}{\sqrt{2}} = \sin 45°$

$\therefore$ $r_0 = 45°$

But $\angle r + \angle r_0 = \angle BAC$ or $\angle r + 45° = 75°$ $\therefore$ $\angle r = 30°$

If i be the angle of incidence at the face AB, [Fig. 3.28] then

$\dfrac{\sin i}{\sin r} = 1·5$ or $\sin i = 1·5 \times \sin r = 1·5 \times \sin 30° = 1·5 \times \frac{1}{2} = 0·75$

$$\therefore \quad i = \sin^{-1} 0·75 = 48°\ 35'$$

Example 4 : *A ray of light traverses a prism of r.i. 1·6 and just undergoes total reflection at the second face. If the refracting angle of the prism be 60° what is the angle of incidence at the first face ? Given sin 38°41' = 0·6250 and* sin $35°48' = 0·5850$.

Ans. If r_1 is the angle of refraction at the first face and r_2 the angle of incidence at the second, then $A = r_1 + r_2$. Since, the ray is totally reflected at the second face, $\sin r_2 = \dfrac{1}{1·6} = 0·6250 = \sin 38°41'$ $\therefore$ $r_2 = 38°41'$.

Now, $A=r_1+r_2$ or $60°=r_1+38°41'$ or, $r_1=60°-38°41'=21°19'$.

suppose, i is the angle of incidence at the first face. Then

$$1{\cdot}6=\frac{\sin i}{\sin r_1}=\frac{\sin i}{\sin 21°19'}=\frac{\sin i}{0{\cdot}3656}$$

Hence $\sin i=1{\cdot}6\times0{\cdot}3656=0{\cdot}5850$ (nearly)$=\sin 35°48'$. $\therefore$ $i=35°48'$.

3.20. Refraction through a thin prism at a small angle of incidence :

A prism is regarded to be thin when its refracting angle ($\angle BAC$ in fig 3.29) is 5° or less. Since the angle of incidence is very small, the angle of refraction r_1 is also small, so that $\sin r_1=r_1$ and $\sin i_1=i_1$. Further since A is very small i_2 and r_2 are also very small and $\sin i_2=i_2$ and $\sin r_2=r_2$.

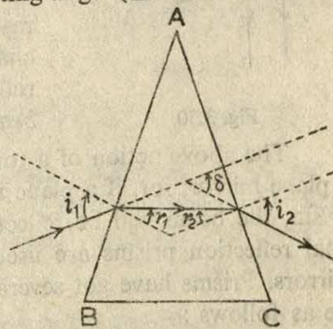


Fig. 3.29

Now, considering the refractions at the two faces of the prism (fig. 3.29), we can write, $\sin i_1=\mu \sin r_1$ or $i_1=\mu r_1$.... (i)

Also, $\sin i_2=\mu \sin r_2$ or $i_2=\mu r_2$....[ii]

We know, $\delta=i_1+i_2-A$.

Substituting the values from equation (i) and (ii), we get,

$$\delta=\mu(r_1+r_2)-A=\mu A-A \qquad [\because \; A=r_1+r_2]$$

or, $\delta=A(\mu-1)$.

Hence, the deviation of ray through a thin prism does not depend upon the angle of incidence provided it is small.

Example : *A glass prism of refracting angle 5° and refractive index 1·52 is placed in contact with another prism of material having r.i. 1·63 with their refracting angles turned in the opposite directions. A ray of light, incident normally on the first prism, passes through the combination without any deviation. What is the refracting angle of the second prism ?*

Ans. In the case of a thin prism, the deviation of a ray δ is given by $\delta=(\mu-1)A$. Since the ray passes out undeviated, the deviation produced by the first prism is equal and opposite to that produced by the second. That is,

$$\delta_1=\delta_2$$

$$\text{or} \quad (\mu_1-1)\,A_1=(\mu_2-1)\,A_2$$

$$\text{or} \quad (1{\cdot}52-1)\times5=(1{\cdot}63-1)A_2$$

$$\text{or} \quad 0{\cdot}52\times5=0{\cdot}63\times A_2$$

$$\text{or} \quad A_2=\frac{5\times0{\cdot}52}{0{\cdot}63}=4°8' \text{ (nearly)}$$

3.21. Some specific uses of prism :

(i) **Total reflection prism :** *ABC* is a right angled isosceles glass prism. A beam of parallel rays is incident normally on the surface *AB*. The rays will enter into the prism without any deviation and will fall on the hypotenuse face *AC* (Fig. 3.30). The angle of incidence of these rays at this face is 45° ; but the critical angle between glass and air is about 41°45′. So, the rays are totally reflected and they fall on the face *BC* normally. The rays, then, emerge from the prism without any further change of direction. Such a prism is called a total reflection prism. It is to be noted that the *original beam of rays is deviated through 90°.*

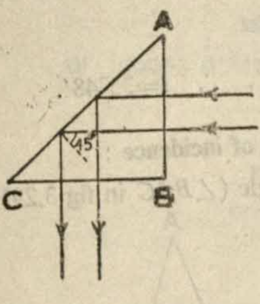


Fig. 3.30

The above action of a total reflection prism is very much similar to that of a plane mirror, for, if a plane mirror be placed in the position *AC* instead of the prism, the beam will be reflected in the same way as before. For this reason, total reflection prisms are used in many optical instruments in place of plane mirrors. Prisms have got several *advantages* over plane mirrors. The advantages are as follows :

(*a*) Due to reflection and refraction at the front and back surfaces of a plane mirror, multiple images are produced and there is a loss of brightness of the image. Since total reflection of light takes place in a prism, the image becomes very bright and there is no confusion due to multiple images.

(*b*) There is a silvering at the back of plane mirrors, which gets tarnished in course of time. The image produced by such a tarnished plane mirror is obviously feeble. Such thing does not happen in a prism because it does not require a silvering. It always gives a bright image.

(*c*) Some light is lost due to scattering in a plane mirror which is almost absent in a prism.

Example 1 : *A ray of light incident normally on one of the faces of a right-angled isosceles prism is found to be totally reflected.* (*i*) *What is the minimum value of the refractive index of the material of the prism* (*ii*) *When the prism is immersed in water, trace the path of the emergent rays for the same incident ray, indicating the values of all angles.* μ *of water*$=4/3$. [*I.I.T. 1973*]

Ans. (i) Let *ABC* be the right-angled isosceles prism and a ray be incident on the face *AB* normally [Fig. 3.31]. The ray passes into the prism straight and is incident at *D* on the face *AC*. From the figure it is clear that, the angle of incidence at $D=45°$. Since the ray is just totally reflected at *D*, the critical angle between the glass and air is evidently 45° *i.e.* $\theta_c=45°$. If μ be the *r.i.* of the material of the prism. $\sin\theta_c=\frac{1}{\mu}$; so $\sin 45°=\frac{1}{\mu}$ or $\mu=\sqrt{2}=1{\cdot}414$.

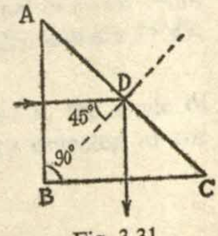


Fig. 3.31

(ii) Now. *r.i.* of glass with respect to water ${}_w\mu_g=\frac{\mu_g}{\mu_w}=\sqrt{2}\times\frac{3}{4}=1{\cdot}06.$

If θ_c' be the critical angle between glass-water interface, then

$$\theta_c'=\sin^{-1}\frac{1}{{}_w\mu_g}=\sin^{-1}\frac{1}{1{\cdot}06}=\sin^{-1}0{\cdot}9434=\sin 70°48'$$

But the angle of incidence of the ray at $D=45°$. Since the angle of incidence is less than the critical angle, the ray will be refracted into water [Fig. 3.32]. It will not undergo total internal reflection. Assuming the angle of refraction to be r, we get from Snell's law,

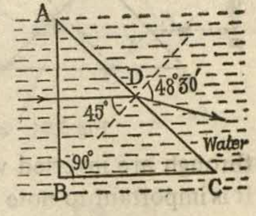


Fig. 3.32

$$\sqrt{2}\times\sin 45°=\tfrac{4}{3}\times\sin r,\ \text{or}\ \sin r=\tfrac{3}{4}=0{\cdot}75=\sin 48°30' \quad \therefore\ r=48°30'.$$

Example 2 : *Three right-angled prisms of refractive indices μ_1, μ_2, μ_3 are fitted together so that the faces of the middle prism are in contact each with one of the outside prisms. If a ray passes through the composite block without any deviation, show that $\mu_1^2+\mu_3^2-\mu_2^2=1$.* [*I.I.T. 1969*]

Ans. *PQRST* is the path of a ray of light through the combination of the prisms. [Fig. 3.33]. As the ray passes out undeviated, the incident ray *PQ* is parallel to the emergent ray at T.

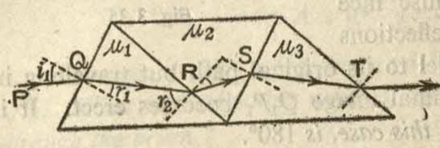


Fig. 3.33

Let the angle of incidence at Q be i_1 and angle of refraction$=r_1$. As the angle of the prism is 90°, the angle of incidence at R is $r_2=90°-r_1$ [$\because\ A=r_1+r_2$]. If r_3 be the angle of refraction at R, the angle of incidence at S is $(90°-r_3)$. Again, if angle of refraction at S be r_4, then the angle of incidence at $T=90°-r_4$. Again since the incident ray PQ and the emergent ray are parallel to each other the angle of emergence at $T=90°-i_1$.

Now considering refraction at Q, $\sin i_1=\mu_1\sin r_1$

$$\text{or}\quad \sin^2 i_1=\mu_1^2\sin^2 r_1 \qquad \text{.. (i)}$$

For refraction at R, $\mu_1\sin(90°-r_1)=\mu_2\sin r_3$

$$\text{or}\quad \mu_1^2\cos^2 r_1=\mu_2^2\sin^2 r_3 \qquad \text{.. (ii)}$$

For refraction at S, $\mu_2\sin(90°-r_3)=\mu_3\sin r_4$

$$\text{or}\quad \mu_2^2\cos^2 r_3=\mu_3^2\sin^2 r_4 \qquad \text{.. (iii)}$$

For refraction at T, $\mu_3\sin(90°-r_4)=\sin(90°-i_1)$

$$\text{or}\quad \mu_3^2\cos^2 r_4=\cos^2 i_1 \qquad \text{.. (iv)}$$

Adding eqns (i) (ii) (iii) & (iv) we get

$$\mu_1^2+\mu_3^2=1+\mu_2^2 \quad \text{or} \quad \mu_1^2+\mu_3^2-\mu_2^2=1.$$

(ii) **Erecting prism** : With the help of this prism, an inverted image may be made erect. It is also a right angled isosceles glass prism (Fig. 3.34). Suppose QP is an inverted image of the candle. Light from the candle parallel to the base or hypotenuse face BC enters the face AB of the prism. Total reflection occurs at the base, because the angle of incidence exceeds the critical angle. As a result the rays passing through are inverted without any change of direction, giving an erect image PQ. It is important to note that *the deviation, here, is nil.*

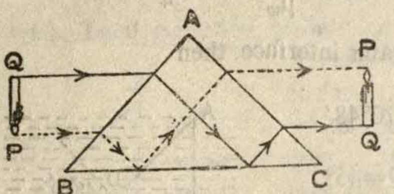


Fig 3.34

In many optical instruments like telescopes, binoculars, periscopes etc., erecting prisms are used in order to make an inverted image erect. There is, however, another way in which the above prism may be used to make an inverted image erect.

Suppose, an inverted image PQ is formed in front of the hypotenuse face BC of the prism ABC (Fig. 3.35). Light from the image PQ is incident perpendicularly on the hypotenuse face and the rays will undergo two internal reflections inside the prism and finally emerge parallel to its original path but travelling in the opposite direction. As a result, the final image Q_1P_1 becomes erect. It is to be noted that *the deviation of the ray, in this case, is* 180°.

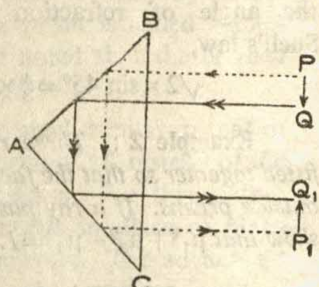


Fig. 3.35

(iii) **Prism Periscope** : A simple periscope, made of plane mirrors, has been described in art 2.8. In improved type of periscopes, however, total reflection prisms are used in place of plane mirrors, because prisms can produce brighter image than mirrors. Fig. 3.36 shows the section of a periscope.

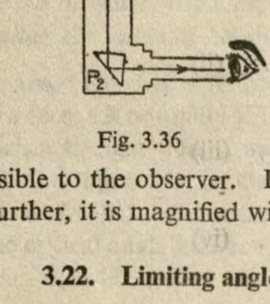


Fig. 3.36

Two right-angled isosceles prisms are fitted in a tube with their hypotenuse faces parallel to each other. A ray of light coming from a distant object—say a tree, is totally reflected by the hypotenuse face of the prism P_1 on to the parallel hypotenuse face of the prism P_2 wherefrom it is again totally reflected parallel to its original path, to the eye of the observer. The object then becomes visible to the observer. Due to total reflection of light, the image is very bright. Further, it is magnified with the help of lenses.

3.22. Limiting angle of incidence for a prism of given angle for no emergence :

Suppose a ray is incident on the surface AB of a prism of given angle A, at an angle of incidence i_1 so that it emerges from the other surface AC just

grazing the surface [Fig. 3.37]. The required limiting angle of incidence in this case is i_1 because at a smaller angle, the ray will be totally reflected at the surface AC and will not emerge from it. The reason is that when i_1 decreases r_1 also decreases but r_2 increases because $r_1+r_2=A$, which is constant.

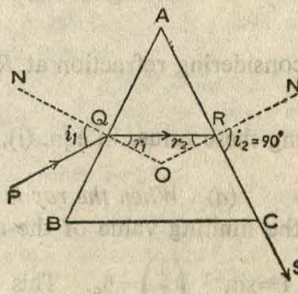


Fig. 3.37

Since the emergent ray grazes the surface AC, $i_2=90°$ and $r_2=\theta_c$, the critical angle.

Also, $A=r_1+r_2=r_1+\theta_c$

$\therefore \quad r_1=A-\theta_c$

Further, $\sin\theta_c=\dfrac{1}{\mu}$ and $\cos\theta_c=\sqrt{1-\sin^2\theta_c}=\dfrac{\sqrt{\mu^2-1}}{\mu}$

Now considering refraction at Q, we have,

$$\begin{aligned}\sin i_1&=\mu\sin r_1=\mu\sin(A-\theta_c)\\&=\mu\sin A.\cos\theta_c-\mu\cos A\sin\theta_c\\&=\mu.\ \sin\ A\times\frac{\sqrt{\mu^2-1}}{\mu}-\mu.\cos A.\frac{1}{\mu}\\&=\sin A\sqrt{\mu^2-1}-\cos A.\end{aligned}$$

Example : *The refracting angle of a prism is 60° and its refractive index is* $\sqrt{\frac{7}{3}}$. *What is the limiting angle of incidence of a ray that will be just transmitted through the prism ?*

Ans. The limiting angle of incidence is given by,

$\sin i_1=\sin A\sqrt{\mu^2-1}-\cos A$. Here, $A=60°$ and $\mu=\sqrt{\frac{7}{3}}$; Hence,

$$\sin i_1=\sin 60°\sqrt{\tfrac{7}{3}-1}-\cos 60°=\frac{\sqrt{3}}{2}\times\frac{2}{\sqrt{3}}-\tfrac{1}{2}=\tfrac{1}{2}\quad\therefore\quad i_1=30°.$$

3.23. Limiting angle of a prism for no emergence :

Although the material of a prism is transparent to light, yet light will not pass under all circumstances, through a prism. It depends upon the refracting angle of the prism. It may be proved that there is a limiting value of the angle of a prism, beyond which light will be refused transmission through the prism, whatever may be the angle of incidence.

Let $PQRS$ be the course of a ray of light through a prism ABC (Fig. 3.37). The ray RS emerges just grazing the surface AC. The angle of incidence at $AB=i_1$ and the angle of refraction$=r_1$. The angle of incidence at $AC=r_2$ and the angle of emergence$=i_2=90°$.

It is clear that $r_2=\theta_c$, the critical angle between glass and air. It is also clear that $\angle A$ is the limiting value of the prism angle which allows the ray to emerge just grazing the surface AC. From the figure, we can write, $A=r_1+r_2$..(i)

Considering refraction at Q, $\sin i_1=\mu \sin r_1$ or, $r_1=\sin^{-1}\left(\frac{\sin i_1}{\mu}\right)$. Similarly, considering refraction at R, we have, $\sin 90°=\mu \sin r_2$; so $r_2=\sin^{-1}\frac{1}{\mu}$. Substituting these values in eqn. (i), we get, $A=\sin^{-1}\left(\frac{\sin i_1}{\mu}\right)+\sin^{-1}\left(\frac{1}{\mu}\right)$ .. .. (ii)

(*a*) *When the ray is incident normally i.e.* $i_1=0$. Under the circumstances, the limiting value of the angle A is obtained by putting $i_1=0$ in eqn. (ii). So, $A=\sin^{-1}\left(\frac{1}{\mu}\right)=\theta_c$. This shows that if the angle of the prism is greater than θ_c then the ray incident normally at AB will not be transmitted through the face AC —it will be totally reflected inside.

(*b*) *When the ray is incident at grazing angle i.e.* $i_1=90°$. Under the circumstances, $A=\sin^{-1}\left(\frac{1}{\mu}\right)+\sin^{-1}\left(\frac{1}{\mu}\right)=2\theta_c$. This shows that if the angle of the prism is greater than $2\theta_c$, the ray incident at 90° on AB will not be transmitted through AC.

Considering the above two cases, we can conclude that *the limiting angle of a prism is twice the critical angle.*

Exercises

Essay type :

1. Prove that the incident and the emergent rays are parallel to each other when the rays pass through a parallel-sided block. What will be the lateral shift of the ray ?

2. (*a*) For two media, '*a*' and '*b*' prove that $_a\mu^b=\frac{r.i. \text{ of the medium } `b'}{r.i. \text{ of the medium } `a'}$

(*b*) The ratio between the wavelengths of incident and refracted waves of light is its refractive index. Give reasons. [*Jt. Entrance 1985*]

3. (i) Find the relation between the apparent depth and the real depth of an object seen perpendicularly through a parallel-sided glass block.

(ii) There is some liquid in a pot. Establish the relation between real depth, and apparent depth of the liquid when viewed normally from above. How will the apparent depth change when viewed slantingly? [*H. S. Exam. 1979*]

4. An object is viewed through a parallel-sided glass plate. If d be the thickness of the plate and μ the *r.i.* of glass, then prove that the apparent displacement of the object towards the observer$=(\mu-1)\frac{d}{\mu}$. Also prove that the apparent displacement of the object is independent of its position below the glass plate. [*H. S. Exam. 1980*]

5. Clearly explain 'total internal reflection' and 'critical angle'. What is the relation between the critical angle and the refractive index of the denser medium ? [*H. S. Exam. 1983*] State whether critical angle is available in the following cases :—(i) rays of light are going from air to glass and (ii) rays of light are going from glass to air.

6. What is a mirage ? Explain, with a neat diagram, how a mirage is formed ? Is it a real or a virtual image ?

7. We, on the surface of the earth, see the sun to describe daily an arc of a circle of 180° in air but a fish in water will see it as an arc of a circle of 98°. Explain.

8. What is a prism ? Cite some specific uses of prisms. A ray of light is incident normally on a refracting face of a prism whose refracting angle is 60°. Draw the path of the ray through the prism. Suppose the critical angle for glass is 42° and that the prism has two faces.

9. Describe briefly the principle of operation of a totally reflecting prism. [*H. S. Exam. 1978, '81*]

10. What is the angle of deviation of a ray in connection with the refraction through a prism ? Find an expression for it and explain, with a diagram, the variation of the angle of deviation with the variation of the angle of incidence. [*H. S. Exam. 1979*]

11. What is the angle of minimum deviation ? Obtain an expression for the refractive index of the material of a prism in terms of the refracting angle of the prism and the angle of minimum deviation. [*H. S. Exam. 1979*]

12. Show that for a prism whose refracting angle is greater than twice the critical angle between the material of the prism and the outside medium, a ray incident at one face will not emerge from the other face, whatever may be the angle of incidence. [*H. S. Exam. 1980*]

Short answer type :

13. What is refraction of light ? Explain, with neatly drawn diagrams, how refraction takes place in the following cases : (a) from air to glass and (b) from water to air.

14. Answer the following questions : (*a*) Why does a straight stick appear bent when partly immersed in water ? (b) Why does a vessel full of water appear much shallower than it actually is ? (c) Why does the sun remain in view for some time after it has already set ? (d) Why does a metallic ball painted black with lampblack appear shining when immersed in water ? (e) Why does a crack in a glass pane appear shining ? (f) An empty test tube, partly immersed and held obliquely in water, appears shining over the immersed part ; why ? (g) On warm sunny days, asphalted roads often appear to be covered with pools of water some distance ahead which disappear when approached. Why ? (h) Why does a glass rod immersed in glycerine become invisible ?

15. What are the laws of refraction ? What is refractive index ? What do you mean by saying that the refractive index of glass is 1·5 ?

16. What is meant by a medium being optically denser than another ? What is its relation with the physical density of the medium ? Arrange the following media according to increasing optical density : (i) glass (ii) turpentine (iii) ice and (iv) water.

17. What is the relation between the *r.i.* of a medium and the velocity of light in it ? In which of the following media, the velocity of light is the greatest and the least ? (i) Air (ii) Water (iii) Glass.

Is the velocity of light in a medium for red light greater or less than that for green light ?

18. In travelling through a prism, a ray of light of colour *A* is found to be more deviated than another ray of colour *B*. Which colour has greater velocity in the prism in this case ?

19. A converging beam of the rays try to meet at a point on a screen but they are intercepted by a parallel-sided glass block. Where will the rays meet now ? Draw a diagram. [*I.I.T. 1974*]

20. What is critical angle ? What is its relation with refractive index ?

21. Why does a piece of diamond appear more glittering than a piece of glass of same shape and size ?

22. Will you call the mirage an image ? If so, is it a real or a virtual image ?

23. If a prism is surrounded by a medium denser than the material of the prism, in which direction will a ray of light be deviated when it is refracted through the prism ?

Objective type :

24. Write 'Yes' or 'No' in the following cases :—

(i) Is the velocity of light same in all media ?——

(ii) Does the refracted ray bend away from the normal in the case of refraction from a rarer to a denser medium ?——

(iii) Is the light totally reflected when it is incident on the surface of a medium other than that through which it is travelling ?——

(iv) Does the refractive index of a medium depend on the angle of incidence of a ray in the medium ?——

(v) Is refraction of light responsible for the twinkling of star ?——

(vi) Is ordinary reflection of light identical with total internal reflection ?——

(vii) A ray of light is incident on a prism of given angle. Will it emerge from the prism in all cases of incidence ?——

(viii) Can total internal reflection of light be applied to deviate a ray through 90° ?

25. Put a $\surd$ mark against the correct answer :

(*a*) If the speed of light in medium no. 1 and medium no. 2 be v_1 and v_2 respectively, then the *r.i.* of medium no. 2 with respect to medium no. 1 is (1) v_1/v_2 (2) v_2/v_1 (3) $\sqrt{v_1/v_2}$.

(*b*) If the speed of light in two media of *r.i.* μ_1 and μ_2 be v_1 and v_2 respectively, then (1) $v_1=v_2$ (2) $\mu_1 v_1=\mu_2 v_2$ (3) $\mu_1 v_2=\mu_2 v_1$ (4) $v_1\mu_1^2=v_2\mu_2^2$.

(*c*) If light enters from vacuum into a substance, its speed decreases by 25% of its previous value. The *r.i.* of the substance is (1) 4/3 (2) 5/3 (3) 3/2 (4) 5/4.

(*d*) A prism of $\mu=1{\cdot}5$ is immersed in water. The deviation of a ray as compared to deviatoin in air (i) remains the same (2) increases (3) decreases (4) is doubled.

(*e*) The correct relation between the critical angle and the refractive index of the denser medium is (1) $\sin\theta_c=\mu$ (2) $\sin\theta_c=1/\mu$ (3) $\sin\theta_c=1/\mu^2$ (4) $\sin\theta_c=\mu^2$.

Numerical Problems :

26. The *r.i.* of water with respect to air is 1·33 and that of an oil with respect to air is 1·45. What are the *r.i.* of oil with respect to water and of water with respect to oil?

[Ans. 1·07 ; 0·9]

27. (i) A picture is stuck at the bottom of a block of glass 4 cm. high. How far will it appear to be raised when viewed perpendicularly ? *r.i.* of glass=1·6. [Ans. 1·5 cm]

(ii) There is a spot on the inside bottom of a pot. A liquid of *r.i.* 1·4 is poured into the pot. How much will the spot appear to be raised when viewed from the top if the depth of the liquid is 3.5 cm ? [*H. S. Exam. 1983*] [Ans. 1·07 cm]

28. An electric bulb is placed in air at a distance of 12′ from the surface of separation of a denser medium of *r.i.* 1·5. It is viewed through the denser medium from a distance of 10′ below the surface of separation. Find the distance where the bulb will now be seen.

[Ans. 28′ away from the eye]

29. When a small lamp is held 1·5 metres above the surface of a tank, the image of the lamp seen by reflection in the surface appears to coincide with the image of the bottom of the tank. If *r.i.* of water is 4/3, calculate the depth of the tank. [Ans. 2 metres]

30. A ray of light travelling with in a rectangular glass block falls on one of the faces of the block at an angle of incidence 30°. Some of the light is reflected internally and the rest emerges into air. Given that the *r.i.* of glass for the light is 1·5, calculate the angle between the internally reflected ray and the emergent ray. Sin 48°40′=0·75. [Ans. 101°20′]

31. A tank contains a slab of glass 8 cm thick and of *r.i.* 1·6. Above this is a depth of 4·5 cm. of a liquid of *r.i.* 1·5 and upon this floats 6 cm. of water ($\mu=\frac{4}{3}$). To an observer looking from above, what is the apparent position of a mark on bottom of the tank.

[Ans. 6 cm. from bottom]

[Hints : Apply the formula : App. depth$=\frac{d_1}{\mu_1}+\frac{d_2}{\mu_2}+\frac{d_3}{\mu_3}$]

32. What is the apparent position of an object below a rectangular block of glass 6 cm. thick if a layer of water 4 cm. thick is on top of the glass ? (μ for glass$=1{\cdot}5$ and for water$=1{\cdot}34$)
[Ans. 3 cm. from bottom]

33. If *r.i.* of a medium with respect to air be $\sqrt{2}$, find the critical angle between them.
[Ans. 45°]

34. A transparent liquid of *r.i.* $\sqrt{3}$ is taken in a beaker and upon it is poured another liquid of *r.i.* $\frac{3}{2}$. What will be the critical angle between the liquids ? [Ans. 60°]

35. A fish is at a depth of '*h*' in a still pond. Prove that the free surface of the pond will appear to the eye of the fish like a plane mirror with a circular hole and the radius of the hole is $\dfrac{h}{\sqrt{\mu^2-1}}$. [*r.i.* of water$=\mu$.]

36. A small fish is 6 ft. below the surface of a pond and 4·5 ft. from the bank. A boy, 5 ft. tall, is standing 8 ft. from the edge of the pond. Assuming that the side of the pond is vertical, how much nearer can the boy move to the edge of the pond before his movement becomes visible to the fish. μ of water $\frac{4}{3}$. [Ans. 1′4″ nearer]

37. Light from a luminous point on the lower face of a rectangular glass slab 3 cm. thick, strikes the upper face and the totally reflected rays outline a circle of 4·8 cm. radius on the lower face. What is the *r.i.* of the glass ? [*Jt. Entrance 1978*] [Ans. 1·6]

38. *ABCD* is the plan of a glass cube. A horizontal beam of light enters the face *AB* at grazing incidence. Show that the angle which any ray emerging from B*C* would make with the normal to *BC* is given by $\sin\alpha=\cot\theta_c$ where θ_c is the critical angle.

39. The refracting angle of a prism is 60° and the angle of minimum deviation of a ray through the prism is 40°. What is the *r.i.* of the material of the prism ? $\sin 50°=0{\cdot}776$?
[Ans. 1·53]

40. The refracting angle of a prism is 90° and the other two angles are 45°. If a ray is incident normally on a refracting surface of the prism, show, by a neatly drawn diagram, how the ray will be refracted ? What will be the deviation in this case ? [Ans. 90°]

41. A ray of light is incident at an angle 60° on one of the refracting surfaces of a prism and is deviated through 30° on emergence from the other surface. If the refracting angle of the prism be 30°, prove that the ray is incident on the second surface normally. Also calculate the *r.i.* of the prism. [*I.I.T. 1978*] [Ans. $\sqrt{3}$]

42. A ray of light is refracted through a prism of angle 70°. If the angle of refraction in the glass at the first face is 28°, what is the angle of incidence in the glass at the second face ?
[Ans. 42°]

[Hints : $A=r_1+r_2$**].**

43. The refracting angle of a prism is 60° and its refractive index is $\sqrt{3}$. If the prism is used at 60° angle of incidence, what will be the deviation of the ray ? [Ans. 60°]

44. A glass ($\mu=1{\cdot}5$) prism is immersed in water ($\mu=1{\cdot}33$). What will be the limiting angle of the prism in that condition so that no ray can emerge from the prism ? [Ans. 125°30′]

45. A certain prism is found to produce a minimum deviation of 51° while it produces a deviation of 62°48′ for two values of angle of incidence, *viz* 40°6′ and 82°42′ respectively. Determine the angle of the prism, the angle of incidence at minimum deviation and the *r.i.* of the prism.
[Ans. 60° ; 55°30′ ; 1·648]

46. Refractive index of the material of a prism is $\sqrt{2}$ and its refracting angle is 75°. At what minimum angle must a ray of light be incident on one of its refracting faces so that it may emerge from the other refracting face ? [*H.S. Exam 1984*] [Ans. 45°]

Harder Problems :

47. A ray of light is incident on a parallel-sided glass block and emerges parallel to the direction of the incident light from the opposite face. If the angle of incidence θ be small, show that the lateral shift x of the ray is given by $x=\dfrac{t.\,\theta\,(\mu-1)}{\mu}$ where t=thickness of the block and μ the *r.i.* of glass.

48. A rectangular glass block of thickness 10 cm. and *r.i.* 1·5 is placed over a small coin. A beaker is filled with water of *r.i.* 4/3 to a height of 10 cm and is placed over the glass block. Find the apparent position of the object when it is viewed at near normal incidence.
[*I.I.T. 1975*] [Ans. 14·16 cm below to the water surface]

49. A rectangular block of glass is placed on a printed page lying on a horizontal surface. Find the minimum value of the refractive index of glass for which the letters on the page are not visible from any of the vertical faces of the block. [*I.I.T. 1979*[[Ans. 1·41]

50. A concave mirror of radius of curvature 1 metre is kept at the bottom of a tank of water. The mirror will produce an image of the sun when it is just overhead. What will be the distances of the image of the sun from the mirror when the tank contains water 80 cm and 40 cm. deep ? [*I.I.T. 1968*] [Ans 50 cm ; 47·5 cm]

51. Calculate the lateral displacement of a ray of light passing through a 15 cm thick slab of glass, the opposite sides of which are parallel to each other, if the angle of incidence of the ray be 60°. *R.I.* of glass=1·5. [*Jt. Entrance 1981*] [Ans. 7·69 cm]

52. A point source S is placed at the bottom of a vessel containing a liquid of *r.i.* 5/3. A person is viewing the source from above the surface. There is an opaque disc of radius 1 cm floating on the surface. The centre of the disc lies vertically above the source S. The liquid from the vessel is drained out through a tap. What is the maximum height of the liquid for which the source cannot at all be seen from above ? [*I.I.T. 1970*] [Ans. 1·33 cm]

53. A coin is placed at the bottom of an empty hemispherical basin. When looked over the edge of the basin, the coin is just invisible, but becomes just visible when the basin is filled up by a liquid. If r be the radius of the coin, show that $r=a(\mu^2-1)/(\mu^2+1)$ where μ=*r.i.* of the liquid and a=radius of the hemisphere.

54. A cube of 1 ft edge is made by a material whose *r.i.* is 1·65. There is an air bubble at the centre of the cube. An opaque circular disc is to be attached at the centre of each face of the cube so that the bubble may not be visible from outside. What should be the minimum radius of the disc ? [Ans. 8·6 inches]

55. A nail is fixed perpendicularly to a circular wooden disc at its centre. The disc is floated in water with the nail downwards. What must be the ratio of the longest possible length of the nail to the radius of the disc, so that the nail is completely invisible from air ? Critical angle for water=48°30′
[*H.S. Exam 1985*] [Ans. $\sqrt{7/3}$]

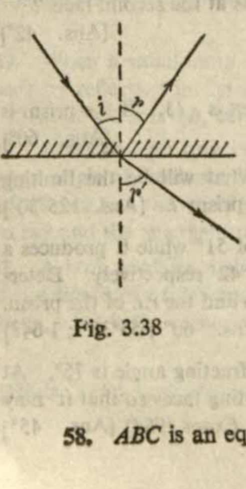


Fig. 3.38

56. A ray of light from a denser medium strikes a rarer medium at an angle of incidence i [Fig. 3·38]. The reflected and the refracted rays make an angle of 90° with each other. The angles of reflection and refraction are r and r'. Show that the critical angle θ_c is given by $\theta_c=\sin^{-1}(\tan r)$ [*I.I.T. 1985*]

57. Light enters a prism of refracting angle A at grazing incidence to emerge at an angle θ with normal. Show that *r.i.* of the material of the prism is :

$$\mu=\left[1+\left(\frac{\cos A+\sin\theta}{\sin A}\right)^2\right]^{\frac{1}{2}}$$
[*Jt. Entrance 1966*]

58. ABC is an equilateral prism. The side BC is covered by a liquid and the other two

sides are open. The *r.i.* of the material of the prism is 1.56. A ray of light is incident on the face *AB* and is refracted to the side *BC*. It is incident on the face *BC* at the critical angle so that it is totally reflected and emerges from the face *AC*. If the angle between the incident ray and the emergent ray be 120°, find the *r.i.* of the liquid. [Ans. 1·35]

59. The refracting angle of a prism is 90°; If α is the angle of minimum deviation and β the angle of deviation of a ray which enters at grazing incidence, then show that $\sin\alpha = \sin^2\beta$ and $\cos\alpha = \mu\cos\beta$, where μ is the *r.i.* of the prism.

60. The *r.i.* of the material of a prism of refracting angle 45° is 1·6 for a certain monochromatic ray. What should be the minimum angle of incidence of this ray on the prism so that no total internal reflection takes place as the ray comes out of the prism ?

[*I.I.T. 1976*] [Ans. 10°6′ nearly)]

61. (a) Find the angle of incidence and also the angle of deviation of a ray of light that passes through a glass prism having refracting angle of 80° symmetrically. μ of glass$=1{\cdot}5$; $\sin 40° = 0{\cdot}6428$; $\sin 74°37 = 0{\cdot}9642$. [Ans. 74°37′ ; 69°14′]

(b) What is is the greatest value of the refractive index for which light can pass in this way through an 80° prism ? What is the corresponding angle of deviation ?

[*Jt. Entrance 1986*] [Ans. 1·56 ; 100°]

62. A parallel beam of light falls normally on the first face of a prism of small angle. The portion of the beam which is refracted at the second surface is deviated through an angle of 1°35′ and the portion which is reflected at the second surface and emerges again at the first surface makes an angle of 8°9′ with the direction of the incident beam. Calculate the angle of the prism and the *r.i.* of the glass. [Ans. 2°30′; 1·63]

4

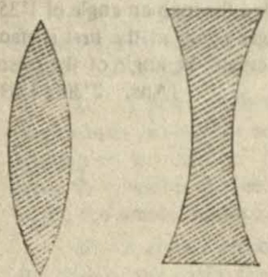
LENSES AND THEIR ACTIONS

4.1. Introduction :

Lenses are known to have been in use for centuries. It was known to the people of ancient times that lenses can bring a parallel beam of rays to a focus. Utilising this property of lenses, burning glasses were invented long ago. In 1857, a glass sphere was constructed on the basis of the above property of a lens, to focus the solar rays on to a paper marked with hours and minutes, and thereby burning it gradually. In this way, the sphere was used to denote time. Lenses are now essential parts of many optical devices, such as microscopes, projectors, spectacles, telescopes, cameras, etc.

4.2. Definition of lenses :

A *lens* is a portion of a transparent medium bounded by two spherical or one spherical and one plane surfaces.



(a) Fig. 4.1 (b)

A lens whose central part is thicker than the edge is called a *convex* or *converging* lens [Fig. 4.1(*a*)] and the lens whose edge is thicker than its central part is called a *concave* or *diverging* lens [Fig. 4.1(*b*)].

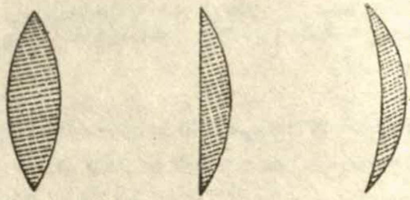
4.3. Different types of lenses :

Depending upon the nature of the surfaces, there may be different types of lenses as given below:

(1) **Double or biconvex** : A lens whose both surfaces are convex is called a double convex lens [Fig. 4.2(*a*)].

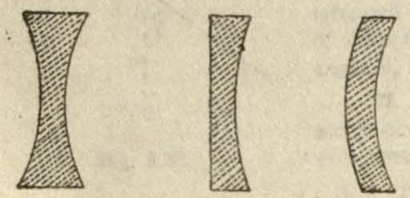
(2) **Plano convex** : A lens whose one surface is convex and the other plane is called a plano-convex lens [Fig. 4.2(*b*)].

(3) **Concavo-convex** : A convex lens having one of its surfaces concave and the other convex is known as a concavo-convex lens [Fig. 4.2(*c*)].



(a) (b) (c)
Fig. 4.2

(4) **Double or bi-concave** : A lens having both the surfaces concave, is called a double concave lens [Fig. 4.3(*a*)].



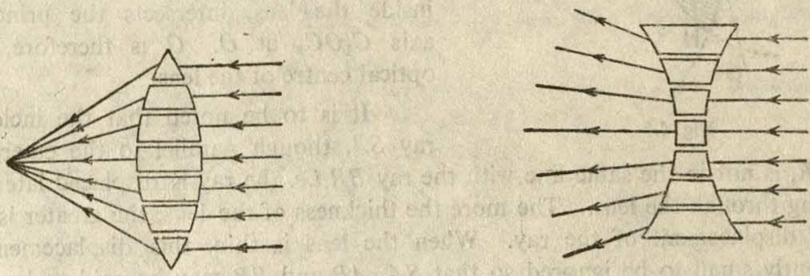
(a) (b) (c)
Fig. 4.3

(5) **Plano concave** : One surface of this lens is concave and the other surface is plane [Fig. 4.3(*b*)].

(6) **Convexo-concave** : It is a concave lens having one surface concave and the other convex. [Fig. 4.3(*c*)].

4.4. Why is a convex lens called a converging and a concave lens a diverging lens ?

A convex lens may be regarded as being made up of a very large number of portions of triangular prisms. If the spherical surfaces of the various truncated prisms are imagined to be produced, the angles of the prism can be seen to increase from zero at the middle to a small value at the edge of the lens [Fig. 4.4]. Now the deviation δ of a ray of light by a small-angle prism is given by $\delta=(\mu-1)\ A$ where A is the angle of the prism. [see art 3.20]. Consequently the truncated prisms corresponding to a position further away from the middle of the lens deviates an incident ray more than those prisms nearer the middle. Thus, a beam of light parallel to the principal axis of a convex lens will, therefore, converge to a point on the axis [Fig. 4.4(a)]. For this reason, a convex lens is called a converging lens.



(a) Fig. 4.4 (b)

Similarly, if a concave lens be regarded as being made up of a large number of truncated prisms, the angles of the prisms increase gradually towards the centre [Fig. 4.4(b)]. A beam of light parallel to the axis will be so deviated towards the bases of the truncated prisms that they will all appear to diverge from a point on the axis. For this reasons, a concave lens is called a diverging lens.

4.5. Some important terms in connection with a lens :

(i) **Centre of curvature :** If the surfaces of a lens are spherical, then each surface is a part of a sphere, the centre of which is called the centre of curvature of that surface. For example, both the surfaces of the lens LN [Fig. 4.5) are spherical. C_1 is the centre of the sphere (shown by dotted lines) of which LMN is a part. Hence, C_1 is the centre of curvature of the surface LMN. Similarly, C_2 is the centre of curvature of the other surface LPN.

If one of the surfaces is plane, its centre of curvature is situated at infinity.

(ii) **Radius of curvature :** The radius of the sphere of which the surface of the lens is a part, is called the radius of curvature of the surface. Thus, C_1M is the radius of curvature of the surface LMN and C_2P that of the surface LPN (Fig. 4.5).

(iii) **Principal axix :** The principal axis of a lens is the line joining the centres of curvature of its surfaces. In fig. 4.5, C_1OC_2 is the principal axis of the lens LN.

If one of the surfaces of the lens is plane, then a perpendicular drawn from the centre of curvature of the spherical surface to the plane surface is called the principal axis of the lens.

(iv) **Optical centre** : Optical centre of a lens is a point on the principal axis, fixed with respect to the lens, so that all rays, whose paths within the lens pass through this point, must have their corresponding incident and emergent rays parallel to each other.

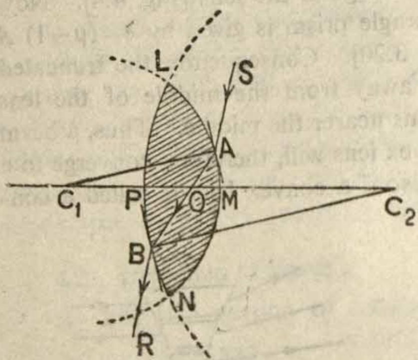


Fig. 4.5

In fig. 4.5, a ray of light SA is inci- at A on the surface LMN and emerges from the point B of the surface LPN in a direction BR parallel to the incident direction SA. The path of the ray AB inside the lens, intersects the principal axis C_1OC_2 at O. O is therefore, the optical centre of the lens.

It is to be noted that the incident ray SA, though parallel to the emergent ray BR, is not in the same line with the ray BR *i.e.* the ray is displaced laterally in going through the lens. The more the thickness of the lens, the greater is the lateral displacement of the ray. When the lens is thin, this displacement is sufficiently small to be ignored so that SA, AB and BR may be said to be one straight line. For this reason, the optical centre of a thin lens may be defined in the following way :

Optical centre of a thin lens is a point on its axis so that all rays passing through it are undeviated.

[**N.B.** If the radii of curvature of the surfaces of a lens are equal, the optical centre will be equidistant from the surfaces of the lens. If the radii of curvature are, however, not equal or one of the surfaces of the lens is plane, the optical centre will not be equidistant from the surfaces].

Optical centre is a fixed point : Depending upon the shape of the lens, the optical centre is a fixed point for the given lens. It may be proved in the following way.

In fig. 4.5, two tangent planes have been drawn at A ad B to the surfaces of the lens. The planes will be parallel for, we know, an incident ray and its corresponding emergent ray become parallel to each other, if they are refracted by a parallel-sided block. Since, the incident ray SA and the emergent ray BR are parallel, the portion of the lens there may be regarded as a parallel-sided block. So the tangent planes at A and B are parallel. Draw the straight lines C_1A and C_2B ; C_1A is the radius of curvature of the surface LMN and is perpendicular to the tangent plane drawn at A. Similarly C_2B is the radius of curvature of the surface LPN and is perpendicular to the tangent plane drawn at B. Hence, C_1A is parallel to C_2B.

For this reason, $\Delta^s C_1AO$ and C_2BO are similar.

$$\therefore \quad \frac{OC_1}{OC_2}=\frac{C_1A}{C_2B}=\frac{C_1M}{C_2P} \quad [C_1A=C_1M \text{ being radii of the same sphere}]$$

$$\text{or} \quad \frac{C_1M-OC_1}{C_2P-OC_2}=\frac{C_1M}{C_2P} \quad [C_2B=C_2P \text{ being radii of the same sphere}]$$

$$\text{or} \quad \frac{OM}{OP}=\frac{C_1M}{C_2P}$$

If the radius of curvature of the surface LMN be r_1 and that of the surface LPN be r_2 then $C_1M=r_1$ and $C_2P=r_2$; in that case, $\frac{OM}{OP}=\frac{r_1}{r_2}$

So, the optical centre O divides the straight line PM into two parts which are in the same ratio as the radii of curvature of the surfaces of the lens. Since the radii of curvature of a given lens are fixed, the position of the optical centre is also fixed.

It is to be borne in mind that the optical centre of a lens may be inside or outside the material of the lens. In the case of a convexo-concave lens, for example, the optical centre O lies outside the material [Fig. 4.6]. Whatever, may be the position, its distance from any surface of the lens is proportional to the radius of the curvature of the surface *i.e.* $\frac{OM}{OP}=\frac{r_1}{r_2}$

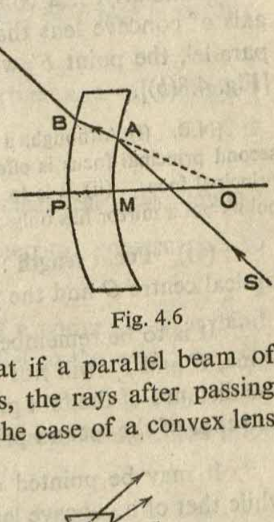
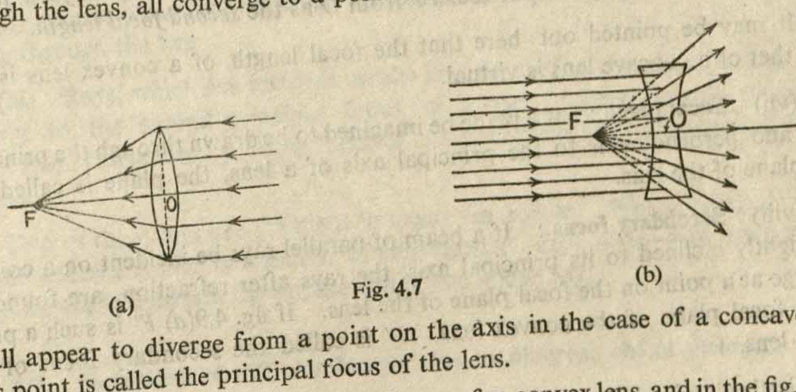


Fig. 4.6

If, however, $r_1=r_2$ then $OM=OP$ *i.e.* for a lens of equal radius of curvature the optical centre is equidistant from both the surfaces.

(v) **Principal focus :** We have already seen that if a parallel beam of light, parallel to the principal axis, is incident on a lens, the rays after passing through the lens, all converge to a point on the axis in the case of a convex lens

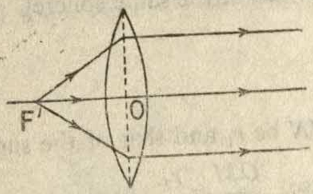


(a) Fig. 4.7 (b)

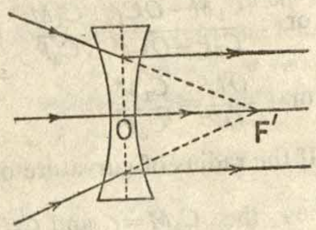
or all appear to diverge from a point on the axis in the case of a concave lens. This point is called the principal focus of the lens.

In fig. 4.7(a), F is the principal focus of a convex lens and in the fig. 4.7(b), F is the principal focus of a concave lens.

The principal focus mentioned above is called the *second principal* focus. There is yet another principal focus called the *first principal focus* which is explained below.



(a)



(b)

Fig. 4.8

Consider a point F' on the axis of a convex lens such that a diverging beam of rays from the point F' after being refracted by the lens, is rendered parallel to the axis [Fig. 4.8(*a*)]. The point F' is called the first principal focus of the convex lens.

Similarly, if a converging beam of rays be so directed to a point F' on the axis of concave lens that after refraction through the lens, the beam is rendered parallel, the point F' will be called the first principal focus of the concave lens. [Fig. 4.8(*b*)].

[N.B. (i) Although, a lens has two principal focii, yet, in the formation of images, the second principal focus is effective. For this reason, simple 'focus' of a lens means the second principal focus. (ii) It is to be noted that as a lens has two refracting surfaces it has two focal points but a mirror has only one focal point because it has only one reflecting surface.]

(vi) **Focal length :** The focal length of a lens is the distance between the optical centre O and the principal focus F or F'.

It is to be remembered that if the media on both sides of the lens are not the same, F and F' will not be equidistant from the optical centre O. In that case, the distance of the first principal focus F' from O is known as the *first focal length* and that of the second principal focus F from O as the *second focal length.*

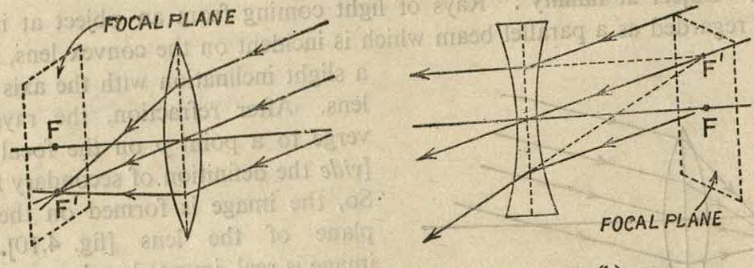
It may be pointed out here that the focal length of a convex lens is real while that of a concave lens is virtual.

(vii) **Focal plane :** If a plane be imagined to be drawn through the principal focus and perpendicular to the principal axis of a lens, the plane is called the focal plane of the lens.

(viii) **Secondary focus :** If a beam of parallel rays be incident on a convex lens slightly inclined to its principal axis, the rays after refraction, are found to converge at a point on the focal plane of the lens. If fig. 4.9(*a*) F' is such a point on the focal plane of the convex lens. F' is called the secondary focus of the convex lens.

Similarly, if a beam of parallel rays be incident on a concave lens slightly inclined to the axis, the rays after refraction through the lens, appear to diverge

from a point on the focal plane of the lens. In fig. 4.9(*b*) F' is such a point on the focal plane of the concave lens. F' is called the secondary focus of the concave lens.



(a) Fig. 4.9 (b)

It is to be remembered that the principal focus of a given lens (concave or convex) is a fixed point but the secondary focus is not.

(ix) **Aperture** : The diameter of the circular boundary of a lens is usually regarded as the aperture of the lens.

In this book, we shall consider lenses of small aperture and of negligible thickness.

4.6. Determination of the position of image by geometrical construction :

Three different types of rays may be used in geometrical construction to locate the image formed by a lens :

(i) Rays which pass through the principal focus of a convex lens or tend to pass through the focus of a concave lens, emerge parallel to the principal axis after refraction through the lens.

(ii) Rays, which pass through the optical centre, emerge undeviated after passing through the lens.

(iii) Rays, which are incident on the lens parallel to the principal axis, will converge to the second principal focus of a convex lens or will appear to diverge from the second principal focus of a concave lens after refraction through the lens.

Two of these rays are sufficient to locate an image. Which particular pair is to be selected is merely a matter of convenience. Figs. 4.10—4.16 are series of diagrams which illustrate the application of the above rays.

4.7. Formation of different images due to different object distances :

The position, nature and size of an image change with the change of position of the object. We shall now discuss the types of images formed as the object is

moved progressively along the principal axis from a very far away point to a point between the lens and the principal focus.

(A) **Convex lens :**

(i) **Object at infinity :** Rays of light coming from an object at infinity, may be regarded as a parallel beam which is incident on the convex lens, having a slight inclination with the axis of the lens. After refraction, the rays converge to a point *p* on the focal plane [*vide* the definition of secondary focus]. So, the image is formed on the focal plane of the lens [fig. 4.10]. The image is real, inverted and very small in size. This property of a convex lens is utilised in the objective of an astronomical telescope.

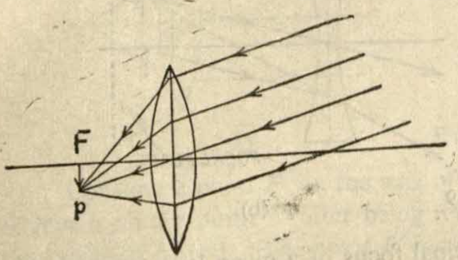


Fig. 4.10

(ii) **Object beyond 2f :** *PQ* is the object. Rays of light *PL* and *PO*, starting from the point *P* of the object, after refraction actually converge at *p*. A perpendicular *pq* drawn from *p* on the axis will give the position of the image [Fig. 4.11]. From the figure, it is clear that the image is formed between *f* and 2*f*. It is real, inverted and smaller than the object. This property of a convex lens is used in a camera.

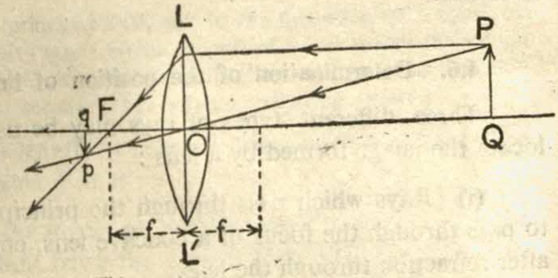


Fig. 4.11

(iii) **Object at 2f :** Following usual method of ray drawing it is seen that the image is also formed at distance 2*f* from the lens [Fig. 4.12]. The image is real, inverted but of same size as the object. In a terrestrial telescope, a convex lens is used in this way to make an inverted image erect.

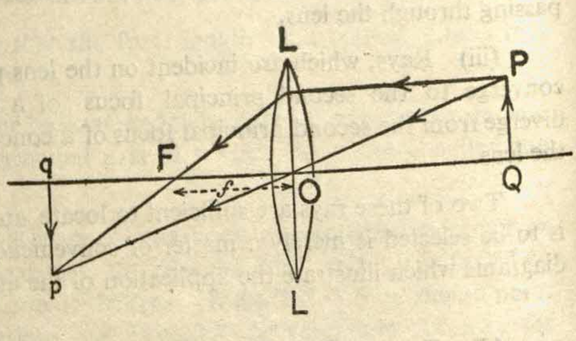


Fig. 4.12

(iv) **Object between f and 2f :** *PQ* is the object placed somewhere between *f* and 2*f* from the lens. If the image is located by geometrical construction it will be found that the image is formed beyond 2*f* of the lens [Fig. 4.13]. The image is real, inverted and larger in size than the object. A convex lens is used in this way in magic lanterns, in the objective of a microscope etc.

(v) **Object at the focus :** In fig. 4.14, an object PQ is shown placed at the focus of a convex lens. Rays of light, coming from the object, after refraction

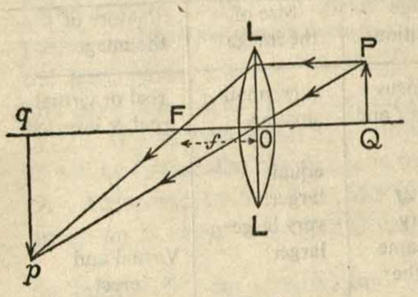


Fig. 4.13

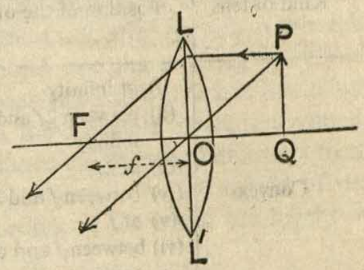


Fig. 4.14

through the lens, will be rendered parallel. The refracted rays will form image at infinity. The image will be magnified enormously. A convex lens is used in this way in instruments like spectrometer to produce a parallel beam of rays.

(vi) **Object between the lens and f :** An object PQ is situated somewhere between the lens and the focus F, of a convex lens. Rays of light from P after refraction, form a diverging beam and do not actually meet at any point. On the other hand, the refracted rays, when produced backward, appear to diverge from p. It is, therefore, the virtual image of P. A perpendicular

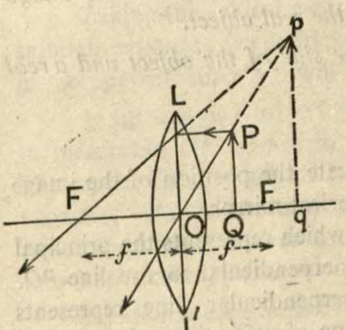


Fig. 4.15

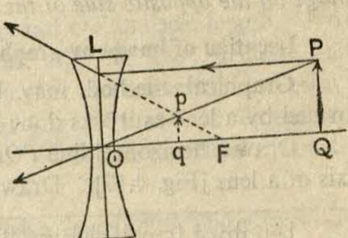


Fig. 4.16

pq on the axis will give the full image. From the fig. 4.15 it is seen that the image is formed on the same side of the lens as the object. It is virtual, erect and larger in size than the object. This property of a convex lens is utilised in the construction of a magnifying glass, the eye-pieces of telescopes and microscopes.

(B) **Concave lens :** Wherever the object may be situated the image formed by a concave lens is always virtual, erect and smaller than the object and is situated between the focus and the lens. Fig 4.16 shows the formation of an image by a concave lens.

To keep the above results in memory, they may be tabulated in the following way :

Kind of lens	Position of the object	Image position	Size of the image	Nature of the image
Convex	(i) at infinity	at the focus	very small	real or virtual
	(ii) between $2f$ and infinity	between f and $2f$	smaller	real & inverted
	(iii) at $2f$	at $2f$	equal	,,
	(iv) between f and $2f$	beyond $2f$	larger	,,
	(v) at f	at infinity	very large	,,
	(vi) between f and optical centre	on the same side as the object	larger	Virtual and erect
Concave	(i) at infinity	at the focus	Very small	Virtual and erect
	(ii) Anywhere in front of the lens	Always between the focus and optical centre	Smaller	,,

Remember the following :

(*i*) *A concave lens always produces a virtual image of a real object and the image is smaller in size than the real object.*

(*ii*) *A convex lens produces both the real and virtual images. The image may be greater than, equal to or less than the size of the real object.*

(*iii*) *A virtual image is always produced on the side of the object and a real image on the opposite side of the object.*

Location of image by graphical method :

Graphical method may be employed to locate the position of the image formed by a lens as it was done in the case of a spherical mirror.

Draw a horizontal line *POP* on a graph paper which represents the principal axis of a lens [Fig. 4.17]. Draw another line *LOL'* perpendicular to the line *PO*.

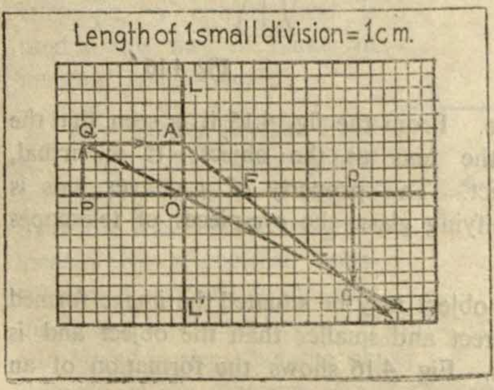


Fig. 4.17

This perpendicular line represents the plane of refraction of the lens through its optical centre *O*. Let the lens be convex, having a focal length of 5 cm. If the scale is so selected that 1 small division is equal to 1 cm then a point *F* taken at a distance of 5 small divisions from *O* will give the position of the focus of the lens.

Suppose the image of an object 4 cm long and situated at a distance 8 cm from the lens

is to be located. To get the position of the object, take a point P at a distance of 8 small divisions from O and draw a perpendicular line PQ which is 4 small divisions high. Two incident rays from the point Q are taken—one going parallel to the axis of the lens is incident on the lens at A and the other passes straight through the optical centre O. The first ray, after refraction through the lens, passes through the focus F and the second ray passes straight without any deviation. The two emergent rays intersect each other at q which is the image of Q. If a perpendicular qp is drawn on the axis, the complete image is obtained. It will be found that the image is formed at a distance of 13·3 small divisions from O. According to the scale, the image distance=13·3 cm. Again, height of the image pq is about 6·6 small divisions. So, according to the scale, the height of the image=6·6 cm (nearly) and magnification$=\frac{6 \cdot 6}{4}=1 \cdot 65$ (nearly).

The same results will be obtained if calculations are made with the help of the lens formula. It goes without saying that the above graphical method is also applicable in the case of a concave lens.

4.8. Convention of sign :

While drawing the images of an object placed at different positions we have seen in the preceding article that the image is sometimes formed on one side of the lens and sometimes on the other. To specify different image distances and object distances, suitable sign convention need be adopted. The usual convention is as follows.

Taking the optical centre of the lens as the origin, distances measured in the same direction as the incident light will be counted *negative* and those measured in the opposite direction will be counted *positive*.

In fig. 4.6(a), F is the focus of the convex lens and OF its focal length. Now, in measuring OF we are to proceed along the direction of the incident light. Hence, the focal length of a convex lens, according to the convention of sign, is negative. But in the case of a concave lens, in going from O to F, we are to proceed against the direction of the incident light. The focal length of a concave lens is, therefore, positive.

In 1934, the Physical Society of London recommended a new convention of sign. It is as follows :

(i) All real distances are reckoned as positive ;

(ii) All virtual distances are reckoned as negative.

According to this new convention, the focal length of a convex lens is positive while that of a concave lens is negative. In this book, however, we shall follow the old convention.

4.9. General formula for lenses :

When a lens forms an image of an object the distances of the object and the image measured from the optical centre of the lens are called the object distance and the image distance respectively. They are generally denoted by 'u' and 'v' while the focal length of the lens is denoted by 'f'. All these

quantities are inter-related and the relation is known as the general lens formula. In the following way, we can establish the general lens formula.

(i) **Convex lens producing a real image :**

LOL' is a thin convex lens of small aperture and PQ is an object standing perpendicular to the axis in front of the lens [Fig. 4.18(*a*)]. The image pq is located by geometrical construction according to the art. 4.6. The image is real and inverted. Now, $\Delta^s pqF$ and RFO are similar. So,

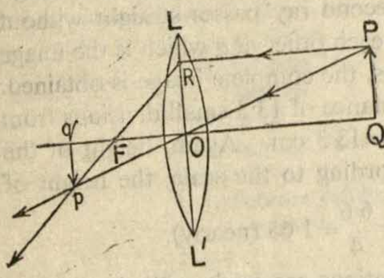


Fig. 4.18(a)

$$\frac{pq}{Fq} = \frac{RO}{OF} = \frac{PQ}{OF} \quad [\because \; PQ = RO]$$

$$\therefore \; \frac{pq}{PQ} = \frac{Fq}{OF} \quad \ldots \quad (i)$$

Again, $\Delta^s qpO$ and QPO are also similar, So,

$$\frac{pq}{Oq} = \frac{PQ}{OQ} \quad \therefore \quad \frac{pq}{PQ} = \frac{Oq}{OQ} \quad \ldots \quad (ii)$$

Comparing eqns, (i) and (ii), we have.

$$\frac{Fq}{OF} = \frac{Oq}{OQ} \quad \text{or,} \quad \frac{Oq - OF}{OF} = \frac{Oq}{OQ} \quad \ldots \quad (iii)$$

According to fig. 4.16(*a*), object distance $\rightarrow OQ = +u$
image distance $\rightarrow Oq = -v$
focal length $\rightarrow OF = -f$

Substituting these values in eqns. (iii) we get,

$$\frac{-v-(-f)}{-f} = \frac{-v}{u}. \quad \text{or,} \quad \frac{f-v}{-f} = \frac{-v}{u} \quad \text{or,} \quad uf - uv = vf$$

Dividing both sides by uvf, we get,

$$\frac{1}{v} - \frac{1}{f} = \frac{1}{u} \quad \text{or,} \quad \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

This is the general formula of a lens.

(ii) **Concave lens producing a virtual image :**

Consider an object PQ standing in front of a thin concave lens LOL' of small aperture. The image pq is located by geometrical construction according to the art 4.6. The image is virtual and erect [Fig. 4.18(*b*)]. Now, Δ^s pqF and RFO are similar.

$$\text{So,} \quad \frac{pq}{qF} = \frac{RO}{OF} = \frac{PQ}{OF} \quad [\because \; PQ = RO]$$

$$\therefore \; \frac{pq}{PQ} = \frac{qF}{OF} \quad \ldots \quad (i)$$

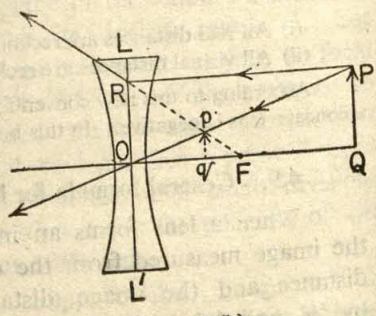


Fig. 4.18(b)

Again from similar $\Delta^s qpO$ and QPO,

we get, $\frac{pq}{Oq}=\frac{PQ}{OQ}$

$\therefore \frac{pq}{PQ}=\frac{Oq}{OQ}$ .. (ii)

Comparing eqns. (i) and (ii), we have,

$\frac{qF}{OF}=\frac{Oq}{OQ}$ or, $\frac{OF-Oq}{OF}=\frac{Oq}{OQ}$ .. (iii)

According to the fig. 4.18(b),

object distance$\rightarrow OQ=+u$

image distance$\rightarrow Oq=+v$

focal length$\rightarrow OF=+f$

Substituting these values in eqn. (iii), we get, $\frac{f-v}{f}=\frac{v}{u}$ or, $uf-uv=vf$.

Dividing both sides by uvf, we get, $\frac{1}{v}-\frac{1}{f}=\frac{1}{u}$ or, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

4.10. Linear magnification :

The ratio of the linear size of the image to that of the object is called the linear magnification.

Hence, linear magnification $(m)=\frac{\text{Linear size of the image}}{\text{,, ,, ,, ,, object}}$

From fig. 4.18(a), $m=\frac{pq}{PQ}=\frac{Oq}{OQ}=\frac{v}{u}$

Similarly, from fig. 4.18(b), $m=\frac{pq}{PQ}=\frac{Oq}{OQ}=\frac{v}{u}$

So, for any lens, the linear magnification $m=\frac{v}{u}=\frac{\text{image distance}}{\text{object distance}}$

According to the convention of sign, in the case of a convex lens [fig. 4.18(a)] u is $+ve$ but v is $-ve$, and the image is inverted. Again, in the case of a concave lens [Fig. 4.18(b)], both u and v are $+ve$ and the image is erect. So, we can say that *negative magnification means inverted image and the positive magnification erect image.*

Relation between m, u and v :

We have, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$; multiplying both sides by v, we get, $1-\frac{v}{u}=\frac{v}{f}$

$\therefore 1-m=\frac{v}{f}$ or $m=1-\frac{v}{f}=\frac{f-v}{f}$

Again multiplying both sides by u, we get, $m=\frac{f}{u+f}$.

Example 1 : *An object is placed at a distance of (a) 50 cm. and (b) 15 cm. from a convex lens. If the focal length of the lens is 20 cm., what will be the positions of the images ? If the object is 2 cm. long, what will be the sizes of the images ? What is the displacement of the image ?*

Ans. (*a*) Here $u=+50$ cm. ; $f=-20$ cm. (convex)

We know, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$; here $\frac{1}{v}-\frac{1}{50}=-\frac{1}{20}$

$$\therefore \quad \frac{1}{v}=-\frac{1}{20}+\frac{1}{50}=-\frac{3}{100} ; \therefore \quad v=-\frac{100}{3}=-33{\cdot}3 \text{ cm.}$$

i.e. the image is formed at a distance 33·3 cm. from the lens on the other side of the object. Here, magnification $m=\frac{v}{u}=\frac{100/3}{50}=\frac{2}{3}$

So, the size of the image=size of the object × magnification $=2\times\frac{2}{3}=1{\cdot}33$ cm.

(*b*) Here, $u=+15$ cm. ; $f=-20$ cm.

From, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ we have, $\frac{1}{v}-\frac{1}{15}=-\frac{1}{20}$ or, $\frac{1}{v}=\frac{1}{15}-\frac{1}{20}=\frac{1}{60}$

$$\therefore \quad v=+60 \text{ cm.}$$

i.e. the image is formed on the same side as the object and at a distance of 60 cm. from the lens. In this case, $m=\frac{v}{u}=\frac{60}{15}=4$

$\therefore$ the size of the image=size of the object × magnification $=4\times2=8$ cm.

The displacement of the image=33+60=93 cm.

Example 2 : *When a point source ts placed 30 cm. away from a lens, an image is formed on the other side of the lens and 10 cm. from it. What kind of lens is it ? What is its focal length ?*

Ans. Since the image is formed on the other side of the lens, the image is real and the lens is convex ; for real image is formed only by a convex lens.

Here $u=30$ cm. ; $v=-10$ cm. (real image) ; $f=$?

We know, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad \therefore \quad -\frac{1}{10}-\frac{1}{30}=\frac{1}{f}$ or, $-\frac{4}{30}=\frac{1}{f}$

$$\therefore \quad f=-\frac{30}{4}=-7{\cdot}5 \text{ cm.}$$

Example 3 : *An object 5 cm. high is placedp erpendicularly in front of a convex lens. An image 25 cm. high is formed on a screen 100 cm. away from the lens. Calculate the focal length of the lens.*

Ans. Here, magnification, $m=\frac{25}{5}=5$; but $m=\frac{v}{u}=5$

or, $v=5u$. But $v=100$ cm. (given) $\therefore$ $u=20$ cm.

Since the image is real (as it is formed on a screen), the image distance is $-ve$.

So, in the present case, $v=-100$ cm. ; $u=20$ cm. $f=$?

From the lens equation, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, we have $-\frac{1}{100}-\frac{1}{20}=\frac{1}{f}$

$$\text{or,} \quad -\frac{6}{100}=\frac{1}{f} \quad \therefore \; f=-\frac{100}{6}=-16{\cdot}6 \text{ cm.}$$

Example 4 : *An object is placed 30 cm. in front of a convex lens of focal length 10 cm. Where will be the image formed ? State the nature of the image. How many times is the image magnified or diminished ?*

Ans. Here $u=+30$ cm. ; $f=-10$ cm. (convex) ; $v=$?

We know, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ or, $\frac{1}{v}-\frac{1}{30}=-\frac{1}{10}$ or, $\frac{1}{v}=\frac{1}{30}-\frac{1}{10}=-\frac{2}{30}$

$$\therefore \; v=-15 \text{ cm.}$$

i.e. the image is formed on the other side of the lens at a distance of 15 cm. The image is real and inverted. Further, magnification, $m=\frac{v}{u}=\frac{15}{30}=\frac{1}{2}$ *i.e.* the image is half the size of the object.

Example 5 : *It is found that when an object is placed 50 cm. in front of a lens, the image is formed 200 cm. on the other side of it. Find the displacement of the image if the object is moved 10 cm. away from the lens.*

Ans. Since the image is formed on the other side of the lens, the lens is convex. Now, $v=-200$ cm. ; $u=+50$ cm. From lens equation, we get

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad \text{or,} \quad -\frac{1}{200}-\frac{1}{50}=\frac{1}{f} \quad \text{or,} \quad -\frac{5}{200}=\frac{1}{f} \quad \therefore \; f=-40 \text{ cm.}$$

That the lens is convex is confirmed by the negative focal length. In the second case, $u=+60$ cm ; $f=-40$ cm. ; $v=$?

From lens equation $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ we have, $\frac{1}{v}-\frac{1}{60}=-\frac{1}{40}$

$$\text{or,} \quad \frac{1}{v}=\frac{1}{60}-\frac{1}{40}=-\frac{1}{120} \quad \therefore \; v=-120 \text{ cm.}$$

The image is now formed on the other side of the lens at a distance of 120 cm. So, the displacement of the image$=(200-120)=80$ cm. towards the lens.

Example 6 : *A point object is placed at a distance of 12 cm. on the axis of a convex lens of focal length 10 cm. On the other side of the lens, a convex mirror is placed at a distance 10 cm. from the lens such that the image formed by the combination coincides with the object itself. What is the focal length of the convex mirror ?* [*I.I.T. 1976*]

Ans. The point object P is placed 12 cm. away from the optical centre O of the convex lens L. MO_1 is a convex mirror at a distance 10 cm. from the lens. Now, if the rays from P, falling on the lens, are so refracted that they fall on the mirror normally, then the rays will retrace their path and the image will be formed

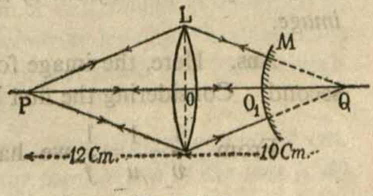


Fig. 4.19

at P. [Fig. 4.19]. If the rays falling on the convex mirror are directed towards the centre of curvature Q of the mirror, then they will be incident on the mirror normally.

Here P and Q will act as conjugate focii for the convex lens. *i.e.* $u=+12$ cm. ; $f=-10$ cm. ; $v=$?

From conjugate relationship, we get $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ or, $\frac{1}{v}-\frac{1}{12}=-\frac{1}{10}$

$$\therefore \quad \frac{1}{v}=\frac{1}{12}-\frac{1}{10}=-\frac{1}{60} \quad \text{or} \quad v=-60 \text{ cm}$$

[Negative sign shows that the point Q lies on the right of the lens.]

So, $OQ=60$ cm. $\therefore$ $O_1Q=60-10=50$ cm.

Hence the radius of curvature of the mirror $O_1Q=50$ cm and its focal length$=\frac{50}{2}=25$ cm.

Example 7 : *Find the length of the image of a straight filament 3 cm. long placed along the principal axis of a thin convex lens of 12 cm. focal length with its near end 21 cm. from the lens.*

Ans. Considering the near end of the filament first, we get $u=21$ cm. $f=-12$ cm.

From $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$; we get $\frac{1}{v}-\frac{1}{21}=-\frac{1}{12}$ or, $\frac{1}{v}=\frac{1}{21}-\frac{1}{12}$ or, $\frac{1}{v}=-\frac{1}{28}$

$$\therefore \quad v=-28 \text{ cm.}$$

So, the image of the near end of the filament is formed at a distance of 28 cm. from the lens on the other side.

Considering now the furthest end of the filament, $u=21+3=24$ cm. ; $f=-12$ cm. Again from $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ we have, $\frac{1}{v}-\frac{1}{24}=-\frac{1}{12}$

$$\text{or,} \quad \frac{1}{v}=\frac{1}{24}-\frac{1}{12}=-\frac{1}{24} \quad \therefore \quad v=-24 \text{ cm.}$$

So, the image of the furthest end of the filament is formed at a distance of 24 cm. from the lens on the other side.

$\therefore$ The length of the image$=28-24=4$ cm.

Example 8 : *Two convex lenses of focal length 20 cm. are situated 10 cm. apart and have a common axis. An object 5 cm. in height is placed on the axis at a distance of 15 cm. in front of the first lens. Find the size and position of the final image.*

Ans. Here, the image formed by the first lens will act as the object for the second. Considering the first lens, $u=15$ cm. ; $f=-20$ cm. ; $v=$?

From $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ we have, $\frac{1}{v}-\frac{1}{15}=-\frac{1}{20}$ or, $\frac{1}{v}=\frac{1}{15}-\frac{1}{20}=\frac{1}{60}$

$$\therefore \quad v=+60 \text{ cm.}$$

The image is therefore, formed on the same side as the object (since the image distance is $+ve$) and at a distance 60 cm. from the lens.

The distance apart between the lenses being 10 cm., the distance of this image from the second lens$=60+10=70$ cm. This is the object distance for the second lens.

Now from $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ we get, $\frac{1}{v}-\frac{1}{70}=-\frac{1}{20}$ or, $\frac{1}{v}=\frac{1}{70}-\frac{1}{20}=-\frac{5}{140}=-\frac{1}{28}$

$v=-28$ cm. *i.e.*, the final image is formed on the right of the second lens (*i.e.* on the opposite side of the object) and at a distance 28 cm. from the second lens.

Now, magnification by the 1st. lens$=\frac{v}{u}=\frac{60}{15}=4$

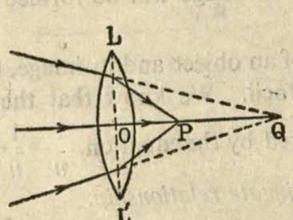
and „ „ „ 2nd „ $=\frac{v}{u}=\frac{28}{70}=\frac{2}{5}$

$\therefore$ total magnification$=4\times\frac{2}{5}=\frac{8}{5}$

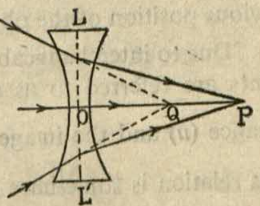
The size of the final image=size of the object$\times$total magnification $=5\times\frac{8}{5}=8$ cm.

4.11. Virtual object and its image :

It is not a fact that objects are always real. Sometimes rays of light are incident on a lens in such a way that a real object is not available but the rays produce a virtual object. In fig. 4.20(i) & (ii), a beam of converging rays tend to converge on the point Q but before they converge, they are intercepted by the lens. In this case, Q acts as a virtual object. In the case of a convex lens, which is a *converging lens*, the rays after passing through the lens, are more converged and form a real image at P which is nearer to the lens than Q.



(i)



(ii)

Fig. 4.20

In the case of a concave lens, which is a *diverging lens*, the rays after passing through the lens, become a little diverging and form a real image at P which is further away from the lens than Q. Note that the concave lens here is producing a real image. But if the object distance OQ be greater than the focal length of the concave lens, the image again becomes virtual.

Example 1 : *A beam of converging rays, trying to converge at a point 10 cm. behind a convex lens, is intercepted by the lens. If the focal length of the lens is 40 cm, where will the rays converge ?*

Ans. See fig. 4.20(i). Here, the virtual object distance $(u)=OQ=10$ cm. (being virtual, $-ve$ sign should be put before it). ; $f=-40$ cm (convex)

we know, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$; Here $\frac{1}{v}-\left(-\frac{1}{10}\right)=-\frac{1}{40}$ or, $\frac{1}{v}+\frac{1}{10}=-\frac{1}{40}$

$$\therefore \quad v=-8 \text{ cm}.$$

i.e. the rays will converge at a point (P) 8 cm away from the lens. The negative sign shows that the image is real.

Example 2 : *A converging beam of light is incident on a diverging lens of focal length 20 cm. If the beam, in absence of the lens, converges to a point 5 cm. behind the lens, find the position of the image formed by the lens.*

Ans. The point where the beam tends to converge [the point Q in fig. 4.20(ii)] acts as a virtual object point. So, here $OQ=u=-5$ cm ; $f=+20$ cm (concave).

Now, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ or $\frac{1}{v}-\left(-\frac{1}{5}\right)=\frac{1}{20}$ or $\frac{1}{v}=\frac{1}{20}-\frac{1}{5}=-\frac{3}{20}$

$$\therefore \quad v=-6{\cdot}6 \text{ cm}.$$

i.e. the rays will actually converge at a point (P) 6·6 cm away ($OP=6{\cdot}6$ cm) from the lens and a real image is formed there.

4.12. Conjugate pair of focii :

Since the path of a ray of light is reversible, the position of an object point lying on the principal axis of a lens and the position of its image formed by the lens are interchangeable. In other words, when a lens forms a real image of an object, the image will be formed at the position of the object if the object is shifted to the first position of the image. But such is not the case with a virtual image. In the case of a virtual image, however, the incident rays are to be so directed that they tend to converge at the position of the virtual image in absence of the lens. Then, after refraction, the image will be formed at the previous position of the object.

Due to interchangeability of the positions of an object and its image, the two points are referred to as a pair of conjugate focii. We know that the object distance (u) and the image distance (v) are related by the equation, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$.

This relation is sometimes referred to as the *conjugate relationship.*

Newton's relation : Newton showed that conjugate points obey the relation $xx'=f^2$, where x and x' are their distances from respective focii on the same side of the lens. The relation can be proved by taking the case of the converging lens in fig. 4.21, where $OP=u=x+f$ and $OQ=v=x'+f$. Substituting in lens equation for real image,

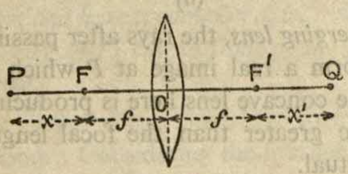


Fig. 4.21

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

we get $\frac{1}{x'+f}+\frac{1}{x+f}=\frac{1}{f}$ $\therefore$ $f(x+x'+2f)=(x'+f)(x+f)$

or, $f(x+x'+2f)=(xx'+x'f+fx+f^2)$

or, $xx'=f^2$

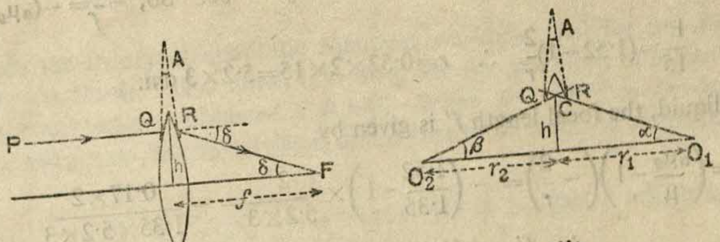
Since $x'=f^2/x$, it follows that x' increases as x decreases. Thus, the image Q recedes from the focus F' and hence away from the lens when the object P approaches the lens. Further, from fig. 4.21 it can be shown that the transverse magnification m is given by $m=\frac{x'}{f}=\frac{f}{x}$.

The equation $xx'=f^2$ is known as *Newton's equation* of conjugate points.

4.13. Focal length of a lens in different media :

If the radii of curvature of the two surfaces of a glass lens be r_1 and r_2 and the refractive index of glass with respect to air be ${}_a\mu_g$, then it can be shown that the focal length of the lens is given by : $\frac{1}{f}=({}_a\mu_g-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$; for a double convex lens $\frac{1}{f}=({}_a\mu_g-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$

The above relation for f can be found by using the deviation formula due to a small angle prism. Consider a ray PQ parallel to the principal axis of the



(i) (ii)

Fig. 4.22

lens at a height h above it [Fig. 4.22]. This ray, after emergence, passes through the focus F and undergoes a small deviation δ given by $\delta=h/f$.. .. .. (i)

This is the deviation through a prism of small angle A formed by the tangents at Q and R to the surfaces of the lens as shown in the diagram. Now, for a small-angle prism, we know $\delta=({}_a\mu_g-1)\,A$, where ${}_a\mu_g$ is the *r.i.* of the material of the prism. (See art 3.20).

From (i), $\frac{h}{f}=({}_a\mu_g-1)A$ or $\frac{1}{f}=({}_a\mu_g-1)\frac{A}{h}$ .. .. (ii)

Now, if O_1 and O_2 be the centres of curvature of the two surfaces of the lens then, O_1Q and O_2R are normals at Q and R respectively (Fig 4.22(ii)]. From simple geometry $\angle RCO_1=\alpha+\beta=\angle A$, where α and β are the angles with the principal axis at O_1 and O_2 respectively. But $\alpha=h/r_1$ and $\beta=h/r_2$ [Fig. 4.22(ii)]

$$\therefore \quad A=\frac{h}{r_1}+\frac{h}{r_2} \quad \text{or} \quad \frac{A}{h}=\frac{1}{r_1}+\frac{1}{r_2}$$

Substituting in eqn. (ii), we get $\frac{1}{f}=({}_a\mu_g-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$

If, however, the lens is placed in a medium other than air, having a refractive index μ with respect to air, then, the focal length f of the lens is given by :

$$\frac{1}{f}=\left(\frac{{}_a\mu_g-\mu}{\mu}\right)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)=\left(\frac{{}_a\mu_g}{\mu}-1\right)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

This shows that the focal length of a lens increases if the lens is surrounded by a medium denser than air.

Example 1 : *The focal length of an equi-convex lens is 15 cm. and the refractive index of its material (i.e. glass) is 1·52. What will be its focal length when it is fully immersed in a liquid of refractive index 1·35 ?*

Ans. For a glass lens in air, $\frac{1}{f}=({}_a\mu_g-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$. As the lens is equi-convex, $r_1=-r$ (say) and $r_2=+r$, and $f=-ve$. So, $-\frac{1}{f}=-({}_a\mu_g-1)\,\frac{2}{r}$

or, $\frac{1}{15}=(1{\cdot}52-1)\frac{2}{r} \quad \therefore \quad r=0{\cdot}52\times2\times15=5{\cdot}2\times3$ cm.

In liquid, the focal length f' is given by,

$$\frac{1}{f'}=\left(\frac{{}_a\mu_g}{\mu}-1\right)\left(-\frac{2}{r}\right)=-\left(\frac{1{\cdot}52}{1{\cdot}35}-1\right)\times\frac{2}{5{\cdot}2\times3}=-\frac{0{\cdot}17\times2}{1{\cdot}35\times5{\cdot}2\times3}$$

$$\therefore \quad f'=-61{\cdot}94 \text{ cm.}$$

Example 2 : *If the r.i. of glass with respect to air be $\frac{3}{2}$ and of water with respect to air be $\frac{4}{3}$, find the ratio of the focal lengths of a glass lens kept immersed in air and in water.*

Ans. Let the focal length of the glass lens in air be f_1 and in water f_2. If the radii of curvature be r_1 and r_2 then,

$$\frac{1}{f_1}=({}_a\mu_g-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right) \text{ and } \frac{1}{f_2}=\left(\frac{{}_a\mu_g}{{}_a\mu_w}-1\right)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

$$\therefore \quad \frac{f_1}{f_2}=\frac{{}_a\mu_g-{}_a\mu_w}{{}_a\mu_w({}_a\mu_g-1)}=\frac{\frac{3}{2}-\frac{4}{3}}{\frac{4}{3}(\frac{3}{2}-1)}=\frac{1}{6}\times\frac{6}{4}=\frac{1}{4}$$

4.14. A few problems in connection with a convex lens and real image :

(*a*) *Prove that for a fixed object and a fixed screen, there are in general, two positions of a convex lens between them, for each of which, a sharp image of the object is cast on the screen.*

Let S be the screen and O the object fixed in their positions [Fig. 4.23]. Let L be one position of a convex lens which produces a sharp image on the screen S. Suppose the distance between the object and the screen$=D$; the object distance$=u$ and the image distance$=v$. As the image is real, v is $-ve$ and as the lens is convex, f is also $-ve$.

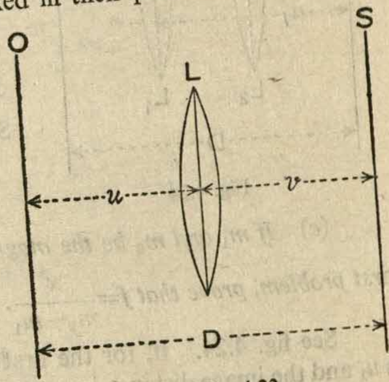


Fig. 4.23

From the equation of the lens,

$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, we have, $-\frac{1}{v}-\frac{1}{u}=-\frac{1}{f}$

or $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ or, $\frac{1}{D-u}+\frac{1}{u}=\frac{1}{f}$

or $\frac{D}{(D-u)u}=\frac{1}{f}$ or, $u^2-D.u+D.f.=0$

This is a quadratic equation. Solving the equation, we get two real roots of u which are,

$$u_1=\frac{D+\sqrt{D^2-4D.f}}{2} \text{ and } u_2=\frac{D-\sqrt{D^2-4D.f}}{2}$$

From the theory of quadratic equation, we know that for two real roots, $D^2>4.D.f.$ or $D>4f.$ This shows that there are two positions of a lens for each of which a real image is obtained if $D>4f.$ When $D^2=4D.f.$ or $D=4f$, the two real roots merge into one value which means that there is only a single position of the lens.

When $D^2<4Df$ or $D<4f$, the two roots will be *imaginary*, which means that in no position, the lens will give a real image.

This shows that to obtain a real image, the distance between the object and the screen must at least be equal to $4f$, where f is the focal length of the lens. If, at any time, it is difficult to obtain a real image on a screen when a converging lens is used, possible causes may be (i) the object is nearer to the lens than its focal length or (ii) the distance between the screen and object is less than four times the focal length of the lens.

(*b*) *Prove that if the displacement of the lens between the first and the second position in the first problem be x and the distance between the object and the screen D, then the focal length f of the lens is given by* $f=\frac{D^2-x^2}{4D}$.

Let L_1 and L_2 be the two positions of the lens [Fig. 4.24]. If u_1 and u_2 be the object distances in the two cases, we have seen earlier, that

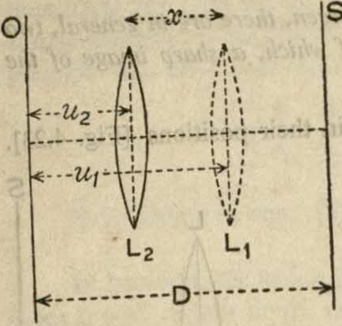


Fig. 4.24

$$u_1=\tfrac{1}{2}(D+\sqrt{D^2-4Df})$$

$$\text{and } u_2=\tfrac{1}{2}(D-\sqrt{D^2-4D.f})$$

$$\therefore\ u_1-u_2=\tfrac{1}{2}[(D+\sqrt{D^2-4Df})$$

$$-(D-\sqrt{D^2-4Df})]=\sqrt{D^2-4Df}$$

$$\text{or, } x=\sqrt{D^2-4Df} \qquad [\because\ u_1-u_2=x]$$

Squaring, $x^2=D^2-4D.f$

$$\text{or } f=\frac{D^2-x^2}{4D}.$$

(c) *If m_1 and m_2 be the magnifications in the two positions of the lens in the first problem, prove that* $f=\dfrac{x}{m_2-m_1}$.

See fig. 4.24. If, for the first position L_1 of the lens the object distance is u_1 and the image distance is v_1, then since the image is real, we can write,

$$\frac{1}{v_1}+\frac{1}{u_1}=\frac{1}{f} \text{ or, } 1+\frac{v_1}{u_1}=\frac{v_1}{f} \quad \therefore\ 1+m_1=\frac{v_1}{f}$$

Similarly, for the second position L_2 of the lens, $1+m_2=\dfrac{v_2}{f}$

$$\text{Subtracting, } m_2-m_1=\frac{v_2-v_1}{f}=\frac{x}{f} \quad [\because\ v_2-v_1=x]$$

$$\therefore\ f=\frac{x}{m_2-m_1}.$$

(d) *Prove that if the sizes of the images in the two positions of the lens in the first problem be d_1 and d_2, then the size of the object,* $d=\sqrt{d_1d_2}$.

$$\text{See fig. 4.24 Here, } m_1=\frac{d_1}{d}=\frac{v_1}{u_1}=\frac{D-u_1}{u_1}$$

$$\text{Similarly, } m_2=\frac{d_2}{d}=\frac{v_2}{u_2}=\frac{D-u_2}{u_2}$$

$$\text{Multiplying, } \frac{d_1d_2}{d^2}=\frac{(D-u_1)(D-u_2)}{u_1u_2}=\frac{D^2-D(u_1+u_2)+u_1u_2}{u_1u_2}$$

$$=\frac{D_2-D(u_1+v_1)+u_1u_2}{u_1u_2} \quad [\because\ u_2=v_1]$$

$$=\frac{D^2-D^2+u_1u_2}{u_1u_2}=1 \quad [\because\ u_1+v_1=D]$$

$$\therefore\ d_1d_2=d^2 \text{ or } d=\sqrt{d_1d_2}$$

(*e*) *Prove that the minimum distance between an object and its real image formed by a convex lens is four times the focal length of the lens.*

Let u and v be the object and image distances respectively. As the image is real it is formed on the other side of the lens than the object and hence, the distance between them $l=u+v$.

If l is minimum, $\frac{dl}{du}=0$ or $\frac{d}{du}(u+v)=0$ or $\frac{dv}{du}=-1$

Now, for a real image, we know $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$.

Differentiating, we get, $-\frac{1}{v^2}\frac{dv}{du}-\frac{1}{u^2}=0$ $\quad\therefore\quad \frac{dv}{du}=-\frac{v^2}{u^2}$

So, for l to be minimum, $-\frac{v^2}{u^2}=-1$ or $u=v$.

Putting this condition in $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, we get $\frac{2}{u}=\frac{1}{f}$ $\quad\therefore\quad u=v=2f$.

Hence, $l_{min}=(u+v)=2f+2f=4f$.

Example : *An object is placed at a distance of D from a screen. A convex lens forms an image of the object on the screen. When the lens is shifte dthrough a distance x towards the screen, another image is formed on the same screen. Prove that the ratio of the sizes of the two images is equal to* $\left(\frac{D+x}{D-x}\right)^2$.

Ans. Let u be the object distance in the first position of the lens. The corresponding image distance$=D-u$. Since the image is real, we can write,

$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ or $\frac{1}{D-u}+\frac{1}{u}=\frac{1}{f}$ or $\frac{D}{(D-u)u}=\frac{1}{f}$ or, $u^2-D.u+D.f=0$.

From this quadratic equation we get the following values of u :

$u_1=\frac{1}{2}(D-\sqrt{D^2-4D.f})$ and $u_2=\frac{1}{2}(D+\sqrt{D^2+4D.f})$ .. (i)

According to the problem,

$$x=u_2-u_1=\frac{D+\sqrt{D^2+4Df}}{2}-\frac{D-\sqrt{D^2-4Df}}{2}=\sqrt{D^2-4Df}$$

Putting this value in eqn. (i), $u_1=\frac{D-x}{2}$ and $u_2=\frac{D+x}{2}$ $\quad\therefore\quad u_1+u_2=D$

Since v_1 and v_2 are the two image distances in the two positions of the lens, we have $u_1+v_1=D=u_2+v_2$ $\quad\therefore\quad u_1+v_1=D=u_1+u_2$ or $v_1=u_2$. Similarly, $u_1=v_2$

So, $u_1=v_2=\frac{D-x}{2}$ and $u_2=v_1=\frac{D+x}{2}$

If the linear size of the object be I and that of the first image I_1 then,

$$\frac{I_1}{I}=\frac{v_1}{u_1}=\frac{D+x}{D-x}$$

If, again, I_2 be the size of the second image, $\frac{I_2}{I}=\frac{v_2}{u_2}=\frac{D-x}{D+x}$

$$\therefore\ \frac{I_1}{I_2}=\frac{I_1}{I}\div\frac{I_2}{I}=\left(\frac{D+x}{D-x}\right)\div\left(\frac{D-x}{D+x}\right)=\left(\frac{D+x}{D-x}\right)^2$$

4.15. Power of a lens :

Consider two lenses—one having a greater focal length than the other. If a beam of parallel rays falls on the lenses, the rays will be brought to focus at the focal point of the lenses. In the case of the lens of shorter focal length, the point of convergence of the rays will be nearer to the lens than the other. In such cases, the first lens is said to have a greater power than the second.

Definition : *The power of a lens means its power of convergence in the case of a convex lens, or its power of divergence in the case of a concave lens.*

The convergence or divergence, as the case may be, produced by a lens will be greater, the shorter the focal length. So, the reciprocal of the focal length $\left(\frac{1}{f}\right)$ is taken as a measure of the power of a lens.

A lens having a focal length of 100 cm, (*i.e.* 1 metre) is said to have unit power called a *Dioptre*. Opticians, however regard the *power of a convex lens to be positive and that of a concave lens to be negative.*

So, to express the power of a lens in dioptres, express the focal length in metres and get the reciprocal. Thus, a convex lens of focal length 25 cm. has a power$=+\frac{1}{\frac{25}{100}}=+4$ dioptres. Alternatively, a lens having power 2 dioptres, has a focal length$=\frac{100}{2}=50$ cm.

4.16. Simple method of identification of lenses :

A convex lens, we know, forms a virtual, magnified and erect image of an object held very close to the lens while a concave lens forms a virtual, diminished and erect image of the same object. Hence, a lens may be indentified in a simple way by holding a finger, say, very close to the lens and looking for its image from the other side of the lens. If the image is magnified, the lens is convex. If the image is diminished the lens is concave.

4.17. Determination of the focal length of a convex lens by U-V method :

By pins : Fix a convex lens L in a lens-holder and place it on a table. Put a pin P on the left of the lens and adjust its height so that the tip of the pin is in same level with the axis of the lens. Looking through the lens from the right,

an inverted image p of the pin P will be visible [Fig. 4.25]. Now place another pin Q on the right of the lens so that its tip coincides with the tip of the inverted image p and there is no parallax between them. When the parallax is avoided, the two tips will move together if the eye is slightly moved to and fro. In this case, the pin P is the object and the pin Q is the position of its image. Their distances from the lens are respectively 'u' and 'v'. Then, from the equation $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ the focal length 'f' can be found out. Since the image is real, v is to be considered negative.

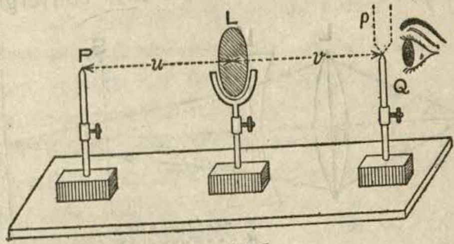


Fig. 4.25

By keeping the lens L or the pin P at various positions, the experiment should be repeated several times and the mean value of 'f' should be taken.

4.18. Determination of the focal length of a convex lens by a plane mirror :

If an object-point O be situated at the focus of a convex lens L, then the rays diverging from the object after refraction through the lens will be rendered parallel to the axis of the lens. If, now, a plane mirror M be held at the back of the lens perpendicular to the axis, the rays will be reflected back as a parallel beam, which after passing through the lens again, will converge at the focal plane of the lens and will form an image O' there. Based upon this principle, there is a simple method for determining the focal length of a convex lens.

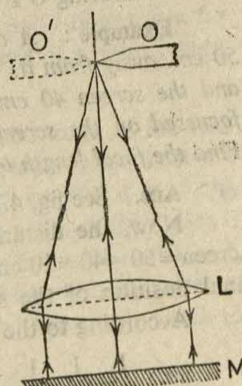


Fig. 4.26

Experiment : Put a convex lens L on a plane mirror M placed horizontally on a table. Fix a pin in a suitable holder so that the tip of the pin O coincides with the axis of the lens and is at some distance above the lens. Looking from above, the tip of the pin and its image will be visible. Adjust the height of the pin until there is no parallax between the tip O and its image O'. Measure the distance of the tip of the pin from the surface of the lens. This gives the focal length of the lens.

4.19. Determination of the focal length of a concave lens :

Mount a convex lens L on an optical bench and adjust the position of a paper-screen S such that a sharp image P' of the illuminated cross-wire

P is obtained on the screen. Note the position of the screen. Now introduce the concave lens L' under test in between the screen and the convex lens but nearer to the convex lens. Then the rays converging to P' become less

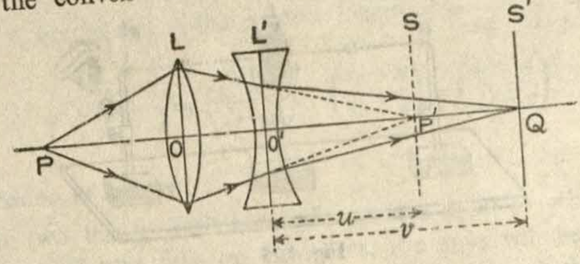


Fig. 4.27

convergent. The screen is to be moved further to S' to obtain the sharp image Q again [Fig. 4.27]. Note the position of S'. Then, for the concave lens, the first image P' behaves as a virtual object and Q is its real image.

If f be the focal length of the concave lens, then, $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$.

If the present case, both u and v are negative. Therefore, the formula becomes $\frac{1}{f}=-\frac{1}{v}-\left(-\frac{1}{u}\right)=\frac{1}{u}-\frac{1}{v}$.

Now, $u=O'P'$ and $v=O'Q$. $\therefore \frac{1}{f}=\frac{1}{O'P'}-\frac{1}{O'Q}$

Measuring $O'P'$ and $O'Q$, the focal length f can be found out.

Example : *A convex lens produces an image of a small object on a screen 50 cm. away from it. A concave lens is then placed between the convex lens and the screen 40 cm. away from the convex lens. In order to bring the image focussed on the screen again, the screen has to be removed 5 cm. further away. Find the focal length of the concave lens.*

Ans. See fig. 4.27. Here, $LS=50$ cm. ; $OO'=40$ cm. and $SS'=5$ cm.

Now, the distance between the concave lens and the first position of the screen$=50-40=10$ cm. And, the distance between the concave lens and the final position of the screen$=10+5=15$ cm.

According to the fig. 4.27, $u=-10$ cm. and $v=-15$ cm.

So, $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}=-\frac{1}{15}+\frac{1}{10}=\frac{1}{30}$ $\therefore$ $f=30$ cm.

*4.20. Spherical aberration in a lens :

The formula of lenses that we got in art 4.9 was derived on the assumptions that the aperture of the lens was very small and that the rays were incident on a narrow region around the axis of the lens *i.e.* rays were *paraxial*. But in practice, the lenses that are used are not always of small aperture and the rays used are not always confined to the paraxial region. In

such cases, the image formed by a single lens is defective. This defect is known as *spherical aberration*.

Consider a convex lens of fairly big aperture on which a beam of parallel rays, coming from a distant object situated on the axis of the lens, is incident in a direction parallel to the axis of the lens [Fig. 4.28]. Those rays which are incident near the axis—known as *paraxial rays*—after refraction are all found to converge at a point F_c which is called the paraxial focal point. But the rays falling near the margin on the outer zone of the lens, known as *marginal rays*, after refraction, are found to intersect the axis at various points. In fig. 4.28 the most marginal rays have been shown to intersect the axis at F_r which is known as the marginal focal point. The other rays falling on the lens at various other points, after refraction, intersect the axis at points lying between F_c and F_r. Why does it happen so ?

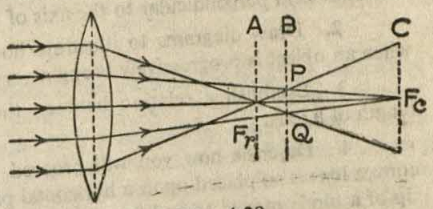


Fig. 4.28

We know that a lens may be regarded as made up of a large number of truncated prisms. The convex lens shown in fig. 4.28 may also be supposed to consist of small prisms whose refracting angles gradually increase from central part to the marginal part. We, also, know that the deviation of a ray passing through a prism increases with the increase of its refracting angle. As a result, the rays emerging from the paraxial region of the lens will be deviated less than those emerging from the marginal region. Due to this difference of deviation between the marginal and the paraxial rays, they are found to intersect the axis at different points lying between F_c and F_r. For this reason, a point object at a far away point on the axis of the lens gets an extended image instead of a point image. This defect in image is called the spherical aberration.

It is to be noted that all rays emerging from a particular zone of the lens meet the axis at one single point but rays emerging from different zones meet the axis at different points. Consequently, if a screen be held on the way of the emergent rays, perpendicular to the axis, as at A—a circular patch of light with a bright centre will be obtained. The diameter of the circular patch will be minimum when the screen is situated at B. This circular patch of minimum diameter is referred to as *the circle of least confusion* and it is taken as the closest approach to the point image of the point object.

There are various ways of removing spherical aberration, the simplest of which is a combination of convex lenses where two convex lenses are used co-axially with a separation equal to the difference of their focal lengths *i.e.* $x=f_1-f_2$ where x is the separation and f_1, f_2 are the focal lengths.

Exercises

Essay type :

1. Explain with neat diagrams how a convex lens forms a real image and a concave lens a virtual image. What properties of a lens would you use to locate the position of an image of an object held perpendicular to the axis of a lens ? Illustrate it with neatly drawn diagrams.

2. Draw diagrams to illustrate how the position, nature and size of an image change when an object is progressively brought very near to a convex lens from a far away point.

3. Establish a relation between the object distance, the image distance and the focal length of a lens. [*H. S. Exam. 1979*]

4. Describe how you will proceed to determine the focal length of a convex lens. A convex lens is so placed upon a horizontal plane mirror that the axis of the lens is vertical. The tip of a pin is moved to and fro along the axis of the lens. Where will the tip of the pin and its image formed by the lens coincide ? Give reasons for your answer.

5. Show that the minimum distance between an object and its real image formed by a convex lens is four times the focal length of the lens.

6. From the expression ot deviation caused by a small-angle prism, show that the focal length of a bi-convex lens is given by $\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$

7. There is an object on one side and a screen on the other side of a convex lens. Show that there are two positions of the convex lens for which a real image of the object is formed on the screen if the distance between the object of the screen is more than four times the focal length of the lens.

8. What is spherical aberration in a lens ? Explain its origin. What is the method for producing minimum spherical aberration ?

Short answer type :

9. What is a lens ? What is the difference between a converging and a diverging lens ?

10. Explain with diagrams why a convex lens is called a converging lens and a concave lens a diverging lens.

11. Explain the following terms : (i) centre of curvature (ii) optical centre (iii) focus (iv) first focal length (v) second focal length (vi) aperture.

12. What type of lens would you use and where the object is to be placed in order to obtain (i) a real but magnified image (ii) a virtual but magnified image (iii) a real but diminished image (iv) a virtual but diminished image (v) a real image of same size. Draw neat diagrams in each case and mention the practical applications.

13. (a) Show, with the help of diagrams how a convex lens forms image of an object when the object is (i) between principal focus and the pole (ii) at $2f$ from the lens (iii) between $2f$ and infinity. [*H. S. Exam 1981*]

(b) A concave lens of *r.i.* μ is immersed in a liquid whose *r.i.* is (i) greater than (ii) equal to (iii) less than μ. If a parallel beam is incident on the lens, draw diagrams to explain the nature of emergent beam in each case. [*I.I.T. 1973*]

14. Does the focal length of a lens depend on the medium in which the lens is immersed ? If so how ?

15. A convex lens ($\mu=1{\cdot}5$) is immersed in water ($\mu=1{\cdot}33$). Will the focal length change ? If so, how ? [*Jt. Entrance 1984*]

A concave lens made of a material of *r.i.* μ is immersed in a medium whose *r.i.* is greater than μ. Trace the path of the emergent rays when a parallel beam of light is incident on the lens. [*Jt. Entrance 1986*]

16. What is power of a lens ? What is 'dioptre' ?

17. A spherical mirror has a single focal point but a lens has two focal points. What is the reason of this difference ?

18. Will the two focal lengths of a lens be equal if the media on two sides of a lens are not the same ?

Objective type :

19. State which one is correct :

(i) Will a convex lens behave as a concave lens if the medium surrounding the lens be denser than that of the lens ? [Ans. No ; Yes ; it will cease to be a lens.]

(ii) A glass lens has a focal length of 20 cm. It is immersed in water. Will its focal length increase ? [Ans. No ; Yes ; it will remain 20 cm.]

(iii) On one side of a lens there is air and on the other water ; will the first focal length be equal to the second focal length ? [Ans. No ; Yes ;]

(iv) A lens has 25 cm. focal length. What is its power in dioptres ? [Ans. $+4D$; $-4D$; $+40D$.]

(v) A lens is producing an image of same size as the object. What is the nature of the lens ? [Ans. Concave, convex ; plano-convex.]

(vi) A convex lens A of focal length 80 cm and a concave lens B of focal length 5 cm are kept along the same axis with a distance x between them. If a parallel beam of light falling on A leaves B as a parallel beam, what is the value of x ? [Ans. 15 cm, 75 cm, 85 cm.]

Numerical Problems :

20. An object 2 cm. high, is placed at a distance of (i) 50 cm. and (ii) 15 cm. respectively from a concave lens of local length 20 cm. Find the positions and heights of the images in the two cases. [Ans. (i) 14·3 cm ; 0·57 cm ; (ii) 8·57 cm ; 1·14 cm]

21. Find the position, nature and size of the image of an object, 1 inch high, placed in front of a convex lens, at a distance of twice the focal length of the lens. [Ans. twice the focal length, real, 1″]

22. A virtual image is produced by a lens when an object is placed 20 inches from the lens. The size of the image is $\frac{2}{3}$ that of the object. Determine the position of the image, the nature and focal length of the lens. [Ans. 13·3″, concave ; 40″]

23. A convex lens forms a real image of double the size than the object when the object is placed 15 cm. from the lens. How far the object is to be placed so that virtual image of double the size may be produced by the same lens ? [Ans. 5 cm.]

24. An object 2″ inches high, is placed from a convex lens (focal length=7 inches) at distances of (*a*) 4 inches (*b*) 10 inches respectively. Find the position, nature and size of the image. [Ans. (*a*) $9\frac{1}{3}$″ virtual ; $4\frac{2}{3}$″ (*b*) $3\frac{1}{3}$″ real ; $4\frac{2}{3}$″]

25. An object 3 cm. high placed 10 cm. away from a concave lens of focal length 20 cm. Calculate the position, height and nature of the image formed. [Ans. 6·6 cm. ; 2 cm. ; virtual]

26. What will happen when the following rays are incident on a convex lens of focal length 20 cm. :—(i) a parallel beam of rays (ii) a diverging beam of rays coming from a point 20 cm. away from the lens (iii) a diverging beam of rays coming from a point 5 cm. away from the lens (iv) a converging beam of rays tending to converge at a point 20 cm. behind the lens.

Taking the mean ray of the beam coincident with the axis of the lens, draw a neat diagram in each case.

[Ans. (i) converge at the focus (ii) emerge as a parallel beam (iii) produce a virtual image at a distance 6.6 cm. (tv) converge at a point 10 cm. from the lens.]

27. An upright image of an object is formed by two convex lenses of same focal length. Show that the distance between the lenses must, at least, be double of the focal length of any lens. Show, in a diagram, the paths of the rays.

28. A magnified image is to be cast on a screen 10 metres away from a convex lens. If the magnification be 20, 'what would be the focal length of the lens ? [Ans. 47·6 cm.]

29. The image of a real object in a diverging lens of focal length 10 cm. is formed 4 cm. from the lens. Find the object distance and the magnification. [Ans. 6·66 cm ; 0·6]

30. Two convex lenses of focal lengths 3 cm. and 4 cm. respectively are placed at a distance of 8 cm. apart and an object 1 cm. high is situated on their common axis 4 cm. in front of the lens of smaller focal length. Calculate the position and size of the final image.
[Ans. 2 cm. away on the right of the second lens ; 1·5 cm.]

31. Two convex lenses, each of focal length f are at a distance $3f$ apart. Where should an object be placed so that the lens system may produce a real image ? [Ans. $u > 2f$ or $< 3f/2$]

32. A glass ($\mu = \frac{3}{2}$) lens of focal length 12 cm. is immersed in water ($\mu = \frac{4}{3}$). What will be its focal length then ? [Ans. 48 cm.]

33. A glass lens has a focal length of 5 cm. in air. What will be its focal length in water ? *R.I.* of glass is 1·51 and that of water is 1·33. [*I.I.T. 1977*] [Ans. 19·39 cm.]

34. A convergent beam of light passes through a divergent lens of focal length 20 cm. and is brought to a focus at a point 15 cm. from the lens. Find the position of the point at which this beam would have been focussed in absence of the lens. [Ans. 8·57 cm.]

35. A concave lens of focal length 15 cm. is placed in the path of a converging beam of light 3 cm. in front of the converging point of the beam. Find the position of the point where the beam actually converges after refraction through the lens.
[*H. S. Exam 1984*] [Ans. 3·75 cm on the sight side of the lens]

36. A convex lens of 5 inches focal length forms an image of an arrow which lies *along* the axis of the lens with its middle point 9·5 inches from the lens. The length of the arrow is 1 inch. Find the length of its image. [Ans. 1·25 inches]

37. Light from an object passes through a thin converging lens, of focal length 20 cm. placed 24 cm. from the object and then through a thin diverging lens of focal length 50 cm and the final real image is formed 62·5 cm away from the concave lens. Find (i) the position of the image due to the first lens (ii) distance between the lenses (iii) magnification of the final image.
[Ans. (i) 120 cm. from the convex lens (ii) 92·2 cm. (iii) 2·2]

38. A convex lens formed a real image of an object at a distance 20 cm. from it. When a concave lens was placed 4 cm. away from the convex lens towards the screen, the image moved 10 cm. further away. What was the focal length of the concave lens ? [Ans. 37·5 cm.]

39. (i) A convex lens forms an image of an object 'n' times magnified. Prove that the object distance$=(n+1)\dfrac{f}{n}$; where f=focal length of the lens. [*H. S. Exam. 1983*]

(ii) The distance between an object and a thin divergent lens is m times greater than the focal length of the lens. How many times smaller will be the image than the object ?
$\left[\text{Ans. } \dfrac{1}{m+1}\right]$

40. An object is placed in front of a convex lens at such a distance away that the lens forms a real image of same size. Then the object is moved 16 cm. towards the lens. The image still remains real but is magnified three times. Calculate the focal length of the lens. [Ans. 24 cm.]

Harder Problems :

41. A convex lens placed at a certain distance away from an object produces a real image of magnification m_1. When the object is moved through a distance x away from the lens, the image is still real but of magnification m_2. Prove that the focal length 'f' of the lens is given by

$$f = \frac{x}{\dfrac{1}{m_1} - \dfrac{1}{m_2}}$$

42. An image 1 cm. long of an object is formed on a screen by a convex lens. Keeping the object and screen fixed, the lens is moved until another image is formed on a screen. If this image is 0·75 cm. long, what is the length of the object ? [Ans. 0·87 cm.]

43. An object is placed a certain distance away from a screen. Keeping a convex lens between them, it is found that two positions of the lens are available, for each of which a sharp image of the object is cast on the screen. If x be the displacement of the lens and m_1 and m_2 be the magnifications of the image in the two positions of the lens, then prove that $f=\dfrac{x}{m_1-m_2}$

[*H. S. Exam. 1983*]

44. A beam of light, converging to a point 10 cm. behind a converging lens, is incident on the lens. Find the position of the image point if the lens has a focal length of 40 cm. [Ans. 8 cm. from the lens]

45. An object is at a distance D cm. from a screen. A convex lens of focal length f forms an image of the object on the screen. When the lens is shifted through a distance x cm., another sharp image is formed on the screen. Show that $x=\sqrt{D(D-4f)}$

46. A virtual image of an object is formed when the object is placed at a distance of 30 cm. from a lens. The magnification of the image is 2/3. Find the position of the image and focal length of the lens. What is the nature of the lens ?

[*H. S. Exam. 1981*] [Ans. 20 cm. ; 60 cm. ; concave]

47. An object is placed at 20 cm. left of a convex lens of focal length 10 cm. If a concave mirror of focal length 5 cm. is placed 30 cm. to the right of the lens, find the magnification and nature of the final image. [*I.I.T. 1974*] [1 ; real and erect]

48. An object is placed at a distance of 8 cm. on the axis of and one side of a convex lens of focal length 16 cm. A second convex lens of focal length 10 cm. is placed co-axially on the other side of it and at a distance of 5 cm. from the first lens. Find the position and magnification of the final image formed by the lens combination.

[*H. S. Exam. 1981*] [Ans. 19·09 cm from the second lens ; 1·8]

49. A pin is placed 10 cm. in front of a convex lens of focal length 20 cm. made of a material of *r.i.* 1·5. The surface of the lens further away from the pin is silvered and has a radius of curvature 22 cm. Determine the position of the final image. Is the image real or virtual ?

[*I.I.T. 1978*] [Ans. 132 cm ; real]

50. A convex lens of focal length f produced an image of an object m times magnified on a screen. If x be the distance between the object and the screen, show that $f=\dfrac{x.m.}{(1+m)^2}$

51. When an object is placed 60 cm. away from a lens, its image is formed on the other side of the lens at a distance of 300 cm. from the lens. If the object is moved by 20 cm. towards the lens, by how much would the position of the image move ?

[*H. S. Exam. 1978*] [Ans. 200 cm. from the lens on the side of the object]

52. The convex surface of a thin concavo-convex lens of glass of *r.i.* 1·5 has a radius of curvature 20 cm. The concave surface has a radius of curvature 60 cm. The convex side is silvered and placed on a horizontal surface. (i) Where should a pin be placed on the optic axis so that its image is formed at the same place ? (ii) If the concave part is filled with water of *r.i.* $\frac{4}{3}$, find the distance through which the pin should be moved so that the image of the pin again coincides with the pin. [*I.I.T. 1981*] [Ans. (i) 15 cm. (ii) 3·4 cm]

53. A plano-convex lens has a thickness of 4 cm. when placed on a horizontal table with the curved surface in contact with it, the apparent depth of the bottom-most point of the lens is found to be 3 cm. If the lens is inverted such that the plane face is in contact with the table, the apparent depth of the centre of the plane face of the lens is found to be 25/8 cm. Find the focal length of the lens. [*I.I.T. 1984*] [Ans. 80 cm.]

54. The numerical value of the focal length of a thin convex lens is f and an object is at a distance u $(u>f)$ from the lens ; a plane mirror is placed behind the lens perpendicular to the

axis at a distance f from the lens. The image of the object after reflection at the mirror and a second refraction through the lens is at a distance v in front of the lens. Prove that $u+v=2f$.

[Hints : Let v_1 be the image distance for 1st refraction. Then

$$\frac{1}{v_1}-\frac{1}{u}=-\frac{1}{f} \text{ or } \frac{1}{v_1}=\frac{1}{u}-\frac{1}{f}=\frac{f-u}{uf} \quad \therefore \quad v_1=-\frac{uf}{u-f}$$

$-ve$ sign shows that the image is real and on the side of the plane mirror. The distance of this image from the mirror$=f-\frac{uf}{u-f}=-\frac{f^2}{u-f}=\frac{f^2}{f-u}$

$\therefore$ The plane mirror forms an image of it at $\frac{f^2}{f-u}$ *behind* it. So the distance of this image from the lens$=f+\frac{f^2}{f-u}=\frac{2f^2-fu}{f-u}$. If v' be the final image by the lens (rays now come from left to the right) $\frac{1}{v'}-\frac{f-u}{2f^2-fu}=-\frac{1}{f}$ or $\frac{1}{v'}=\frac{f-u}{2f^2-fu}-\frac{1}{f}=-\frac{1}{2f-u}$.

Since the rays are now proceeding from left and v' is $-ve$ it shows that the image is formed at a distance $2f-u$ in front of the lens. Since this distance is v, we have $v=2f-u$ or $u+v=2f$].

55. Show that a thin diverging lens of focal length $+f$ followed by a thin convergent lens of focal length $-f$ will bring a parallel beam of light to a focus beyond the second lens provided that the separation of the lenses a satisfies $0<a<f$. Does this property change if the lenses are interchanged ? What happens when $a=0$?

56. Two thin lenses, one having $f=-12$ cm. and other $f=+10$ cm. are separated by 7 cm. A small object is placed 43.5 cm from the centre of the lens system on the principal axis first on one side and next on the other side. Find the location of the final image in each case.

[24 cm on the right of second lens ; 60 cm on the left of 1st lens]

57. The radius of curvature of the convex face of a plano-convex lens is 12 cm. and its *r.i.* is 1·5. Find the focal length of the lens. The plane surface of the lens is now silvered. At what distance from the lens will parallel rays incident on convex face converge ? Sketch the ray diagram to locate the image, when a point object is placed on the axis, 20 cm. from the lens.

[*I.I.T. 1979*] [Ans. (i) 24 cm. (ii) 12 cm. in front of the lens (iii) 30 cm. on the other side]

58. A convex lens B of focal length 20 cm. is placed at a distance of 30 cm. to the right of an identical lens A. A point object is placed at a distance of 30 cm. to the left of A on the common axis of the two lenses. Where should a convex mirror of radius of curvature 7 cm. be placed so that the final image coincides with the object ? [*I.I.T. 1971*] [Ans. 5 cm. on the right of B]

59. When a small object is placed at A on the axis of a convex lens of focal length f, the image is erect. When the object is moved to B, the image is inverted but of the same size as before. If m be the magnification, show that $AB=\frac{2f}{m}$.

60. The image of a square hole on a screen illuminated by light is obtained on another screen with the help of a converging lens. The distance of the illuminated square from the lens is 40 cm. The area of the image is 9 times that of the square hole. Calculate the position of the image and the focal length of the lens. [Ans. 120 cm. ; 30 cm.]

5

DISPERSION OF LIGHT AND SPECTRUM

5.1. Dispersion of light :

The ancients were aware of the brilliant hues produced when sunlight passed through transparent gems and crystals, but it was not until the middle of the seventeenth century that Sir Isaac Newton investigated the problem systematically.

Experiment : In an opaque screen, a small circular hole *H* is made, through which white sunlight is allowed to pass and fall on a prism *P*. (Fig. 5.1). After emergence from the prism, when the rays fall on another screen *S*, the rays are found to spread out into a band of colours. The sequence of colours in the band is the same as one seen in the rainbow, with red at one end and violet at the other. The other intermediate colours are orange, yellow, green, blue and indigo. The sequence of position of the colours can readily be obtained from the word VIBGYOR formed by taking the initial words of the colours. The band of colours is called a *spectrum* and the spreading process is called *dispersion*.

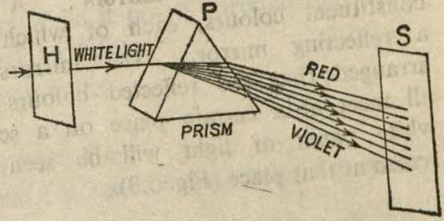


Fig. 5.1

From the spectrum so formed it is seen that the deviations of different colours are different, red having the least deviation while violet having the most. Difference in deviation is known as refrangibility. The refrangibility of yellow being intermediate between the refrangibilities of red and violet, the yellow is sometimes referred to as the mean colour.

When white light passes through a prism, why does it get dispersed into seven different colours ? Light of different colours have different velocities in a dispersive medium like glass. The velocity of red light is the greatest and that of violet the least. As a result, in crossing the width of the prism, rays of different colours take different times and get separated when they come out of the prism. In air or in vacuum, the velocity of all the coloured rays is the same. So, white light is not dispersed in vacuum or air.

5.2. Composite nature of white light :

The phenomenon of dispersion of white light proves that white light is a mixture of seven different colours and hence it is a *composite* or *compound* light. If any of the seven colours of the spectrum is again allowed to pass through another similar prism, no further splitting is found to occur ; the light retains its colour which shows that (i) *each colour of the spectrum is monochromatic* (*mono*—single ; *chrome*—colour) and (ii) *the colours are not introduced by the prism but are components of white light.*

Composite nature of white light is well established if all the seven colours of the spectrum, on recombination, can produce white light again. This can however, be done in the following ways :

(i) **By two similar prisms** : P and Q are two identical prisms, made of same material, placed side by side with their refracting angles turned in the opposite directions. The spectrum formed by the prism P out of the white light incident on it, is allowed to pass through the prism Q. It will be seen that the prism Q has combined the different colours into white light on the screen S (Fig. 5.2).

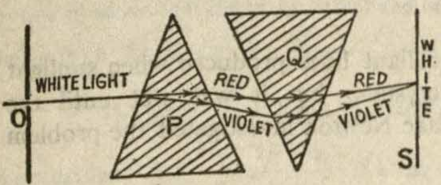


Fig. 5.2

(ii) **By several mirrors** : A prism disperses white light into seven constituent colours, each of which falls on a reflecting mirror. The mirrors are so arranged that the reflected colours of light all meet at a certain place on a screen. A white patch of light will be seen on the screen at that place (Fig. 5.3).

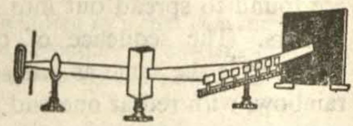


Fig. 5.3

(iii) **By Newton's colour disc** : It is a circular disc of cardboard divided into four equal quadrants. Each quadrant is again divided into unequal sectors painted with colours of the spectrum in their correct proportions and positions. The disc is mounted on a turn-table. When the disc is rotated rapidly, it appears nearly white. The reason is that before the impression due to any one colour has died away it is succeeded by all other colours. Due to this 'persistence of vision' there is a physiological mixture of spectral colours and the disc appears almost white to the eye.

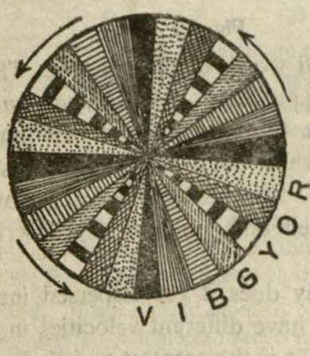


Fig. 5.4

5.3. Sensation of colour :

Sensation of colours depends on the wave length of light. All the seven colours of the spectrum have different wave lengths. Each of them is called *monochromatic* light. The wave length of light is usually expressed in *Angstrom* unit. Its symbol is $A°$. $1A° = 10^{-8}$ cm. The wave lengths of the spectral colours in angstrom unit are as follows :

Red—6800A°
Orange—6300A°
Yellow—5800A°
Green—5300A°
Blue—5050 A°
Indigo—4800A°
Violet—4300A°

From the table, it is seen that red light has the longest and the violet the shortest wave length. Sensation of light to the eye is the strongest in the yellow-green region. Sensation gradually diminishes on both sides of this region. Visibility in the red and the violet regions is almost nil. For this reason, a substance illuminated with yellow light appears brightest while a substance illuminated with red or violet light appears almost dark.

5.4. Angular dispersion and dispersive power :

In art 3.20, we have seen that the deviation undergone by a monochromatic ray of light while passing through a thin prism is given by $\delta=(\mu-1)A$, where A is the angle of the prism and μ its refractive index.

Now, suppose that a ray of white light is incident on a thin prism. The ray will be dispersed into seven different colours while it passes through the prism and the deviation of these different colours will be different.

Now, for the mean yellow colour, we may write,

$$\delta=(\mu-1)A \quad \ldots \quad \ldots \quad \text{(i)}$$

where δ=the deviation of the mean yellow colour

μ=refractive index of the prism for mean yellow colour or mean refractive index.

Similarly, for red and violet light, we have

$$\delta_r=(\mu_r-1)A \quad \ldots \quad \ldots \quad \text{(ii)}$$

and $$\delta_v=(\mu_v-1)A \quad \ldots \quad \ldots \quad \text{(iii)}$$

Now, angular dispersion is defined as the difference between the angles of deviation of the extreme colours of the spectrum, viz, red and violet.

$$\therefore \text{ Angular dispersion}=\delta_v-\delta_r=(\mu_v-\mu_r)A=\frac{(\mu_v-\mu_r)}{\mu-1}(\mu-1)A$$

[multiplying and dividing by $(\mu-1)$]

$$=\frac{(\mu_v-\mu_r)}{\mu-1}.\delta \quad \text{[From eqn. (i)]}$$

or, $\dfrac{\delta_v-\delta_r}{\delta}=\dfrac{\mu_v-\mu_r}{\mu-1}=\omega$, where ω is the *dispersive power* of the material of the prism.

According to differential calculus, the slight increase of *r.i.* of the material from violet to red $(=\mu_v-\mu_r)$ may be called $d\mu$. $\therefore \quad \omega=\dfrac{d\mu}{\mu-1}$

$$\text{Now, angular dispersion}=\frac{(\mu_v-\mu_r)}{\mu-1}.\,\delta=\omega.\delta$$

$$=\text{dispersive power}\times\text{mean deviation.}$$

Example : *A glass prism has refracting angle 8° and its refractive indices for blue and red light are 1·532 and 1·514 respectively. Find the angular dispersion produced by the prism. What is the dispersive power of the material of the prism ?*

Ans. Angular dispersion$=(\mu_b-\mu_r)A=(1\cdot532-1\cdot514)\times8^\circ=0\cdot144^\circ$

Also, dispersive power $\omega=\dfrac{\mu_b-\mu_r}{\mu-1}$ [μ=mean refractive index)

Now, $\mu=\dfrac{\mu_b+\mu_r}{2}=\dfrac{1\cdot532+1\cdot514}{2}=1\cdot523$

$$\therefore\quad \omega=\frac{1\cdot532-1\cdot514}{1\cdot523-1}=\frac{0\cdot018}{0\cdot523}=0\cdot034 \text{ (nearly).}$$

5.5. Impure and pure spectrum :

When white light is ordinarily dispersed by a prism and the dispersed rays are received on a screen, we get an impure spectrum because in the spectrum so formed, the different colours are not distinctly visible nor do the colours occupy their respective positions. The reason for a spectrum being impure is as follows : we cannot isolate a single ray. However small the hole on the screen may be, we always get a number of rays each of which produces its own spectrum. All these spectra, getting overlapped make an impure spectrum on the screen. This may be demonstrated by blowing smoke in the path of the rays emerging from the prism. The smoke appears coloured only at the boundary of the beam, the centre being white near the prism.

Definition : *A spectrum in which different colours of the spectrum occupy distinct and different positions, and are distinctly visible is called a pure spectrum.*

A spectrum in which different colours of the spectrum do not occupy distinct and different positions and are not visible clearly is called an impure spectrum.

5.6. Methods of producing pure spectrum :

(i) S is a very narrow slit illuminated with white light. A convex lens L is so placed that it produces a sharp image S' of the slit S on a screen M [Fig. 5.5]. Now, a prism P is placed between the lens and the screen in the position of minimum deviation of the yellow ray. In this position of the prism, other colours

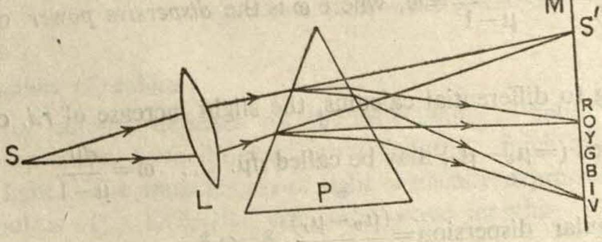


Fig. 5.5

will also come out almost with minimum deviation and there will be not much difference in their paths through the prism. Consequently, the white light will be dispersed by the prism into seven constituent colours which will produce seven distinct images of the slit on the screen.

(ii) A narrow slit S is placed at the focus of a convex lens L_1 and is illuminated by white light. The lens sends out a parallel beam of white rays on to the prism P which is placed in the position of minimum deviation for the yellow ray (Fig. 5.6). When the white ray will be dispersed by the prism, rays of any one colour come out as parallel bundle. On account of the dispersion, the angle of

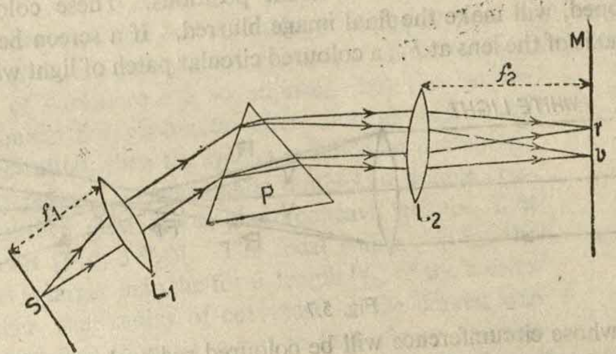


Fig. 5.6

emergence is, however, different for different colours. A second lens L_2 now, focusses the different parallel bundles to different points on the screen M. A pure spectrum will, therefore be formed on the screen.

Conditions of forming pure spectrum : From the above arrangements, it is clear that the following conditions are necessary for producing a pure spectrum :

(i) The slit should be very narrow ; for a wide slit allows a large number of rays to fall upon the prism, each of which forms its own spectrum and all these spectra, on superposition, make an impure spectrum on the screen.

(ii) A convex lens should be used in order to make the incident beam parallel. This will make the deviations of the rays of one particular colour equal.

(iii) The prism should be placed in the position of minimum deviation of the mean yellow ray. Rays of other colours will consequently, come out with minimum deviation.

(iv) Another convex lens should be used in order to form different coloured images of the slit on the screen.

*5.7. Chromatic aberration in a lens :

In establishing the lens formula we assumed, among other things, that monochromatic light was incident on the lens and hence, no question regarding the dispersion of light arose at that time. But we know that a lens may be regarded as a combination of a large number of small prisms and as such the behaviour of a lens towards white light is very much the same as that of a prism. So, when white light passes through a lens close to the edge, it is dispersed in much the same way as it is dispersed when passing through a prism. The violet light, being deviated the most, comes to a focus nearer the lens than the red. The edges of the image, as a result, become tinged with colour and the image becomes defective.

Consider a parallel beam of white rays falling on a convex lens (Fig 5.7). After dispersion, the violet rays will be brought to focus at a point F_v, nearer the lens than the red rays which will be brought to focus at F_r. *So, the focal length of a convex lens is the greatest for red and least for violet ray.* If the lens forms an image of an object illuminated with white light, coloured images of different magnifications will be formed at different positions. These coloured images, being overlapped, will make the final image blurred. If a screen be held at right angles to the axis of the lens at F_v, a coloured circular patch of light will be obtained

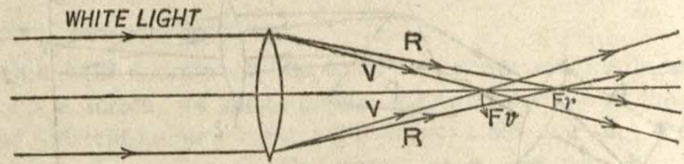


Fig. 5.7

on the screen whose circumference will be coloured red and centre violet. If the screen be similarly placed at F_r, the circumference will be coloured violet and the centre red. The image at both the positions will be coloured. This defect of image due to dispersion is called **chromatic aberration.**

In art 4.13, it has been said that the focal length of a convex lens is given by $\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$ where $\mu=r.i.$ of the lens material and r_1, r_2 are its radii of curvature. From this relation, it is clear that the focal length of the lens will be different for different colours because μ is different for different colours. For example for blue colour $\frac{1}{f_b}=(\mu_b-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$ and for red $\frac{1}{f_r}=(\mu_r-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$.

Now for crown glass $\mu_b=1{\cdot}523$ and $\mu_r=1{\cdot}513$. So, for blue light $\frac{1}{f_b}=(1{\cdot}523-1)$ $(\frac{1}{20}+\frac{1}{15})$ if the radii of curvature are 20 cm and 15 cm respectively. We, therefore, get $f_b=16{\cdot}39$ cm. Similarly, for red light $\frac{1}{f_r}=(1{\cdot}513-1)(\frac{1}{20}+\frac{1}{15})=\frac{1}{16{\cdot}7}$ or $f_r=16{\cdot}70$ cm.

Hence, the difference of the focal lengths of the above lens$=f_r-f_b=$ $16{\cdot}70-16{\cdot}39=0{\cdot}31$ cm.

Chromatic aberration can be minimised in the following ways :—

(i) Two lenses, one convex and one concave, *put in contact* can minimise chromatic aberration if $\frac{\omega_1}{f_1}+\frac{\omega_2}{f_2}=0$ where, ω_1 and ω_2 are the dispersive powers of the lenses and f_1 and f_2 are their mean focal lengths.

(ii) Two convex lenses, made of *same material*, when placed co-axially so that their separation $x=\frac{1}{2}(f_1+f_2)$, where f_1 and f_2 are their mean focal lengths, can minimise chromatic aberration.

Achromatic doublet : We have seen that a convex lens of crown glass and a concave lens of flint glass when kept in contact with each other on the same axis form an achromatic system, provided their focal lengths and dispersive powers satisfy the relation $\frac{\omega_1}{f_1}+\frac{\omega_2}{f_2}=0$. This condition does not, however, contain the radii of curvature of the curved surfaces of the lenses. But we know that the focal length of a lens depends on the radius of curvature of its curved surfaces. If the radii of curvature are so adjusted that the above system minimises the chromatic aberration as well as the spherical aberration, then the system is called an *achromatic doublet*. To prepare an achromatic doublet, a double convex lens of crown glass and a plano-concave lens of flint glass are used [Fig. 5·7(a)]. The focal length (f_1) of the concave lens is larger than the focal length (f_2) of the convex lens. Further the radius of curvature of the curved side of the concave lens is made equal to that of any one side of the convex lens. The two curved surfaces of equal radius of curvature are cemented together by an adhesive, known as canada balsam. This reduces the loss of light due to reflection at the composite surfaces.

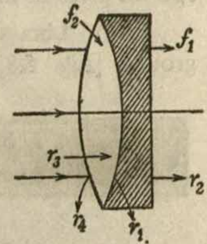


Fig. 5.7(a)

5.8. Types of spectra :

Spectra produced by different substances may be broadly divided into two types : (i) *emission spectra* and (ii) *absorption spectra.*

Emission spectra is again divided into several classes such as (i) *continuous spectra* (ii) *band spectra* (iii) *line spectra.* Similarly, absorption spectra is divided into (i) *dark line spectra* and (ii) *dark band spectra.*

Spectra
- Emission spectra
  - Continuous
  - Band
  - Line
- Absorption spectra
  - Dark line
  - Dark band

We shall now discuss these different types one by one.

5.9. Emission spectra :

A body may be suitably excited to emit light which produces a spectrum known as emission spectrum.

(i) **Continuous spectra :** It presents the appearance of an unbroken band of light, varying in colour from point to point. The intensity shades off on both sides of a certain region of maximum intensity. It is given by incandescent solids such as glowing filament of an electric bulb, white light of an incandescent

vapour, electric arc etc. The region of maximum intensity shifts towards the violet end as the temperature of incandescent body rises.

(ii) **Band spectra** : It consists of bands of light separated by dark spaces. Each band consists of a collection of fine lines, which are closely packed near one edge. Compounds like cyanogen etc., give band spectra. Mercury vapour and nitrogen also give band spectra. If weak electric discharge be passed through nitrogen taken in a discharge tube, nitrogen produces band spectra. Study of band spectra gives us an insight into the molecular arrangement of the substance.

(iii) **Line spectra** : It consists of separate bright lines on a dark background [Fig. 5.8]. Lines may be sharp or diffuse and generally they are of varying intensity.

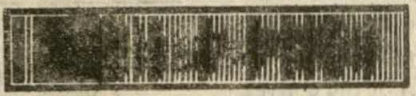


Fig. 5.8

Gases generally give line spectra. Elements in their atomic state also give line spectra. As a matter of fact, line spectra are characteristic of elementary state of the substance. Each element produces its line spectra whose colour and position (*i.e.* wave length) are fixed. For this reason, elements present in any source can be identified by spectral analysis. In recent years it has also been possible to find the approximate amount of each element by measuring the brightness of the lines. These techniques of *spectro-chemical analysis* are taking the place of regular chemical methods in many industries, in medicine, in forensic analysis etc.

5.10. Absorption spectra :

(i) **Dark line spectra** : Dark line absorption spectrum is a continuous spectrum with certain characteristic lines absorbed from it by any absorber. It is known that a gas or a vapour will absorb, at a comparatively lower temperature, those particular kinds of light which it itself emits at a higher temperature. Thus, if white light coming from a carbon arc be spectrosopically examined, it will present a continuous spectrum but if a sodium flame be interposed between the arc and the slit of the spectroscope, the continuous spectrum will be found to be missing two D-lines which are the characteristic lines of sodium light. This type of spectrum is called a darkline absorption spectrum. In the same way, if a beam of white light be allowed to pass through a Bunsen-flame impregnated with lithium salt, the characteristic red line of lithium will be found to be absent in continuous spectrum.

(ii) **Dark-band spectra** : If white light from a strong source is allowed to pass through a coloured solid or liquid, and then examined spectroscopically a part of the spectrum will be blotted out from the continuous spectrum of white light. This is called an absorption band. The resultant spectrum is called a dark-band absorption spectrum.

For example, if white light is made to pass through a plate of blue cobalt glass, the continuous spectrum of white light is found to be crossed by three dark bands, a broad one and two sharper ones in the region from red to green. A

piece of ruby glass when placed in the path of the incident white light will absorb all colours except some portion of the red and in the resultant spectrum, only the red region will be visible and the rest will be dark.

5.11. Solar spectrum :

The solar spectrum, on casual inspection, looks like a continuous spectrum with all the colours occupying their respective positions. But when the spectrum is examined with a spectroscope of high resolving power, it is found to be crossed by innumerable dark lines. The positions of these dark lines are fixed. These dark lines characterise *the solar spectrum as dark-line absorption spectrum.* These lines were first noticed by the Englishman Wollaston in 1802 but they were studied carefully by the German scientist Fraunhofer in 1814, for which the lines were subsequently named as *Fraunhofer lines.* The most important of these lines originally found out by Fraunhofer are named as A, B, C, D, E, etc., according to the English alphabet. The line marked A lies in the extreme red, B and C in the red region, D in the yellow-orange, E in the green, F and G in the blue and H and K in the violet region. He mapped out and counted as many as seven hundred dark lines in the visible and invisible parts of solar spectrum.

Explanation of the origin of Fraunhofer lines : Kirchhoff's law :

If light from a source having a continuous spectrum is examined after it has passed through a sodium flame, the spectrum is found to be crossed by a dark line ; this dark line is in the position corresponding to the bright line emission spectrum obtained with the sodium flame alone. The continuous spectrum with the dark line is naturally characteristic of the absorbing substance, in this case sodium and it is known as an absorption spectrum. An absorption spectrum is obtained when red glass is placed in front of sunlight, as it allows only a narrow band of red rays to pass through the glass.

Kirchhoff's investigations on absorption spectra in 1855 led him to formulate a simple law concerning the emission and absorption of light by a substance. The law states : *A substance which emits light of a certain wavelength at a given temperature can absorb light of same wave length at that temperature.* In other words, a good emitter of a certain wavelength is also a good absorber of that wavelength. From Kirchhoff's law, it follows that if the radiation from a hot source emitting a continuous spectrum is passed through a vapour, the absorption spectrum obtained is deficient in those wave lengths which the vapour would emit if it were raised to the same high temperature. This gives us a satisfactory explanation of the origin of Fraunhofer lines in the solar spectrum.

The sun consists of an intensely hot nucleus called the **photosphere** surrounded by a comparatively cooler gaseous envelop known as **chromosphere.** The temperature of photosphere is about several million degrees of celsius while that of chromosphere is only about several thousand degrees of celsius. Now white light from the nucleus of the sun while passing through the gaseous envelop of comparatively lower temperature, is robbed off the colours which these gaseous ele-

ments would emit when suitably excited. Hence the dark lines as observed in the solar spectrum signify the existence of various elements in the gaseous forms in the chromosphere of the sun. For example, A and B lines in the red region of the solar spectrum signify the presence of oxygen in the solar atmosphere because oxygen in a vacuum tube when electrically excited, would present line spectra consisting of red lines in the positions occupied by A and B lines. D dark lines in the orange-yellow region signify the presence of sodium because sodium vapour presents yellow lines in the same position when common salt is put in a non-luminous Bunsen flame.

Arguing in the same way, other lines signify the presence of iron, calcium, magnesium, hydrogen etc. in the sun's atmosphere. Kirchhoff, on careful study of these dark lines in the solar spectrum, numbering several thousands came to the conclusion that other elements like nickel, barium, copper, zinc etc also exist in the sun but the existence of gold, silver, mercury etc were found to be doubtful.

It is to be pointed out here that all the Fraunhofer lines observed in the solar spectrum are not due to absorption by chromosphere. Some of the dark lines are produced as a result of the absorption by oxygen and water-vapour present in the earth's atmosphere. These are called **telluric lines** and these lines become very faint when observations are taken from a high altitude more than 9000 ft. high.

Some of the most prominent Fraunhofer lines with their wavelengths and the elements responsible for their absorption in the chromosphere are given below :

Lines	Elements	Wavelengths
A	Oxygen	7594A°
B	,,	6867A°
C	Hydrogen	6563A°
D_1	Sodium	5896A°
D_2	,,	5890A°
F	Hydrogen	4861A°
G_1	,,	4341A°
H	Calcium	3969A°
K	,,	3934A°

5.12. Complete spectrum and Angstrom Unit :

In spectroscopy, wavelength is generally expressed in angstrom unit.

$$1 \text{ Angstrom unit } (A^\circ) = 10^{-8} \text{ cm.}$$

A detailed study of the various radiations of different wavelengths in the spectra emitted by other substances reveals that besides visible spectrum, infra-red spectrum and ultra-violet spectrum, there are other radiations of smaller and longer wavelengths which have the properties same as the properties of ordinary

visible light. All these radiations are known as *electromagnetic waves.* All of them have the same velocity as the velocity of visible light ; they exhibit reflection, refraction, interference and other optical properties.

Of the visible spectrum, red has the longest wave length of 7×10^{-5} cm or 7000A° and the violet the shortest wavelength of about 4×10^{-5} cm or 4000A° [Fig 5.9]. Light of wavelengths shorter than violet are called *ultra-violet light* whose wavelengths extend from 4000A° to about 1000A°. Still shorter wavelength rays are known as *X-rays.* X-ray wave-lengths extend from 1400A° to about 0·06A°. The *gamma rays* emitted by radioactive substances have wavelengths even shorter than those of X-rays. Gamma rays of wavelength between 1·4A° and 0·01A° have been detected.

Beyond red, we come across *infra-red* rays of wave-lengths larger than red. Infra-red rays have wave-lengths between 700A° to 0·04 cm. Still greater wavelength radiations or waves are generally called the television waves. Their wavelengths extend from 0·04 cm. to $2{\cdot}5\times10^{6}$ cm.

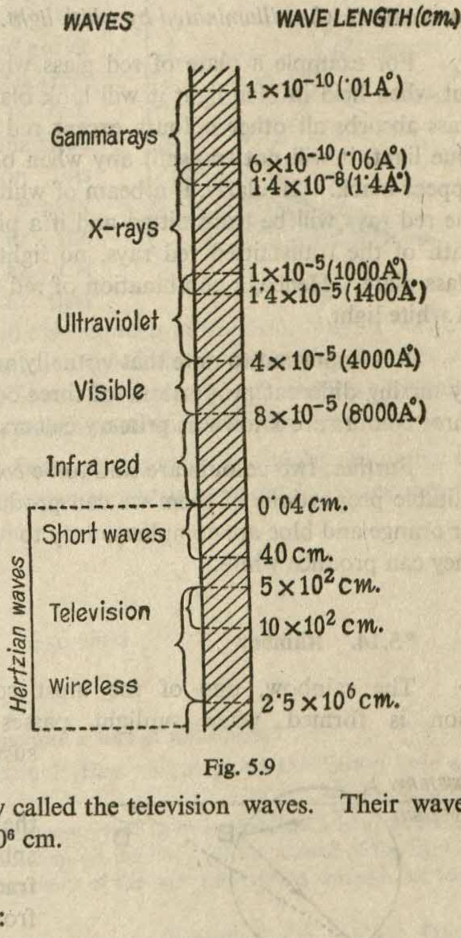


Fig. 5.9

5.13. Colour of different bodies :

We see everyday substances of various colours like red flower, blue paper, green glass etc. How are these colours produced ?

The colour of an opaque object is judged by the colour it can reflect when illuminated by white light. For example, if a piece of red cloth, a red flower or a red stick be held in the red part of the spectrum, the object will appear red ; but when held in the green part, it will appear black because it will absorb all other colours except the one it itself possesses. Hence, an opaque object appears to have the particular colour which it can reflect.

It is to be noted in this connection that *black is no colour ; it is the absence of all colours.* When white light falls on a piece of black cloth, it absorbs all the colours of the white light and reflects none. Hence, it looks black. It is also to be remembered that *white is no colour too ; it is the presence of all colours.*

When white light falls on a piece of white cloth, it reflects all the colours of white light and absorbs none. So, it looks white.

The colour of a transparent body, on the other hand, is judged by the colour it can transmit when illuminated by white light.

For example a plate of red glass when held in white light will appear red but when held in blue light it will look black. This is due to the fact that a red glass absorbs all other colours except red which it transmits. Since it absorbs blue light, it will not transmit any when blue light falls on it. It will, therefore, appear black. Similarly, if a beam of white light be incident on a red glass plate, the red rays will be transmitted and if a piece of blue glass be now placed in the path of the transmitted red rays, no light will be transmitted through the blue glass plate. Hence, a combination of red and blue glass plates will appear black in white light.

It is important to note that virtually any recognisable colour can be produced by mixing different proportions of three colours *viz.* red, green and blue. These three colours are known as **primary colours.**

Further, two colours are said to be *complimentary* to each other if by mixing suitable proportions of them we can produce white. Thus yellow and deep blue or orange and blue are complementary to each other because when mixed together they can produce white.

*5.14. Rainbow :

The rainbow, one of the most commonly observed examples of dispersion, is formed when sunlight passes through myriad droplets of water suspended in the air after a shower.

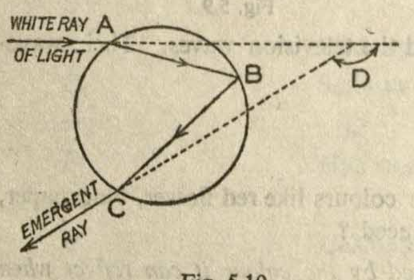


Fig- 5.10

Consider a ray of white light from the sun incident at the point A on a spherical drop of rain. The ray is refracted on entering the drop at A, reflected from the back surface at B and refracted again at C on coming out in air (Fig. 5.10). But while going through, the ray is also dispersed into different colours just as a white ray is dispersed by a prism. It is clear from the figure that the ray, on emergence, suffers certain deviation which is given by the $\angle D$ (see Fig 5.10). It has been found that *the ray which suffers minimum deviation, produces the maximum sensation of colour to the eye.* Calculation shows that the minimum deviation for red ray is about 138° and that for the violet ray is about 140°.

Now, suppose that the rains are falling at one end of the sky and from the opposite end the sun shines upon the falling drops. An observer, turning his back towards the sun, is looking towards the falling drops

(Fig. 5.11). Imagine, for the observer, such an arc of a circle on the sky that water drops situated on the arc may send the sun-rays to the eye of the observer with a deviation of 138°. In that case, the drops will appear intensely red to the eye of the observer. He will see a red coloured arc at that position, because 138° deviation corresponds to the minimum deviation of red.

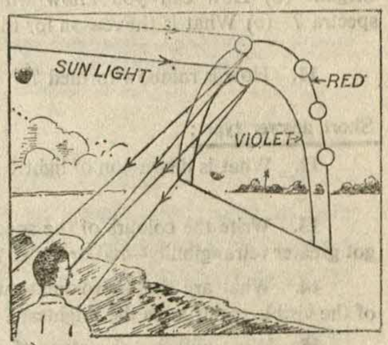


Fig. 5.11

Similarly, if another arc be imagined so that water drops situated on this arc may send the sun-rays to the eye of the observer with a deviation of 140°, the drops will appear intensely violet and the observer will see a violet arc at that position. Arcs of other colours will also be seen in the same way in between the red and violet. The total effect produced by all the drops is what is called a rainbow. This is known as the *primary bow*. A larger and a fainter bow is sometimes seen above the primary bow. This is known as the *secondary bow*.

In a primary rainbow, red arc lies on the outside and violet arc on the inside. In a secondary rainbow, the colour arrangement is reversed *i.e.*, violet arc lies on the outside and red arc on the inside.

Exercises

Essay type :

1. How would you prove the composite nature of white light ?

2. What are pure and impure spectra ? How will you proceed to form a pure spectrum on a screen ? [*cf. H. S. Exam. 1978*]

3. Prove, by means of simple experiments, that colours of different bodies depend on the following : (a) the colour of the light incident on the body (b) the colour of the light reflected by the body, if the body is opaque (c) the colour of the light transmitted through the body if the body is transparent.

4. Describe different types of spectra, giving an illustration of each kind. Describe the general nature of spectra produced by (i) an incandescent solid body (ii) a gas discharge tube at low pressure.

5. What do you mean by chromatic aberration in a lens ? How can it be removed ?

6. What is meant by chromatic aberration of a lens ? Illustrate it by a suitable diagram in the case of a convex lens. [*H. S. Exam. 1981*]

7. What are emission and absorption spectra ? Discuss about line spectra, band spectra and continuous spectra in connection with emission spectra and dark line spectra and dark band spectra in connection with absorption spectra. [*H. S. Exam 1984*]

8. Describe the nature of solar spectrum. What are Fraunhofer lines and telluric lines ? How do they originate ?

9. What are emission and absorption spectra ? How are Franhofer lines produced in the solar spectrum ? [*H. S. Exam. 1981*]

10. (a) What do you mean by continuous spectrum and line spectrum ? Explain their origin. (b) How can you know what elements are present in a source of light from its line spectra ? (e) What is the reason for the origin of dark lines in the solar spectrum ?
[*H. S. Exam. 1984*]

11. How is rainbow formed ?

Short answer type :

12. What is dispersion of light ? What is its reason ? What is a spectrum ?
[*H. S. Exam. 1984*]

13. Write the colours of the spectrum in order of their wave lengths. Which colour has got greater refrangibility—red or violet ?

14. What are the approximate wave length limits of visible spectrum ? Which portion of the visible spectrum is the brightest ?

15. What will be the nature of spectrum of the following sources :—(a) sun (b) electric filament lamp (c) a neon lamp (d) sodium salt illuminated by Bunsen flame.

16. Compare the focal lengths of a glass lens in the case of red and violet light when the lens is (i) convex and (ii) concave.

17. Will the focal length of a thin convex lens increase if it is determined with monochromatic red light instead of blue light ?

[**Hints** : When red light is used, the focal length will increase. Red light has greater wavelength, and hence smaller deviation. The refractive index of glass, for this reason, is smaller for red light than blue light.

Now, $\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$; so $f \propto \frac{1}{\mu-1}$, so as μ decreases, f increases].

18. What is the difference in appearance between a line spectrum and a continuous spectrum ? Why are a number of dark lines seen in the spectrum of light from the sun ?

19. Distinguish between emission spectra and absorption spectra. What is Kirchhoff's law in connection with absorption spectra ?

Objective type :

20. Pick up the correct answer :—(a) What is the process called when a ray of white light is split into different colours while passing through a prism ?
[Ans. refraction, reflection, dispersion].

(b) Which region of the spectrum has zero visibility ? [Ans. Yellow-green, red, violet]

(c) To which type of spectral lines do the Fraunhofer lines belong ?
[Ans. emission line spectra, absorption line spectra, dark band spectra].

(d) How will a red rose appear in blue light ? [Ans. red, blue, black].

(e) Which type of spectra is the characteristic of elementary substances ?
[Ans. band spectra, line spectra, absorption spectra.]

(f) A tailor wishing to match thread for a blue shirt, should do so (i) under a yellow light (ii) under a blue light (iii) near a window (iv) in semi-darkness. Which is correct ?

(g) There is a blue disc on a white background. In red light we observe (i) red disc on a green background (ii) green disc on a red background (iii) black disc on a red background and (iv) red disc cn a black back ground. Which is correct ?

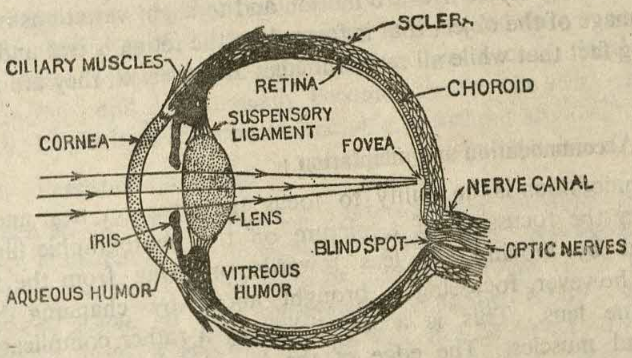
6

HUMAN EYE AND VARIOUS OPTICAL INSTRUMENTS

6.1. Human eye :

Eye is nature's priceless gift to man which has enabled him to enjoy the beauties of form, colour and motion. It is like an exceptionally fine camera with an elaborate lens system on the one side and a sensitive screen, like a photographic film, on the other. The description of the principal parts of an eye is as follows [Fig. 6.1].

The shape of human eye is almost spherical. It can move in a space called the *socket* of the eye. The refracting media of the eye consists of the *cornea*, the *aqueous humor*, the *crystalline lens* and *vitreous humor* and its function is to focus an image of the object to be seen on the sensitive screen. Cornea is the anterior



Fig, 6.1

(*i.e.* front) transparent part of the outer coat of the eye-ball. The rest of the outer coat, called the *sclera* is white and opaque. Aqueous humor is a transparent watery liquid, a bit saline in taste, filling the space between the cornea and the crystalline lens. The eye lens is a transparent biconvex lens with back surface more convex than its front surface. It is held in position by *suspensory ligaments to ciliary muscles.* Vitreous humour is a jelly-like fluid, transparent in nature, filling up the space between the lens and the retina. Like a camera, the eye contains an *iris diaphragm* which opens wider for faint light and closes down to a bare pinhole opening for very bright sun light. It is this iris that contains the pigment determining the colour of the eye. The *pupil* of the eye is the aperture at about the centre of the iris. *Retina*, the light sensitive screen of the eye, is the innermost coat of the eye-ball. It is composed of nervous elements and contains minute structures called *rods* and *cones*. At the very centre of the retina there is a small yellowish looking spot called *fovea centralis* where vision is most sensitive. The region of the retina at which the optic nerve enters the eye is most insensitive to light and is called the *blind spot*. There is a black pigment, called the *choroid*, inside the eyeball. If light falls on any other part of the eye-ball

except the retina, choroid absorbs it. The line obtained by joining the centre of the cornea and the centre of crystaline lens is called the *optic axis*, while the line obtained by joining the fovea centrals and the centre of the eye-lens is called the *visual axis*.

Function of the eye : It has been pointed out before that the refracting media of the eye consists of the cornea, the aqueous humor, the crystalline lens and the vitreous humor. When an object appears before the eye, light from the object is refracted by the above system and an image is formed on the retina. The light pulses on the retina are received by tiny cones and rods whose function seems to change the light into electricity. Each cone and rod is connected with an individual nerve which conducts the electricity to the brain. Just how these electrical impulses are produced by the cones and rods and how they are interpreted by the brain as vision, is still only vaguely understood by the scientists. Experiments seem to indicate that the cones respond only to bright light and are particularly responsible for the detection and distinction of colour, whereas the rods are sensitive to very feeble light, to motion and to slight variations in intensity.

The image of the object that is formed on the retina is real and inverted. It is an amzing fact that while all retinal images are inverted, they are interpreted as being erect.

6.2. Accommodation and adaptation :

Accommodation is the ability to focus the eyes on near and far objects. In a camera the focussing of a picture on the photographic film or plate is accomplished by moving the lens towards or away from the film. In the human eye, however, focussing is brought about by changing the shape of the crystalline lens. This is accomplished by a rather complicated system of ligaments and muscles. The edge of the lens is surrounded by the ciliary muscles which, by contracting, cause the lens to bulge out. This reduces the focal length of the lens, bringing nearby objects to focus on the retina. When the ciliary muscles relax, the suspensory ligaments, being under tension, pull at the edges of the lens thus tending to flatten it. Under these conditions, the focal length increasses, bringing distant objects to focus on the retina.

Accommodation of human eye is, however, limited. It has been found that a normal eye, without being strained, can accommodate objects lying between a point about 25 cm from the eye and infinity. But if the object be brought very close to the eye, it will not be distinct. The eye will have to be strained in order to see it clearly. For this reason, we feel pain and strain in our eye if we try to see very near objects for a pretty long time. For a normal eye, a distance of about 25 cm. is found to be the distance of most distinct vision and it is known as *the least distance of distinct vision.*

The point situated at about 25 cm. from the eye is called the *near point* and the furthest point upto which a normal eye can see without straining the eye is called the *far point*. For a normal eye, the far point is supposed to be situated at infinity. The range between the near point and the far point is called the *visual range*. Objects situated anywhere in this visual range will be visible to the eye.

It has already been mentioned that there is an aperture at about the centre of the iris, known as the pupil. Making the pupil small or large with the help of muscles, less or more light may be allowed to enter into the eye. A man unknowingly makes his eye pupil large in dim light and small in strong light. This power enjoyed by a man is called his *adaptation*. The change of the size of the pupil does not, however, take place immediately with the arrival of light but happens a little later. For this reason when the light of a brightly lit room is switched off at night suddenly, the eyes are at first blinded and cannot see anything but after some time, when the eyes become used to the darkness, the furniture and other things of the room become dimly visible. In bright light, the pupil of the eye is small in size. When the light is switched off, outside light, coming through the windows illuminate the objects in the room very dimly but as the pupil can not grow in size immediately, the small amount of outside light can not enter into the eye which appears to be blinded for a moment. As time is allowed, the pupil gradually becomes larger and the outside light entering into the eye, makes the objects of the room dimly visible.

In the same way, if a man remaining in a pitched dark room for some time, suddenly enters a room with bright light, his eyes become dazzled. After some time when the pupil automatically becomes smaller in size, the eyes become used to the bright light and can see the objects without any difficulty.

6.3. Persistence of vision :

When an eye sees an object, the impression of the object on the retina does not vanish immediately as the light is cut off. The impression on the retina persists for a period of about $\frac{1}{10}$th of a second. This effect is known as the *persistence of vision*. If two separate events happen before an eye within an interval of $\frac{1}{10}$th of a second, the eye will not be able to distinguish them as separate events but will see them as a continuous one. This happens because while the impression of the first event lingers due to persistence of vision, the impression of the second one arrives. If a glowing chip of wood be made to rotate rapidly in a circle, the eye will see a ring of fire due to persistence of vision.

In this connection, it may be pointed out that continuous picture seen in cinema-show is due to this effect.

6.4. The advantage of two eyes :

As with two ears we hear only one sound, so by our two eyes we see only one object although we fix both of our eyes on the object. But it is to be remembered that two eyes produce two different images of the object on the retina but they are mingled into one by an inexplicable process of the brain. The binocular vision is important in judging position and relative distance between two points as well as in having an idea of solidity and depth of an object. Everybody knows how difficult it is to put a thread into a needle by keeping one eye closed, because we cannot judge the relative distance with one occular vision. Also when we see an object with our two eyes, the two images that are produced on the retina are not exactly identical owing to the slight difference in the positions of the two eyes. The right eye sees more of the right side of the

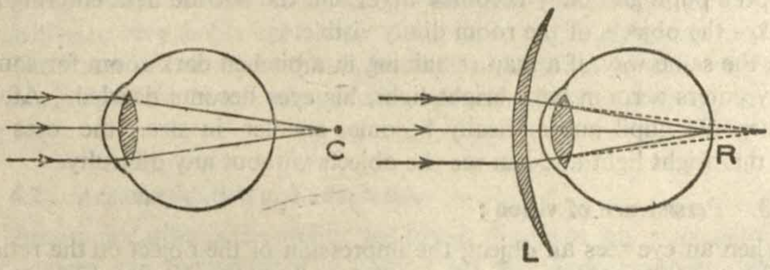
object than the left eye and vice versa and thus an idea of solidity and depth of the object is conveyed to the brain. For all these reasons, services of two eyes are essential, otherwise for mere seeing a thing, one eye would have been sufficient.

6.5. Defects of vision and their remedies :

For a normal eye, the near point is situated at a distance of 25 cm. and the far point at infinity. An object placed anywhere in this extended visual range will be visible to the eye. An eye, having a visual range less than this, is called a defective eye. The two common defects of vision are : (i) *long sight* or *hypermetropia* and (ii) *short sight* or *myopia.*

Besides these there are two more defects. They are known as (i) *astigmatism* and (ii) *presbyopia.*

(i) **Long sight :** For various reasons, the eye-ball of a person may contract in size or the focal length of eye lens may increase. As a result, parallel beam of



(a) Fig. 6.2 (b)

rays coming from a distant object tend to converge at a point, say c, behind the retina [Fig. 6.2(*a*)]. This defect of vision is called long sight. To correct this defect, a converging lens of suitable focal length be held before the eye which will add some convergence to the incoming rays before they meet the eye lens and will thus enable distant objects to be seen in good focus [Fig. 6.2(*b*)].

To see close at hand, the same eye requires the use of a converging lens of still greater power. This is explained in the Fig. 6.3. The near point N' of the long-sighted eye is away from the near-point N of the normal eye. The focal length of the converging lens L held before the eye should be such that rays coming from N after refraction through the lens, may appear to come from N' (Fig. 6.3) From the figure it is clear that for the object at N, N' will act as a virtual image produced by the lens L. If D be the least

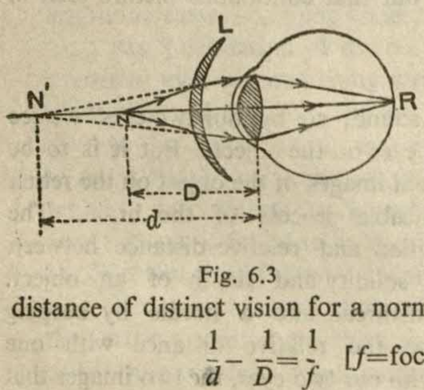


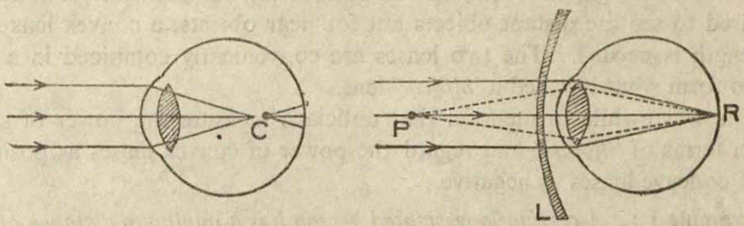
Fig. 6.3

distance of distinct vision for a normal eye and d that for the defective eye then

$$\frac{1}{d}-\frac{1}{D}=\frac{1}{f} \quad [f\text{=focal length of the lens } L]$$

$$\therefore \quad \frac{1}{f}=\frac{1}{D}-\frac{1}{d}=\frac{1}{25}-\frac{1}{d}$$

(ii) **Short sight** : A normal eye, in a relaxed condition, can see clearly a very distant object because parallel rays are brought to focus on the retina by the eye-lens. In an eye suffering from short-sight, this cannot happen. The parallel rays from a distant object after passing through the eye-lens are brought to focus at a point, C say, in front of the retina as shown in the Fig. 6.4(a). Consequently,



(a) Fig. 6.4 (b)

such an eye cannot see distant objects distinctly. This defect is called *short-sight* and it arises either from a decrease in the focal length of the crystalline lens or from an elongation of the eye ball.

To remedy this defect, a diverging lens of suitable focal length is to be used. For, due to this defect, the eye-lens becomes too much convergent and to decrease the convergence, the incoming rays are made a little divergent with the concave lens before they meet the eye lens which, then, brings the rays to focus on the retina [Fig. 6.4(b)].

It is evident from the figure that if the parallel rays from a distant object after passing through the concave lens, appear to diverge from P, the far-point of the defective eye, then the eye will see the object as if placed at P. The focal length of the lens to be used for correction of far-point is evidently equal to the distance between the eye and the defective far-point P.

(iii) **Astigmatism** : It frequently happens that the cornea acquires a greater curvature in one plane than in another. Such irregularities are called astigmatism. An eye suffering from astigmatism can not see with equal distinctness a horizontal and a vertical line placed at a certain distance away. Rays from an object in one plane will be brought to focus by the eye at a different place from rays in another plane imposing a strain on the eye. Draw a few radial lines from a centre on a card and hold it in front of the eye of a man at a certain distance away. If the man fails to see any two perpendicular lines with equal distinctness, he is suffering from astigmatism. To remedy this defect, such eye requires astigmatic spectacle lenses, *i.e.*, lenses that have more curvature in one direction than at right angles. The surfaces of such lenses are not spherical. It is rather difficult to grind and polish such surfaces. These are called *cylindrical lenses* (toric lens) whose curvature compensates for the curvature of the cornea in the particular astigmatic plane.

(iv) **Presbyopia** : As the average person grows older the crystalline lens of the eye tends to harden and the muscles that control it tend to grow weaker, thus making accommodation more and more difficult. The existence of these

conditions is referred to as presbyopia. As a result, old people feel difficulty in seeing near objects but parallel rays from distant objects are easily brought to focus on the retina by the eye lens. It is very often seen that old people, while reading a book or a news-paper, hold it at arm's length because their near-point has moved further away.

When, with age, practically all accommodation has been lost, a concave lens is required to see the distant objects but for near objects, a convex lens of short focal length is needed. The two lenses are conveniently combined in a circular frame to form what is called a '*bifocal*' lens.

It is worthwhile to mention that opticians measure the power of spectacle lenses in terms of '*dioptre*' and regard the power of convex lenses as positive and those of concave lenses as negative.

Example 1 : *A certain long sighted person has a minimum distance of distinct vision (near-point distance) of 150 cm. He wishes to read type at a distance of 25 cm. What glasses should he use ? What is its power in dioptres ?*

Ans. Since the defect is long-sight, a convex lens spectacle is necessary. The focal length of the lenses should be such as to form an image at a distance 150 cm. of an object placed at 25 cm. So, here, $u=25$ cm. and $v=150$ cm.

We know, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$; Here, $\frac{1}{150}-\frac{1}{25}=\frac{1}{f}$ or, $\frac{1}{f}=-\frac{1}{30}$

$$\therefore\ f=-30 \text{ cm.}=-\frac{30}{100} \text{ metres}=-\frac{3}{10} \text{ metres.}$$

$$\therefore \text{ Power of the lens}=+\frac{1}{\frac{3}{10}}=+\frac{10}{3}D=+3{\cdot}3D.$$

Example 2 : *A person with short-sight is able to read print only when held 15 cm. from the eye. What kind of glasses and of what focal length and power are necessary in order that print held at a distance of 25 cm. from the eye may be read clearly ?*

Ans. Since the defect is short-sight, a concave lens is needed. The focal length of the lens should be such that the print held at 25 cm. from the eye may appear to be at 15 cm.

Here, $u=+25$ cm. ; $v=+15$ cm. ; $f=$?

From the relation, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, we have, $\frac{1}{15}-\frac{1}{25}=\frac{1}{f}$. or $\frac{2}{75}=\frac{1}{f}$

$$\therefore\ f=\frac{75}{2} \text{ cm.}=\frac{75}{2\times100} \text{ metres}=\frac{3}{8} \text{ metres.}$$

$$\therefore \text{ Power of the lens}=-\frac{1}{\frac{3}{8}}=-\frac{8}{3}D=-2{\cdot}67D.$$

Example 3 : *A person can focus objects only when they lie between 50 cm. and 300 cm. from his eyes. What spectacles should he use (a) to increase his maximum distance of distinct vision to infinity (b) to reduce his least distance of distinct vision to 25 cm. ? Find this range of distinct vision using each pair.*

Ans. (*a*) To increase the maximum distance of distinct vision from 300 cm. to infinity, the person requires a diverging lens. Assuming that the lens is close to his eye, the focal length $LP=300$ cm. as P is 300 cm. from the eye [Fig. 6.4(*b*)]. One limit of the range of distinct vision is now shifted to infinity. The other limit is the object distance (u) corresponding to an image distance (v) of 50 cm. from the lens, as the person can see clearly objects 50 cm. from his eyes. In this case, $v=50$ cm. ; $f=300$ cm. From the lens equation, we have, $\frac{1}{50}-\frac{1}{u}=\frac{1}{300}$ or $\frac{1}{u}=\frac{1}{50}-\frac{1}{300}=\frac{1}{60}$ $\therefore$ $u=60$ cm.

The range of distinct vision is thus from 60 cm. to infinity.

(*b*) To reduce the least distance of distinct vision from 50 cm. to 25 cm. the person requires a converging lens [Fig. 6.3]. Assuming that the lens is close to the eye, in this case, $u=D=25$ cm. ; $v=d=50$ cm. as the image must be formed 50 cm. from the eye on the same side as the object. The focal length of the lens is given by, $\frac{1}{d}-\frac{1}{D}=-\frac{1}{f}$ or $\frac{1}{f}=\frac{1}{D}-\frac{1}{d}=\frac{1}{25}-\frac{1}{50}=\frac{1}{50}$ $\therefore$ $f=50$ cm.

Objects placed at the focus of this lens appear to come from infinity. The maximum distance of distinct vision u, is given by substituting $v=300$ cm. and $f=-50$ cm. in the lens formula.

Thus, $\frac{1}{300}-\frac{1}{u}=-\frac{1}{50}$ or $\frac{1}{u}=\frac{1}{300}+\frac{1}{50}=\frac{7}{300}$ $\therefore$ $u=\frac{300}{7}=42\frac{6}{7}$ cm.

The range of distinct vision is thus from 25 to $42\frac{6}{7}$ cm.

Example 4 : *A person with hypermetropia requires a $+1D$ spectacle lens to see objects clearly at a great distance. What power spectacles will this person require to see objects at 50 cm. ?*

Ans. Since powers of lenses are additive, a lens can be added to the one already present and the focal length of this additional lens should be such as to render the rays coming from an object 50 cm. away parallel. Clearly the lens should be convex and its focal length should be 50 cm. Hence its power $=+\frac{1}{\frac{50}{100}}=+2D$; So, the power of the spectacle lens the person should use $=+(1\frac{1}{4}+2)=+3\frac{1}{4}D$.

6.6. Photographic camera :

We know that when an object is placed at a distance more than twice the focal length of a convex lens, the lens produces a real, inverted and diminished image of the object. This property of a convex lens is applied in the working of a camera. The following are the principal parts of a camera [Fig. 6.5(*a*,)].

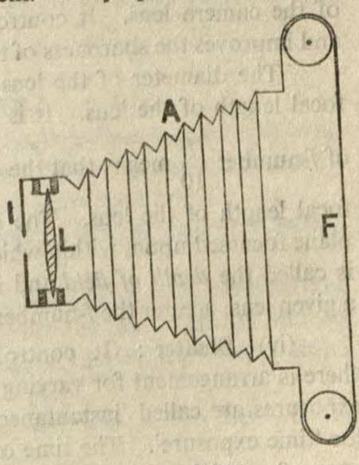


Fig. 6.5(a)

(i) **Light-proof chamber** : The chamber is made either of cloth or of leather and is painted black inside. Its length can be altered by controlling its fold (A).

(ii) **Lens** : In front of the light-proof chamber, there is the camera lens (L). It is a convex lens. In good photographic camera, a combination of lenses is used in order to remove harmful aberrations. Four typical high-quality camera lenses containing several lens components are shown in Fig 6.5(b), Each lens contains an adjustable diaphragm or stop.

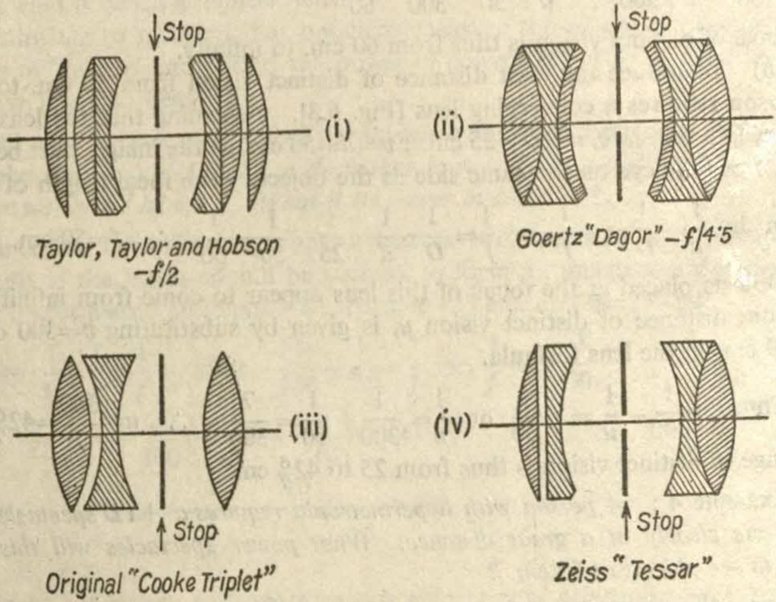


Fig. 6.5(b)

(iii) **Diaphragm** : It is an adjustable shutter (I) controlling the aperture of the camera lens. It controls the intensity of light admitted into the camera and improves the sharpness of the image.

The diameter of the lens aperture is usually expressed as a fraction of the focal length of the lens. It is known as the f-number of the lens. Thus, a lens of f-number $\frac{f}{10}$ means that the diameter of the aperture of the lens is $\frac{1}{10}$ th of the focal length of the lens. The range of distances on the near and far sides of a plane focussed upon, within which the details are imaged with acceptable sharpness is called the *depth of field* and is of particular importance in photography. For a given lens, greater the f-number, the greater is the depth of field.

(iv) **Shutter** : It controls the time of exposure. In modern camera, there is arrangement for varying the time of exposure from 0·1 to 0·01 sec. Such exposures are called 'instantaneous exposures'. There is, of course, arrangement for 'time exposure'. The time of exposure depends on the amount of light in the image formed by the camera lens which again depends on the effective aperture

of the diaphragm. If d be the diameter of the aperture and f the focal length of the lens, the time of exposure is proportional to $(f/d)^2$. The quantity, d/f is, however, called the *relative aperture.*

(v) **Screen** : There is a ground glass plate (F) behind the lens on the other end of the light proof chamber. It is called a screen. Before taking the actual photograph, the length of the chamber is adjusted until a sharp image of the object is formed on this screen.

(vi) **Plate** : It is a clear glass plate having a layer of chemical emulsion on it, which makes the plate light-sensitive.

(vii) **Slide** : It is a wooden casing, flat and light-proof, for holding the photographic plate.

Function of a camera : A sharp image of the object to be photographed is first formed on the ground-glass screen by adjusting the length of the light-proof chamber. The screen is then removed and replaced by the photographic plate or film. Light from the object is allowed to fall on the plate for a short duration depending on the intensity of the light and the relative aperture of the lens. A good photographer can easily find out the proper time of exposure, depending upon the circumstances. Light falling on the plate produces chemical reaction in the silver bromide emulsion and deposits silver at different points of the image in different densities. After this, the plate is developed and fixed in a dark room. For this purpose, the plate is dipped in a solution named 'developer'. This makes the image come out slowly and when it is distinct, the plate is washed in clear water. The plate is then dipped in another solution, called 'hypo'—it is, in reality, a solution of sodium hyposulphite—and washed in clear water.

The image so formed on the plate is a *negative.* A *positive* is printed from the negative by placing a sensitized paper in contact with the negative and exposing the paper to light. It is then dipped in developer and then in hypo. After that it is washed in clear water and dried. In this way, a photograph is prepared.

It goes without saying that photography is a very skilful work. It requires much experience in producing a good photograph.

6.7. Comparison between camera and human eye :

It has already been mentioned that a camera bears a close resemblence with human eye. As a matter of fact, human eye can be called a superfine natural camera. The points of similarity between the two are mentioned below.

(i) In a camera there is a light-proof chamber. The eye ball in a human eye serves as a light-proof chamber.

(ii) There is a convex lens in a camera which forms a real, inverted and diminished image of an object. Similarly, in eye, the cornea, aqueous humor, the crystalline lens and vitreous humor, constitute a converging system which forms a real, inverted and diminished image of an object.

(iii) The diaphragm in a camera controls the aperture of the lens. In the same way, iris in an eye controls the aperture of the lens.

(iv) The shutter regulates the time of exposure in a camera. In human eye, the eye-lid controls the time of exposure.

(v) In a camera, the image is formed on a light sensitive plate or film. In human eye, the image is received on the retina.

(vi) The distance between the lens and the plate, in a camera, is altered for proper focussing by adjusting the fold of the chamber. In an eye, proper focussing is done by the exercise of the power of accomodation.

(vii) The inside of the camera box is painted black ; the eye, similarly, has a coating of black paint, called choroid, inside it.

Visual Instruments

The instruments which assist us in seeing things properly are called visual instruments. In this respect, microscopes, telescopes, binoculars are visual instruments. The apparent size of an object depends upon the angle it subtends at the eye. The greater the angle subtended, the greater will be the apparent size of the object. This angle is called the *visual angle.* It is well known that street lamp-posts appear to be shorter the farther they are although, in fact, all are of the same height.

Visual angle : Consider an object PQ placed at some distance from the eye and suppose θ is the angle subtended (in radians) by it at the eye [Fig. 6.6(a)]. Since the opposite angles at L are equal, it follows that the length $P'Q'=a\theta$ where a is the distance from the eye-lens to the retina. As this distance is constant for an eye, $P'Q' \propto \theta$. We thus arrive at the important conclusion that *the length of the image formed by the eye is proportional to the angle subtended at the eye by the object.* This angle is known as the *visual angle* ; the greater the visual angle, the greater is the apparent size of the object.

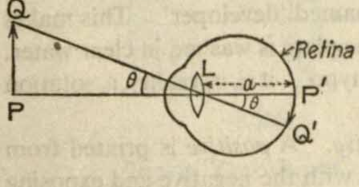
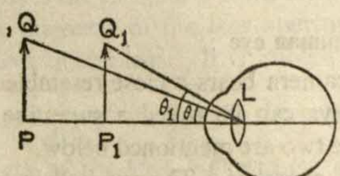


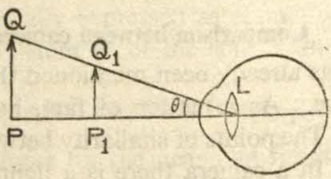
Fig. 6.6(a)

Fig. 6.6(b) illustrates what happens when the object is moved from the position P to P_1 and viewed by the eye in both positions. At P_1, the angle θ_1 subtended at the eye is greater than the visual angle θ subtended at P. Hence



(i)

Fig. 6.6(b)



(ii)

Fig. 6.6(c)

the object appears larger at P_1 than at P although its actual size is the same. Fig. 6.6(c) illustrates the case of two objects at P and P_1 respectively which subtend the same visual angle θ at the eye. The objects thus appear to be of equal size although the object at P is actually bigger than that at P_1.

The visual angle subtended by an object at the eye may be increased by bringing the object close to the eye. But there is a limit to the distance to which objects can be brought closer to the eye because we know that if the distance is less than the least distance of distinct vision, the objects become blurred. The purpose of visual instruments is to increase the apparent size of the object by increasing the visual angle and thereby to enable the eye to see the object distinctly and clearly. For this reason, these instruments are called 'aids to vision.'

6.8. Microscopes :

Microscopes are optical instruments by means of which we see magnified image of very small objects which are not ordinarily visible to the eye with clear definition. There are two types of microscopes, *viz.*, (i) *Simple microscope* or *magnifying glass* or *reading glass* and (ii) *Compound microscope.*

(i) **Simple microscope** : Small objects—like small types, small prints etc.—which are not clearly visible to the eye, may be seen distinctly with the help

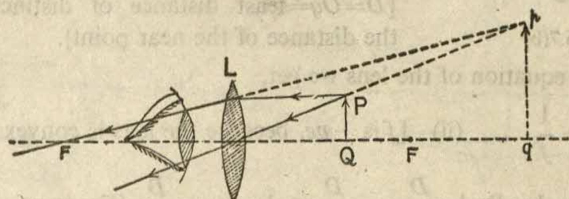


Fig. 6.7(a)

of a simple microscope or a magnifying glass. It consists of a single convex lens of short focal length suitably mounted in a holder.

Action : Suppose PQ is a small object to be magnified by the simple microscope [Fig. 6.7(a)].

A short-focus convex lens is so placed that the object PQ falls within the focal length of the lens. As shown in the figure, the lens, L forms a virtual, erect and magnified image pq. Placing eye on the other side of the lens, this magnified image will be seen instead of the object. If the distance of the lens from the object is so adjusted that the image pq is formed at the least distance of distinct vision, the eye will see the magnified image clearly, without any strain on the eye.

When an object PQ is seen through a convex lens acting as a *magnifying glass,* various coloured virtual images are formed. As each coloured image subtends the same angle at the eye close to the lens the colours received by the eye practically overlap. Thus, the virtual image seen in a magnifying glass is almost free from chromatic aberration.

Magnification : In the case of optical instruments, we are usually concerned with apparent sizes of an object and its image which are determined by the visual angles subtended at the eye by the object and the image respectively. The magnification is, therefore, angular magnification and it is defined as the ratio :

$$\frac{\text{Angle subtended at the eye by the image}}{\text{Angle subtended at the eye by the object placed at the position of the image.}}$$

Now, in a simple microscope, the image is formed at the least distance of distinct vision. So, in this case, the magnification,

$$m=\frac{\text{Angle subtended at the eye by the image formed at the near point}}{\text{Angle subtended at the eye by the object placed at the near point}}$$

Since the eye is placed very near to the lens, the magnification is expressed by,

$$m=\frac{\text{Angle subtended at the lens by the image formed at the near point}}{\text{Angle subtended at the lens by the object placed at the near point}}$$

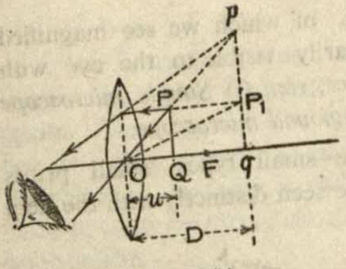


Fig. 6.7(b)

$$=\frac{\angle pOq}{\angle P_1Oq} \quad \text{[Fig. 6.7(}b\text{)]}$$

$$=\frac{\tan \angle pOq}{\tan \angle P_1Oq} \quad \text{[Since angles are small]}$$

$$=\frac{pq}{Oq} \div \frac{P_1q}{Oq}=\frac{pq}{P_1q}=\frac{pq}{PQ}=\frac{Oq}{OQ}=\frac{D}{u} \quad .. \quad \text{(i)}$$

[$D=Oq$=least distance of distinct vision *i.e.* the distance of the near point].

From the equation of the lens we get,

$$\frac{1}{D}-\frac{1}{u}=-\frac{1}{f} \quad .. \quad \text{(ii)} \quad [f \text{ is } -ve. \text{ because the lens is convex.}]$$

Multiplying by D, $1-\frac{D}{u}=-\frac{D}{f}$ or, $1-m=-\frac{D}{f}$ [From eqn.... (i)]

$$\therefore \quad m=1+\frac{D}{f} \quad .. \quad \text{(iii)}$$

From this equation, it is clear that smaller the focal length of the lens, the greater is the magnification. For a normal eye, the distance of the near point $D=25$ cm. $\therefore \quad m=1+\frac{25}{f}$.

If the image is formed at infinity $v=\infty$. From eqn (ii) We get

$$-\frac{1}{u}=-\frac{1}{f} \text{ or } \frac{D}{u}=\frac{D}{f} \quad \therefore \quad m=\frac{D}{f} \quad .. \quad \text{(iv)}$$

So, according to the position of the image, the magnification may have any value between $\left(1+\frac{D}{f}\right)$ and $\frac{D}{f}$.

Example : *A magnifying glass of focal length 2 cm. is used to form an image of a small object at the least distance of distinct vision (=25 cm.). Find the magnification produced and the position where the object is to be placed.*

Ans. For a magnifying glass, $m=1+\frac{D}{f}$; here $f=2$ cm. and $D=25$ cm.

$$\therefore \quad m=1+\frac{25}{2}=13{\cdot}5$$

Further, from the lens equation, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ we have $v=+25$ (image is virtual)

and $f=-2$ cm. (lens is convex). So,

$$\frac{1}{25}-\frac{1}{u}=-\frac{1}{2} \text{ or } \frac{1}{u}=\frac{1}{25}+\frac{1}{2}=\frac{27}{50} \quad \therefore \quad u=\frac{50}{27}=1{\cdot}85 \text{ cm.}$$

The object should be placed 1·85 cm. from the lens.

(ii) **Compound microscope** : If the object under examination is very small, the simple microscope cannot magnify it sufficiently. A more powerful instrument is necessary. A compound microscope can serve the purpose.

Description : It is a combination of two converging lens systems which are fitted co-axially in a tube. The lens (O), nearer the object under examination, is called the **objective** which is of very short focal length. The other lens (E)

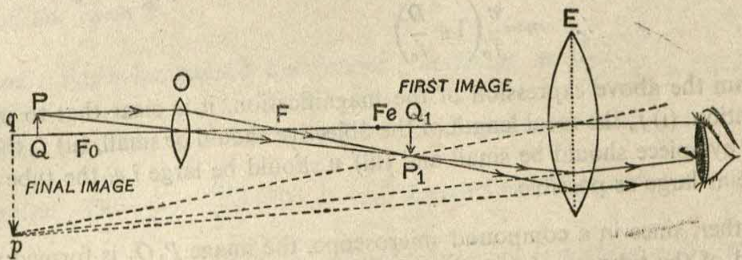


Fig. 6.8

behind which the eye is placed, is called the **eye-piece** which is of moderately short focal length [Fig. 6.8). With the help of a screw provided with the tube, the eye piece can be moved towards or away from the objective.

Action : The object PQ to be examined is brought up close to the objective and placed just beyond the focal point F_0 of the objective which forms a real, inverted and magnified image P_1Q_1 of it inside the tube. This image is not received on a screen but merely exists in space. The eye-piece is now moved with the help of the focussing screw and is placed in such a position that the image P_1Q_1 falls within its focal length. The eye-piece, in this case, acts as a simple microscope and produces an enlarged, inverted and virtual image pq at the near-point of the eye. The eye, placed behind the eye-piece can see distinctly the enlarged image pq of the object PQ.

Magnification : In this instrument, the magnification takes place in two stages—first by the objective and then by the eye-piece. If their magnifications are respectively m_1 and m_2, then the magnification m of the instrument is given by, $m=m_1 \times m_2$.

Now, $m_1=\frac{v}{u}$, where v=the distance of the image P_1Q_1 from the objective O, and u=the distance of the object PQ from the same lens,

It has been pointed out that the eye-piece, in the present case, behaves like a simple microscope. Hence its magnification m_2 is given by, $m_2=1+\frac{D}{f_e}$

where f_e=focal length of the eye-piece. $\therefore\ m=\frac{v}{u}\left(1+\frac{D}{f_e}\right)$

Now, applying the lens equation to the objective, we have,

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f_0}\quad [f_0=\text{focal length of the objective}]$$

$$\text{or,}\quad \frac{v}{u}=\frac{v}{f_0}-1\quad \therefore\ m=\left(\frac{v}{f_0}-1\right)\left(1+\frac{D}{f_e}\right)$$

Again, since f_0 is small compared to v, we can write $\frac{v}{f_0}-1=\frac{v}{f_e}$

$$\therefore\ m=\frac{v}{f_0}\left(1+\frac{D}{f_e}\right)$$

From the above expression of the magnification, it is clear that to increase magnification, (i) f_0 the focal length of the objective should be small, (ii) f_e the focal length of eye-piece should be small and (iii) v should be large *i.e.* the tube length should be as large as possible.

Further, since in a compound microscope, the image P_1Q_1 is formed almost at the end of the tube, $v=L$, where L is the tube length. Also, since f_e is much smaller than D, $1+\frac{D}{f_e}=\frac{D}{f_e}$ (nearly). Consequently, $m=\frac{LD}{f_0f_e}$

This shows that the magnification increases with the increase of the tube length L.

The objective or the eye-piece of a compound microscope is never made of a single convex lens. In order to remove spherical and chromatic aberrations as well as to increase the illumination of the final image, the objective and the eye-piece are always made of a combination of several lenses.

Example 1 : *A compound microscope consists of an objective lens of 1 cm. focal length and an eye-piece of 2·5 cm. focal length. What is the distance between the lenses and what is the magnification if the object is in sharp focus when it is 1·05 cm. from the objective ?*

Ans. For the objective, u=1·05 cm. ; $f=-1$ cm.

we know $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ or, $\frac{1}{v}-\frac{1}{1\cdot05}=-1\quad\therefore\ v=-21$ cm.

i.e. the image is formed on the other side at a distance 21 cm. from the objective. For the eye-piece, v=25 cm. (near point) ; $f=-2\cdot5$ cm.

Again, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ or, $\frac{1}{25}-\frac{1}{u}=-\frac{1}{2\cdot 5}$ $\therefore$ $u=2\cdot 27$ cm.

So, the distance between the lenses$=21+2\cdot 27=23\cdot 27$ cm.

Again, magnification produced by the objective $m_1=\frac{v}{u}=\frac{21}{1\cdot 05}$

and magnification produced by the eye-piece $m_2=1+\frac{D}{f_e}=1+\frac{25}{2\cdot 5}$

$\therefore$ total magnification, $m=\frac{21}{1\cdot 05}\left(1+\frac{25}{2\cdot 5}\right)=220$

Example 2 : *A compound microscope has an objective of focal length 0·5 cm. and an eye-piece of focal length 2·5 cm. If the image is 25 cm. from the eye-piece and the magnification is 330 what is the approximate distance between the centres of the lenses ?*

Ans. For a compound microscope, we know, $m=\frac{L}{f_0}\cdot\frac{D}{f_e}$

Here, $m=330$; $f_0=0\cdot 5$ cm. $f_e=2\cdot 5$ cm. ; $D=25$ cm.

Hence, $330=\frac{L}{0\cdot 5}\times\frac{25}{2\cdot 5}=20\times L$ $\therefore$ $L=16\cdot 5$ cm.

Example 3 : *A model of a compound microscope is made up of two converging lenses of 4 cm. and 12 cm. focal length at a fixed separation of 30 cm. Where must the object be placed so that the final image may be at infinity ? What will be the magnifying power if the microscope is used by a person whose least distance of distinct vision is 25 cm. ?*

Ans. If the final image is at infinity, the image P_1Q_1 formed by the objective O (Fig. 6.8) should be at the focus of the eye-piece E *i.e.* at a distance of 12 cm. from the eye-piece. Hence, the distance of P_1Q_1 from the objective $O=30-12=18$ cm.

Considering the action of the objective, we have, $v=-18$ cm. ; $f=-4$ cm.; $u=$? Now from lens equation $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ we have, $-\frac{1}{18}-\frac{1}{u}=-\frac{1}{4}$

$$\text{or } \frac{1}{u}=\frac{1}{4}-\frac{1}{18}=\frac{7}{36} \therefore u=\frac{36}{7}=5\cdot 14 \text{ cm.}$$

Now, magnification, $m=\frac{v}{f_0}\left(1+\frac{D}{f_e}\right)$ [see page 150]

Here $v=18$ cm. ; $f_0=4$ cm. ; $f_e=12$ cm. and $D=25$ cm.

Hence, $m=\frac{18}{4}\left(1+\frac{25}{12}\right)=13\cdot 9$ (nearly).

6.9. Astronomical Telescope :

It is an instrument used for viewing distant objects like stars, planets etc.

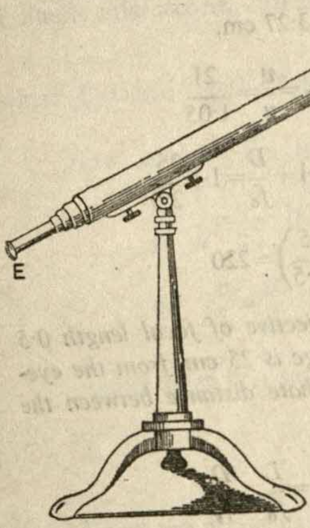


Fig. 6.9

Fig. 6.9 shows a section of the instrument. Like a compound microscope, it consists of an objective and an eye-piece fitted co-axially in a brass tube. The convex lens O is the objective which is turned towards the object under examination. It is a converging system of very long focal length. The eye-piece is also a convex lens E behind which the eye is placed. It has comparatively a small focal length. The eye-piece is movable, being fitted in a draw-tube which can be moved inside the main tube.

Action : For a very distant object, *e.g.* a planet, which is effectively at infinity, rays coming from any point on it are sensibly parallel on reaching the telescope. They are incident on the objective O slightly inclined to the axis and after refraction, they form image FP in the focal plane of the objective. The image FP acts as an object for the eye-piece. If the eye-piece is so adjusted that its distance

Fig. 6.10(a)

from the image FP is equal to its focal length, the emergent rays will be parallel [Fig. 6.10(a)]. A virtual, greatly magnified and inverted image is formed at infinity. This process of focussing the telescope is known as the *focussing for infinity.*

For *ordinary vision* i.e. for near-point focussing, the final image is to be formed at the least distance of distinct vision. For this purpose, the eye-piece is pushed a little inside *i.e.* towards the objective so that the image P_1Q [Fig. 6.10(b)] formed by the objective falls within the focal length of the eye-piece and the eye-piece behaving like a simple microscrope, produces a magnified, virtual and inverted image pq at the least distance of distinct vision.

Because of the optical unsteadiness of the atmosphere, magnification of more than about 1500—2000 are seldom used in astronomy. Even more important to astronomers is the *light gathering* power than magnification of a telescope which

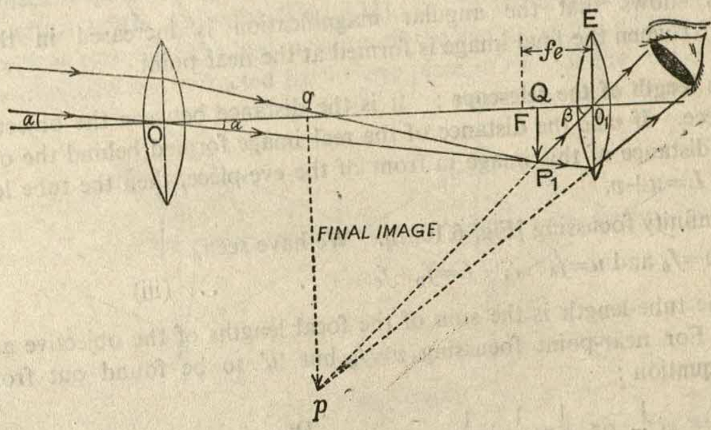


Fig. 6.10(b)

determines how faint a star can be and still to be seen. This depends on the area of the objective and is one reason for making telescopes of large diameter.

Magnification : For any visual instrument, magnification means angular magnification. In the case of a telescope focussed for infinity, [Fig. 6.10(a)]

$$m=\frac{\text{the angle }(\beta)\text{ subtended at the eye by the final image}}{\text{the angle }(\alpha)\text{ subtended at the eye by the object}}$$

Since the object is far away from the instrument, the angle it subtends at the eye is same as that subtended by it at the objective.

$$\therefore \quad m=\frac{\angle FEP}{\angle FOP}=\frac{\tan\angle FEP}{\tan\angle FOP}=\frac{FP}{EF}\div\frac{FP}{OF}=\frac{OF}{EF}=\frac{f_0}{f_e} \quad .. \quad \text{(i)}$$

[f_0=focal length of the objective : f_e=focal length of the eye-piece.]

So, to obtain high magnification, the objective lens should have a long focal length f_0 and the eye-piece a short one, f_e. In this respect, a telescope differs from a compound microscope.

For a telescope focussed for near-point [Fig. 6.10(b)].

$$m=\frac{\beta}{\alpha}=\frac{\tan\beta}{\tan\alpha}=\frac{P_1Q}{O_1Q}\div\frac{P_1Q}{OQ}=\frac{P_1Q}{O_1Q}\times\frac{f_0}{P_1Q}=\frac{f_0}{O_1Q}$$

Now for the eye-piece, P_1Q is the object and pq is the image formed at the near-point ($=D$), So, $\frac{1}{O_1q}-\frac{1}{O_1Q}=-\frac{1}{f_e}$

$$\text{or,} \quad \frac{1}{O_1Q}=\frac{1}{f_e}+\frac{1}{O_1q}=\frac{1}{f_e}+\frac{1}{D} \quad [\because \; O_1q=D]$$

$$\therefore \quad m=f_0\left(\frac{1}{f_e}+\frac{1}{D}\right)=\frac{f_0}{f_e}\left(1+\frac{f_e}{D}\right) \quad .. \quad .. \quad \text{(ii)}$$

For infinity focussing $O_1q=\infty$; because the image is formed at infinity. So, from eqn. (ii), we get. $m=\frac{f_0}{f_e}$ which is the eqn. (i).

This shows that the angular magnification is increased in the ratio $(1+f_e/D):1$ when the final image is formed at the near point.

Tube length of the telescope : It is the distance between the objective and the eye-piece. If v be the distance of the real image formed behind the objective and u the distance of this image in front of the eye-piece, then the tube length L is given by $L=u+v$.

For infinity focussing [Fig. 6.10(*a*)]. We have seen,

$$v=f_0 \text{ and } u=f_e \quad \therefore \quad L=f_0+f_e \quad \ldots \quad \ldots \quad \text{(iii)}$$

i.e. the tube-length is the sum of the focal lengths of the objective and the eye-piece. For near-point focussing $v=f_0$ but 'u' to be found out from the following equation :

$$\frac{1}{D}-\frac{1}{u}=-\frac{1}{f_e} \text{ or, } \frac{1}{u}=\frac{1}{D}+\frac{1}{f_e} \text{ or, } u=\frac{Df_e}{D+f_e}$$

$$\therefore \quad L=f_0+u=f_0+\frac{Df_e}{D+f_e} \quad \ldots \quad \ldots \quad \text{(iv)}$$

Example 1 : *An astronomical telescope consists of an objective of focal length 180 cm. and an eye-piece of focal length 3 cm. Calculate the (i) magnifying power and (ii) the tube length of the instrument when the instrument is focussed for infinity.*

Ans. For infinity focussing, $m=\frac{f_0}{f_e}=\frac{180}{3}=60$ [from eqn. (iii)] and tube length, $L=f_0+f_e=180+3=183$ cm. [from eqn. (ii)]

Example 2 : *A telescope has an objective of focal length 50 cm. and an eye-piece of focal length 5 cm. The least distance of distinct vision is 25 cm. The telescope is focussed for distinct vision on a scale 200 cm. away from the objective. Calculate (i) the separation between the objective and the eye-piece (ii) the magnification produced.* [*I.I.T. 1980*]

Ans. When the telescope is focussed for distinct visions the magnification, according to eqn. (ii) art 6.9 is given by

$$m=\frac{f_0}{f_e}\left(1+\frac{f_e}{D}\right)=\frac{50}{5}\left(1+\frac{5}{25}\right)=\frac{50}{5}\times\frac{6}{5}=12$$

The length of the tube $L.=f_0+\frac{D.f_e}{D+f_e}$ [eqn (iv) of art 6.9]

$$=50+\frac{25\times5}{25+5}=50+\frac{25}{6}=54\tfrac{1}{6} \text{ cm.}$$

6.10. Eye ring and its relation to angular magnification :

Suppose AB and CD are the diameters of the apertures of the objective and the eye-piece respectively of a telescope. Here, the eye-piece will form an image of the aperture of the objective [Fig. 6.11]. In the figure EF is the image. From the figure, it is clear that the best position of the eye to receive all the rays coming from the objective and refracted by the eye-piece is EF. If the size of the image

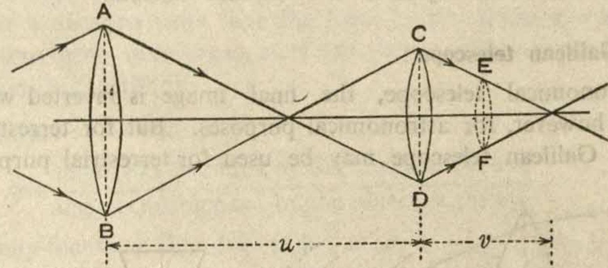


Fig. 6.11

EF happens to be equal to the size of the eye-pupil, then all the rays will enter into the eye and make the image brightest. EF is called the *exit pupil* of the telescope. A metallic ring, having diameter equal to EF, is usually placed at the position of EF. This metallic ring is known as the *eye-ring*. The diameter AB of the aperture of the objective is known as the *entrance pupil* of the telescope.

When the telescope is focussed for infinity, we have seen that the magnification, $m=\frac{f_0}{f_e}=\frac{\text{focal length of the objective}}{\text{,, ,, ,, ,, eye-piece}}$

Now suppose u=the distance of the objective from the eye-piece

and v= ,, ,, ,, ,, image EF of the objective from the eye-piece.

Then, for the real image EF, we can write $\frac{1}{v}+\frac{1}{u}=\frac{1}{f_e}$

But, when the instrument is focussed for inifinity, $u=L=f_0+f_e$

So, $\frac{1}{v}+\frac{1}{f_0+f_e}=\frac{1}{f_e}$ or $\frac{1}{v}=\frac{1}{f_e}-\frac{1}{f_0+f_e}=\frac{f_0}{f_e(f_0+f_e)}$ $\therefore$ $v=\frac{f_e(f_0+f_e)}{f_0}$

Again, suppose D=diameter of the aperture of the objective (AB)

d= ,, ,, ,, ,, ,, ,, eye-ring (EF)

Then $\frac{D}{d}=\frac{u}{v}=\frac{(f_0+f_e)\cdot f_0}{f_e(f_0+f_e)}=\frac{f_0}{f_e}=m$

So, we can write, magnification $m=\frac{D}{d}=\frac{\text{diameter of the objective}}{\text{,, ,, ,, eye-ring}}$

Example : *The focal lengths of the objective and the eye-piece of a telescope focussed for infinity are 60 cm. and 6 cm. respectively. If the eye of an observer is supposed to be very near the eye-piece and the diameter of eye pupil be 2·8 mm, what should be the diameter of the objective lens in order that all the rays coming*

from the objective may enter the eye ? Suppose the instrument is focussed towards a distant star.

Ans. Since the instrument is focussed for infinity, $m=\frac{f_0}{f_e}=\frac{60}{6}=10.$

Again $m=\frac{\text{Diameter of the objective}}{\text{,, ,, ,, eye-ring}}$ or $10=\frac{\text{Diameter of the objective}}{2{\cdot}8}$

or Diameter of the objective$=2{\cdot}8\times10$ mm$=2{\cdot}8$ cm.

6.11. Galilean telescope :

In astronomical telescope, the final image is inverted which is of no consequence, however, for astronomical purposes. But for terrestrial use, it is not suitable. Galilean telescope may be used for terrestrial purposes. It was

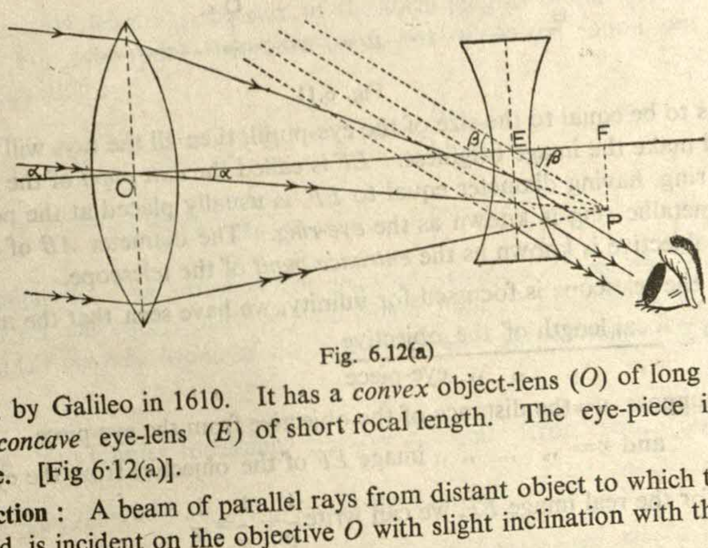


Fig. 6.12(a)

invented by Galileo in 1610. It has a *convex* object-lens (*O*) of long focal length and a *concave* eye-lens (*E*) of short focal length. The eye-piece is, as usual, movable. [Fig 6·12(a)].

Action : A beam of parallel rays from distant object to which the telescope is turned, is incident on the objective *O* with slight inclination with the axis. [Fig.

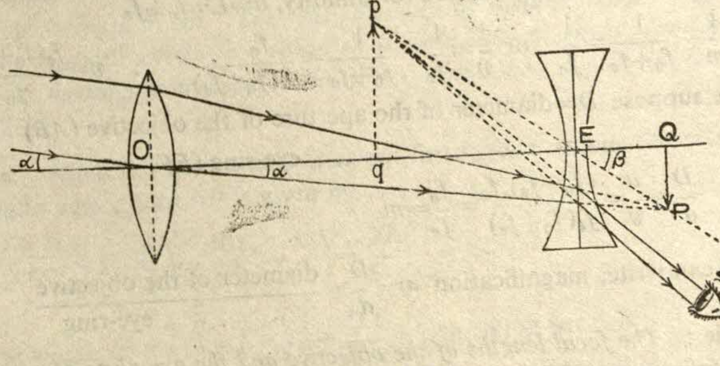


Fig. 6.12(b)

6.12(*a*)]. After refraction, the rays try to form a real image *FP* on the focal plane of the objective but before they do so, they are intercepted by the eye-piece *E*.

Here FP may be regarded as a virtual object for the eye-piece. If the eye-piece be so adjusted that it is at a distance from FP equal to its focal length, then the emergent rays will be parallel and a greatly magnified image will be formed at infinity. This sort of focussing is evidently known as the focussing for infinity.

For near-point focussing, however, the eye-piece is pushed a little towards the objective so that the image PQ formed by the objective (virtual object for the eye-piece) is at a distance more than the focal length of the eye-piece. In this case, the eye-piece forms an enlarged, erect and virtual image pq at the least distance of distinct vision [Fig. 6.12 (*b*)].

Magnification : As before

$$m=\frac{\text{angle }(\beta)\text{ subtended by the image at the eye}}{\text{angle }(\alpha)\text{ subtended by the object at the eye}}$$

For infinity focussing [Fig. 6.12 (a)].

$$m=\frac{\beta}{\alpha}=\frac{\angle PEF}{\angle POF}=\frac{\tan\angle PEF}{\tan\angle POF}=\frac{OF}{EF}=\frac{f_0}{f_e}$$

In this case, the tube length $L=OF-EF=f_0-f_e$

For near-point focussing, [Fig. 6.12 (b)].

$$m=\frac{\beta}{\alpha}=\frac{\tan\beta}{\tan\alpha}=\frac{QP}{EQ}\div\frac{QP}{OQ}=\frac{OQ}{EQ}=\frac{f_0}{EQ}$$

Now, for the eye-piece, QP is the virtual object and pq its real image.

Applying lens equation, we get, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

or, $\frac{1}{Eq}-\frac{1}{-EQ}=\frac{1}{f_e}$ or, $\frac{1}{D}+\frac{1}{EQ}=\frac{1}{f_e}$ [$\because$ $Eq=D$] $\therefore$ $\frac{1}{EQ}=\frac{1}{f_e}-\frac{1}{D}$

So, $m=f_0\left(\frac{1}{f_e}-\frac{1}{D}\right)=\frac{f_0}{f_e}\left(1-\frac{f_e}{D}\right)$

Here, the tube-length $L=OQ-EQ=f_0-\frac{f_e.D}{D-f_e}$

Example : *A galilean telescope has an object glass of 12 cm. focal length and an eye-lens of 5 cm focal length. It is focussed on a distant object so that the final image seen by the eye appears to be situated at a distance of 30 cm. from the eye lens. Determine the magnification produced.*

Ans. For a Galilean telescope focussed for near point, the magnification m is given by, $m=\frac{f_0}{f_e}\left(1-\frac{f_e}{D}\right)$; Here, $f_0=12$ cm. ; $f_e=5$ cm. and $D=30$ cm ;

hence $m=\frac{12}{5}\left(1-\frac{5}{30}\right)=\frac{12}{5}\times\frac{5}{6}=2.$

6.12. Comparative study of astronomical and Galilean telescopes :

Similarities : (i) Both the telescopes may be used for viewing distant objects. (ii) Both of them have an objective and an eyepiece. (iii) The focal length and aperture of the objective in both cases are larger than those of the eye-piece. (iv) Both of them are refracting telescopes, (v) Both the instruments have the same expression for their magnification.

Dissimilarities : (i) The eye-piece in an astronomical telescope is a convex lens whereas the eye-piece of a Galilean telescope is a concave lens, (ii) For infinity focussing, the tube-length of a Galilean telescope is (f_0-f_e) and that for an astronomical telescope is (f_0+f_e). So, Galilean telescopes have shorter length than astronomical telescopes and are therefore, more handy. (iii) Galilean telescope gives erect image but astronomical telescope gives inverted image. For this reason, Galilean telescope is more suitable than astronomical telescope for terrestrial uses. (iv) The magnifying power and the field of view are very limited in the case of a Galilean telescope than in the case of an astronomical telescope. (v) Cross-wires cannot be fitted in a Galilean telescope. So, no measurement is possible with such telescope whereas astronomical telescopes can be fitted with crosswires. (vi) Common defects observed in an image such as chromatic and spherical aberrations can be remedied by using a compound eye-piece in an astronomical telescope ; but no such eye-piece can be used in a Galilean telescope. Consequently the image seen through a Galilean telescope is not free from the aberrations.

6.13. Terrestrial telescope :

In an ordinary astronomical telescope, the final image is inverted with respect to the object. If it is used for any terrestrial purpose, the object will appear inverted through the instrument. This, undoubtedly, is very inconvenient. All heavenly bodies being spherical, no such inconvenience is caused due to the image being inverted because spherical objects when inverted, remains spherical. An astronomical telescope with arrangement for erecting the image gives rise to the terrestrial telescope. Fig 6.13 with the ray-diagram shows how the image is made erect.

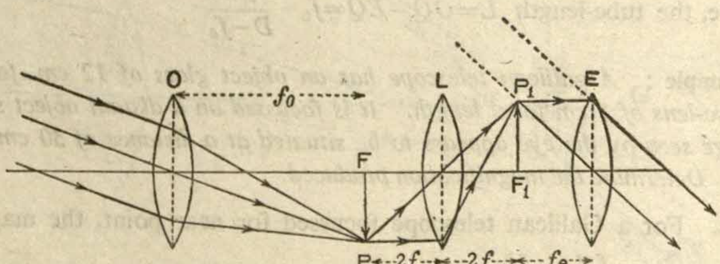


Fig. 6.13

In this case, the objective O forms an image PF of the distant object at its focal plane, the image being inverted. An additional convex lens L is introduced and situated from PF at a distance twice its own focal length $(2f)$. We know that in

such a case, the lens L will produce an image P_1F_1 which is erect and of the same size as PF and is formed at an equal distance on the other side of the lens L. This lens L is called the *erecting lens*. The eye-piece E is now adjusted to give a virtual, erect image at the least distance of distinct vision. The final image is thus erect with respect to the object and the instrument may, therefore, be used for terrestrial purpose.

It is to be noted that the function of the erecting lens is simply to erect the inverted image PF. As FP and F_1P_1 are of equal size, the lens L does not add anything to the magnification of the instrument, which is f_0/f_e as in the case of an ordinary astronomical telescope. But due to absorption and reflection of light by the erecting lens, some loss of light takes place which affects the illumination of the final image. Further, due to the introduction of the erecting lens, the tube length of the telescope is increased by four times the focal length of the erecting lens. The tube length of a terrestrial telescope$=f_0+f_e+4f$ but that for an astronomical telescope$=f_0+f_e$.

6.14. Reflecting telescopes :

A telescope in which a *mirror* is used as the *objective* is called a reflecting telescope. The earliest reflecting telescope was invented by Gregory in 1663. The most common type of reflecting telescopes are due to Newton and Cassegrain.

(a) **Newtonian reflecting telescope :** It was invented by Sir Issac Newton in 1668. It consists of a curved parabolic mirror C [Fig. 6.14] and a total reflecting prism M held at 45° with the axis of the parabolic mirror. When a parallel pencil of rays from a distant object is incident along the axis of the telescope, they are reflected by the parabolic mirror C and the reflected rays tend to converge to F which is the focus of the parabolic mirror. But before actual convergence, they are intercepted by the prism M which deflects the rays through 90° and the rays come to a focus at F_1.

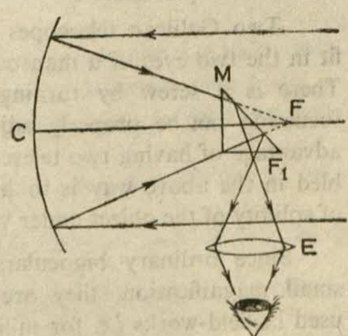


Fig 6.14

The image formed at F_1 is viewed by the eye-piece E mounted in a small side tube. The eye-piece is so adjusted that F_1 coincides with the focus of the eye-piece. Consequently, the rays coming from F_1 after passing through the eye-piece emerge as a parallel beam. Therefore, a magnified image of the distant object is formed at infinity. This sort of focussing is known as focussing for infinity. For ordinary vision, the eye-piece is pushed a little towards F_1.

The magnification of the telescope, as in the case of a refracting telescope, is obtained from the ratio of the focal length of the parabolic mirror to the focal length of the eye-piece.

(b) **Cassegrain's reflecting telescope :**

The difference between a Newtonian telescope and a Cassegrain telescope is that the total reflecting prism M of the former is replaced by a convex mirror in the latter. Rays from a distant object after reflection from a large concave mirror C fall on the convex mirror M and are allowed to converge at I. The final image is formed at I which can be seen through the eye-piece E, rays passing through a small hole in the concave mirror. The 200 inch Mount Polomar telescope is a Cassegrain type of reflecting telescope.

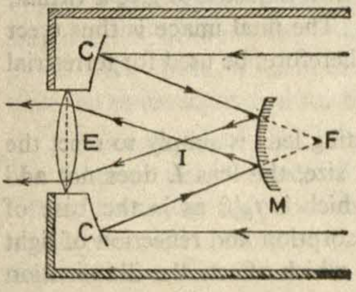


Fig. 6.15

6.15. Advantages of a reflecting telescope over a refracting one :

(i) Reflecting telescopes are free from both spherical and chromatic aberrations. Because the mirror is parabolic, spherical aberration is absent and because there is no refraction taking place, the question of chromatic aberration does not come in.

(ii) Mounting and manufacture of mirrors are easier than those of lenses of corresponding wide aperture.

(iii) There is some difficulty of obtaining a large mass of glass free from imperfections for the purpose of making an objective ; but for a reflecting telescope, the problem is a simple one because it needs only a perfect reflecting surface which is easily available.

6.16. Binoculars :

Two Galilean telescopes mounted together with their axes parallel just to fit in the two eyes of a man constitute an opera glass or a binocular (Fig. 6.16). There is a screw by turning which the focussing can be properly adjusted. The advantage of having two telescopes assembled in the above way is to have an idea of solidity of the object under view.

Since ordinary binoculars have but small magnification, they are no longer used in field-works *i.e.* for military operations, hunting expedition etc, for which prism-binoculars are used instead. In a prism binocular, rays coming from the objective are allowed to fall on an erecting prism which reflects the rays so that the rays travel the entire length of the tube. They are again reflected by a second erecting prism and finally pass through the eye-piece. Since, the rays travel the length of the tube three times, an objective of long focal length can be used. This gives a much higher magnification than for the same length of the tube without prisms.

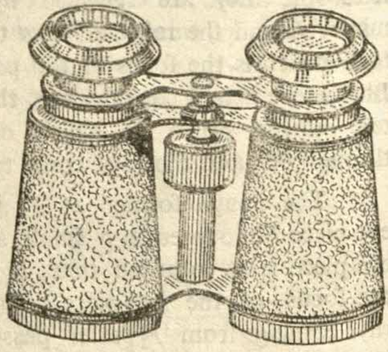


Fig. 6.16

Exerciscs

Essay type :

1. Write a short note on human eye. What is accomodation of the eye ?

2. What are the common defects of vision ? Explain how the defects can be removed by spectacles ?

3. What are short sight and long sight ? What are the reasons of these defects ? How can they be remedied ?

4. Describe the different parts of a photographic camera. Compare and contrast the optical arrangements of human eye with those of a photographic camera. [*Jt. Entrance 1983*]

5. How can you construct a simple microscope by means of a single convex lens ? What is meant by its magnification ?

6. Describe a compound microscope. Explain, with the help of a neat diagram, how it produces a magnified image. Deduce an expression for its magnification. [*H.S. Exam 1980,'82*]

7. Prove that the magnification produced by a compound microscope is proportional to the tube length.

8. Describe an astronomical telescope. Drawing a neat diagram, explain how (i) the objective and (ii) the eye-piece form an image of a distant object. [*H. S. Exam. 1979*]

9. What is the eye-ring of a telescope ? Find the size and the position of the eye-ring in terms of the diameter of the objective and the focal lengths of the objective and the eye-piece when the telescope is focussed for infinity.

10. What is a Galilean telescope ? Explain its action with the help of a ray diagram and obtain an expression for its magnification.

11. Describe a binocular. What are its uses ?

12. State the difference in behaviour between an astronomical telescope and a Galilean telescope. What is the difference between a Galilean telescope and a binocular ?

Short answer type :

13. What is the advantage of binocular vision ? What does persistence of vision mean ?

14. What do you mean by least distance of distinct vision ? What is its value ?

15. What do you mean by (i) accomodation (ii) adaptation (iii) near point and (iv) far point ?

16. What is the most common type of defect of vision that old people suffer from ? What is the remedy of this defect ?

17. (i) Street lamp-posts appear to be shorter the farther they are away, although all are of same height. Why does it happen so ?

(ii) When the lights of a brightly lit room are suddenly put out, the people of the room are blinded for a moment. Why ?

18. A person wears bifocal converging spectacles, one surface of each lens being spherical and the other cylindrical. State the defects in his vision and explain how the spectacles correct them.

19. Answer the following questions briefly : (*a*) How will you arrange two convex lenses of focal lengths 5 cm. and 30 cm. to form an astronomical telescope ? (*b*) If it forms a clear image of a star, what will be the distance between the lenses ? (*c*) Why is the aperture of the objective lens greater than that of the eye-piece ? (*d*) What change in the image will take place if half of the aperture of the objective lens is covered by a sheet of opaque paper ? (*e*) Which lens of an astronomical telescope forms a real image and which lens a virtual image ? (*f*) Why is the final image inverted ?

Objective type :

20. Write *W* for the incorrect statements and *R* for the correct ones :

(*a*) The cones of human eye respond to strong light and cause sensation of colour and colour difference while the rods are sensitive to feeble light and movement of objects. ———

(*b*) Concave lenses are used to remedy long sight defect and convex lens for short sight defect. ———

(*c*) The focal length of the eye-piece of an astronomical telescope is longer than that of the objective. ———

(*d*) Galilean telescopes are used in binoculars because they form erect images. ———

(*e*) The length of the image of an object formed by the eye is proportional to the visual angle subtended at the eye by the object. ———

Numerical Problems :

21. A short-sight man can read printed matter distinctly when it is held at 15 cm. from his eyes. Find the focal length of the glasses which he must use if he wishes to read with ease a book at distance of 60 cm. [Ans. Concave lens, 20 cm.]

22. The near-point of a person is at a distance of 200 cm. from his eyes. What spectacle should he use in order to read a print 20 cm. way ? [Ans. Convex lens ; $f=22{\cdot}2$ cm.]

23. The near-point of a person suffering from long-sight is 100 cm. away but the far-point is normal. What type of spectacle should he use to normalise his near-point ? Where will be the far-point situated ? [Ans. Convex ; $f=33\frac{1}{3}$ cm.]

24. A boy, suffering from long sight, uses spectacles of $+1{\cdot}75$ *D* power to see distant objects. What spectacles should he use to read books at a distance of 40 cm. ? [Ans. $+4{\cdot}25$ *D*]

25. A certain person can see clearly objects at distances between 20 cm. and 200 cm. from his eyes. What spectacles are required to enable him to see distant objects clearly and what will be his least distance of distinct vision when he is wearing them ?
[Ans. Concave ; $f=200$ cm. ; 22·22 cm.]

26. In order to correct his near point to 25 cm. a man is given spectacles with converging lenses of 50 cm. focal length and to correct his far point to infinity he is given diverging lenses of 200 cm. focal length. Ignoring the separation between lens and eye find the distances of his near and far point when not wearing the spectacles. [Ans. near point=50 cm. ; far point=200 cm.]

27. The ratio of the magnifying power of a convex lens used as a simple microscope when it is used to throw an image at the least distance of distinct vision to that when it is used to throw an image at infinity is 1·20. If the focal length of the lens is 4·8 cm, find the least distance of distinct vision. [Ans. 24 cm]

28. Two convex lenses of focal lengths 1 cm. and 6 cm. respectively are arranged to form a microscope. A small object is placed 1·2 cm. from the object glass. If the image seen appears to be 25 cm. from the eye-piece what is the distance between the object glass and the eye-piece ?
[Ans. 10·8 cm.]

29. An astronomical telescope consisting of an objective of focal length 60 cm. and an eye-piece of focal length 3 cm. is focussed on the moon so that the final image is formed at the least distance of distinct vision (25 cm.) from the eye-piece. Assuming that the diameter of the moon subtends an angle of $\frac{1}{2}°$ at the objective, calculate (*a*) the angular magnification and (*b*) actual size of the image seen. [Ans. (*a*) 22·4 (*b*) diameter=4·9 cm.]

30. A simple astronomical telescope is made by two convex lenses of focal lengths 1 ft and 2 inches. If a man sees the moon with this instrument, what will be the distance between the lenses ? [Ans. $13\frac{5}{8}$ inch]

31. The focal lengths of the objective and the eye-piece of a compound microscope are 2 cm. and 7 cm. respectively. How far from the objective an object should be placed so that the image is formed at a distance of 25 cm. in front of the eye-piece ? The distance between the lenses is 20 cm. [Ans. 2·32 cm.]

Harder Problems :

32. Photographs of the ground are taken from an aircraft flying at an altitude of 2000 metres by a camera with a lens of focal length 50 cm. The size of the film in the camera is 18 cm $\times$ 18 cm. What area of the ground can be photographed by this camera at any one time ? [*I. I. T.* 1976] [Ans. 0·52 sq. km]

33. Howrah bridge is photographed from an aeroplane flying at an altitude of 500 metre by a camera with a lens of focal length 40 cm. On the photograph the length of the bridge appears to be 6 cm. What is the actual length of the bridge ? [Ans. 75 metre]

34. A telescope is prepared with a convex lens of 20 cm. focal length as objective and another convex lens of 5 cm. focal length as eye-piece. Find the magnification when (i) the eye is focussed for paralled rays (ii) the eye is focussed at the least distance of distinct vision at 25 cm away. [Ans. (i) 4 (ii) 4·8]

35. The focal lengths of the two convex lenses of a telescope are 30 cm and 3 cm respectively and the distance between them is 33 cm. The instrument is focussed at the moon. If the moon subtends an angle of 30′ at the objective, find the angle subtended by the image of the moon formed by the instrument at the eye of the observer. [Ans. 5°]

36. An astronomical telescope consists of two convex lenses of focal lengths 60 cm and 6 cm. Find the distance between the two lenses when the telescope is used by a normal eye to look at the moon. How much movement of the eye-piece is necessary to focus an object at a distance of 12 metre without altering the accomodation of the eye ? [Ans. 66 cm ; 3·13 cm]

37. A small astronomical telescope has an objective of focal length 50 cm and an eye-piece of focal length 5 cm. It is focussed on the sun so that the final image is formed at a point 25 cm. from the eye-piece. If the diameter of the sun subtends an angle of 32′ at the objective, calculate the angular magnification and the actual size of the image seen. [*Jt. Entrance 1981*] [Ans. 12 ; 2·79 cm]

38. A person adjusted a microscope whose eye-piece has focal length 5 cm. The least distance of distinct vision of the person is 25 cm. Another person whose eye-sight is defective, adjusted the eye-piece for his least distance of distinct vision, by moving it 5 cm. away. What type of defect of vision he has ? What type of lens and of what focal length should he use to make his near point situated at 25 cm. ? [Longsight ; $f = +38{\cdot}9$ cm]

39. A convex lens forms a real image of a small object on a screen placed at a distance of 50 cm. from the lens. The observer then holds one of its spectacle glasses between the convex lens and the screen and 5 cm away from the lens. The screen is now shifted 15 cm nearer to the lens to refocus the image. Find the focal length of the spectacle glass . Is the man long or short sighted ? [*Jt. Entrance 1974*] [90 cm ; long sighted]

40. A compound microscope is composed of an objective and an eye-piece of focal lengths 0·5 cm. and 1·5 cm. respectively. Assuming that the least distance of distinct vision is 25 cm, calculate the spacing required between the objective and the eye-piece in order that the magnification is 500. [Ans. 16·07 cm.]

41. The object glass of a microscope has a focal length of 1 inch and the eye-piece of focal length $\frac{10}{9}$ inches. The lenses are fixed 4 inches apart and the microscope is focussed on an object so as to form a final image at 10 inches from the eye-piece. Calculate the magnifying power of the instrument and the position of the object in front of the objective. [Ans. 20 ; 1·5 inches]

42. The magnifying power of a telescope in normal adjustment is 20 and the focal length of the eye-piece is 5 cm. What is the magnifying power obtained when the system is adjusted so that the final image of a distant object is formed 25 cm from the eye-piece ? [Ans. 24]

Harder Problems :

32. Photographs of the ground are taken from an aircraft flying at an altitude of 2000 metres by a camera with a lens of focal length 50 cm. The size of the film in the camera is 18 cm × 18 cm. What area of the ground can be photographed by this camera at any one time?
[I. I. T. 1976] [Ans. 0·52 sq. km]

33. Howrah bridge is photographed from an aeroplane flying at an altitude of 2000 metre by a camera with a lens of focal length 40 cm. On the photograph the length of the bridge appears to be 5 cm. What is the actual length of the bridge?
[Ans. 250 metre]

34. A telescope is prepared with a convex lens of 30 cm. focal length as objective and another convex lens of 5 cm. focal length as eye-piece. Find the magnification when (i) the eye is focussed for parallel rays (ii) the eye is focussed at the least distance of distinct vision at 25 cm away.
[Ans. (i) 6 (ii) 7·2]

35. The focal lengths of the two convex lenses of a telescope are 54 cm and 3 cm respectively and the distance between them is 57 cm. The instrument is focussed at the moon. If the moon subtends an angle of 30' at the objective, find the angle subtended by the image of the moon formed by the instrument at the eye of the observer.
[Ans. 9°]

36. An astronomical telescope consists of two convex lenses of focal lengths 60 cm and 6 cm. Find the distance between the two lenses when the telescope is used by a normal eye to look at the moon. How much movement of the eye-piece is necessary to focus an object at a distance of 12 metre without altering the accommodation of the eye? [Ans. 66 cm ; 3·13 cm]

37. A small astronomical telescope has an objective of focal length 50 cm and an eye-piece of focal length 5 cm. It is focussed on the sun so that the final image is formed at a point 25 cm from the eye-piece. If the diameter of the sun subtends an angle of 32' at the objective, calculate the angular magnification and the actual size of the image seen.
[H. S. (Tripura) 1981] [Ans. 12; 2·79 cm]

38. A person adjusted a microscope whose eye-piece has focal length 5 cm. The least distance of distinct vision of the person is 25 cm. Another person whose eye-sight is defective, adjusted the eye-piece for his least distance of distinct vision by moving it 5 cm away. What type of defect of vision he has? What type of lens and of what focal length should he use to make his near point situated at 25 cm?
[Hypermetropia ; $f = +$[illegible] cm]

39. A convex lens forms a real image of a small object on a screen placed at a distance of 30 cm from the lens. The observer then holds one of his spectacle glasses between the convex lens and the screen and, 5 cm away from the lens. The screen is now shifted 15 cm away for the image to refocus the image. Find the focal length of the spectacle glass. Is the man long- or short-sighted?
[H. Entrance 1974] [90 cm ; long-sighted]

40. A compound microscope is composed of an objective and an eye-piece of focal length 0·5 cm. and 1·5 cm. respectively. Assuming that the least distance of distinct vision is 25 cm, calculate the spacing required between the objective and the eye-piece in order that the magnification is 500.
[Ans. 16·07 cm.]

41. The object glass of a microscope has a focal length of 1 inch and the eye-piece of focal length 2 inches. The lenses are fixed 6 inches apart and the microscope is focussed on an object so as to form a final image at 10 inches from the eye-piece. Calculate the magnifying power of the instrument and the position of the object in front of the objective. [Ans. 20 ; 1·5 inches]

42. The magnifying power of a telescope in normal adjustment is 20 and the focal length of the eye-piece is 5 cm. What is the magnifying power obtained when the system is adjusted so that the final image of a distant object is formed 25 cm from the eye-piece? [Ans. 24]

MAGNETISM

1

MAGNET AND ITS PROPERTIES

1.1. Natural magnets and magnetism :

More than twenty five centuries ago, people knew that a certain form of iron ore, now known as *magnetite* had the property of attracting small pieces of iron. These ores were abundantly available in the province of Magnesia in Asia minor and the name magnetite probably came from the name of the province. The ore 'magnetite' is called magnet because it has the property of attracting iron.

The ore is called *natural magnet* because it is available in nature. The property by virtue of which it can attract a piece of iron is called **magnetism.** Besides attractive property, a magnet has also a **directive property** *i.e.* a splinter of this rock, hung by a thread, would always set itself in the north-south direction. If disturbed, it will again come to the north-south position after a few to-and-fro oscillations. For this reason, this ore is sometimes known as *lodestone* or *leading stone.* Long ago, the Chinese were said to have discovered this directive property of the lodestone and they used it as a compass for guiding their ships. So, a natural magnet has two distinct properties *viz.* (i) **attractive property** and (ii) **directive property.**

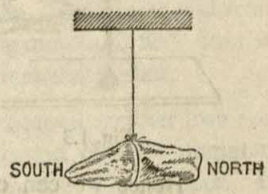


Fig. 1.1

Magnetism, it may be mentioned, is a physical property and not a chemical one. When a piece of iron is magnetised, the piece does not undergo any change of shape, colour, weight or mass nor any change in the position of the centre of gravity.

1.2. Some important definitions :

(i) **Poles of a magnet** : *Poles of a magnet are definite regions at the ends of the magnet where the attracting power of the magnet appears to be most concentrated.* This fact may be demonstrated by dipping a bar magnet into iron filings. On withdrawing the bar from the filings, it will be found that a good amount of filings cling together at the two ends [Fig 1.2] but almost no filings collect at the middle portion of the bar.

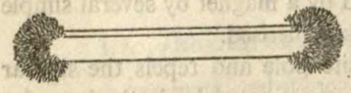


Fig. 1.2

These points are called **poles** of the magnet which for all practical purposes can be regarded as points somewhere near but not at the end of the magnet. When a magnet is suspended freely, a particular pole of the magnet always points towards the north and the other towards the south. For this reason, the former is called the **north-seeking** or simply the **north pole and the latter the south-seeking** or simply **south pole.**

(ii) **Magnetic axis :** The line obtained by joining the poles of a magnet is called the axis of the magnet.

(iii) **Neutral line :** A line drawn at right angles to the axis of a magnet through a point midway between the poles where there is no magnetic attraction, is referred to as the neutral or equatorial line of the magnet.

(iv) **Effective length or the magnetic length :** The distance between the two poles of a magnet is called the effective or the magnetic length of the magnet. This length is about $\frac{5}{6}$th of the actual length of the bar.

1.3. Action of magnetic poles on each other :

Take a magnetic needle and a bar-magnet with its north and south poles marked. Bring the north pole of the bar magnet near the north pole of the megnetic needle as shown in Fig. 1.3. It will be found that the poles are repelling each other. Next bring the north pole of the bar-magnet near the south pole of the needle. Attraction will take place. Same results will be obtained by similar experiments with the south pole of the bar magnet.

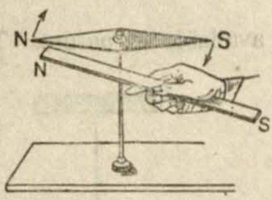


Fig. 1.3

Hence we can conclude that *like poles repel each other and unlike poles attract.*

1.4. Magnet, Magnetic and Non magnetic substances :

(1) A substance which can attract metals like iron or steel and always points to a particular direction when suspended freely is called a magnet.

Substances which are attracted by a magnet are called magnetic substances. Apart from iron and steel, cobalt and nickel are good magnetic substances.

Substances which are neither attracted nor repelled by a magnet are called non-magnetic substances ; for example, glass, paper, wood etc.

(2) A magnet has two poles one of which always directs towards the north and the other to the south when the magnet is suspended freely.

A magnetic or a non-magnetic substance has no poles and does not point to any particular direction when suspended freely.

(3) A magnetic substance can be magnetised by a magnet by several simple processes but a non-magnetic substance cannot be magnetised.

(4) A pole of a magnet attracts the opposite pole and repels the similar pole of another magnet but a magnetic substance is attracted by both the poles of the magnet.

1.5. Permanent and temporary magnets :

Definition : *A permanent magnet is one which, once magnetised, can retain its magnetism for a long time.* For example, steel or tungsten steel (an alloy of steel and tungsten), once magnetised retains its magnetism for a pretty long time.

A temporary magnet is one which, once magnetised cannot retain its magnetism for a long time. For example a rod of soft iron can readily be magnetised and very strongly too, but it cannot retain its magnetism for a long time.

Whenever we intend to mention about permanent magnets, iron and steel come to our mind first. But in recent years, a vast range of special magnetic alloys had been developed which can serve the purpose of permanent magnets very well. *Alnico*, an alloy of aluminium, cobalt and nickel (1% aluminium, 20% nickel, 5% cobalt and rest iron) is a good magnet and can be used as a permanent magnet. Also, *stalloy*, an alloy of iron and silicon (5% silicon, 95% iron), *Permalloy*, an alloy of iron and nickel (50% iron and 50% nickel), *Mumetal*, an alloy of iron, nickel and copper (22% iron, 73% nickel and 5% copper) are good permanent magnets.

From the above list, it appears that alloys having iron as an ingradient are good permanent magnets. But that is not always the case. Manganese and iron are magnetic substances, but an alloy made of 12% manganese and 88% iron is found to be non-magnetic. This alloy is called **Hadfield manganese steel.**

On the other hand, an alloy may be prepared containing neither iron nor nickel nor cobalt yet it exhibits strong magnetism. Conrad Heuslar prepared such an alloy by mixing 24% manganese, 16% aluminium and 60% copper. It is called **Heuslar alloy** and it behaves like a magnet.

1.6. Repulsion is a surer test of magnetisation :

Take a magnetic needle. Now take a bar to test whether it is magnetised or not. Bring one end of the bar close to the north pole of the needle. *If there be attraction, the conclusion is not definite.* It may be that the bar is magnetised and if so, then the end of the bar in question is a south pole. On the other hand, the bar may be a piece of iron which is always attracted by a magnet. *But if repulsion is observed, the conclusion is definite,* because repulsion takes place only in the case of two similar poles. Hence the bar is magnetised and the end of the bar in question is a north pole. Thus, *repulsion is a surer test of magnetic condition of a body than attraction.*

1.7. Different methods for making artificial magnets :

A magnetic substance can be converted into an artificial magnet, mainly by two methods : (a) *method of touch* and (b) *electrical method.*

The method of touch is, again, divided into three classes ; viz (i) *method of single touch* (ii) *method of separate touch* and (iii) *method of double touch.*

(i) **Method of single touch** : Take a suitable rectangular bar of steel which is to be magnetised and place it on a table. Bring one pole of a bar magnet in contact with one end of the steel bar. Move the bar magnet along the length

of the steel bar from one end to the other end keeping the magnet always parallel to its first position [Fig. 1.4]. This means that if the magnet is held at an angle of 30°, it is to be kept always inclined at an angle of 30° while it is moved along the steel bar. Then lift the bar magnet and place it on the first end and repeat the rubbing in the same way for several times. Then turn the steel bar over so that downside now becomes upside. Repeat the rubbing in the similar way for several times. The steel bar will be found to be magnetised.

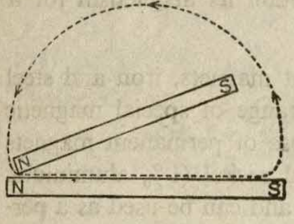


Fig. 1.4

The end of the steel bar where the stroking magnet left it, will be found to acquire polarity opposite to the polarity of the stroking pole. Thus if a north pole be used for stroking, then the end of the steel bar where it leaves the bar will be *S*-pole and the first end will acquire a north pole. This method develops weak magnetism in a bar.

(ii) **Method of separate touch** : Place a rectangular steel bar to be magnetised on a table and place two opposite poles of two bar magnets at the middle of the steel bar as shown in Fig. 1.5. Draw them simultaneously apart from the middle to the extreme ends. Lift them and place them again at the middle in the similar way as before and repeat the rubbing for several times. Turn the steel bar over and repeat the process.

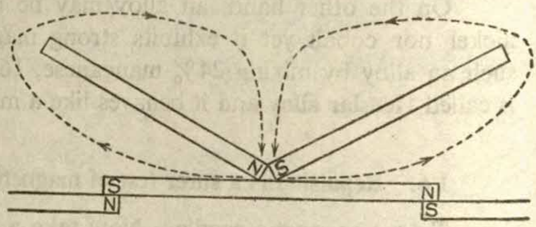


Fig. 1.5

The steel bar will be found to be magnetised with polarities at the ends which are opposite to those of the stroking poles leaving the respective ends. The steel bar will acquire stronger magnetisation if the bar is supported on the opposite poles of two other bar magnets so that the poles of the lower magnets are similar to the stroking poles as shown in the figure.

(iii) **Method of double touch** : Place the steel bar to be magnetised so that it is supported by the opposite poles of two bar magnets as shown in Fig 1.6. Fasten a piece of compressed cork or a piece of wood between the opposite poles of two other bar magnets so that the poles are kept at a fixed distance apart. Place them at the middle of the steel bar such that the poles of the magnets below and those of the stroking magnets above are similar. Move them together from the

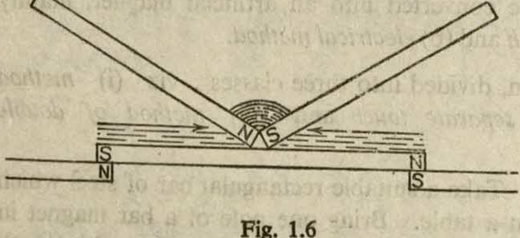


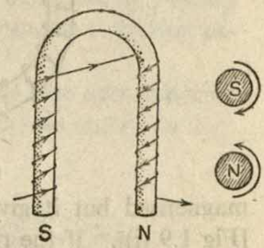
Fig. 1.6

middle to one extreme end and then back to the other extrimity. Repeat the rubbing several times ending at the middle so that each half of the steel bar is rubbed an equal number of times. Repeat the process by turning the steel bar upside down.

The steel bar will be found to be magnetised with polarities at the ends opposite to the nearest stroking pole as shown in the Fig. 1.6. Of all the processes of rubbing, this method makes the most powerful magnets.

(iv) **Electrical method** : Take a bar of steel over which is wound a coil of insulated copper wire. Instead of a straight bar it may be bent in the form of a horse-shoe [Fig 1.7]. When a current is passed through the coil of wire, the steel bar will be found to acquire magnetism.

To determine the polarity developed, look at one end of the bar and trace the direction of current flowing through the coil at that end. If it be counter-clockwise, then that end will be a north pole; if it be clockwise, then that end will be a south pole as shown in Fig. 1.7 (b).



(a) Fig. 1.7 (b)

1.8. Magnets with more than two poles : consequent poles :

Due to faulty magnetisation, a magnet is sometimes found to have developed more than two poles. For example, in the method of separate touch, if both the stroking poles are *N*-poles, then the steel bar will develop two south poles at the two ends with a north pole at the middle [Fig. 1.8]. The extra pole that is developed at the middle is called a *consequent pole*.

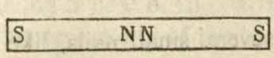


Fig. 1.8

Thus, if both the ends of a bar magnet show repulsion when brought, one after another, near a particular pole (say north pole) of a magnetic needle, then it is clear that the bar magnet has acquired similar polarities at the two ends and an opposite polarity at the middle. In other words, the bar magnet has consequent poles.

Consequent poles may also develop in a bar due to faulty electrical method. Insulated copper wires are wound on a steel bar in such a manner that the windings on one half of the bar are in one direction while on the other half the windings are on the opposite direction. If a strong current is now sent through the wire with the help of a battery, the bar will be magnetised. Looking at the right end of the bar, the current appears to flow clockwise. So, a *S*-pole is created at that end. Also looking at the left end the current flows clockwise, creating again a *S*-pole at that end. In this condition a north pole will be created at the middle of the bar.

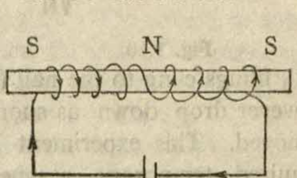
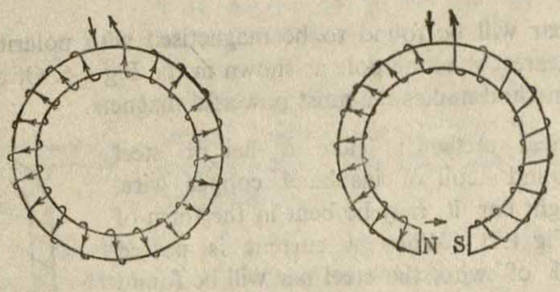


Fig. 1.8(a)

1.9. Magnets with no poles :

If a ring of iron be magnetised by passing electric current through a copper wire wound round the ring or by rubbing a magnet round the ring, the ring becomes



(i) Fig. 1.9 (ii)

magnetised but it gives no external evidence of the existence of free polarity [Fig 1.9 (i)]. If the ring be cut anywhere, a north pole appears at one cut-end and a south pole at the other [Fig 1.9 (ii)].

1.10. Magnetic induction :

We know, that a magnetic substance can be magnetised by friction and electric current. There is another simple way of magnetising a substance. It has been found that when a piece of unmagnetised steel is placed either near to or in contact with a pole of a magnet and then removed, it acquires magnetism. This is called **magnetic induction.** The following experiments will illustrate the phenomenon very well.

Experiments : (i) A steel bar magnet will pick up several small nails, like a chain from one of its poles (Fig. 1.10). If the bar magnet be now carefully removed from above, the whole chain breaks up. Each nail attracted the one next to it only as long as the magnet was near and we say that magnetism was induced in the nails by the bar magnet.

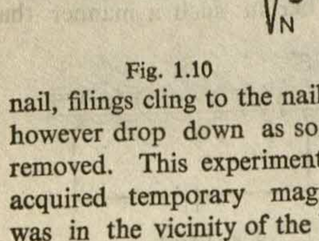


Fig. 1.10

(ii) When a soft-iron nail is dipped in iron filings and removed, filings do not cling to the nail. But if a bar magnet be held above the nail, filings cling to the nail (Fig. 1.11). Filings will, however drop down as soon as the bar magnet is removed. This experiment shows that the nail acquired temporary magnetism as long as it was in the vicinity of the bar magnet.

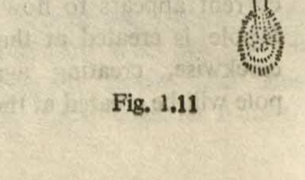


Fig. 1.11

Definition : *Temporary magnetism acquired by a magnetic substance under the influence of a strong permanent magnet is called induced magnetism and the phenomenon is known as magnetic induction.*

Now insert a piece of paper, glass or wood in the space between the bar magnet and the nail (Fig. 1.11). The iron filings will continue to cling to the nail which shows that non-magnetic substances like wood, paper etc. cannot disturb or prevent magnetic induction.

1.11. Nature of polarity in induced magnetism :

A bar of soft iron may be magnetised by induction. The nature of polarity developed at the two ends of the bar may be ascertained from the following experiment.

Take a magnetic needle and place a bar magnet along a line perpendicular to the axis of the needle and passing through one of its poles as shown in Fig. 1.4. Adjust the distance between the needle and the bar magnet such that the needle is not affected by the magnet.

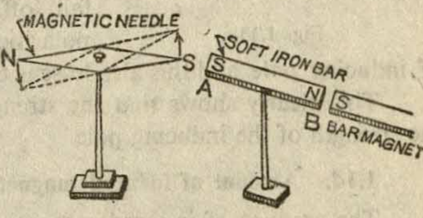


Fig. 1.12

Suppose, *S*-poles of the needle and the bar magnet face each other (Fig. 1.12). Now insert a bar of soft iron *AB* between the bar magnet and the magnetic needle. The *S* pole of the needle will at once, repel. This shows the bar *AB* has acquired induced magnetism under the influence of the bar-magnet and that the end *A* has developed *S* polarity. Obviously the end *B* has developed *N* polarity. So we see that the inducing *S* pole of the bar magnet has induced opposite polarity *i.e. N* polarity at the nearest end *B* of the bar *AB* and similar polarity *i.e. S* polarity at the furthest end A.

On reversing the bar magnet so that its *N* pole is nearest to *B* it will be found that the nearest end B has got *S*-pole and the furthest end *A* has *N* pole.

From this result, we can generalise by saying that the *induced pole nearest to the inducing pole is of opposite sign and the induced pole furthest from the inducing pole is of same sign as that of the inducing pole.*

1.12. Induction precedes attraction :

We know that a soft iron bar is attracted by a magnet. The reason of this attraction is magnetic induction. When the bar is held near the pole of the magnet, opposite polarity is developed at the near end of the soft-iron bar due to induction and then attraction takes place between these two opposite polarities. There is of course, a repulsion between the inducing pole and the induced like pole. But the opposite poles being nearer, the attraction predominates over the repulsion. This is why it is said that induction precedes attraction.

1.13. Amount of induction depends upon the strength of the inducing pole :

Place a soft-iron bar on the edge of a table with some portion of it projecting outside. Keep two strong magnets, one over the other, with their like poles facing each other and place the combination behind the soft-iron bar as shown

in the fig. 1.13. The bar will acquire induced magnetism under the influence of the combination of strong magnets and a chain of soft-iron nails will be supported by its projecting end. Now slowly move away one of the strong magnets. Some of the nails will be found to drop down. It shows that with the decrease of the strength of the inducing pole, the strength of the induced pole also decreases.

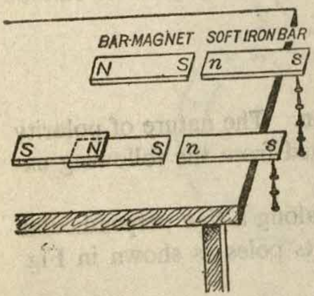


Fig. 1.13

Now, reverse the magent that was removed earlier. Placing it on the first magnet, bring the opposite polarities closer slowly. [Lower part of fig. 1.13]. Remaining nails will be found to fall off one by one. Approach of opposite polarities means gradual lessening of the strength of inducing pole and this also lessens the strength of the induced pole.

This clearly shows that the strength of the induced magnetism depends on the strength of the inducing pole.

1.14. Amount of induced magnetism :

The amount of induced magnetism depends on the following factors :—

(i) Strength of the inducing pole ; greater the strength of the inducing pole, the greater is the amount of induced magnetism.

(ii) The nature of induced material ; under similar circumstances, soft-iron is found to acquire greater amount of induced magnetism than steel. Cobalt and nickel acquire still less amount.

(iii) Distance between the induced and the inducing pole ; less the distance, the greater is the amount of induction.

(iv) The medium between the induced and the inducing pole.

1.15. Change of polarity due to induction :

Suppose, *N*-pole of a strong magnet is quickly brought very close to the *N*-pole of a magnetic needle (or of a weak magnet). According to the laws of attraction and repulsion, the similar poles should repel each other and the magnetic needle should move away. Instead, the needle may very often be seen to be attracted. The reason of this attraction is the reversal of polarity of the needle or of the weak magnet due to induction. The strong magnet will induce a south polarity on the *N*-pole of the weak magnet and this induced polarity, being stronger will predominate over the *N*-pole of the weak magnet. The same thing will happen at the other end of the weak magnet. In other words, the polarities of the weak magnet under the influence of the strong magnet will be reversed and attraction will take place.

Generally, when the strong magnet is removed, the weak magnet regains its own polarity but reversal of polarity, on some occasions, may be permanent. This fact should be borne in mind while testing the polarity of a magnet by a magnetic needle. The two poles should never be brought closer quickly. They should be brought closer slowly from a distance.

1.16. Factors responsible for destruction or weakening of magnetism :

The following factors are responsible for destruction or weakening of magnetism of a magnet :

(i) If two bar-magnets are left side by side with their like poles facing each other, then each pole will induce opposite polarity on the other and the magnetism of both will gradually become weak.

(ii) Induction of earth's magnetism also weakens the strength of a magnet. If, for example, a magnet be suspended at the northern hemisphere with its *S*-pole pointing downward, the earth's magnetism will induce opposite polarity on it and the magnet will gradually lose its magnetism.

(iii) A magnet becomes weak if it is twisted, hammered or allowed to drop repeatedly from a certain height. Soft-iron magnets are affected more than steel magnets by such rough treatment.

(iv) The magnetism of a magnet is weakend if the magnet is heated. It loses all its magnetism if it is heated beyond a particular temperature. This temperature is known as **Curie point.** It is different for different magnetic materials. Experiments show that curie-point for iron is about 770°C and for nickel 400°C. If a magnet be heated to a temperature below the curie-point, it partly loses its magnetism but regains it fully when it is cooled to its initial temperature.

(v) Alternating current is also responsible for demagnetising a magnet. The hair-spring of a watch is made of steel. If a watch is brought near a strong magnet, the hair spring may be magnetised and in that case, the watch will not keep correct time. In order to demagnetise the hair-spring, the watch is held near a coil through which alternating current is fed. After some time, the watch is removed. It will keep correct time now.

1.17. Magnetic keepers :

A bar magnet tends to become weaker with age, due to *self-demagnetisation* which is caused by the poles at the ends trying to induce opposite polarities on each other. Similarly, a horse-shoe magnet left to itself, tends to become weaker due to the same reason.

In order to prevent self-demagnetisation, bar magnets are kept in pairs, with their opposite poles adjacent and with small pieces of soft iron, called

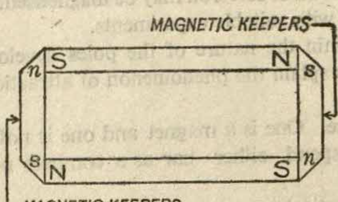


Fig. 1.14

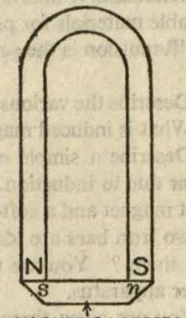


Fig. 1.15

keepers placed across their ends (Fig. 1.14). These keepers become strong

induced magnets and form a closed chain of polarity as shown in fig. 1.14. The poles at each end of the magnet, are thereby protected against self-demagnetising effect.

In the case of a horse-shoe magnet also, a small piece of soft-iron is placed across the poles (Fig. 1.15). Here also a closed chain of polarity is formed and the effect of self-demagnetisation is neutralised.

1.18. Simple identification of magnet, magnetic substance and non-magnetic substance :

Consider three rods *A*, *B* and *C* having identical external appearance and colour. One of them is a magnet, other is made of some magnetic substance and the third is of non-magnetic substance. How will you identify them without taking help of another magnet ?

Touch each one of them, one by one, with the other two. One of the rods will be found which will not be attracted by the other two. Evidently, this rod is made of non-magnetic substance. Let the rod be *C*. The reason is that non-magneatic substances are not attracted by a magnet.

Take the rod, say *A* in hand and touch one of its ends along the length of the other rod *B*. If attraction is felt all along the length of the bar *B*, the bar *A* is a magnet and the bar *B* is made of magnetic substance. On the other hand, if attraction is felt only at the two ends of the bar *B*, the rod *A* is made of magnetic substance and the rod *B* is a magnet. The reason is that the attractive force of a magnet is the strongest at its ends and feeblest at the middle.

In this way, the above rods may be identified without the help of any other substance.

Exercises

Essay type :

1. What are the differences, magnetically speaking between a piece of soft iron, a piece of brass and a piece of load stone ?

2. What are temporary and permanent magnets ? What is their difference ? What are the suitable materials for preparing permanent and temporary magnets ?

3. "Repulsion is the surer test of magnetisation than attraction"—Explain. [*H. S. Exam. 1978*]

4. Describe the various methods by which a rod of soft iron may be magnetised.

5. What is induced magnetism ? Explain with suitable experiments.

6. Describe a simple experiment to ascertain the nature of the poles developed at the ends of a bar due to induction. How would you explain the phenomenon of attraction between a permanent magnet and a soft-iron bar ?

7. Two iron bars are identical in appearance. One is a magnet and one is not. How can you identify them ? You are not permitted to suspend either bar as a compass needle or to use any other apparatus.

8. You are given three bars of exactly identical appearance. You are told that one of them is a magnet, the other is made of some magnetic material and the third is made of some non-magnetic substance. Without the help of any other thing, how will you identify them ? [*H. S. Exam. 1980*]

Short answer type :

9. What are Hadfield manganese steel and Heuslar alloy ?

10. What do you mean by consequent pole ? State the reason of its development. [*Jt. Entrance 1982*]

11. Two iron bars always attract, no matter the combination in which their ends are brought near to each other. Can you conclude that one of the bars must be unmagnetised ?

12. What is the nature of polarities developed by induction ?

13. A number of soft-iron nails can be suspended, one after another, like a chain from one pole of a magnet. But the nails fall off as soon as the magnet is removed. Why ?

14. How will the strength of induced pole depend on the following factors : (*a*) strength of the inducing pole and (*b*) the distance between the induced and the inducing poles.

15. How can you reverse the magnetism of a compass needle ?

16. Answer the following questions briefly : (*a*) What are the factors on which the induction depends ? (*b*) *N*-pole of a magnet may be attracted by the *N*-pole of a strong magnet. How is this possible ? (*c*) Vertical rods of steel and iron are found to acquire feeble magnetism. Why ? (*d*) What kind of polarities will be developed in such rods in northern hemisphere ? (*e*) Is it possible to have a magnet without poles ?

17. After magnetising a steel rod, it is found that both ends of the rod repel the north pole of a magnetic needle. How is it possible ?

18. Explain : (i) Pole (ii) Neutral axis (iii) Magnetic length.

19. What is demagnetisation ? How can it take place ? What is Curie point ?

20. The north pole of a strong magnet *A* is slowly brought near the north pole of a freely suspended weak magnet *B*. How will the north pole of the magnet *B* behave (i) when the magnet *A* is far away from the magnet *B* and (ii) when the magnet *A* is very near to the magnet *B*.

21. A nail is placed at rest on a smooth table near a strong magnet. It is released and attracted to the magnet. What is the source of the kinetic energy of the nail ?

22. What is a magnetic keeper ? It is found that a horse-shoe magnet retains its magnetism better when a piece of soft-iron is placed across its poles than when there is no soft-iron.—Explain the reason. [*cf. H. S. Exam 1980*]

23. The Curie-point of nickel is 350°C—Explain.

Objective type :

24. Fill in the gaps with suitable words :

(*a*) A——magnet is one which, once magnetised, can retain its magnetism for a long time. (temporary, permanent, horse-shoe)

(*b*) Temporary magnetism acquired by a magnetic substance under the influence of a strong permanent magnet is called——magnetism. (permanent, para, induced)

(*c*) The ore magnetite is called —— because it has the property of attracting iron. (lodestone, ferromagnet, magnet)

2

MAGNETIC FIELD AND MAGNETIC LINES OF FORCE

2.1. Magnetic field :

The space surrounding a magnet in which magnetic force is exerted is called a *magnetic field.* Theoretically the field extends upto infinity but practically it becomes inappreciable after a certain distance.

2.2. Force of attraction or repulsion between two magnetic poles : Coulomb's law :

Consider two magnetic poles of strength m_1 and m_2 respectively at a distance r from each other (Fig. 2.1). If they are like poles, they will repel each other but if they are unlike poles, they will attract each other. What is this force of attraction or repulsion ? It will be obtained from *Coulomb's law.* The law states that *the force acting between two magnetic poles is directly proportional to the product of their pole strengths and inversely proportional to the square of the distance between them.*

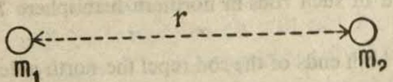


Fig. 2.1

If F be the force, then according to the Coulomb's law, $F \propto m_1 m_2$ when r is constant and $F \propto \frac{1}{r^2}$ when m_1 and m_2 are constants. Combining these two relations,

$$F \propto \frac{m_1 m_2}{r^2} \quad \text{or,} \quad F = \frac{1}{\mu} \cdot \frac{m_1 m_2}{r^2} \text{ where } \mu \text{ is a constant.}$$

The constant 'μ' depends upon the medium in which the poles are situated and is known as the '*permeability*' of the medium. In general, the value of this constant is taken 1 for air (strictly speaking vacuum) or for any other non-magnetic material.

2.3. Pole of unit strength :

From the above equation, we get the definition of a pole of unit strength. When $F=1$ dyne, $\mu=1$ (air), $r=1$ cm. and $m_1=m_2$, then $m_2=m_1=1$.

Definition : *If two identical poles, placed 1 cm. apart in air repel each other with a force of 1 dyne, each is said to have unit strength and is called a unit pole.*

According to the above definition of unit pole, two magnetic poles of strength m_1 and m_2 placed in air will exert a force on each other which is given by

$$F = \frac{m_1 . m_2}{r^2}$$

According to the C.G.S. system, a pole of strength 50, for example, means that it will exert a force of 50 dynes on a unit pole placed 1 cm. from it in air.

2.4. Intensity of magnetic field :

Definition : *The intensity of magnetic field at a point is defined as the force experienced by a unit north pole placed at that point.*

What will, therefore, be the intensity at a point, due to a pole of strength m, at a distance r from it? If a unit north pole be placed at a distance r, the force on it, according to the art 2.2 is given by, $F=\frac{m\times 1}{\mu r^2}=\frac{m}{\mu r^2}$ *i.e.* the intensity at a distance r is $m/\mu r^2$.

It is clear from this expression that the field intensity is different at different points in a non-uniform magnetic field.

The unit of intensity is called **Oersted (Oe).** Formerly, it was called '*Gauss*'.

According to the definition, the intensity at a point in a magnetic field is 1 oersted if a unit north pole placed at that point experiences a force of 1 dyne. Similarly, if a pole of strength m be placed in a uniform magnetic field of intensity H, the pole will experience a force $f=mH$.

It is evident that intensity is a vector quantity. For this reason it is sometimes called *magnetic field vector.*

[**Note** : 'Magnetic field strength' and 'magnetic intensity' are synonymous with the intensity of a magnetic field.]

Example 1 : *What force is exerted between two magnetic poles of strength 32 and 36 at a distance 12 cm. from one another in air* ?

Ans. Here, $m_1=32$; $m_2=36$ and $r=12$ cm.

We know, $F=\frac{m_1 m_2}{r^2}$ $\therefore$ $F=\frac{32\times 36}{(12)^2}=8$ dynes.

Example 2 : *Two poles, one of which is 5 times as strong as the other, exert on each other a force equal to the weight of 80 mg, when placed 10 cm. apart in air. Find the strength of each pole.*

Ans. Here $F=80$ mg-wt.$=\frac{80}{1000}$ gm-wt. $=\frac{80}{1000}\times 980$ dynes$=\frac{98\times 8}{10}$ dynes.

Further $m_1=5m_2$ and $r=10$ cm. ; We know $F=\frac{m_1 m_2}{r^2}$

$\therefore$ $\frac{98\times 8}{10}=\frac{5m_2\times m_2}{10\times 10}$ or, $m_2{}^2=98\times 16$ $\therefore$ $m_2=28\sqrt{2}=39{\cdot}48$ units.

Hence $m_1=5\times m_2=39{\cdot}48\times 5=197{\cdot}4$ units.

Example 3 : *What is the intensity at a point 10 cm. away in air from a magnetic pole of strength 100 units.*

Ans. We know if F be the intensity at a distance r from a pole of strength m, then $F=\frac{m}{r^2}$. Here $m=100$ and $r=10$ cm. $\therefore$ $F=\frac{100}{10\times 10}=1$ Oe.

Example 4 : *Two like poles of strengths 16 and 25 units are placed 9 cm. apart. At which point on the line joining the two is the intensity of the field zero* ?

Ans. Since the two poles are like, it is evident that the point should be between them and nearer to the pole of less strength. Let the point be at a distance x cm. from the pole of strength 16.

The intensity at the point due to the pole of strength $16=\frac{16}{x^2}$

and " " " " " " " " " " " $25=\frac{25}{(9-x)^2}$

$$\therefore \quad \frac{16}{x^2}=\frac{25}{(9-x)^2} \quad \text{or,} \quad \frac{4}{x}=\frac{5}{9-x}$$

or, $36-4x=5x \quad \therefore \quad x=4$ cm.

Hence, the point is 4 cm. from the pole of strength 16 or 5 cm. from the other pole.

Example 5 : *Two identical magnetised needles, each of mass 4 gm and length 20 cm are suspended with their south poles together. In equilibrium, the two north poles move apart until the distance between them is 4 cm. Find the pole strength of each needle. Assume that the poles are concentrated at the ends of the needle.*

[*Jt. Entrance 1985*]

Ans. Let m be the pole strength of each needle. The force of repulsion between the N-poles $=\frac{m^2}{d^2}=\frac{m^2}{16}$ [Fig. 2.1(a)].

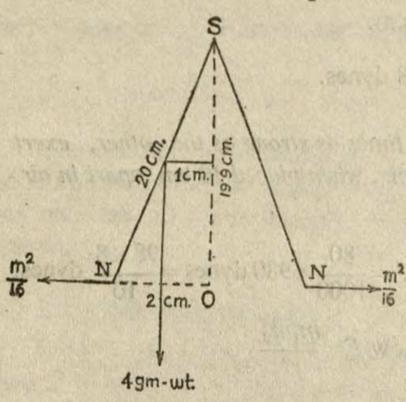


Fig. 2.1(a)

The weight of the needle=4 gm. wt$=4\times 980$ dynes is acting vertically downwards from the middle-point of the needle. The distance $NO=\frac{1}{2}NN=2$ cm. Since the system is in equilibrium, the moments of the forces about the point S (say), will be equal to zero *i.e.*

$$\frac{m^2}{16}\times 19{\cdot}9=4\times 980\times 1$$

$$[SO=\sqrt{400-4}=19{\cdot}9 \text{ cm.}]$$

$$\text{or } m^2=\frac{4\times 980\times 16}{19{\cdot}9}$$

$$\therefore \quad m=56{\cdot}14 \text{ units (nearly)}$$

2.5. A freely suspended magnet in a uniform magnetic field : Magnetic moment :

Consider a magnet NS of pole strength 'm' and effective length $2l$ (pole to pole distance) suspended freely in a uniform magnetic field of strength H. If the magnet is left to itself, it will direct itself in the direction of the magnetic field.

Suppose, the magnet is deflected through an angle θ (Fig. 2.2). In this position the poles of the magnet will be acted on by two equal, parallel but oppositely directed forces mH and mH as shown in the figure. These forces evidently constitute a couple which tends to bring the magnet in its initial position.

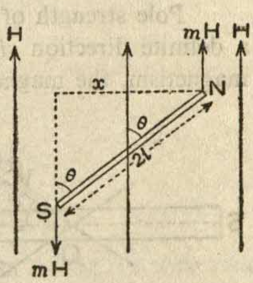


Fig. 2.2

Now, moment of the couple$=mH\times x$

But $\frac{x}{2l}=\sin\theta$ or $x=2l.\sin\theta$

$\therefore$ the moment of the couple$=mH\times 2l\sin\theta$ ....(i)

So, it is found that when the magnet is deflected through an angle θ from its position of rest, a couple acts on it whose moment$=2mlH\sin\theta$.

Magnetic moment : If $\theta=90°$ the moment of the couple$=mH\times 2l\sin 90°$ $=2mlH$.

If, further, $H=1$, the moment$=2ml$=pole strength × effective length. This moment is called the *magnetic moment* of the magnet.

Definition : *The moment of a magnet (or the magnetic moment) is equal to the moment of the mechanical couple required to keep the magnet at right angles to a field of unit intensity.*

Example 1 : *Calculate the moment of the couple required to keep a bar-magnet of moment 200 c.g.s. unit in a uniform field of 0·39 oersted at an angle of 30° with the direction of the field.* [*I.I.T. 1961*]

Ans. We know that moment of the couple required to keep a bar-magnet of moment M in a uniform field of intensity H at an angle θ is $MH\sin\theta$.

Here, $M=200$ c.g.s. $H=0{\cdot}39$ Oe ; $\theta=30°$

$\therefore$ the required moment of the couple $=MH.\sin\theta$

$$=200\times 0{\cdot}39\times\sin 30°$$

$$=200\times 0{\cdot}39\times\tfrac{1}{2}=39 \text{ dyne.-cm.}$$

Example 2 : *A magnetised wire of steel 31·4 cm. long has a pole strength of 5 c.g.s. unit. It is then bent in the form of a semi-circle. Calculate the magnetic moment of this magnet.* [*I.I.T. 1965*]

Ans. Magnetic moment=pole strength × magnetic length. Magnetic length means the linear distance between the poles. When the wire is bent in the form of a semi-circle, the magnetic length is equal to the diameter of the circle. Now, for a semi-circle, the length of the circumference$=\pi r$

$\therefore$ $\pi r=31{\cdot}4$ or $r=\frac{31{\cdot}4}{\pi}=\frac{31{\cdot}4}{3{\cdot}14}=10$ cm.

$\therefore$ Magnetic moment=pole strength × diameter

$$=5\times 2r=5\times 20=100 \text{ c.g.s. units,}$$

2.6. Magnetic moment is a vector quantity :

Pole strength of a magnet is a scalar quantity but the magnetic length has a definite direction *viz* from the south pole to the north pole. In all cases of magnetism, the magnetic axis and hence the magnetic length are attributed with the above direction. For this reason magnetic moment has also a definite direction from the south pole to the north pole *i.e.* it is a vector quantity. Like all other vector quantities, magnetic moment of a magnet may be resolved or compounded.

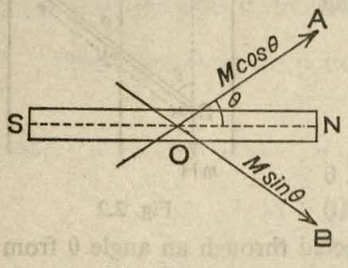


Fig. 2.2(a)

Consider a bar magnet *SN* of magnetic moment *M* [Fig. 2.2 (a)]. The moment may be resolved along any two perpendicular directions, say *OA* and *OB*. If *OA* is inclined at an angle θ with the axis of the magnet, the component along $OA=M\cos\theta$ and the component along $OB=M.\sin\theta$. We may now consider as if the magnet *SN* is absent and instead there are two magnets of moments $M\cos\theta$ and $M\sin\theta$ respectively in the positions *OA* and *OB*.

2.7. Intensity of magnetic field due to a bar-magnet :

We shall discuss the intensity of magnetic field due to a bar-magnet in two standard positions *viz* (i) *end-on position* or *axial position* and (ii) *broad-side on position* or *equatorial position.*

(i) End-on position or axial position :

NS is a small bar-magnet having length $2l$ and pole strength m. A point *P* is taken along the axis of the bar magnet at a distance *d* from the centre *O* of the magnet [Fig. 2.3 (a)]. The position of *P* is called the *end-on* or the *axial position.* We are to find out the intensity at *P* due to both the poles of the bar-magnet.

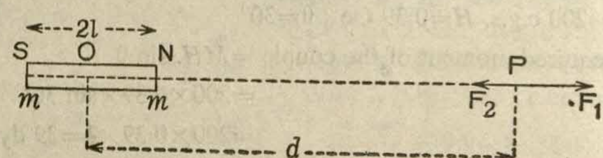


Fig. 2.3(a)

From the definition of intensity, we know that if we place a unit north pole at *P*, then the force experienced by it due to the bar-magnet *NS* will be the intensity at *P*. Now, the force of repulsion F_1 exerted by N-pole of the bar-magnet on the unit north pole situated at *P* is given by, $F_1=\dfrac{m\times1}{(NP)^2}=\dfrac{m}{(d-l)^2}$

Again the force of attraction F_2 exerted by the S-pole of the bar-magnet on unit north pole situated at *P* is given by, $F_2=\dfrac{m\times1}{(SP)^2}=\dfrac{m}{(d+l)^2}$

Since, the forces F_1 and F_2 act along the same line but in the opposite directions and since N pole is nearer to P, we have, $F_1 > F_2$.

So, the resultant force on unit pole at $P = F_1 - F_2$

$$= \frac{m}{(d-l)^2} - \frac{m}{(d+l)^2} = \frac{4m.l.d.}{(d^2-l^2)^2} = \frac{2M.d}{(d^2-l^2)^2}$$

where $M = 2\ m.l$ = magnetic moment of the bar-magnet.

$\therefore$ The magnetic intensity at P is given by, $F = \dfrac{2\ M.d}{(d^2-l^2)^2}$ .. (i)

If the bar magnet is small and the point P is taken at a long distance away, $d \gg l$, so that l^2 may be neglected compared to d^2.

In that case, $F = \dfrac{2M.d.}{d^4} = \dfrac{2\ M}{d^3}$ .. .. .. (ii)

(ii) **Broad-side-on or equatorial position :**

As before NS is a bar-magnet, having length $2l$ and pole strength m. P is a point at a distance d from its centre on the perpendicular drawn on the axis of the bar-magnet through the centre [Fig. 2.3 (b)]. The position of P is called the *broad-side on* or the *equatorial position*. We are to find the magnetic intensity at P. Let us suppose that a unit north pole is placed at P.

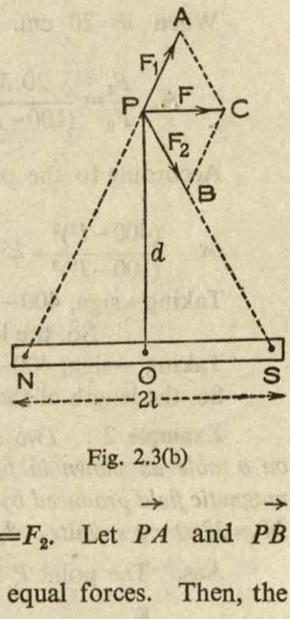


Fig. 2.3(b)

Now, the force of repulsion F_1 on the unit pole exerted by the N-pole of the bar-magnet is

$F_1 = \dfrac{m \times 1}{(PN)^2}$ acting along $\overrightarrow{PA}$.

Also, the force of attraction F_2 on the unit pole exerted by S-pole of the bar magnet is

$F_2 = \dfrac{m \times 1}{(PS)^2}$ acting along $\overrightarrow{PB}$. Since $PN = PS$, we have $F_1 = F_2$. Let $\overrightarrow{PA}$ and $\overrightarrow{PB}$ represent, in magnitude and direction, these two equal forces. Then, the diagonal $\overrightarrow{PC}$ of the parallelogram $PACB$ will represent the resultant force, F.

Now, Δ^s APC and PNS are similar ; so $\dfrac{PC}{PA} = \dfrac{NS}{PN}$ or $\dfrac{F}{m/(PN)^2} = \dfrac{2l}{PN}$

$\therefore$ $F = \dfrac{2ml}{(PN)^3} = \dfrac{M}{(PN)^3}$, where $M = 2\ ml$ = magnetic moment of the bar-magnet. But, $(PN)^3 = (d^2+l^2)^{\frac{3}{2}}$ $\therefore$ $F = \dfrac{M}{(d^2+l^2)^{\frac{3}{2}}}$ .. (iii)

If the magnet be small and the point P is taken at a long distance away $d \gg l$, so that l^2 may be neglected in comparison with d^2. In that case,

$$F=\frac{M}{d^3} \quad \cdots \quad \cdots \quad \text{(iv)}$$

Comparing equations (ii) and (iv), we can say that *the intensity due to a small bar-magnet at an end-on position is twice that due to the same magnet at broad-side on position at the same distance.*

Example 1 : *The ratio of the intensities of magnetic fields at two points situated at distances 10 cm. and 20 cm. from the centre of a magnet along its axis is 12·5 : 1. What is the length of the magnet ?*

Ans. The magnetic intensity at an axial point due to a bar-magnet is given by, $F=\frac{2\,M.d}{(d^2-l^2)^2}$.

When $d=10$ cm ; $F_1=\frac{2\,M\times 10}{(100-l^2)^2}=\frac{20.M}{(100-l^2)^2}$

When $d=20$ cm. $F_2=\frac{2\,M\times 20}{(400-l^2)^2}=\frac{40.M}{(400-l^2)^2}$

$\therefore \quad \frac{F_1}{F_2}=\frac{20\,M}{(100-l^2)^2}\times\frac{(400-l^2)^2}{40\,M}=\frac{(400-l^2)^2}{2(100-l^2)^2}$

According to the problem, $\frac{(400-l^2)^2}{2(100-l^2)^2}=12{\cdot}5$

or $\frac{(400-l^2)^2}{(100-l^2)^2}=25=(5)^2$ so, $\frac{400-l^2}{100-l^2}=\pm 5$

Taking $+$sign, $400-l^2=500-5l^2$ or $4l^2=100 \quad \therefore \quad l=5$ cm.

So, the length of the bar-magnet$=5\times 2=10$ cm.

Taking $-$sign, $400-l^2=-500+5l^2$ or, $6l^2=900 \quad \therefore \quad l=5\sqrt{6}$ cm.

So, the length of the bar-magnet$=5\sqrt{6}\times 2=24{\cdot}4$ cm. (nearly)

Example 2 : *Two short magnets of magnetic moments M_1 and M_2 are fixed on a table as shown in fig. 2.4. What will be the direction and magnitude of the magnetic field produced by these magnets at the point P ? $M_1=2700$ c.g.s. units ; $M_2=3200$ c.g.s. units ; $d_1=30$ cm. $d_2=40$ cm.* [*I.I.T. 1976*]

Ans. The point P is situated on the broad-side on position (or equatorial position) with respect to both the magnets. Now the field intensity at P due to the magnet of moment M_1 is given by $F_1=\frac{M_1}{d_1^3}$ and it acts parallel to the length of the magnet [Fig. 2.4]. Similarly, the field intensity F_2 due to the other magnet is $F_2=\frac{M_2}{d_2^3}$ and it acts parallel to the length of the second magnet. Since the two intensities are at right angles to each other, the resultant field F is given by

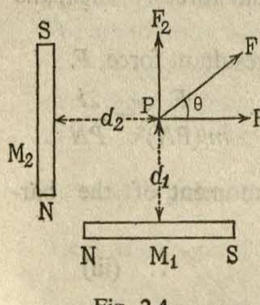


Fig. 2.4

$$F=\sqrt{F_1{}^2+F_2{}^2}=\sqrt{\left(\frac{M_1}{d_1{}^3}\right)^2+\left(\frac{M_2}{d_2{}^3}\right)^2}=\sqrt{\left(\frac{2700}{27000}\right)^2+\left(\frac{3200}{64000}\right)^2}$$

$$=\frac{\sqrt{5}}{20}\text{ Oe}=\frac{2\cdot24}{20}=0\cdot112\text{ Oe.}$$

If the resultant field acts in a direction making an angle θ with the direction of F_1, then $\tan\theta=\frac{F_2}{F_1}=\frac{M_2/M_1}{d_2{}^3/d_1{}^3}=\frac{3200}{64000}\times\frac{27000}{2700}=\frac{1}{2}\quad\therefore\quad\theta=\tan^{-1}\frac{1}{2}$

Example 3 : *The poles of a bar-magnet, each of strength 14·4 units, are 30 cm. apart. The bar-magnet is pivoted without friction at the centre of the bar. The bar has been kept inclined at an angle of 30° to a magnetic field of intensity 0·25 Oe by the application af a force F at a point 12 cm. away from the pivot perpendicular to the axis of the bar. Find the magnitude of the force.*

Ans. *AB* is the bar-magnet. It is pivoted at *O*, the middle point of the bar. *H* is the direction of the magnetic field [Fig. 2.5]. The two poles will experience forces *mH* and *mH* in the opposite directions. The force *F* is applied perpendicular to the magnetic axis *AB* at *C*, 12 cm. away from *O*. Since the bar is at rest, the algebraic sum of the moments of all the forces about any point will be zero. Hence taking moments about *O*

$$mH\times EO+mH\times GO-F\times CO=0$$

or $\quad mHl\cos30°+mHl\cos30°=F\times12$

$$[AO=BO=l]$$

or $\quad 2\times14\cdot4\times0\cdot25\times15\times\frac{\sqrt{3}}{2}=F\times12$

or $\quad 54\sqrt{3}=F\times12$ or $F=7\cdot78$ dynes (nearly)

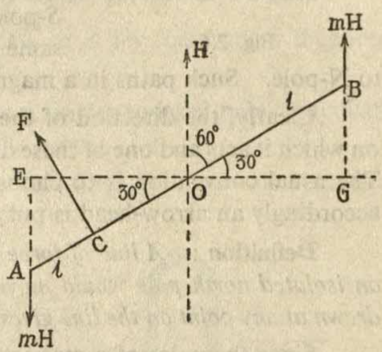


Fig. 2.5

2.8. Two poles of a magnet are of equal strength :

We can show, by the following simple experiment, that the poles of a magnet are of equal strength.

Place a bar-magnet on a piece of cork and float it in water. The bar-magnet, after a few oscillations, will point towards north and south. If the magnet be slightly disturbed, it will rotate a little but will not move sideways and will finally come back to the initial north-south position. This shows that the magnet is acted on by a couple instead of a single resultant force, because we know that a couple produces rotational motion while a single resultant force produces a translational motion. Suppose, the two poles of the magnet are of strength *m* and *m'*. Since the magnet is freely floating in the earth's magnetic field, the forces acting on the poles are *mH* and *m'H*, where *H* is the horizontal intensity of the earth's magnetic field.

Since a couple is constituted of two parallel, equal but oppositely directed forces, it follows that *mH=m'H i.e. m=m'*. Hence, the two poles of a magnet are of equal strength.

2.9. Magnetic lines of force :

By the following simple experiment, it can be shown that the magnetic force in the field of a bar magnet acts along a particular direction.

Experiment : A bar-magnet *NS* rests on the edge of a glass trough containing water. Float a magnetised knitting needle vertically in water on a fairly large piece of cork with its *n*-pole projecting a little out of the water. When the needle is held near the *N*-pole of the magnet and then released, it is repelled and travels towards the *S*-pole along a curved path (Fig. 2.6). If the needle is released from the same point, it will move along the same curved path repeatedly, but when released from different points, it will follow different paths (shown by the dotted lines in the figure). If the experiment is repeated with the *S*-pole of the needle uppermost, it travels along the same path but in opposite direction *i.e.* from *S*-pole to N-pole. Such paths in a magnetic field are known as *lines of force.*

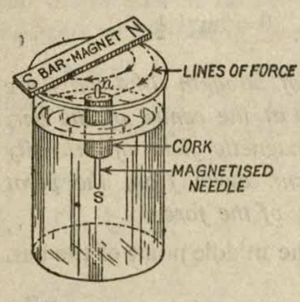


Fig. 2.6

Clearly, the direction of the force along a line of force depends on the pole on which it acts and one of these directions has to be chosen as a standard direction. The usual convention is to choose the direction of the force on a north pole and accordingly an arrow-head is put on each line of force.

Definition : *A line of force is defined as a line in a magnetic field along which an isolated north pole would move if free to travel and it is such that the tangent drawn at any point on the line gives the direction of the resultant force at that point.*

Since the poles of a magnet are of equal strength, the *N*-pole is urged along a line of force in one direction while the *S*-pole is urged with an equal force in the opposite direction. The result is that a small magnet (say a compass needle) sets itself along the line of force without any bodily motion in any direction.

It has been pointed out earlier that a line of force starts from *N*-pole of a magnet and terminates on the *S*-pole. From this it may appear as if the line of force is a discontinuous line. As a matter of fact, it is not ; it is supposed to exist inside the magnet too and its direction inside the magnet is from south to north pole [Fig. 2.7]. So, *a line of force is a closed curve.*

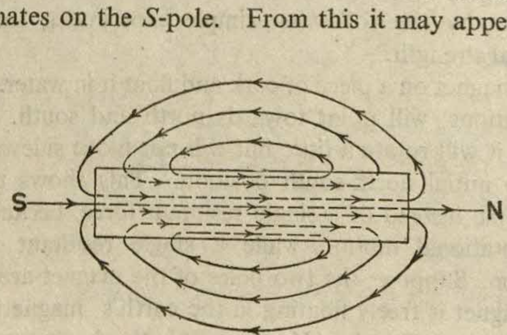


Fig. 2.7

2.10. Properties of magnetic lines of force :

In order to explain different magnetic phenomena, the following properties are attributed to the lines of force :

(i) A line of force emerges from the north pole of a magnet and terminates on the south pole.

(ii) Each line of force has a tendency to contract longitudinally like a stretched elastic thread and they exert lateral pressure.

(iii) No two lines of force cut each other ; because in that case there would be two directions of resultant magnetic force at a given point which is not possible.

(iv) Lines of force emerge normally from a north pole and enter normally into a south pole.

2.11. Pictures of lines of force due to some permanent magnets :

The nature of lines of force produced by a few permanent magnets is shown in the adjoining diagrams.

Fig. 2.8(i) shows the lines of force due to a single, isolated pole, if available. The lines of force are radial with the pole at the centre.

Figs. 2.8 (ii) and 2.8 (iii) illustrate the lines of force due to a bar magnet and a horse-shoe magnet respectively.

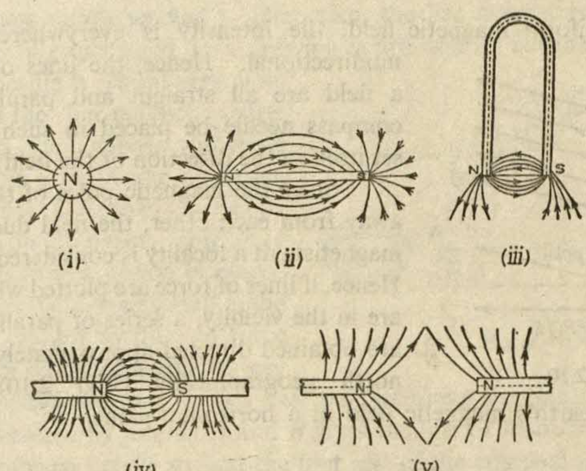


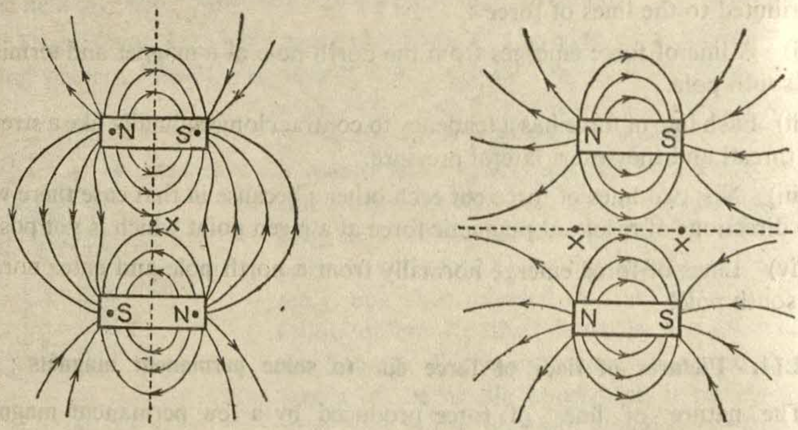
Fig. 2.8

How the lines of force are produced when two unlike and like poles face each other are depicted in the figs 2.8 (iv) and 2.8 (v) respectively.

Two bar magnets with their axes parallel :

Fig. 2·9 (i) shows the lines of force due to two bar magnets placed with their axes parallel and with their unlike poles near each other. Note that a neutral point (×) appears on the perpendicular bisector of the axes equidistant from the

axes of the bar magnets. Fig. 2·9 (ii) shows the lines of force due to two bar magnets placed with their axes parallel and with their like poles near each other.



(i) Fig. 2.9 (ii)

Here two neutral points (× and ×) appear on a line equidistant from the axes of the magnets.

2.12. Lines of force due to a uniform magnetic field :

In a uniform magnetic field, the intensity is everywhere the same and unidirectional. Hence, the lines of force of such a field are all straight and parallel lines. If a compass needle be placed in such a field, it will set itself in the direction of the field everywhere.

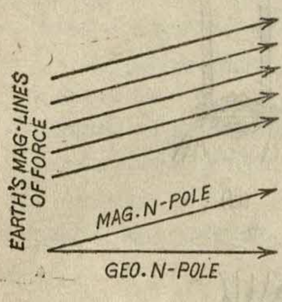


Fig. 2.10

Since the magnetic poles of the earth are far away from each other, the field due to the earth's magnetism at a locality is considered to be uniform. Hence, if lines of force are plotted when no magnets are in the vicinity, a series of parallel straight lines are obtained directed approximately from south to north geographically (Fig. 2.10). These lines represent the earth's magnetic field in a horizontal plane.

2.13. Mapping of lines of force :

Magnetic lines of force may be mapped in two ways :— (i) By iron filings and (ii) By compass needle.

(i) **Iron filings method :** Fix a thin sheet of paper on a drawing board by means of board pins and place a bar-magnet at the centre of the paper [Fig 2.11(i)]. Sprinkle some iron filings on the paper around the magnet and slightly tap the board. It will be found that filings have arranged themselves in a particular pattern. Due to inductive action of the bar magnet, each filing becomes a minute

magnet and sets itself in the direction of the lines of force. Tapping of board facilitates the filings to arrange themselves.

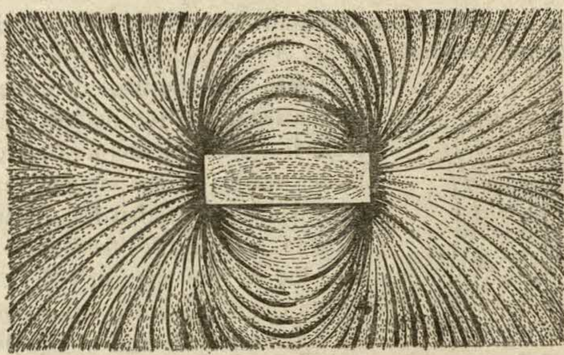


Fig. 2.11 (i)

To get a permanent picture of the lines of force, a thin and uniform coating of paraffin is to be given on the paper beforehand. When the coating becomes cold, the filings are to be sprinkled. When the filings have arranged themselves along the lines of force, the paper should be heated a little. Due to heat, wax will melt and the filings will be embeded in the wax. When the paper cools, the wax solidifies and we get a permanent picture of the magnetic lines of force.

(ii) **Compass needle method** : Here also, the bar magnet is placed at the centre of a piece of paper fixed on a drawing board and its outline is drawn. A small compass needle is placed very close to the *N*-pole of the bar-magnet. After a few oscillations, the compass will come to rest pointing at a particular direction. Its two ends *A* and *B* are then marked by putting two dots on the paper. [Fig 2.11 (ii)]. When a compass needle is placed freely in a magnetic field, it sets itself in the direction of the tangent to lines of force there. Since the compass needle is very small, it may be assumed that its axis has been set along the lines of force. So, the small line obtained by joining *A* and *B* gives the line of force at the position. Now shift the compass needle so that the first end of the compass is placed on *B*. Another mark *C* is then put on the other end of the compass needle. Repeat the process until the line traced out returns to the magnet. A smooth curve is drawn through the points obtained. In this way, other lines of force can be drawn so as to fill up the whole field. Fig. 2.12(a) and (b) show such mapping of lines.

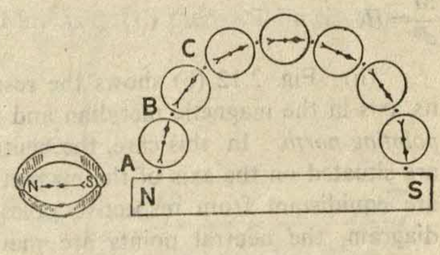


Fig. 2.11(ii)

2.14. Change of lines of force of a bar magnet due to earth's magnetic field :

The lines of force discussed in art 2.10 are due to one single magnet or due to two magnetic poles. Effect of earth's magnetism was not considered in those cases.

As a matter of fact, the pattern of lines of force changes on account of earth's magnetism. The resultant pattern of lines of force when a magnet's field is added to that of the earth depends on the direction in which the magnet is placed.

(i) Fig. 2.12(a) shows the resultant field when a magnet is lying with its axis in the magnetic meridian and its *N-pole pointing north.* From the figure it is seen that the lines of force at right angles to the length of the magnet and at a distance from it are all parallel and almost straight, but those nearer to the magnet are parallel and curved. It shows that the distant lines of force are due to the earth's field while the nearer ones are due to the magnet's field. Further, two points (marked with a cross sign) are available on *equatorial* line and at equal distances from the magnet where there is no line of force. Those points are called **neutral points.** At the neutral points the earth's field and the magnet's field are exactly equal and opposite and the resultant field is zero.

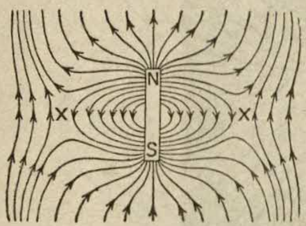


Fig. 2.12(a)

Definition : *A neutral point is defined as a point at which the resultant magnetic field due to a magnet and the earth's horizontal magnetic field is zero.*

At the neutral point $F=H$ or $\dfrac{M}{(d^2+l^2)^{\frac{3}{2}}}=H$ where d is the distance of the neutral point from the centre of the bar magnet. If the bar magnet is short, $\dfrac{M}{d^3}=H.$

(ii) Fig. 2.12 (b) shows the resultant field when the magnet is lying with its axis in the magnetic meridian and its *S-pole pointing north.* In this case, the neutral points are situated on the axis of the magnet and they are equidistant from respective poles. In the diagram, the neutral points are marked with a cross sign.

At the neutral point $F=H$ or $\dfrac{2Md}{(d^2-l^2)^2}=H,$ where d is the distance of the neutral point from the centre of the bar-magnet.

If the bar magnet is short, $\dfrac{2M}{d^3}=H.$

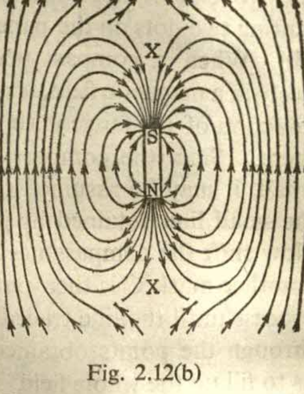


Fig. 2.12(b)

Example 1 : *A bar-magnet 8 cm. long is placed in a horizontal plane along the magnetic meridian with north pole pointing north. Neutral point is found to be situated at a distance 5 cm. from the centre of the magnet. If the earth's horizontal intensity be 0·346 Oe, find the pole strength of the magnet.*

Ans. As the magnet has its north pole pointing north, the neutral point will be situated at an equatorial position, where the intensity is given by
$$F=\frac{M}{(d^2+l^2)^{\frac{3}{2}}}=\frac{M}{(25+16)^{\frac{3}{2}}}=\frac{M}{41\sqrt{41}}=\frac{M}{262\cdot5} \qquad [l=8/2=4 \text{ cm.}]$$
If m be the pole strength, $M=2.m.l.=8\times m$

According to the problem. $F=0\cdot346$ Oe.
$$\therefore\ 0\cdot346=\frac{8\times m}{262\cdot5} \quad \therefore \quad m=\frac{262\cdot5\times0\cdot346}{8}=11\cdot4 \text{ units.}$$

Example 2 : *A small magnet is kept in the magnetic meridian with north pole pointing south. Neutral point is found along the axis of the magnet at a distance 24 cm. from the centre of the bar. What will be the intensity at a distance 20 cm. north of the south pole of the magnet ? $H=0\cdot18$ Oe.*

Ans. As the neutral point is on the axial position, we can write
$$F=\frac{2M}{d^3} \quad \text{or} \quad 0\cdot18=\frac{2M}{(24)^3} \quad \therefore \quad M=0\cdot09\times(24)^3$$
If F_1 be the intensity at a point distant 20 cm. at the end-on position,
$$F_1=\frac{2M}{(20)^3}=\frac{2\times0\cdot09\times(24)^3}{(20)^3}=0\cdot31 \text{ Oe.}$$

Example 3 : *The two poles of a bar-magnet are 10 cm. apart. It is placed in the magnetic meridian with its north pole pointing south. A neutral point is obtained at a distance of 20 cm. from the nearer pole. What will be the resultant field intensity at a point 10 cm. from the centre of the magnet and on the perpendicular bisector of the magnet ? $H=0\cdot4$ Oe.*

Ans. If P be the neutral point, the field intensity (F) there will be equal to H, the horizontal component of earth's magnetic field. Since the north pole of the bar magnet points south, the neutral point P is situated along the axis of the magnet [Fig. 2.13]. Now field intensity F at an axial point is
$$F=\frac{2\times2ml\times d}{(d^2-l^2)^2}$$

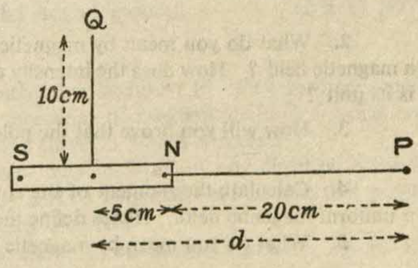


Fig. 2.13

Here, $2l=10$ cm ; $d=20+5=25$ cm ;

So, field intensity at P is
$$F=\frac{2\times10\times m\times25}{\{(25)^2-(5)^2\}^2}=\frac{20\times25\times m}{(30\times20)^2}$$
$$\text{So,} \quad \frac{20\times25\times m}{(30\times20)^2}=0\cdot4 \quad \text{or} \quad m=\frac{0\cdot4\times600\times600}{20\times25}=288 \text{ units}$$

Again at the point Q on the perpendicular bisector and at a distance 10 cm. from the centre of the magnet, the field intensity is given by
$$F_1=\frac{M}{(d^2+l^2)^{\frac{3}{2}}}=\frac{2ml}{(d^2+l^2)^{\frac{3}{2}}}=\frac{288\times10}{\{(10)^2+(5)^2\}^{\frac{3}{2}}}=\frac{288\times10}{125\times5\sqrt{5}}=2\cdot07 \text{ Oe}$$

2.15. Magnetic screen :

If a soft iron ring be placed in the magnetic field of a bar-magnet or between the poles of two magnets facing each other, all the lines of force are found to pass through the ring and no lines cross the space inside. The inside of the ring is, therefore, protected against any magnetic effect and a magnetic needle placed there will remain undisturbed.

When a magnetic needle is held in front of a pole of a magnet, the needle is deflected but if a sheet of soft iron be interposed between them, the needle remains undeflected. The reason is that the lines of force pass through the sheet (Fig. 2.14) and can not extend on the other side. For this peculiar behaviour of soft-iron, the ring or the sheet mentioned above, is called a *magnetic screen.*

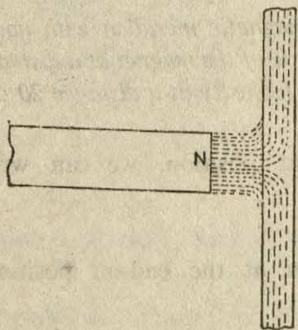


Fig. 2.14

Magnetic screen is put to practical use. For example, delicate measuring instruments like galvanometers etc. which are liable to be affected by stray external magnetic fields can be protected by a magnetic screen. Costly wrist watches are given a soft iron ring round it to make it 'magnet-proof'.

Exercises

Essay type :

1. State Coulomb's law relating the forces acting between two poles and hence define unit magnetic pole. What do you mean by intensity of magnetic field ?
[*H S. Exam. 1978, '80*]

2. What do you mean by magnetic field ? What is meant by the intensity at a point in a magnetic field ? How does the intensity at a point depend on the distance of the point ? What is its unit ?

3. How will you prove that the poles of a magnet are of equal strength ?
[*H. S. Exam. 1980*]

4. Calculate the moment of the couple acting on a magnet which is suspended freely in a uniform magnetic field. Hence define the magnetic moment of a magnet.

5. What do you mean by magnetic lines of force ? State the properties of lines of force.
[*H. S. Exam. 1982*]

6. How would you shield a small magnetic needle from the influence of earth's magnetism ? Draw a neat diagram and show the lines of force.

7. Find the intensity (i) at a point along the axis (ii) at a point on the perpendicular bisector of the axis of a bar magnet.

Short answer type :

8. Write, in brief, what you know about the following :—(i) Magnetic field, (ii) Magnetic lines of force. [*H. S. Exam. 1978*] (iii) Neutral point.

9. A compass needle always sets in one particular direction at any given place in a magnetic field but sets in different directions when placed at different places. What is the reason of it ?
[**Hints :** The magnetic field directions are different at different points.]

10. Indicate the pattern and the direction of the lines of force in the following case :— (i) an isolated single pole, (ii) a bar-magnet, (iii) two like poles facing each other.

11. Draw the lines of force around a bar magnet kept along the magnetic meridian with (i) its *N*-pole pointing north and (ii) its *S*-pole pointing north. Mark the neutral points in each case. [*H. S. Exam. 1979*]

12. (*a*) It is found that the neutral points lie along the axis of a bar magnet kept on a table. What is the orientation of the magnet with respect to the magnetic meridian ? [*I.I.T. 1974*]

(*b*) Draw lines of force around two bar magnets placed with their axes parallel when (i) their like poles are near to each other (ii) their unlike poles are near each other. Also locate the neutral points in both the cases.

13. The material of the bob of a simple pendulum is iron. If a magnetic pole is situated just below the bob, how will the time-period be affected ? Explain with reason. [*H. S. Exam 1984*]

14. A south pole and a north pole are facing each other and a ring of soft iron is placed between the two. Draw the pattern of lines of force before and after the placement of the ring.

15. How can sensitive instruments be protected against sudden external magnetic field ? What is a 'magnet proof' watch ?

16. Define lines of force due to a magnet. What are their properties ? What is neutral point ?

17. A magnetic needle is floating on a piece of cork in the steady water of a lake in northern hemisphere. Will the needle slowly move with the cork towards the north of the lake ?

[**Hints** : No ; the magnetic north pole of the earth will exert equal but opposite forces on the two poles of the needle. So the needle will remain at rest.]

18. A magnetic wire has a length $2l$ and pole strength m. If the wire is broken into two equal parts, what will be the magnetic moment of each part ?

19. A bar magnet is fixed rigidly with its axis parallel to the magnetic meridian. Another bar magnet is kept parallel to the first one at such a position that their centres lie on their perpendicular bisector. If the second magnet is free to move, state whether it will have (i) a linear motion (ii) a rotational motion (iii) both kinds of motion. [*I. I. T. 1977*]

Objective type :

20. (*a*) Two lines of force do not intersect each other because (i) they contract longitudinally like a stretched elastic thread, (ii) two directions of magnetic field at a point are available which is absurd, (ii) lines of force are closed curves. Which is correct ?

(*b*) A compass needle placed at a neutral point may set itself at any direction because (i) there is no electric field at that point, (ii) only earth's magnetic field acts at that point, (iii) the resultant field at that point is uniform. Which is correct ?

(*c*) The field strength at a distance 4 cm. from a single pole of strength 32 units is (i) 8 Oe, (ii) 2 Oe, (iii) 16 Oe. Which is correct ?

Numerical Problems :

21. Two identical thin bar magnets each of length l and pole strength m are placed at right angles to each other, with the north pole of one touching the south pole of the other. Evaluate the magnetic moment of the system. [*I.I.T. 1967*]

[Ans. $\sqrt{2}\,ml$. making 45° with the axis of any magnet.]

22. A magnet 8 cm. long is placed in a magnetic field of intensity $H=0{\cdot}18$; its pole strength is 5. Calculate the moment of the couple required to deflect the magnet at right angles to the direction of the magnetic field. [Ans. 7·2 units]

23. The strength of two magnetic poles are 40 and 60 and they are at a distance of 10 cm. in air. What is the force acting between the poles ? [Ans. 24 dynes]

24. A magnetic pole is eight times stronger than another pole and they exert a force of 500 mg-wt upon each other when separated by distance 10 cm. in air. Find the strength of each pole. [Ans. 78 ; 624]

25. Find the intensity of magnetic field at a distance of 5 cm. from a magnetic pole of strength 100 units ? [Ans. 4 units]

26. Two identical poles are 10 cm. apart in air. Calculate the strength of each pole if the mutual force of repulsion between them be 81 dynes [*H. S. Exam. 1982*] [Ans. 90 units]

27. Two magnetic poles of strength 16 and 25 units are 9 cm. apart. At what point on the line joining the poles will the field intensity be zero ? [Ans. 4 cm. from 16 unit pole]

28. Two north poles repel each other with a force of 2·5 dynes when their distance apart in air is 2 cm. What should be the distance between them when the force is 3·6 dynes ? Find also the repulsive force when the distance between them is 5 cm. [Ans. 1·66 cm. ; 0·4 dyne]

29. Find the intensity of magnetic field at a point 20 cm. from the centre of a magnet of length 10 cm. and pole strength 50 unit, situated on the axis of the bar-magnet.

[Ans. 0·142 Oe (nearly)]

30. A bar-magnet has its two poles separated by 10 cm. It is kept in the magnetic meridian with its north pole pointing south. Neutral point is found at a distance 20 cm. from the nearest pole. What will be the resultant intensity at a point on the perpendicular besector and at a distance of 10 cm. from the centre of the baı ? $H=0{\cdot}4$ Oe. [Ans. 2·07 Oe]

31. The ratio of intensities at two axial points at distances 10 cm. and 20 cm. from the centre of a bar-magnet is 18 : 1. Find the length of the magnet.

[Ans. 12·64 cm. or 23·9 cm.]

32. Two short bar-magnets of moments 108 and 192 units are placed along two lines perpendicular to each other. Find the intensity at the point of intersection of the lines, centres of the magnets being 30 and 40 cm. from this point. [Ans. 0·01 Oe]

Harder Problems :

33. A magnetic wire of moment M is bent in the form of L. If the two arms of the bent wire are 6 cm and 4 cm, find the new moment. [Ans. 0·65 M (nearly)]

34. The magnetic moment of a magnetised steel were of length l is M. If the wire is now bent in the form of a semicircle, show that its persent moment is $\frac{2M}{\pi}$.

35. A bar magnet is placed horizontally in the magnetic meridian with its south pole towards the north and at a point distant 10 cm. from the south pole, the resultant field strength is zero. If the distance between the poles of the magnet is 10 cm, and the value of the horizontal field due to the earth is 0·2 Oe find the pole strength of the magnet.

[*Jt. Entrance 1981*] [Ans. 33·7 (nearly)]

36. A bar magnet of magnetic length 12 cm. and pole strength 25 c.g.s. unit is placed along the magnetic meridian at a place with its N-pole facing the north. If the horizontal component of earth's magnetic field at the place be 0·3 Oersted, obtain the positions of neutral points.

[*Jt. Entrance 1982*] [Ans. 8 cm. away on the perpendicular bisector]

37. A neutral point is found on the axis (extended) of a bar-magnet at a point 7 cm from the nearer pole. The poles of the bar-magnet are 4 cm apart. and the horizontal component of earth's magnetism is 0·36 Oe. Estimate the pole strength of the magnet.

[*Jt. Entrance 1979*] [Ans. 29·65 unit]

38. The poles of a bar-magnet are 10 cm. apart and the moment of the magnet is 1000 unit. A north pole of strength 200 units is kept on the axis of the bar magnet at a distance 25 cm. from the centre of the bar. What force will the bar-magnet exert on the pole ? [Ans. 27·7 dynes]

39. A magnetic needle of moment 900 unit and pole strength 50 unit is so pivoted that it can freely rotate in a horizontal plane. The horizontal component of earth's magnetic intensity

is 0·36 Oe. The magnetic needle is kept at rest inclined to the magnetic meridian at an angle 30° by pulling a thread towards east, which is fixed to the needle. What is the tension in the thread ? What is the length of the needle ? [Ans. 20·78 dynes ; 18 cm.]

40. In order to keep a magnet inclined to 30° with the meridian of a uniform magnetic field of intensity 0·362 Oe, a couple of moment 2896 dyne-cm is required. Calculate the magnetic moment of the magnet. [Ans. 16×10^3 units]

41. When two magnetic poles of different pole-strengths are placed 4 cm. apart, the force acting between them is $\frac{7}{4}$ dynes. The difference of their pole strengths is 3 units. Determine their pole strengths. [Ans. 7 and 4 units]

42. Two short magnets of magnetic moments M_1 and M_2 are fixed on a table as shown in fig. 2·15. M_1=2700 c.g.s. ; M_2=4000 units ; d_1=30 cm ; (i) What should be the distance d_2 such that P is a neutral point ? (ii) Indicate on a diagram the N and S poles of the magnets (Horizontal component of the earth's field=0·3 c.g.s. units) [*I. I. T. 1976*]

[Ans. (i) 20 cm. when the north poles of M_1 and M_2 are facing north. (ii) 15·54 cm. when the north pole M_1 is facing south but the north pole of M_2 is facing north]

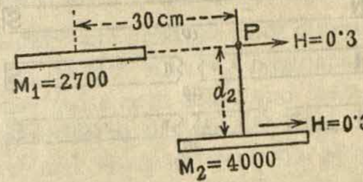


Fig. 2.15

43. Two bar magnets M_1 and M_2 are of equal length. The pole strength of M_1 is twice that of M_2. They are set at right angles to each other with their *n*-poles in contact. If the combination is floated on a piece of cork, find the angle that M_1 will make with the meridian in equilibrium. [Ans. 26·5° (nearly)]

44. Two magnets of equal mass are joined at right angles to each other. Magnet N_1S_1 has a magnetic moment 3 times that of N_2S_2. The arrangement is pivoted so that it is free to rotate in the horizontal plane. What angle the magnet N_1S_1 will make with the magnetic meridian when the system comes to rest ? [*I. I. T. 1975*] [Ans. $\tan^{-1}\frac{1}{3}$]

45. Three small magnets have their centres at the corners of an equilateral triangle ABC. and are free to rotate about their centres. The magnet at A is in equilibrium under the action of the magnets at B and C with its length parallel to BC while the lengths of the magnets at B and C are perpendicular to AB and AC. Prove that under this condition, the moments of the magnets at B and C are equal.

3

MOLECULAR THEORY OF MAGNETISM

3.1. Isolation of a single pole is impossible :

A magnet, artificial or natural, has two poles. It is not possible to isolate one pole of a magnet from the other. If a bar magnet is cut into two, it is found that a new pole now exists near each cut-end, opposite in kind to the pole at the other end of the piece (Fig. 3.1). No matter how many times the cutting process is carried out, this is found to happen always ; it is impossible to have a piece of steel with only one pole on it. This suggests that when the limit is reached in such a cutting-up process, each piece would still be equivalent to a bar-magnet having two poles.

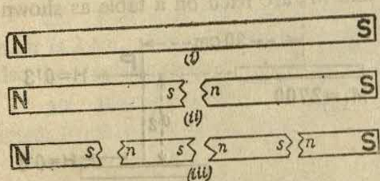


Fig. 3.1

3.2. Theory of magnetism :

From the preceding article we come to know that if the process of cutting-up of a magnet could be carried out indefinitely, then, theoretically, single molecule would eventually be reached and it is quite reasonable to suppose that these would also be magnets with two poles. Wilhelm Weber proposed the *molecular theory of magnetism* from the fact that each individual molecule of a magnetic substance behaves like a full-fledged magnet. The theory was later developed by Ewing. Due to this reason, it is also known as Weber-Ewing molecular theory. According to this theory it is said that not only the molecules of a magnet but even the molecules of a magnetic substance are complete magnets in themselves. In an unmagnetised piece of iron, the molecules are present in all possible directions and form closed chains. Therefore, the speciman does not exhibit any magnetic property. In the case of a magnet however, the molecular magnets are regularly arranged with their north poles all pointing in a given direction while the south poles in the opposite direction.

In an unmagnetised piece of iron, say, these molecules are arranged in random directions and form closed chains [Fig. 3.2]. Consequently, *N*-pole of a molecule get neutralised by the *S*-pole of an adjoining molecule facing it. Such closed chains do not permit manifestation of magnetic properties in the specimen. For this reason, a magnetic specimen does not exhibit free polarities or any other magnetic property until it is magnetised.

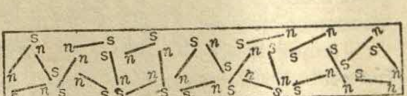


Fig. 3.2

But when the specimen is brought close to the pole (say, *S*-pole), of a strong magnet the closed magnetic chains of the molecules break down under the

influence of the strong pole and the n-pole of the molecules being attracted by the stroking pole, turn towards it [Fig. 3.3]. If the specimen is stroked repeatedly by the strong pole, more and more molecules are pulled into line with their like poles all pointing to the same direction. The molecules thus form *open chains* along the length of the specimen which becomes a magnet with free poles at the two ends (Fig. 3.4).

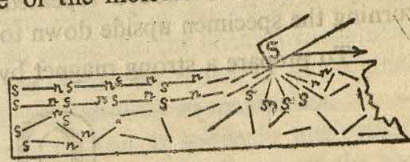


Fig. 3.3

At the middle of the specimen, the molecules are arranged with opposite poles facing each other. Consequently, they neutralise each other and there is no magnetism at the middle of the bar. At each end, however, free poles of like nature exist which give rise to polarities at the two ends of the bar.

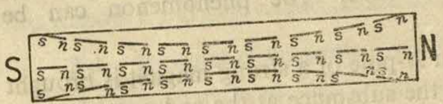


Fig. 3.4

On account of the mutual repulsion between free molecular poles at the ends of the open chains, the chains are not exactly parallel but fan out a little at ends. This is the reason why the poles of a bar magnet are never located exactly at its ends. Furthermore, since each open chain has one free molecular pole at each end the total amounts of free north and south polarity in a magnet are equal.

Modern theory : According to the modern conception, a magnetic substance consists of numerous magnetic regions, very small in size, known as **domains.** Length of each domain may vary form 10^{-3} to 10^{-5} cm. and may consist of about 10^{12} to 10^{15} atoms. Ordinarily, the magnetic axes of the domains are oriented in different directions and hence the specimen does not exhibit any magnetic property.

3.3. Explanation of some magnetic phenomena according to the molecular theory :

(i) *Magnetisation by rubbing* : We know that artificial magnets can be prepared by rubbing. The process can be explained by the molecular theory in the following way :

In single touch method, for example, when the stroking S-pole touches one end of the steel bar, the molecules near the point turn round with their n-poles pointing towards the stroking pole and s-poles in the opposite direction (Fig. 3.3). As the stroking pole rubs the length of the specimen, more and more molecules line up in the above way along the point of contact. Finally when the stroking S-pole leaves the specimen, some of the molecules turn with their n-poles pointing towards that end and s-pole in the opposite way. With repeated stroking of the specimen, almost all the molecules are arranged in this way and the bar exhibits free polarity at its ends.

The other touch methods (double and divided) may also be explained in the same way. It is to be remembered that touch methods only affect the surface molecules ; its effect cannot penetrate deep into the specimen. This is why the

specimen is to be rubbed repeatedly. Rubbing should preferably be done by turning the specimen upside down too.

To prepare a strong magnet by the method of stroking, several thin sheets, instead of a single thick bar, should be magnetised and then the sheets should be bolted together with their similar poles in the same direction. Such magnets are called *laminated magnets* or *magnetic battery* (Fig. 3.5).

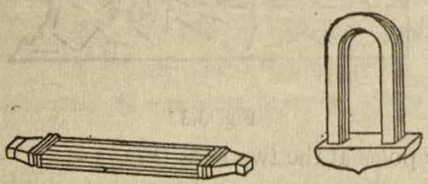


Fig. 3.5

(ii) *Magnetic induction* : Magnetic induction has been described in art 1.10. The phenomenon can be explained by the theory of magnetism.

During the process of induction, when N-pole of a strong magnet is brought near a magnetic substance, the molecules of the substance at the end nearer to the pole come under the influence of the pole. The south poles of the magnetic molecules being attracted by the strong N-pole of the magnet, turn towards the pole and the north poles turn in the opposite direction. Due to such orientation of the molecular magnets near the end of the magnetic substance, other nearby molecules are also affected and they, too orient themselves in the like manner. As a result, a large number of molecules at the end nearer to the inducing N-pole get their south poles turned towards the inducing pole while a large number of molecules at the furthest end get their north poles turned towards that end. Consequently, free south pole is developed at the nearer end and free north pole at the furthest end.

If the specimen is made of soft iron, the above orientations of the molecules break down as soon as the inducing N-pole is removed from near the specimen and closed chains are again formed. For this reason, the specimen is found to lose its magnetism immediately after the removal of the inducing pole.

(iii) *Magnetic saturation* : When a piece of iron or steel is magnetised it is found that as the magnetisation is continued, the magnetism in the specimen gradually increases. The strength of the magnetism in the specimen finally reaches a maximum value, known as *saturation point*. Beyond the saturation point, the magnetism does not increase, even if the process of magnetisation is continued. This can be explained by the molecular theory of magnetism.

When the magnetising forces are applied on the specimen, the closed chains of molecular magnets are broken, some of which turn in a particular direction. A feeble magnetism is, therefore, developed in the specimen. As the force of magnetisation is increased, more and more molecules orient themselves in the given direction and finally when all the molecular magnets arrange themselves in a line, the stage of saturation is reached. Furher magnetisation does not increase magnetism because no more closed chain of molecules exist in the specimen.

(iv) *Curie point* : Curie point has been mentioned in art 1.16. The existence of curie point in a magnetic specimen may be explained by the theory of magnetism.

When a magnet is heated, agitation and hence the mean kinetic energy of the molecular magnets increases. When the temperature of the specimen reaches the curie point, the agitation increases to such an extent that the linear arrangement of the moleeules is disturbed violently and the molecules return to the orginal closed chain formation. In that condition, the magnet loses its magnetism.

(v) *Magnetostriction* : When a long rod of iron is suddenly magnetised by a strong electric current, a metallic sound is heard and the length of the rod increases a little. If the magnet be of horse-shoe shape, the gap between its poles decreases. This phenomenon is known as *magnetostriction.*

According to theory of magnetism, the closed chains of molecules are suddenly broken and the molecules are set parallel to the length of the rod. For this reason, a metallic sound is heard.

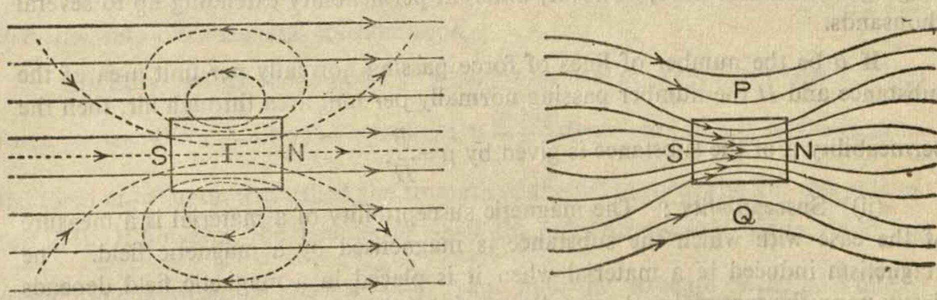
3.4. Presence of magnetic substance in a magnetic field :

A uniform magnetic field is represented by straight and parallel lines of force. If any magnetic substance like iron, steel, nickel etc. be placed in such uniform field, the lines of force undergo a considerable change. When placed in a uniform magnetic field, the molecular magnets (or the domains) of the specimen partly become aligned with the lines of force of the field. Their own lines of force, then become oriented in the same direction. Consequently, two types of lines of force are, then developed inside the specimen :

(i) *The lines of force due to external inducing field.* These lines will be present even if a non-magnetic substance is kept instead of the specimen. These are known as simply *lines of force.*

(ii) *The lines of force due to the induced magnetism* of the specimen. These are known as *lines of magnetisation.*

Consider a cylindrical bar of a magnetic substance placed in a uniform field of intensity H. If α be the cross-sectional area of the bar, the number of lines of force crossing the bar is αH. As the bar is of magnetic substance, it will be



(a) Fig. 3.6 (b)

magnetised by induction and poles will be developed at its ends. It will then create some additional lines of force in air as well as within itself [Fig. 3.6(a)]. In the diagram continuous parallel lines denote the lines of force due the external magnetising field and the broken lines denote its own lines of magnetisation. It is to be noted that both types of lines of force are almost side by side and unidirectional inside the material of the specimen [Fig. 3.6 (b)]. The combined lines of

force within the material are known as *lines of induction.* Outside the specimen, in air, the lines of force and the lines of magnetisation are oppositely directed. Due to this difference, the field intensity inside and outside the specimen becomes different. Lines of induction inside the specimen are very much crowded and thereby make the field intensity very high ; while those outside the specimen are sparse and hence the intensity there is very low (as at P and Q).

If M be the moment of the induced magnetism in the cylindrical rod, then its *intensity of magnetisation* $I=\frac{M}{V}$, where V is the volume of the rod.

Again $M=m.l.$ and $V=\alpha l$ where m is the pole strength of the bar magnet and l its magnetic length. Therefore, $I=\frac{ml}{\alpha.l}=\frac{m}{\alpha}=\frac{\text{pole strength.}}{\text{area.}}$

Hence, *intensity of magnetisation may alternatively be defined as the pole strength developed per unit area of the cross-section of the bar.*

3.5. Some special properties of a magnetic substance :

(i) **Permeability :** We have already seen that when a soft iron ring is placed in the field of a bar-magnet the lines of force are drawn into the iron and concentrate through it. Iron is, therefore, said to be more permeable to magnetic lines of force than air. As a matter of fact, any magnetic substance, placed in a magnetic field, is found to concentrate more lines of force within it than air. The permeability of a substance is the ratio of the number of lines of foce passing normally through unit area of the substance to those of the air. For example, the permeability of a magnetic substance is 100—this means that the number of lines of force passing normally through unit area of the substance is 100 times the number of lines of force passing through air. The permeability of magnetic substances is greater than those of ordinary non-magnetic substances. Different magnetic substances have, however, different permeability extending up to several thousands.

If B be the number of lines of force passing normally per unit area of the substance and H the number passing normally per unit area through air, then the permeability μ of the substance is given by $\mu=\frac{B}{H}$.

(ii) **Susceptibility :** The magnetic susceptibility of a material is a measure of the ease with which the substance is magnetised by a magnetic field. The magnetism induced in a material when it is placed in a magnetic field depends firstly upon the material and secondly upon the magnetising field. But in a given magnetising field, different materials are found to acquire different amount of magnetism. It has been found experimentally that soft-iron acquires a greater amount of magnetism than steel in a given magnetising field. For this reason, soft-iron is said to have more susceptibility than steel. Mumetal, an alloy of nickel, iron and copper (73% nickel, 22% iron and 5% copper), has high susceptibility.

If I be the intensity of magnetisation developed parallel to the lines of force of a uniform magnetic field of intensity H, then the susceptibility K of the substance is given by $K=\frac{I}{H}$. or $I=KH$. It may be proved that $\mu=1+4\pi K$.

(iii) **Retentivity and Coercivity** : Take two identical rods—one of soft-iron and the other of steel—and magnetise them by the same magnetising field. On removal of the magnetising force, it is found that under *certain circumstances*, the soft-iron specimen, like steel, retains almost all the magnetism. Experiment shows that about 90% of the magnetism is retained by them. But if the rods be stirred a little, soft-iron rod loses practically all the megnetism in it while the steel rod does not show any change. This shows that both soft-iron and steel have same retentivity but their coercivity is different. Soft-iron has much less coercivity than steel because soft-iron cannot withstand a slight force applied on it for removal of its magnetism. Under ordinary conditions, the retentivity of soft-iron, is however less than that of steel.

The above properties of magnetic substances are taken into consideration while preparing temporary and permanent magnets used in different appliances and instruments, and accordingly selection is made between iron, steel and other varieties of alloy. For example, a material meant for permanent magnet should have high retentivity and coercivity. For this reason, *tungsten steel*, *cobalt steel*, *alnico*, *ticonal* (an alloy of titanium, cobalt, nickel and aluminium) are used for preparing permanent magnets. For preparing electromagnets, a material having low retentivity and low coercivity is suitable. *Soft-iron* or *stalloy* (5% silicon and 95% iron), is therefore, used for this purpose. For the 'core' of a transformer, *permalloy* (50% iron and 50% nickel) and *transformer steel* (4% silicon and rest soft iron) are used because they have high permeability.

(iv) **Magnetic induction** : We now introduce a new term, *magnetic induction* into our magnetic consideration.

Consider two magnetic poles of strength m_1 and m_2 kept d cm. apart. The force between them in air or in a vacuum, is $\frac{m_1 m_2}{d^2}$ dynes and if m_2 is a unit pole, the force m_1/d^2 upon it is called the strength of the field (say H) due to the pole m_1 (art 2.4). If however, the medium in which the poles are situated has permeability μ, the force between the poles is $\frac{m_1 m_2}{\mu d^2}$ dynes and the strength of the field due to m_1 is $m_1/\mu d^2$. Thus, the strength of the field depends on the medium in which it is placed. It is very important to have a quantity which is fixed for a given pole and distance, independently of the medium. This quantity is called the *magnetic induction* (B) and we see from the above that it must be μ times the strength of the field or $B=\mu H$

Now, for a distance d from a given pole $H=\dfrac{m}{\mu d^2}$

$$\therefore \quad B=\mu \frac{m}{\mu . d^2}=\frac{m}{d^2}$$

B is therefore independent of the medium, thus fulfilling the condition required. Thus, *magnetic induction may be defined as a quantity equal to* μ *times the strength of magnetic field at any point.*

3.6. Force between plane poles in contact :

Consider the space between the plane poles of two bar magnets placed closed together as in Fig. 3.7. If the faces are sufficiently close together the magnetic field at any point between them is uniform and each polar face may, for all practical purposes, be considered to be infinite. Let I be the intensity of magnetisation of each magnet. It may be proved, by the application of Gauss's theorem that the strength of the field at any point between the polar faces due to the polar sheet N or S is $2\pi I$.

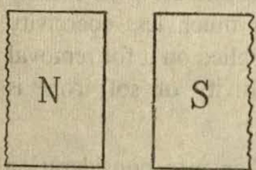


Fig. 3.7

When the poles are very close together each is situated in the field of the other. Thus, the field due to N is $2\pi I$ and each square centimetre of S has an amount of pole I on it and therefore experiences a force$=2\pi I\times I=2\pi I^2$.

If α be the cross-section of the polar face, then if the two poles are in contact, there is a force $2\pi\alpha . I^2$, causing them to cling together.

It is not necessary that the two magnets in contact should be permanent magnets; one may be a permanent magnet and the other a piece of soft iron which will be magnetised by induction by the permanent magnet. The force between the two polar faces will be same as before, provided the intensities of magnetisation in them are equal. If, they are not, the force will be $2\pi\alpha I_1 I_2$ where I_1 and I_2 are the intensities of magnetisation respectively on two sides of the plane of contact.

Example : *Two long soft iron rods of area of cross-section 2·5 sqcm. are placed end to end and in contact. They are situated in a uniform magnetic field of intensity 30 Oe. If the permeability of iron be 150, what is the force required to separate the rods ?*

Ans. The force required to separate the rods $P=2\pi\alpha I^2$

Now, $I=K.H$ and $1+4\pi K=\mu$. $\therefore$ $I=\dfrac{(\mu-1)}{4\pi}.H=\dfrac{(150-1)}{4\pi}\times 30=\dfrac{149\times 30}{4\pi}$

$$\therefore P=2\pi\times 2{\cdot}5\times\left(\frac{149\times 30}{4\pi}\right)^2 \text{dynes}=\frac{5\times(149\times 30)^2}{16\pi}=1{\cdot}9\times 10^6 \text{ dynes.}$$

3.7. Paramagnetic, dia-magnetic and ferro-magnetic substances :

Experimenting with a strong magnet, Faraday found that there are some substances which are attracted by the magnet but there are a few which are

repelled. Those substances which are attracted by magnets are called *paramagnetic* and those repelled are called *diamagnetic* substances. Iron, aluminium, nickel, cobalt etc. are attracted by a magnet. Hence they are paramagnetic substances. Bismuth, antimony, zinc etc., on the other hand, are repelled by a magnet. So they are diamagnetic substances.

Of the parmagnetic substances, some show remarkable magnetic properties, as for example, soft-iron and steel. They are known as *ferromagnetic* substances.

(a) **Paramagnetic substances** : Paramagnetic substances have high permeability but low and positive susceptibility. They tend to move from weaker to stronger parts of the magnetising field and are attracted by a magnet. *Iron, aluminium, manganese, platinum, chromium, salt solution of iron and oxygen* are a few paramagnetic substances.

Properties : (i) A paramagnetic material in the form of a thin bar, if suspended in a magnetic field, sets itself with its longer axis parallel to the direction of the magnetic field (Fig. 3.8). There is a greater concentration of the magnetic lines of force through the material of the specimen.

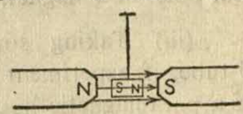


Fig. 3.8

(ii) If some paramagnetic liquid be poured in a U-tube, the level of the liquid in two arms of the U-tube will be equal. If one of the limbs be now placed between the pole pieces of a strong magnet, the liquid in the limb shows a rise in level (Fig. 3.9).

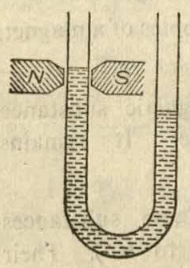


Fig. 3.9

(iii) Take some paramagnetic liquid in a watch glass and put it over the pole-pieces of an electromagnet placed close to each other. The liquid will be found to have collected in a heap at the middle. If, on the other hand, the pole-pieces are further apart, an opposite effect will be observed. The liquid will collect in two heaps over the two pole-pieces (Fig. 3.10). The reason is that when the pole-pieces are near to each other, the magnetic field is the strongest at the middle but when placed further apart, the field is the strongest over the poles.

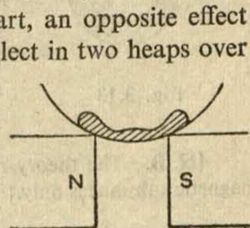


Fig. 3. 10

(iv) A paramagnetic gas, when allowed to ascend between the pole-pieces of a magnet, spreads along the field.

(v) The permeability of paramagnetic substances varies inversely as its absolute temperature. With the increase of the temperature, the permeability decreases and at a given temperature the permeability becomes negative *i.e.* the paramagnetic substance becomes diamagnetic.

(b) **Diamagnetic substances :** Diamagnetic substances have low permeability and negative susceptibility. They tend to move from stronger to weaker parts of the magnetising field and are repelled by a magnet. The magnetism of diamagnetic substances does not depend on temperature. *Bismuth, antimony, phosphorus, copper, alcohol, mercury, gold, hydrogen, water* etc. are a few diamagnetic substances. As a matter of fact, all substances except the para- and ferro-magnetic substances, are dia-magnetic.

Properties : (i) When a thin bar of diamagnetic substance is suspended in a magnetic field, it sets itself at right angles to the direction of the field (fig. 3.11). This shows that a diamagnetic substance moves from stronger to weaker parts of the field. The concentration of magnetic lines of force is more outside than inside the specimen.

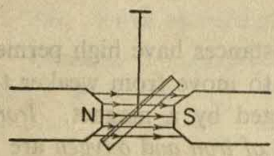


Fig. 3.11

(ii) A diamagnetic liquid kept in a watch glass and placed over two very near poles of a magnet, shows a depression in the middle. (Fig. 3.12).

(iii) Taking some diamagnetic liquid in a U-tube, if experiment be carried out as in the case of a paramagnetic liquid described earlier, the liquid in the limb between the poles will be found to have gone down and the liquid in the other limb up (Fig. 3.13).

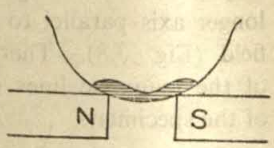


Fig. 3.12

(iv) If a diamagnetic gas be allowed to ascend between the poles of a magnet, the gas spreads across the field.

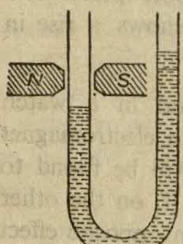


Fig. 3.13

(v) The permeability of a diamagnetic substance does not change with its temperature. It remains constant.

(c) **Ferro-magnetic substances :** These substances have very high permeability (between 1 to 10^6). Their susceptibility is also very high and positive. They are attracted strongly by a magnet. *Iron, steel, cobalt, nickel* and *alloys* made of these metals are good ferro-magnetic substances.

[N. B. The theory of magnetism described in art 3.2 is applicable in the case of ferro-magnetic substances only.]

Properties : All the properties of paramagnetic substances are present in a marked degree in ferro-magnetic substances. The susceptibility of these substances, like paramagnetic substances, varies with its absolute temperature. At a given higher temperature, known as *critical temperature* or *curie point*, the ferro-magnetism disappears and the substance becomes paramagnetic. Experiments show that curie point for iron is about 770°C, for nickel about 400°C and for cobalt about 1100°C.

3.8. Comparison between ferro-, para- and dia-magnetic substances :

Ferro-magnetics	Para-magnetics	Dia-magnetics
1. They are strongly attracted by magnets.	They are feebly attracted by magnets.	They are feebly repelled by magnets.
2. Magnetic permeability is of high order $(1-10^6)$.	Magnetic permeability is of low order $(1-1{\cdot}001)$.	Magnetic permeability is less than 1.
3. Magnetic susceptibility is positive and of high order.	Magnetic susceptibility is positive but of low order.	Magnetic susceptibility is negative and of low order.
4. When placed in a magnetic field, lines of force crowd too much in it.	When placed in a magnetic field, lines of force crowd into the specimen to some extent.	When placed in a magnetic field lines of force try to avoid the specimen.
5. Susceptibility changes with absolute temperature but does not obey any simple law.	Susceptibility is inversely proportional to absolute temperature.	Susceptibility does not depend on temperature.
6. Curie point is available.	Curie point is not available.	Curie point is not available.
7. Ferro-magnetic substance tends to go to stronger region of the field from weaker region.	Para-magnetic substance tends to go the stronger region of the field from the weaker region.	Diamagnetic substance tends to go to the weaker region of the field from the stronger region.
8. The magnetic induction B is much greater than the outside magnetising field H.	The magnetic induction B is slightly greater than the outside magnetising field H.	The magnetic induction B is less than the outside magnetising field H.
9. They are crystalline solids.	They are solid, liquid or gas.	They are solid, liquid or gas.

Exerciscs

Essay type :

1. Narrate briefly the molecular theory of magnetism. How can the magnetisation by friction be explained with the help of this theory ? [*H. S. Exam. 1978*]

2. Discuss in brief the molecular theory of magnetism and explain magnetic induction in the light of this theory. [*H. S. Exam. 1983*]

3. Distinguish between paramagnetic, diamagnetic and ferromagnetic substances. When does a ferromagnetic substance transform into a para-magnetic substance ? [*H. S. Exam. 1982*]

4. Write what you know about : (i) Permeability (ii) Susceptibility (ii) Retentivity (iv) Coercivity. [*cf. H. S. Exam. 1979*]

Short answer type :

5. It is impossible to produce a magnet having one pole ? What is the reason of it ?

6. What will you notice if you go on breaking up a magnet into smaller fragments ? What conclusion can you draw from it ?

7. What are paramagnetic, diamagnetic and ferro-magnetic substances ? How can you differentiate between para and diamagnetic substances ? [*cf. H. S. Exam. 1979*; *'81*]

8. To which category do the following substances belong : platinum, bismuth, steel, aluminium, zinc, water and nickel ?

9. What is magnetostriction ? How is it explained ?

10. For making a permanent magnet, which one would you prefer—steel or soft iron ? Give reasons.

11. Which type of magnetic materials would you select in making (i) electromagnet (ii) core of a transformer (iii) a bar magnet ?

12. What is the harm if the electromagnet of a calling bell be made of steel ?

13. What is the difference between lines of force and lines of induction ? What is intensity of magnetisation ?

14. Two substances A and B have relative permeabilities slightly greater and slightly less than unity respectively. What does this signify about their magnetic properties ? To what group of magnetic substances do A and B each belong ?

15. How will the lines of force inside and outside a specimen of magnetic substance behave when the specimen is placed in a uniform magnetic field ?

Numerical Problems :

16. Calculate the strength of the field near a plane sheet of magnetic pole of strength 8·5 per sq. cm. [Ans. 53·4 Oe]

17. A bar of steel of length 23 cm. breadth 1·2 cm. and thickness 0·5 cm. is placed in a magnetic field of 7·5 Oe and parallel to its length. Find the magnetic moment of the bar if its permeability is 640.

[**Hints** : Magnetic moment $= I.\alpha.l.$ where $I=$intensity of magnetisation ; $\alpha=$area of cross-section and $l=$length] [Ans. 5260 units]

18. Find the magnetic moment of a bar of iron of length 10 cm. and cross-section 0·5 sq cm, if it is uniformly magnetised in the direction of its length to an intensity of 500 Oe. [Ans. 2500 units]

19. When an iron wire of length 40 cm. and cross-section 0·005 sq. cm. is placed along a field of strength 0·5 Oe its magnetic moment becomes 2. Calculate the intensity of magnetisation and the susceptibility. [Ans. 10 ; 20]

20. Two long soft iron rods of area of cross-section 3·14 sq. cm. are placed end to end and in contact. They are situated in a uniform magnetic field of strength 20 Oe. If the permeability of iron is 101, what is the force required to separate the rods ?

[Ans. $0·5 \times 10^6$ dynes]

4

TERRESTRIAL MAGNETISM

4.1. The earth is a huge magnet :

We know that a freely suspended magnet or a compass needle always lines up in a north-south direction. If it is disturbed, it again comes to the same north-south position after a few oscillations. It appears that some external force causes the needle to point in a particular direction. Having noticed this, Sir William Gilbert, a physician to Queen Elizabath, came to the conclusion that the earth is a huge magnet. He said that a magnetic needle could be influenced by a magnet only and since there is no magnet near about the needle, it was the earth's magnetic field which caused the needle to point in the north-southe direction. To prove his view-point, Gilbert shaped a lodestone into a sphere and demonstrated that a small compass placed at any spot of the globe always pointed, as it did on the earth, toward the North pole. Furthermore, it was known that a magnetic substance, buried into the earth for a long time, acquired feeble magnetism under the influence of the earth's magnetic field. For all these reasons, scientists are, now, of opinion that the earth behaves like a huge magnet.

Although the cause for the earth's magnetism is not completely understood, several reasonable theories have been proposed. The earth's magnetism, like an ordinary bar-magnet, has two polarities. The magnetic poles of the earth are known as *dip poles.* A magnet freely suspended about its centre of gravity will remain exactly vertical at two places on the earth. Those two places are the dip poles of the earth's magnetism. The *North Magnetic pole* is located in Bothia Felix in far northern Canada and is about 1500 miles away from the North Geographic pole. The *South Magnetic pole* is located almost diametrically opposite in the southern hemisphere and is about 1400 miles away from the South Geographic pole. The positions of the poles, however, are continuously, though slowly changing.

If we call the magnetic pole of the earth near the geographical *N*-pole as north magnetic pole, then the end of the magnetic needle which points towards the north is, in fact, the south pole *i.e.* opposite to the pole of the earth and we often call it the *north-seeking* pole of the needle. The other pole of the needle is accordingly called the *south-seeking* pole. For brevity, the north-seeking pole of the needle is marked as north pole and the south-seeking pole as the south pole.

To avoid confusion, it was the practice to call the magnetic pole of the earth near the geographical *N*-pole as *blue pole* and the other pole of the earth as *red pole.*

Magnetic meridian plane : The magnetic meridian plane at a place means an imaginary vertical plane passing through the place as well as the magnetic north and south poles of the earth (Fig. 4.1). If a line be imagined on the plane joining the place with the magnetic south and north

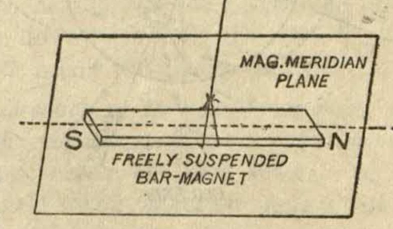


Fig. 4.1

poles of the earth, then the line is called the magnetic meridian line. The axis of a freely suspended magnet at a place passes through the magnetic meridian line at that place.

Geographical meridian plane : The geographical meridian plane at a place means an imaginary vertical plane passing through the place as well as the geographic south and north poles of the earth. If a line be imagined on the plane joining the place with the geographic poles of the earth, then the line is called the geographic meridian line.

4.2. Magnetisation by the earth :

That the earth is a huge magnet is also corroborated by the fact that like other magnets, the earth is capable of magnetisation, although feebly. This can be proved by taking magnetic materials of high susceptibility like soft-iron, permalloy, mu-metal etc. and suspending them either horizontally or vertically in the magnetic meridian for some time. These materials will be found to have acquired feeble magnetism.

Very often we come across peculiar phenomena due to magnetisation by the earth. Iron beams pointing north and south and vertical iron railings and pillars are generally found to be feebly magnetised. North pole is found to develop at the lower end of vertical iron or steel bars, railings etc in northern hemisphere and south pole in southern hemisphere, due to earth's magnetism. Ships made of steel plates acquire feeble magnetism due to the earth during hammering and rivetting. Magnetic mines, devised by the Germans in World War II, were acted on by this feeble magnetism of the ships and destroyed the ships by terrible explosion.

4.3. Lines of force of earth's magnetic field :

Since the earth behaves like a huge magnet, it has its own magnetic field. The magnetic poles of the earth are a little far away from its geographical poles. If we suppose that the earth's magnetism is due to a huge bar-magnet hidden inside the earth, then the bar-magnet should make an angle of about $11\frac{1}{2}°$* with the geographical axis of the earth as shown in fig. 4.2. The axis of the imaginary magnet NS is called the *geomagnetic axis* and the points where the geomagnetic axis intersects the earth are called the *magnetic axis poles*. The end of the bar-magnet pointing toward the north magnetic pole of the earth, is a south pole and the other end is a north-pole because we know that

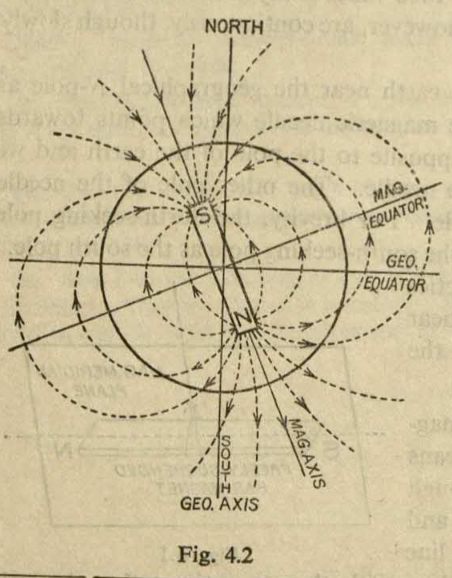


Fig. 4.2

*[Encyclopedia Britannica—Volume 21]

opposite polarities keep themselves near to each other by the force of attraction. We also know that lines of force due to a bar-magnet originate from the *N*-pole and terminate on the *S*-pole of the magnet. If the earth's magnetism is supposed to be produced by an imaginary bar-magnet of above description, then its lines of force will be as shown in fig. 4.2. It is to be noted that earth's lines of force originate from the *S*-pole and terminate on the *N*-pole of the earth. This is because the poles of the imaginary bar-magnet are situated opposite to the poles of earth's magnetism.

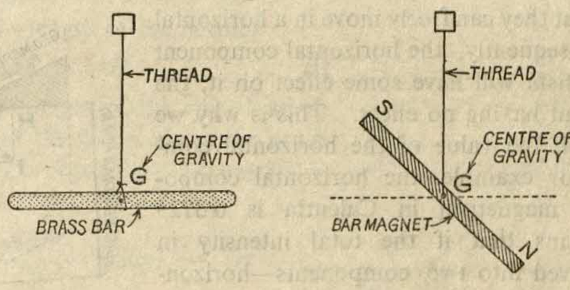
Hence, *if the lines of force due to a bar-magnet are shown to be directed from N-pole to S-pole, those due to earth's magnetism will be opposite i.e. from S-pole to N-pole.*

It is to be noted that dip poles and the magnetic axis poles of the earth are not situated at the same place.

4.4. Elements of earth's magnetism :

In order to specify the earth's magnetic field completely at a place, it is necessary to know (*i*) *the magnetic dip* (*ii*) *the magnetic declination* and (*iii*) *the horizontal component of the earth's magnetic intensity.* These are known as the elements of earth's magnetism. We shall now discuss these elements one by one.

(i) **Magnetic dip** : A rod of non-magnetic substance like brass, when suspended by a thread from its centre of gravity remains always horizontal [Fig. 4.3(*a*)]. But a magnetic needle or a bar-magnet similarly suspended will not come to rest in a horizontal position but will dip down at some angle with the horizontal [Fig. 4.3 (*b*)]. This is because the needle is acted on by the earth's magnetism and



(a) Fig. 4.3 (b)

places itself in the direction of the earth's magnetic field at that place. If the needle be taken gradually towards northern hemisphere, the north pole of the needle will be found to dip more and more downwards. On the other hand, in the southern hemisphere, the south pole of the needle will dip downwards.

Definition : *The angle between the direction of the earth's magnetic field and the horizontal measured in the plane of the magnetic meridian, is called the dip.*

In fig. 4.4 a freely suspended magnetic needle has been shown. Its axis gives the direction of the earth's magnetic field which makes an angle θ with the horizontal plane passing through the point of suspension. The dip at that place is, therefore, θ.

Dip at Calcutta is 30°N—this means that the north pole of a freely suspended magnetic needle at Calcutta will dip down and the axis of the needle will make an angle 30° with the horizontal plane passing through its point of suspension.

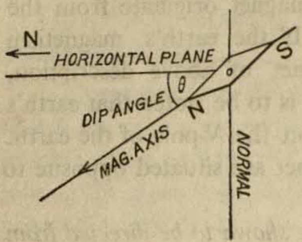


Fig. 4.4

Dip at different places of the earth is different. At the poles the dip is 90° while at the equator, it is 0°.

(ii) **Magnetic declination** : It has been pointed out earlier that the earth's magnetic and the geographical poles do not coincide. Consequently, the magnetic meridian plane and the geographic meridian plane at a place may not coincide. As a matter of fact, *the declination* or *the magnetic variation at a place is the angle between these two planes at the place.* For example, the declination at a place is 1° 15′ E—this means that the above two planes at the place make an angle of 1°15′ between them and that the north pole of a suspended magnetic needle, capable of rotating freely in a horizonal plane, moves away from the geographic meridian towards east through the above angle. The place where the magnetic and the geographic meridians coincide, has zero declination.

(iii) **Horizontal component of earth's magnetic intensity** : If the total intensity of the earth's magnetic field at a place be resolved into two components—one horizontal and the other vertical, then the horizontal component is known as the horizontal component of earth's magnetic intensity.

Generally in a laboratory, we use magnetic needles which are pivoted in such a manner that they can freely move in a horizontal plane only. Consequently, the horizontal component of earth's magnetism will have some effect on it, the vertical component having no effect. This is why we require, in general, the value of the horizontal component only. For example, the horizontal component of earth's magnetism in Calcutta is 0·3725 C.G.S.—this means that if the total intensity in Calcutta be resolved into two components—horizontal and vertical, the former will be 0·3725 C.G.S.

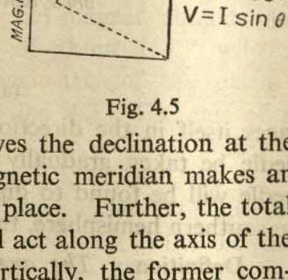


Fig. 4.5

Fig 4.5 shows the magnetic and geographic meridians at a place. The angle δ between them gives the declination at the place. If a magnetic needle, remaining in the magnetic meridian makes an angle θ with the horizontal, then θ will be the dip at the place. Further, the total intensity I of the earth's magnetism at the place will act along the axis of the needle. If the intensity is resolved horizontally and vertically, the former component $H = I\cos\theta$ and the latter component $V = I\sin\theta$

$$\therefore\quad \frac{V}{H} = \frac{I\sin\theta}{I\cos\theta} = \tan\theta.$$

Also, $H^2 + V^2 = I^2(\sin^2\theta + \cos^2\theta) = I^2 \qquad \therefore\quad I = \sqrt{H^2 + V^2}.$

Example 1 : *At a place the angle of dip is 45° and the total intensity of the earth's magnetism is 0·4 Oe. Find the horizontal and vertical components of the earth's magnetism at the place.*

Ans. We know, $\frac{V}{H}=\tan\theta=\tan 45°=1 \quad \therefore \quad V=H.$

Also, $I=\sqrt{H^2+V^2}=\sqrt{2}.H \quad \therefore \quad 0{\cdot}4=\sqrt{2}H \quad$ or $\quad H=\frac{0{\cdot}4}{\sqrt{2}}=0{\cdot}28$ Oe.

Hence $H=V=0{\cdot}28$ Oe.

Example 2 : *The total intensity of earth's magnetic field at a place A is* 0·5 *and dip angle is* 30°. *At another place B, the corresponding values are* 0·55 *and* 45°. *Compare the horizontal intensities at those places.*

Ans. Suppose H_A, H_B and I_A, I_B are respectively the horizontal intensities and total intensities at A and B. Then, $H_A=I_A\cos\theta_1$ and $H_B=I_B.\cos\theta_2$

$$\therefore \quad \frac{H_A}{H_B}=\frac{I_A.\cos\theta_1}{I_B.\cos\theta_2}=\frac{0.5\cos 30°}{0{\cdot}55\cos 45°}=\frac{0{\cdot}5\times\sqrt{3}\times\sqrt{2}}{2\times 0{\cdot}55\times 1}=\frac{11}{10}$$

Example 3 : *When the upper tip of a magnetic dip needle is loaded with a small mass, the dip recorded is found to drop from* 45° *to* 30°. *The total magnetic intensity at the place is* 0·42 *Oe and the pole strength of the magnetic needle is* 200 *c.g.s. units. What is the mass attached* ? [*I.I.T. 1977*]

Ans. When the angle of dip is 45°, it can be written from fig 4.6

$m\times 2l\times H\sin 45°=m\times 2l\times V\cos 45°$

[$2l$=length of the needle]

or, $\quad V=H\tan 45°=H \qquad \ldots$ (i)

Let W be the weight of the mass loaded. Taking moment of the forces about the point O, we get, $m\times 2l\times H\sin 30°+Wl\cos 30°=m\times 2l\times V\cos 30°$

or $\quad mH+\frac{\sqrt{3}W}{2}=\sqrt{3}mV=\sqrt{3}mH$

[From relation (i)]

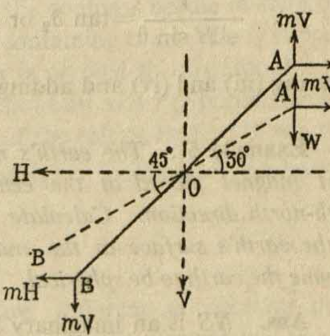


Fig. 4.6

or $\quad \frac{\sqrt{3}}{2}W=(\sqrt{3}-1)mH$

$$\therefore \quad W=\frac{2(\sqrt{3}-1)mH}{\sqrt{3}}=\frac{2(1{\cdot}732-1)mH}{1{\cdot}732}=\frac{2\times 200\times 0{\cdot}732}{1{\cdot}732}H \quad \ldots \text{(ii)}$$

Again $\quad I=\sqrt{H^2+V^2}=\sqrt{2}H \quad$ or $\quad H=\frac{I}{\sqrt{2}}=\frac{0{\cdot}42}{1{\cdot}414}=0{\cdot}297$

Putting this value in eqn. (ii), $W=\frac{2\times 200\times 0{\cdot}732\times 0{\cdot}297}{1{\cdot}732}$dyne

Hence, the reqd. mass$=\frac{2\times 200\times 0{\cdot}732\times 0{\cdot}297}{1{\cdot}732\times 980}gm=0{\cdot}051$ gm

Example 4 : *If δ_1 be the angle of inclination of the magnetic axis of a magnetic needle with horizontal at any vertical plane and δ_2 is that in another vertical plane at right angle to the previous one, prove that the true angle of dip δ is given by*

$$cot^2\delta = cot^2\delta_1 + cot^2\delta_2$$ [*Jt. Entrance 1984*]

Ans. δ_1 is the dip at the first place [Fig. 4.7]. PQ is the magnetic meridian line. P_1Q_1 is the horizontal line when the needle shows the apparent dip δ_1. In this condition, OC gives the direction of the total intensity of earth's magnetic field. OA represents the vertical component of the total intensity. The vertical component will, however, remain unchanged in both the planes. Let H_1 be the horizontal component in the first position.

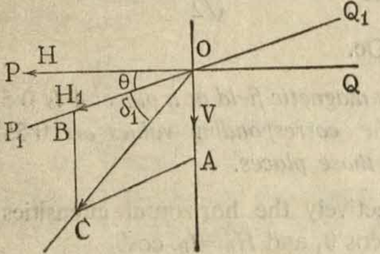


Fig. 4.7

Then $\dfrac{V}{H_1} = \tan\delta_1$ .. (i)

If true horizontal component of earth's field be H, then $H_1 = H\cos\theta$ .. (ii)

From (i) and (ii), we get $\dfrac{V}{H\cos\theta} = \tan\delta_1$ or $\cot\delta_1 = \dfrac{H\cos\theta}{V}$ .. (iii)

In the vertical plane perpendicular to the first vertical plane, the apparent dip$=\delta_2$; θ will have a value equal to $(\theta+90°)$. In this case, $\dfrac{V}{H_2} = \tan\delta_2$ and $H_2 = H\cos(\theta+90°) = -H\sin\theta$.

$$\therefore \quad \frac{V}{-H\sin\theta} = \tan\delta_2 \text{ or } \cot\delta_2 = -\frac{H.\sin\theta}{V} \quad .. \quad \text{(iv)}$$

Squaring (iii) and (iv) and adding, $\cot^2\delta_1 + \cot^2\delta_2 = \dfrac{H^2}{V^2} = \cot^2\delta \left[\because \cot\delta = \dfrac{H}{V}\right]$

Example 5 : *The earth's magnetic field may be considered to be due to a short magnet placed at the centre of the earth and oriented along the magnetic south-north direction. Calculate the ratio of the magnitude of the magnetic field on the earth's surface at the magnetic equator to that at the magnetic poles. Assume the earth to be spherical.* [*I.I.T. 1970*]

Ans. NS is an imaginary bar magnet placed at the centre of the earth [Fig 4.8]. If r be the radius of the earth and M the moment of the imaginary bar magnet, then the horizontal intensity and vertical intensity of the earth's field on the surface of the earth at a latitude λ are given respectively by $H = \dfrac{M\cos\lambda}{r^3}$ and $V = \dfrac{2M\sin\lambda}{r^3}$. Hence total magnetic intensity

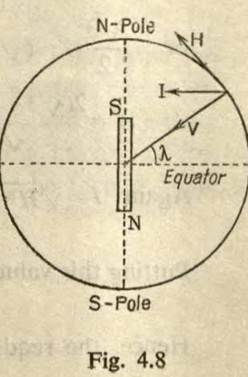


Fig. 4.8

$$I = \sqrt{H^2+V^2} = \sqrt{\frac{M^2\cos^2\lambda}{r^6} + \frac{4M^2\sin^2\lambda}{r^6}}$$

$=\frac{M}{r^3}\sqrt{1+3\sin^2\lambda}$. Since $\lambda=0°$ at the equator, $I_1=\frac{M}{r^3}$

Again at the poles $\lambda=90°$, $\therefore$ $I_2=\frac{M}{r^3}\sqrt{1+3(\sin 90°)^2}=\frac{2M}{r^3}$ $\therefore$ $\frac{I_1}{I_2}=\frac{1}{2}$

4.5. Mariner's Compass :

It has been mentioned earlier that a magnet has a directive property. Using this property of a magnet, a compass has been devised which is used by the sailors in guiding the course of a ship in sea.

The simplest form of a mariner's compass is shown in fig. 4.9. It consists of a circular card having one or more magnetic needles fastened beneath it. The card rotates with the rotation of the needles. The upper surface of the card is marked out by radii into thirty two divisions which are called *points of compass*. One of these divisions is marked *N* with a crown. It indicates the magnetic north because it comes immediately over *N*-pole of the needle. The needle and the card are supported on a sharp metal pivot by means of an agate cap which is fixed to the centre of the needle.

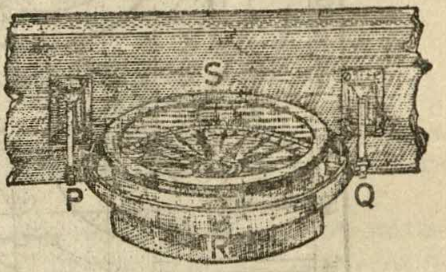


Fig. 4.9

In order to ensure a horizontal position of the compass needle in spite of the rolling and pitching of the ship, the circular box containing the needle is supported on '*gimbals*'. This means that the compass box is pivoted to turn about an axis *RS* within a ring, while the ring itself can turn about an axis *PQ* perpendicular to *RS*. This prevents the compass from partaking of the rolling motion of the ship.

The mariners ascertain the direction by noting the position of the crown in in the card.

4.6. Magnetic maps :

The elements of earth's magnetism are different at different places. On the other hand, there are places where one particular element has the same value. Magnetic maps are prepared to show different features of the elements of earth's magnetism. The values of the elements, however, change with time. Consequently, new maps are prepared from time to time. Magnetic maps are very important to the sailors. Fig. 4.10 shows a magnetic map which contains the following lines.

(i) **Isogonic lines :** Places on the surface of the earth, which have equal declination, are joined by a line, known as isogonic lines. The thin continuous lines in fig. 4.10 are isogonic lines.

(ii) **Agonic lines :** Lines joining points of zero declination are called

agonic lines. The thick continuous lines in the fig. 4.10, having 0° written by their sides are agonic lines.

(iii) **Isoclinic lines :** Lines joining points of equal dip are called isoclinic lines. In fig. 4.10, broken lines represent isoclinic lines.

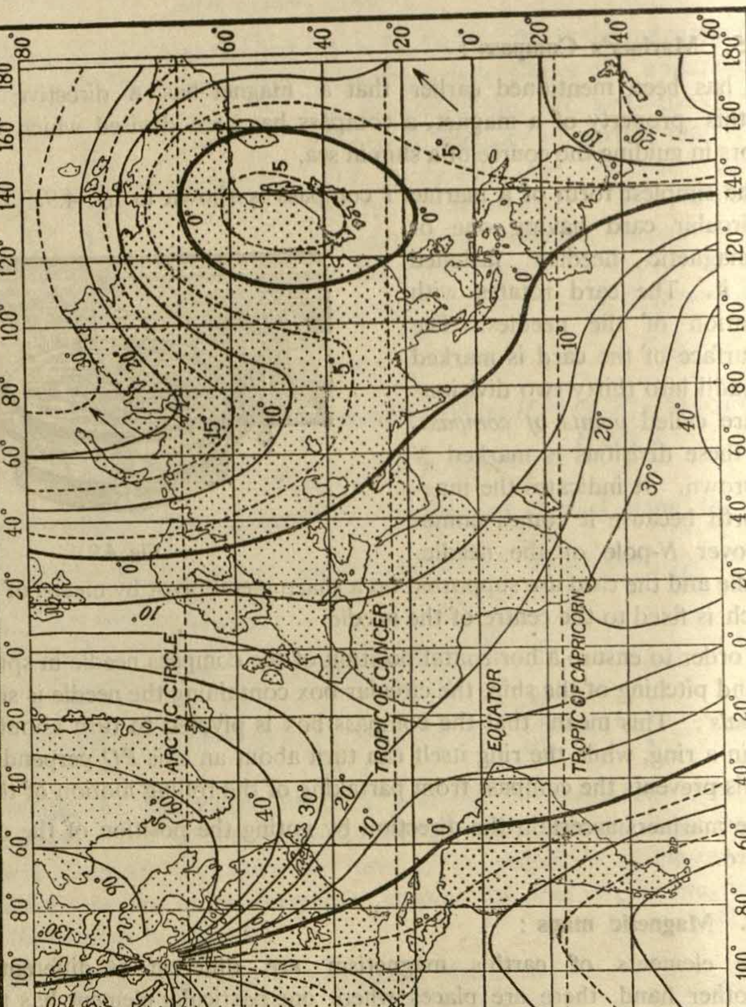


Fig. 4.10

(iv) **Aclinic lines :** The line joining points where the dip is zero, is called the aclinic line. Magnetic equator is an aclinic line.

(v) **Isodynamic lines :** Lines joining points where the values of the horizontal component of earth's magnetic intensity are equal, are called isodynamic lines.

(vi) **Duperrey's lines :** In addition to the above lines, there are lines which indicate the direction of the magnetic meridians, called Duperrey's lines (Fig. 4.11). The magnetic meridians converge to two points called the magnetic poles of the earth.

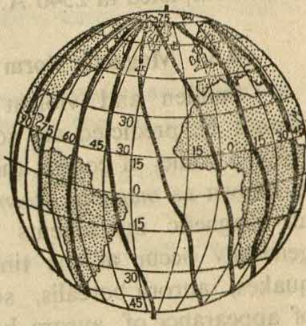


Fig. 4.11

4.7. Changes in the magnetic elements of the earth :

A systematic recording of the magnetic elements of the earth (*viz*, the horizontal intensity, the dip and the declination) at a place shows that they are not constant quantities but they undergo variations. These variations may be classified as follows :

(i) **Daily variations :** All the magnetic elements undergo fairly regular daily variations, but as these are small, special instruments are necessary to record them. They are found to be different at different hours. These variations are, however, not the same from day to day ; on specially 'quiet' days, the changes are almost nil and on some other days, the variations are maximum. It is said that the daily variations are due to electric currents in the upper regions of the atmosphere, but the explanation is by no means complete.

(ii) **Annual variations :** The magnetic elements also have periodic yearly changes. They undergo cyclic changes during the year. Thus, the declination is found to have a maximum value in February and a minimum value in August.

(iii) **Secular variations :** The periodic variations of magnetic elements at a place are characterised as secular variation. By periodic variation we mean that these quantities will attain different values at different intervals of time but finally return to the original value after a definite period of time, the period being a very long one of the order of several hundred years. It has been found that the north magnetic pole turns round the geographical north pole in a circle of radius 17° and completes one full rotation in about 960 years, causing a secular variation of declination at London as shown in the following table.

Year		Declination	
1580	—	11°15′E	
1622	—	6° 5′E	
1659	—	0° 0′	(compass needle pointed true north)
1823	—	24°30′W	
1938	—	11°1′ W	
1952	—	9° 3′W	
2139	—	0° 0′	(compass needle pointed due north)
2540	—	11°15′E	

The last two occasions are yet to come and the cycle of 960 years is expected to be completed in 2540 A.D.

4.8. Magnetic storm :

Sudden and violent changes in the values of magnetic elements, which cannot be predicted are sometimes produced simultaneously all over the world. After a while, of course, the values return to the normal state. This phenomenon is known as *magnetic storm*. During magnetic storm, the magnetic needles of all magnetic instruments are found to behave abnormally. Magnetic storms generally occur at the time of natural calamity like volcanic eruptions, earthquakes, aurora borealis, sunspot etc. During this time—specially at the time of appearance of aurora borealis and sunspot, large quantity of tiny charged particles is thrown at a great speed towards the earth, and they cause a sudden and violent change in the values of magnetic elements. Magnetic storm disturbs the radio communication, telegraph and telephone system.

Magnetic elements at different places

Place	Horizontal component (H)	Declination (δ)	Dip (θ)
Calcutta	0·372 Oe	44′ E	30°N
Delhi	0·340 „	2°2′ E	40°56′N
Bombay	0·72 „	32′ E	25°N
Madras	0·369 „	10′ W	34°37′N
London	0·180 „	10° W	67°N

Exercises

Essay type :

1. "The earth is a huge magnet"—What are the arguments in favour of this statement ? What is the difference between the dip poles and the magnetic axis poles of the earth ?
2. Name and explain the elements of earth's magnetism. [*H. S. Exam. 1978*]
3. What are the three elements of the earth's magnetic field ? Describe the nature of earth's magnetic field.
4. Describe and explain the action of a mariner's compass. Does it show the true north direction ?
5. What is a magnetic map ? What are its uses ? What is magnetic storm ?

Short answer type :

6. The dip at Calcutta is 30°N—what does this statement mean ? At what places the dip is 0° and 90° ? [*H. S. Exam. 1978*]
7. A thin uniform wooden rod, suspended from its centre of gravity, comes to rest in a horizontal position but a thin uniform magnetic rod similarly suspended remains inclined to the horizon. What is the reason of it ? What is the significance of the inclination ? What information do you get about the magnetic intensity of the earth from the inclined position of the magnetic rod ?

8. Vertical steel pillars used in the construction of buildings in Australia are found to develop north polarity at the lower end. Why ?

9. A compass needle is kept on a piece of cork and the cork is floated on water. Will it move towards the north ? or to the south ?

10. What is a 'magnetic mine' ?

11. Draw a diagram showing, in general, the magnetic lines of force around the earth. The lines of force due to a bar-magnet originate at its north pole but the lines of force due to the earth's magnetism originate at its south pole. Why is this difference ?

12. The horizontal component of earth's magnetic intensity in Calcutta is 0·3725 C.G.S.—Explain the statement fully. Why do we require only the horizontal component in our magnetic measurements ?

13. A small magnet is freely suspended in magnetic meridian. Where will it be exactly vertical or horizontal ?

Objective type :

14. (*a*) The angle made by the axis of a freely suspended magnetic needle with the horizontal plane passing through the point of suspension, at a place is called (i) the dip pole (ii) the angle of declination (iii) angle of dip at that place. Which is correct ?

(*b*) The lower end of a vertical pillar kept at the southern hemisphere of the earth acquires (i) north polarity (ii) south polarity (iii) no polarity. Which is correct ?

(*c*) At a place where dip is 45°, the horizontal component of earth's magnetic field and vertical component are (i) unequal (ii) equal (iii) in the ratio 1 : 2. Which is correct ?

(*d*) The magnetic equator is an (i) aclinic line (ii) Isoclinic line (iii) agonic line. Which is correct ?

(*e*) If H and V are the horizontal and vertical components of earth's magnetic field at a place where dip is 60°, then (i) $V=H$ (ii) $V=\sqrt{3}H$ (iii) $H=\sqrt{3}.V$. Which is correct ?

(*f*) A line passing through places having zero value of magnetic dip is called (i) isoclinic line (ii) aclinic line (iii) agonic line (iv) isogonic line. Which is correct ?

(*g*) The mariner's compass is provided with gimbals arrangement so as to (i) give a direct value of dip (ii) give a direct value of declination (iii) keep the needle always horizontal. Which is correct ?

(*h*) The magnetic field due to the earth is closely equivalent to that due to (i) a large magnet equal to the diameter of the earth (ii) a magnetic dipole at the centre of the earth (iii) a large coil carrying a current (iv) neither of the above. Which is correct ?

(*i*) The magnetic compass is not useful for navigation near the magnetic poles because (i) the magnetic field near the poles is zero (ii) the magnetic field near the poles is almost vertical (iii) at low temperature the compass needle loses its magnetic properties (iv) none of the above. Which is correct ?

Numerical problems :

15. At a place, the dip is 45° and the total intensity of earth's magnetism is 0·8 Oe. What are the horizontal and vertical components ? [Ans. 0·564 Oe each]

16. The declination and dip at a place are 25° and 45° respectively. The horizontal component of earth's field there is 0·3 Oe. What will be the horizontal component and the vertical component at the geographical meridian ? cos 25°=0·9063. [Ans. 0·272 Oe ; 0·3 Oe]

[**Hints :** See Fig. 4.5 $\delta=25°$ and $\theta=45°$; if H' and V' are the required components,
$H'=H.\cos\delta=0{\cdot}3.\cos 25°=0{\cdot}3\times0{\cdot}9063=0{\cdot}272$ Oe
$V'=V=H.\tan\theta=0{\cdot}3\times\tan 45°=0{\cdot}3$ Oe.]

17. If the horizontal component of earth's magnetic field at Calcutta be 0·35 oersted and the angle of dip be 30°, calculate the intensity of earth's magnetic field at Calcutta.
[*H. S. Exam. 1980*] [Ans. 0·40 Oe]

18. A magnetic needle freely suspended in the magnetic meridian at a place about its centre of gravity lies inclined at 30° to the horizontal. If the horizontal intensity of the earth's

magnetic field at the place is 0·36 oersted, what is the intensity of the terrestrial magnetic field at the place ? [*H. S. Exam. 1984*] [Ans. 0·42 Oe]

19. Find the value of the vertical component of the earth's magnetic field at a place where the angle of dip is 60° and H=0·2 Oe. [Ans. 0·346 Oe]

20. The vertical and horizontal components of the earth's magnetic field at a place are equal. What is the angle of dip at the place ? [Ans. 45°]

Harder problems :

21. At a place, the dip needle makes an angle of 45° with the horizontal. If a mass of 1 gm is loaded on the upper end of the needle, the dip angle is reduced to 30°. What mass will make the needle horizontal ? [Ans. 2·3 gm]

[**Hints** : In the 1st case, $2mlH \sin 45° = 2mlV \cos 45°$
In the 2nd case, $2mlH \sin 30° + 1.g.l \cos 30° = 2mlV \cos 30°$
In the 3rd case, $m'gl = 2mlV$ [m'=reqd mass]

22. A magnetic needle of mass 10 gm. has a magnetic moment of 28 unit. If the needle is to be maintained horizontal in the northern hemisphere, where should the pivot rest relative to the c.g. of the needle ? The vertical component of earth's magnetic field is 0·5 Oe. [Ans. 0·0143 mm]

23. When a mass of 1 gm is placed on the upper end of a dip needle, the dip angle reduces to 30° from 60°. If the total intensity of earth's magnetic field there be 0·42 Oe, find the pole strength of the needle. [Ans. 1885 units]

24. Earth's magnetic field may be imagined to be due to a small bar magnet located at the centre of the earth. If the magnetic field at a point on the magnetic equator is 0·3 oersted, what is the magnetic moment of such a bar magnet ? What is the value of the magnetic field at the north pole ? The radius of the earth=$6{\cdot}4 \times 10^8$ cm. [*I.I.T. 1973*] [Ans. $78{\cdot}65 \times 10^{24}$; 1·5 Oe]

25. A magnetic needle of pole strength 50 units and 20 cm. in length is capable of rotating about its central pivot. Calculate the weight required to be placed at the end of the needle to make it horizontal at a place where H=0·2 Oe and angle of dip=45° (g=1000 cm/s²). [Ans. 20 mg]

26. In an experiment for finding dip at a place it is found that the apparent dip in one place is 30° and that in a plane at right angles to the find plane is 20°. Find the true dip at the place. [Ans. 17° (nearly)]

[**Hints** : Apply the formula $\cot^2\delta = \cot^2\delta_1 + \cot^2\delta_2$ where δ, δ_1 and δ_2 are the true and the two apparent dips respectively. See example no. 4, page 212].

27. A magnetic needle of mass 7·5 gm has a magnetic moment of 98 units. If the needle is to be maintained horizontal in the northern hemispheres where should the point of support lie relative to its centre of gravity ? Assume that the vertical component of the earth's magnetic field is 25 Oe. [*Jt. Entrance 1986*] [Ans. $33{\cdot}3 \times 10^{-2}$ mm]

ELECTROSTATICS

1

GENERAL FACTS OF ELECTRIFICATION AND ELECTROSTATIC INDUCTION.

1.1. Introduction :

As far back as the sixth century B.C. the Greek philosopher Thales casually noticed that *amber*, the hard sap of a kind of pine tree, when rubbed with silk, exhibits the property of attracting very light bodies like straws, paper etc. You may have noticed that after combing dry hair with a guttapercha or celluloid comb in winter, the comb attracts small pieces of paper. No systematic study of the above phenomenon was, however, known to have been made until about 1600 A.D. when Dr. Gilbert, a physician to Queen Elizabeth of England, carried out detailed investigation about the peculiar behaviour of amber. He found that besides amber many other substances possess the same property. The Greek equivalent for the word 'amber' is 'electron' and probably Dr. Gilbert named the phenomenon as '*electrification*' from this Greek word. These substances which like silk-rubbed amber, can attract other light substances, are called electrified bodies. This electricity remains confined in the body and cannot move from one place to another. For this reason, it is called *static electricity*.

1.2. Electrification by rubbing :

Electricity can be produced by simply rubbing two bodies. Make a piece of glass rod and a piece of silk dry and warm by sun rays. Rub the glass rod smartly with the piece of silk and hold the rod before small pieces of paper. The glass rod will attract the pieces of paper. The glass rod is thus electrified.

Given opportunity, frictional electricity may accumulate in a body to a dangerous extent. When a truck full of petrol moves speedily, the petrol is jerked violently and electric charges are developed due to rubbing. Gradual accumulation of charge, in this way, may cause a sparking. As petrol is extremely inflammable, it may create the danger of a violent explosion. To prevent such accident, charges are not allowed to accumulate. A metallic chain is allowed to dangle on the road from the body of the truck. Frictional charges, as soon as they are produced, leak away to the earth through the chain and cannot accumulate.

1.3. Conductor and non-conductor (or insulator) :

If a brass-bar rubbed with silk, flannel or cat's skin be held in hand and presented before small pieces of paper, the rod will not attract the pieces *i.e.* the rod will not be electrified. But rods of glass, shellac, ebonite etc. when similarly rubbed are easily electrified. Noticing this fact, the ancient scientists came to the conclusion that there are some substances which cannot be electrified. But later on, the conclusion was found to be incorrect. As a matter of fact, all substances, when suitably rubbed, may be electrified. Why, then the brass rod in the above experiment was not electrified ?

The answer to the question is that the brass rod was electrified but the electricity passed readily along the brass rod to the earth through the body of the experimenter and hence no sign of electrification was perceptible. Had the brass rod been held with a wooden handle, the rod would have retained its electrification because electricity cannot pass through wood. We may, therefore, conclude that there are two classes of substances *viz* (i) substances through which electricity can easily pass and they are called **conductors** and (ii) substances through which electricity cannot easily pass and they are called **non-conductors** or **insulators.**

Metals, in general, are good conductors of electricity. Of these again, copper, silver and aluminium are notably very good conductors. You may have seen that house-hold wiring is made of copper wires. Nowadays, of course, aluminium wires are being used in many cases, instead of copper wires. Besides metals, earth, human body, carbon, coal, mercury etc. are examples of conductors of electricity.

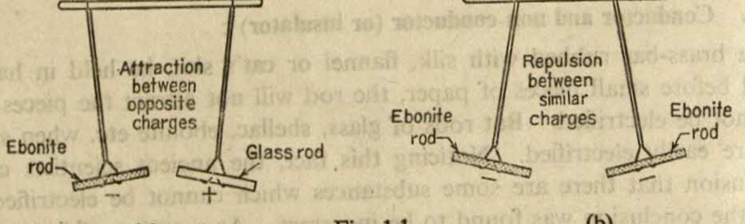
Dry air, glass, paper, wax, wood, ebonite, porcelain, bakelite etc. are non-conductors or insulators. You must have seen that telegraph, telephone or power supply line wires are supported on porcelain knobs. They are not directly connected to electric posts : because in the case of direct connection, leakage of electricity may take place through the posts, making the posts fatal to human life. Porcelain, being a non-conductor, prevents such leakage of electricity. The connecting wires used in laboratories for electrical experiments are covered by silk or cotton which are insulators. As a result, no harm will be caused if two connecting wires touch each other. The wires are known as insulated wires.

It is to be remembered that no *substance is a perfect insulator*. The substances mentioned earlier as insulators are such that electricity can pass comparatively with much difficulty through them.

[N.B. Incidentally, dryness is an important factor in electrical experiments. Moisture being a conductor of electricity, moist apparatus and instruments are not conducive to good electrical experiments. For successful results, all instruments used in electrical experiments must be perfectly dry. In rainy season, the atmosphere becomes damp and in winter, it becomes dry. Electrical experiments are also performed better in winter than in rainy season.]

1.4. Two opposite kinds of electric charge :

Let a glass rod be electrified by rubbing it with silk and suspended from a stirrup [Fig. 1.1(a)] by a silk thread. An ebonite rod is charged by rubbing it



(a) Fig. 1.1 (b)

with a cat's fur and is similarly suspended by the side of the glass rod. The rods will be found to *attract* each other. Again two ebonite rods charged by rubbing

against a cat's fur, are suspended side by side from another stirrup [Fig. 1.1(b)]. The rods will be found to repel each other.

So the above experiment illustrates that the glass and the ebonite rods are endowed with opposite kinds of electricity in the sense that attraction takes place in the case of ebonite and glass while repulsion in the case of ebonite and ebonite.

We may, therefore, conclude that *like charges repel each other and unlike charges attract*.

The scientists unanimously decided that the electricity developed in glass due to rubbing with silk will be reckoned as *positive* while that developed in ebonite due to rubbing with cat's fur will be reckoned as *negative*. Positive and negative are simple names and they have no further significance except that they denote two opposite kinds of electricity.

It should be borne in mind that positive electricity is not always developed in glass or negative electricity in ebonite. The table below shows that the same substance may develope both positive and negative electricity when suitable rubbers are used. The following list has been prepared such that if any two bodies be chosen, the one standing first becomes positive, the other negative when they are rubbed against each other.

1. Fur	6. Ebonite
2. Glass	7. Sealing wax
3. Silk	8. Amber
4. Human body	9. Resin
5. Metals	10. Sulphur

Thus if a metal be rubbed with fur, it becomes negatively charged; if rubbed with resin, it becomes positively charged.

1.5. Repulsion is a surer test of electrification than attraction :

From experiments described in art 1.4 it is found that like charges repel each other but unlike charges or a charged and an uncharged body attract each other.

Now, if by bringing a body A near a charged body B, attraction is noticed, the conclusion regarding the condition of electrification of A is not definite. The body A may either be uncharged or if charged, it is charged with opposite kind of electricity. But if the two bodies are found to repel each other, the conclusion is definite. The body A is charged and charged with same kind of electricity as B because repulsion takes place only between two like charges.

Thus, repulsion is a surer test of electrification than attraction.

1.6. Instrument for detection of electric charge :

A gold-leaf electroscope is a suitable instrument for detecting as well as ascertaining the nature of charge in a body.

Description : Fig. 1.2. shows a form of a gold-leaf electroscope. Two thin strips of gold-leaf (L,L) are fastened at one end of a long narrow rod (P) of metal, The rod passes into a metal vessel with glass window, through an ebonite plug which closes the mouth of the vessel. The leaves may be made of any light metal like aluminium instead of gold. There is a metal disc D attached to the end of the rod P projecting out of the vessel. In some instruments, a knob is given instead of a disc. Two tin-foils (t,t) are pasted along the glass wall of the vessel opposite to the leaves and the foils are in contact with the metallic base of the vessel. Some calcium chloride, which is a drying agent, is kept in a pot inside the vessel to keep the air dry. Moist air creates disturbance in the working of the leaves.

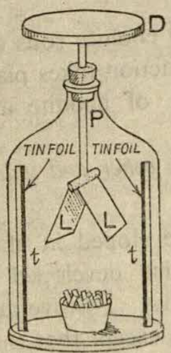


Fig. 1.2

Charging the electroscope by conduction : In order to charge the gold-leaf electroscope, say positively, by conduction, the metal disc D is touched by a glass rod rubbed with silk. Some of the positive charge of the glass rod flows on to and distributes itself over the gold-leaf and the supporting rod. When the glass rod is taken away, the electroscope retains its acquired charge, which distributing itself more or less uniformly over the leaves, causes the leaves to stand out as shown in fig 1.2. In this way, a gold-leaf electroscope may be charged positively by conduction. The more charge given to the electroscope, the higher the gold-leaves are repelled.

To charge the electroscope negatively, an ebonite rod rubbed with flannel, is similarly brought in contact with the disc D. Some of the negative charge of the ebonite rod will flow to the leaves which will remain diverged.

This method of charging an electroscope has, however, one serious disadvantage. If the charged rod contains too much of charge, then immediately after contact, the leaves will diverge so much that they may fall down from the support. Further, this method often gives charge opposite to that expected. It is better to use the method of induction which is described later.

Use of gold-leaf electroscope : To detect the presence of charge on a body, the body is brought near the disc of an electroscope. If the body is charged, the leaves of the electroscope will diverge and from the amount of divergence, an approximate idea of the intensity of charge may be obtained. If the body is uncharged the leaves will not diverge.

To ascertain the sign of charge on a body, *a charged gold-leaf electroscope* is to be taken. Suppose, we take a positively charged gold-leaf electroscope, whose leaves stand out being repelled by positive charges. If, now, the charged body under examination, be brought near the disc D of the electroscope and an increased divergence of the leaves is observed, the inference is that the body has the same type of charge as the electroscope. On the other hand, a decrease in the divergence indicates that the body has opposite type of charge as the electroscope.

Similar experiments may be done with an electroscope charged initially with negative charge.

From these experiments we may conclude that *an increase in divergence of the leaves indicates that the charge on the electroscope and the test charge are of the same kind.*

We can not assume, however, that a decrease in divergence necessarily means that the charge on the electroscope and the test charge are of opposite kind. An uncharged body brought near the cap of an electroscope, produces a decrease in the divergence. It follows that the only sure test for the sign of charge on a body is to secure an increased divergence. The results may be summarised in a table as shown below.

Charge on the electroscope	Charge under test	Effect on divergence of the leaves
−	−	Increase
+	+	,,
−	+	Decrease
+	−	,,
+ or −	uncharged body	,,

1.7. Friction produces both kinds of electricity in equal amount :

In the process of rubbing or friction, there is simultaneous development of both kinds of electricity in equal quantities. It is demonstrated by the following simple experiment. [Fig. 1.3]

An ebonite rod with a flannel cap fitting at one of its ends is taken. The cap is provided with a silk thread so that by pulling the thread the cap can be separated from the rod without touching the rod with hand. Both are well dried and the cap is then smartly rubbed around the rod. The combination is then presented before a gold-leaf electroscope. The leaves of the electroscope do not diverge.

Fig. 1.3

Now remove the cap by pulling the thread and present the ebonite rod and the cap separately to the electroscope. The leaves diverge equally in both cases showing that the rod and the cap are charged with equal amount of electricity.

Further, that the electricities developed in the rod and the cap are of opposite kind is proved by the fact that when they are brought together before the electroscope, no divergence of leaves is produced. The effect of one is evidently neutralised by that of the other.

1.8. Proof-plane :

Fig. 1.4

To detect the presence of charge on a body and for some other purposes, transference of some charge from a charged body to the electroscope is necessary. In this respect, a proof-plane is very helpful. It consists of a small metallic disc provided with an insulating handle made of ebonite, glass or any other insulator (Fig. 1.4). When the disc is brought in contact with a charged body, it collects some charge from the body. Then it is presented before the disc of a gold-leaf electroscope. The leaves diverge and indicate that the body under test is charged. Proof-plane is generally used if a body is heavily charged or if the body cannot be conveniently moved.

1.9. Electronic theory of electricity :

Theories were advanced from time to time to explain different phenomena exhibited by electrified bodies. Superseding all of them, the present accepted theory which is due to the celebrated physicist Sir J.J. Thomson and others, is known as the electronic theory.

The charging of bodies and other allied phenomena are now understood in terms of the structure of the atoms composing the body. Every atom consists of a compact central *nucleus* carrying a positive charge, around which are distributed a number of *electrons*, each negatively charged (Fig. 1.5). The nucleus owes its positive charge to the fact that it contains particles called *protons*. All protons are alike and all carry the same amount of + charge. All electrons are alike and each carries a −charge, equal in amount to the +charge of a proton. Besides protons, nucleus of an atom also contains *neutrons* which are uncharged particles. In its normal condition, the atom is electrically neutral—there are just as many electrons outside the nucleus as there are protons in it. Atoms of different elements contain different number of electrons which revolve in different shells about their respective nuclei. The simplest atom—the hydrogen atom—contains one electron outside the nucleus and one proton in the nucleus. The atom of the second element—helium—has two extra-nuclear electrons and two protons and two neutrons in the nucleus. Generally, the **atomic number** of an element signifies the number of electrons or protons in the atom and the **mass number** signifies the total number of particles in the nucleus. For example, if Z be the atomic number of an element, its atom consists of Z number of electrons outside the nucleus or Z number of protons in its nucleus. If again, A be the mass number of the atom its nucleus contains A number of particles (protons+neutrons). Hence, the number of neutrons in its nucleus$=A-Z$.

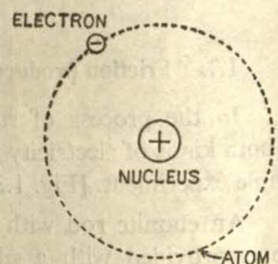


Fig. 1.5

Each atom, in its normal condition, contains as many electrons as there are

protons in the nucleus. The particles in the nucleus are attracted to each other with a great force and it is not easy to disrupt them. Such is not the case with the electrons which can be detached from an atom by easy means. If, somehow, an atom holds more or less than its normal number of electrons, the atom becomes charged with negative or positive electricity respectively. This, in brief, is the electronic theory of electricity [for details, see Modern Physics at the end.].

Electron exists in the atom of every element. It is a fundamental constituent of matter. It is the lightest of all particles, having the minimum amount of electric charge. Its mass is 9×10^{-28} gm. and charge is $4{\cdot}8036\times10^{-10}$ e.s.u. This charge, being the smallest of all known charges, is considered as a unit.

1.10. Explanation of frictional electricity according to the electronic theory :

Every atom, it has been pointed out earlier, contains requisite number of electrons to neutralise the positive charge of its nucleus. But it has been found that an atom has a tendency to attract, at the same time, an additional number of electrons over the normal quota. This tendency is of course, different in different atoms. So, when two different meterials, such as glass and silk, are put in good contact by rubbing, the glass gives up some electrons to the silk because silk has greater tendency of attracting electrons than glass. Thus, the silk now has a negative charge while the glass has a *deficiency* of negative charge which means it has a positive charge. *Charges are, therefore, not produced by rubbing* ; *they are merely separated out.* Similarly when an ebonite rod is rubbed with flannel, electrons from flannel flow to ebonite because ebonite has a greater tendency of attracting electrons than flannel. As a result, ebonite rod becomes negatively charged and the flannel positively.

We also know that due to rubbing, equal amounts of opposite charges are developed. This is also apparent from the above theory because the number of electrons lost by one body is equal to the number gained by the other. Hence, opposite charges of equal amount will be developed in two bodies when they are rubbed.

The difference between a conductor and an insulator, according to the electronic theory, is that an atom of a conductor possesses a large number of free electrons which are capable of random motion like the molecules of a gas while in the atom of an insulator, there are practically no free electrons.

1.11. Electrostatic induction :

In studying magnetism, it was found that a piece of iron could become a magnet by induction merely by being brought near a permanent magnet. In a similar way, charges can be induced in a neutral body by bringing it near a charged one. This phenomenon is known as *electrostatic induction.*

Charge a glass rod *A* with positive electricity by rubbing it with silk and hold it near an uncharged conductor *BC* [Fig. 1.6.(a)]. The conductor *BC* will be charged by induction by the charge of the rod *A*. It can be proved by the following experiment.

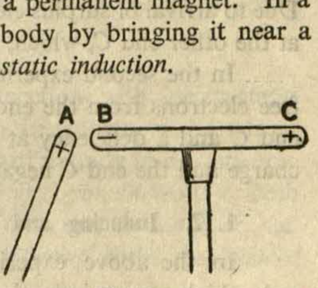


Fig. 1.6(a)

Bring a proof-plane in contact with the end *B* of the conductor and then present the proof-plane near the disc of an uncharged gold-leaf electroscope. The leaves of the electroscope will diverge, showing that the end *B* is charged. If the end *C* is similarly tested, it will also be found to be charged. But if the proof-plane be touched at the middle of the conductor *BC* and then brought near an uncharged electroscope, the leaves will not diverge. From this we may conclude that due to induction both the ends of the conductor *BC* get electrified but there is no electrification at the middle of the conductor.

Now, the question arises as to what kind of charge the ends *B* and *C* get if the glass rod *A* is positively charged ? The answer may be obtained from the following experiment.

Take a *negatively charged* gold-leaf electroscope. Keeping the charged rod *A* near the conductor *BC*, touch a proof-plane at the end *B* and then bring it near the electroscope. The divergence of the leaves will be found to increase, indicating that the end *B* has got negative charge *i.e.* charge opposite to that of the glass rod. When the proof-plane is touched at the end *C* and then presented before a *positively charged* gold-leaf electroscope, the divergence of the leaves will increase, showing that the end *C* is charged positively. If, instead of the glass rod, an ebonite rod rubbed with flannel be held near the conductor *BC*, the end *B* will get positive charge and the end *C* negative charge due to induction [Fig. 1.6. (b)], because the ebonite rod, when rubbed with flannel carries negative charge.

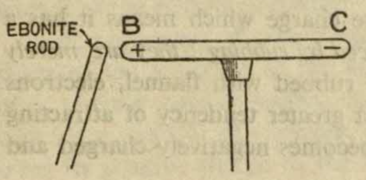


Fig. 1.6(b)

From the above experiments we can generalise by saying that *the end of the conductor nearer the charged body acquires charge opposite to the charged body and the end remote from the charged body, acquires charges similar to the charged body, and that there will be no charge at the middle of the conductor.*

Explanation according to electronic theory :

The production of charge due to induction can be easily explained by electronic theory. Every conductor contains a large number of free electrons which normally move from one atom to another. In the first experiment, the positive charge of the glass rod *A* attracts the free electrons of the conductor to the end *B*, which, therefore, gets a surplus of electrons and becomes negatively charged. Due to arrival of surplus electrons at the end *B*, there occurs a deficiency of electrons at the other end *C*, which, therefore, gets positive charge.

In the second experiment, the negative charge of the ebonite rod repels the free electrons from the end *B* to the end *C*, creating a surplus of electrons at the end *C* and a deficiency at the end *B*. Consequently, the end *B* acquires positive charge and the end *C* negative charge.

1.12. Inducing and induced charges : Free and bound charges :

In the above experiments, the charge of the glass rod or of the ebonite rod, which causes induction in the conductor *BC*, is called the *inducing charge*.

The charge that the conductor BC acquires under the influence of the charged rod, is called the *induced charge.*

Again the charge developed at the furthest end C is often called *free induced charge* and that at the near end B, is called *bound induced charge.* The significance of their calling so, is that if the end C is momentarily touched with a finger, the charge at C is at once removed but the charge at B is held bound by the opposite charge of the inducing body.

Generally, the amount of induced charge is less than that of the inducing charge ; but under certain circumstances, they may be equal (see 'Faraday's ice-pail experiment in art 1.15).

1.13. Induction develops simultaneously both kinds of electricity in equal amounts :

Two metal spheres B and C, supported on insulating stands, are touching each other when a positively charged rod A is brought close to one of them (Fig. 1.7). Induction of charge will take place in the spheres B and C. If the sphere C is now moved away and then the rod A is removed from the vicinity, both spheres are found to be charged, the sphere B negatively and the sphere C positively.

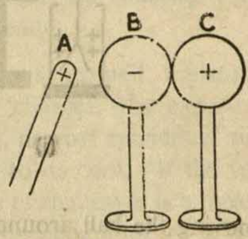


Fig. 1.7

Now, again bring the spheres in contact with each other and keep the charged rod A in their vicinity for some time, and then remove the rod. On examination it will be seen that neither of the spheres has any charge. The negative charge of the sphere B has completely neutralised the positive charge of the sphere C. This shows that equal amounts of charge were induced in the spheres.

1.14. Induction precedes attraction :

We know that a charged body attracts an uncharged body. What is the reason of this attraction ?

When the uncharged body is brought near a charged body, induction takes place, the end of the uncharged body nearest to the charged body getting opposite kind of charge and the end remote from the charged body getting similar kind of charge. The attraction between the two nearer opposite charges is evidently greater than the repulsion between the two further similar charges. As a result the inducing body attracts the induced body. This is why it is said that induction precedes attraction. It may be pointed out in this connection that similar thing happens in the case of magnets also.

1.15. Faraday's ice-pail experiment :

Faraday established some important facts in connection with electrostatic induction by an experiment, which, later on, came to be known as Faraday's ice-pail experiment, although the experiment proper had nothing to do with ice. The facts he established were :—(i) *Induction develops simultaneously both kinds of electricity in equal amounts* and (*ii*) *When the induction is complete, i.e. when the induced body is completely covered by the inducing body, the induced charge becomes equal to the inducing charge.*

Experiment : A hollow metal container (an ice pail) is placed on an insulating stand B and a gold-leaf electroscope (D) is connected to the container by connecting wires [Fig. 1.8(i)]. A metallic ball A is charged positively from another source and then lowered into the pail. It will be seen that as the ball enters the pail the leaves of the electroscope begin to diverge more and more [Fig. 1.8(i)]. When the ball A is well within the pail, the divergence of the leaves becomes the greatest. Upon

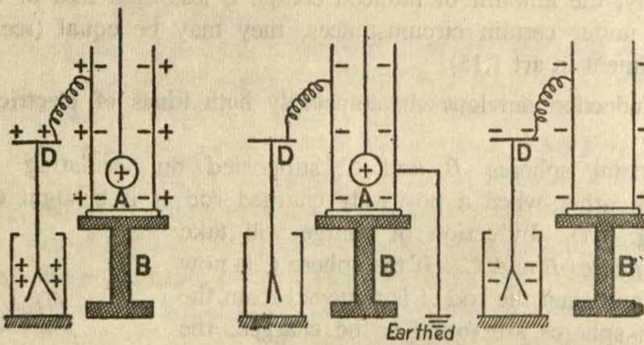


Fig. 1.8

moving the ball around inside the pail, no change in the divergence is shown by the electroscope leaves. In this condition, the induction is said to be complete or full. Due to induction the inside surface of the pail acquires negative charge and the outside surface positive charge. The electroscope, being in direct contact with the outside surface of the pail shares the positive charge and its leaves diverge. To test whether the electroscope has actually shared the positive charge, a positively charged proof-plane may be brought in the vicinity of the disc D of the electroscope, in which case an increased divergence of its leaves will be noticed. This proves conclusively that some of the positive charge of the outside surface of the pail has flown to the electroscope.

Now, touch the pail momentarily with fingers. The free induced positive charge on the outside surface will at once go to the earth. Consequently, the leaves of the electroscope will collapse. [Fig. 1.8(ii)]. But the bound induced negative charge on the inside surface of the container remains. Now, remove the ball A, without touching the pail. According to the laws of induction, the bound induced negative charge should now distribute itself over the whole of the pail and also over the electroscope. Actually, the leaves of the electroscope are found to diverge again and this divergence is equal to the previous divergence [Fig. 1.8(iii)]. That the electroscope has now actually acquired negative charge can be shown by a proof-plane in a manner described earlier. So, this experiment proves that induction develops simultaneously two kinds of charges in equal amounts.

To establish the second part, discharge the electroscope by momentarily touching it with fingers. The leaves of the electroscope will collapse. Again

insert the positively charged ball *A* inside the pail. The leaves of the electroscope will diverge more and more until when the ball *A* is well within the pail, the divergence becomes maximum. The induction now, becomes complete. If the pail is now touched with fingers, the free induced positive charge on the outside surface goes to the earth and leaves of the electroscope collapse. The bound induced negative charge on the inside surface of the pail remains. Now touch the ball *A* with the inside surface (Fig. 1.9). The leaves of the gold-leaf electroscope will remain collapsed as before. After the ball *A* has been removed, the inner surface of the pail and the ball are found to be completely free of charge. The positive charge of the ball *A* and the induced negative charge on the inner surface of the pail have neutralised each other, showing that the *inducing charge is equal to the induced charge, when the induction is complete.*

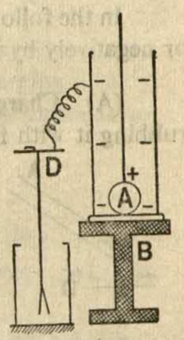


Fig. 1.9

Comparison and collection of charges : Faraday's ice-pail experiment affords us a method of comparing quantities of electric charges. The experiment shows that if a charged body is lowered well inside a tall, narrow cylindrical vessel then it gives to the outside of the vessel a charge equal to its own. If the vessel is connected to the cap of an electroscope, the divergence of the leaves is a measure of the charge on the body. Thus, we can compare the magnitude of charges, without removing them from the bodies which carry them ; we merely lower those bodies, in turn, into a tall insulated vessel, connected to an electroscope and observe the divergence of the leaves in two cases.

Sometimes we may wish to discharge a conductor completely, without letting its charge run to the earth. We can do this by letting the conductor touch the bottom of a tall can on an insulating stand. The whole of the body's charge is then transferred to the outside of the can.

1.16. Comparison between magnetic and electrostatic induction :

(i) As magnetic induction develops two opposite kinds of polarity, so in electrostatic induction, two opposite kinds of charge are developed.

(ii) In the case of electrostatic induction the induced charge disappears as soon as the inducing body is removed but in the case of magnetic induction, the induced magnetism lingers for some time.

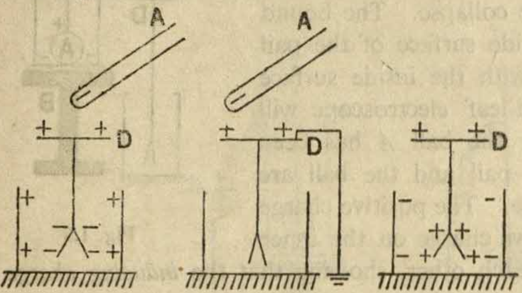
(iii) In the case of electrostatic induction, the induced body should be kept separated from the inducing body but magnetic induction takes place even when the two are in contact.

(iv) In electrostatic induction, the two opposite kinds of induced charges may be separated from each other but in magnetic induction, the two opposite kinds of induced polarities can not be separated from each other.

1.17. Charging a gold-leaf electroscope by induction :

In the following way, a gold-leaf electroscope may be charged either positively or negatively by induction.

(A) **Charging positively :** (i) Charge an ebonite rod (*A*) negatively by rubbing it with flannel and bring the rod near the disc (*D*) of the electroscope to be charged. The free electrons of the disc will be repelled to the leaves by the surplus electrons of the rod *A*. The disc, having thus a deficiency of electrons, acquires positive charge and the leaves having an excess of electrons, acquire negative charge. The leaves, as a result, diverge [Fig. 1.10(i)].



(i) (ii) (iii)
Fig. 1.10

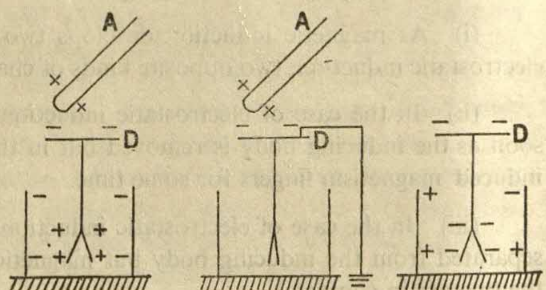
(ii) Keeping the rod *A* in its position, touch the disc *D* of the electroscope momentarily by hand. The surplus electrons of the leaves will, at once, be repelled to the ground and the leaves will collapse [Fig. 1.10(ii)].

(iii) Now, remove the rod *A*. The bound positive charge on the disc *D* will now distribute itself over the electroscope and the leaves getting this charge, will again diverge [Fig. 1.10(iii)].

In this way, with the help of a *negatively* charged rod an electroscope may be charged *positively* by induction.

(B) **Charging negatively :** (i) Charge a glass rod (*A*) positively by rubbing it with silk and hold it near the disc (*D*) of an electroscope. The positive charge of the rod will attract the free electrons of the electroscope to its disc. As a result there will be a surplus of electrons in the disc *D*, which will, therefore, get negative charge and the leaves, having a deficiency of electrons, get positive charge. The leaves of the electroscope, consequently, diverge [Fig. 1.11(i)].

(ii) Keeping the rod *A* in its position, touch the disc of the electroscope for a moment. The leaves will attract electrons from the ground and will have its positive charge neutralised. The leaves will then collapse [Fig. 1.11(ii)].



(i) (ii) (iii)
Fig. 1.11

(iii) Now, remove the rod *A*. The bound negative charge of the disc distributes itself throughout

the electroscope, and the leaves, getting this negative charge, diverge again [Fig. 1.11(iii)].

In this way, with the help of a *positively* charged rod, an electroscope may be charged *negatively* by induction. From the above method of charging a body by induction, one thing is apparent viz., *the induced body gets charge opposite to the charge of the inducing body.*

1.18. Function of the tin plates of a gold-leaf electroscope :

In describing a gold-leaf electroscope (art 1.6), it has been said that a pair of tin plates are pasted along the wall of the vessel in front of the leaves. What is the function of these plates ?

When the gold-leaf electroscope is charged by induction, the leaves acquire similar kinds of charge. The leaves, in their turn, induce opposite charges on the respective tin plates and similar charges on the outside surface of the vessel. The charges on the outside surface go to the ground because the outside surface is in contact with the ground. Consequently the opposite induced charges on the tin plate attract the gold-leaves and help to produce appreciable divergence of the leaves. This makes the instrument sensitive. In fig. 1.10 the leaves have been shown to acquire positive charge and the tin plates negative charge. In Fig. 1.11, however, the leaves, have got negative charge and the tin plates positive charge. In both cases attraction between the plates and the leaves takes place and the divergence becomes appreciable.

1.19. Effect of a charged rod on a charged gold-leaf electroscope :

In art 1.6, it has been described how a gold-leaf electroscope can be used to test the sign of the charge on a body. It may be recalled that a charge of the same kind as that on the electroscope causes an increase while a charge of opposite kind causes a decrease in the divergence of the leaves. We can now explain this in terms of the electronic theory.

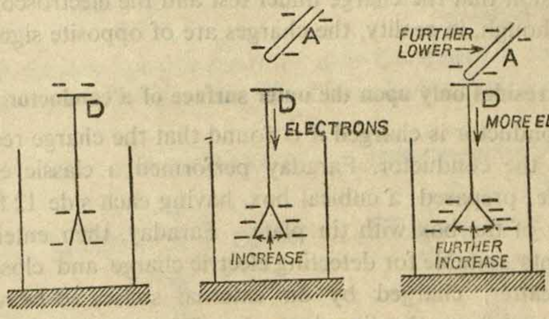


Fig. 1.12

Take the fig. 1.12. A negatively charged rod *A* has been placed in the vicinity of a negatively charged electroscope and the figure shows how the divergence of the leaves has increased. In the same way, the divergence will increase if positively charged rod is brought close to the disc of a positively charged electroscope.

But fig. 1.13 shows what happens when a *strong* negative charge is brought near the disc of a positively charged electroscope. When the charged rod *A* is high above the disc *D*, some of the free electrons of the disc will be repelled to

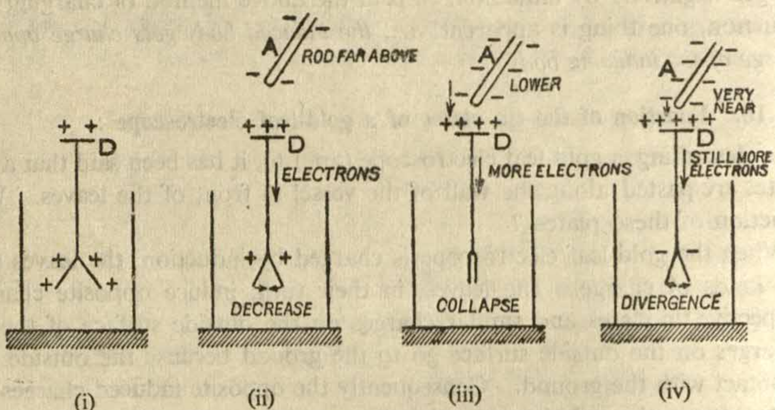


Fig. 1.13

the leaves, which will partly neutralise the positive charge of the leaves. So, the leaves will collapse a little [Fig. 1.13(ii)]. As the charged rod is gradually lowered more electrons will be repelled to the leaves, until eventually the leaves collapse completely [Fig. 1.13(iii)], when the positive charge of the leaves becomes exactly neutralised. After this stage has been reached, a further lowering of the charged rod *A* will cause the leaves to diverge again, since the leaves now acquire an excess of electrons [Fig. 1.13(iv)].

It is, therefore, clear that a charged body should be brought from a *sufficient height* slowly down towards the disc of a gold-leaf electroscope so that the initial decrease in the divergence of the leaves (if any) may not be overlooked. Otherwise, if the observer notices only the final increase in divergence, he may be led to the wrong conclusion that the charge under test and the electroscope charge are of the same kind although, in reality, the charges are of opposite sign.

1.20. Charge resides only upon the outer surface of a conductor :

Whenever a conductor is charged it is found that the charge resides only on the outer surface of the conductor. Faraday performed a classic experiment in this connection. He prepared a cubical box, having each side 12 ft. long. He wrapped the outside of the box with tin plate. Faraday, then entered the box with some instruments suitable for detecting electric charge and closed the door. The box was, thereafter, charged by an external source so heavily that long electric sparks came out from the tin plate ; but Faraday did not find any trace of electricity inside the box.

The effect can also be demonstrated in the laboratory by the following experiments :

(1) **Faraday's butterfly net experiment** : A conical shaped muslin or cotton bag *A* is fastened to a brass ring as shown in the fig. 1.14. The arrangement is

held on an insulating stand. Two silk threads are attached to the apex of the net so that with their help, the net can be pulled inside out. The net is strongly electrified. A proof-plane is brought in contact with the inside surface of the net. The proof-plane shows no sign of electrification when brought near the disc of an electroscope. When the same proof-plane touches the outer surface of the net, it shows that the outer surface is strongly charged.

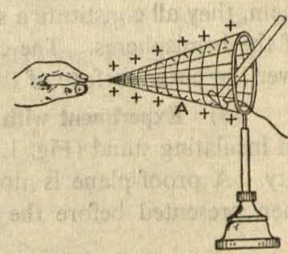


Fig. 1.14

The net is then pulled inside out, so that the outer surface now becomes the inner surface of the net. When the proof-plane touches the new inner surface of the net, it shows no sign of electrification but the new outer surface, on being tested by the proof-plane, is found to be strongly charged.

[N.B. When Faraday performed the experiment, he used a butterfly net. Hence the name of the experiment.]

(2) **Biot's experiment :** *A* is a metallic sphere, supported by an insulating stand *D*. *B* and *C* are two thin hemispherical cups which fit exactly round the sphere *A*. Insulating handles are provided to each of the hemispherical cups [Fig. 1.15(*a*)]. The sphere *A* is first charged and afterwards the hemispheres are fitted over it but *no contact* is made between the hemispheres and the sphere *A*. On removing the hemispheres by insulating handles, they are found to have no charge, all the charge, being retained by the sphere *A*.

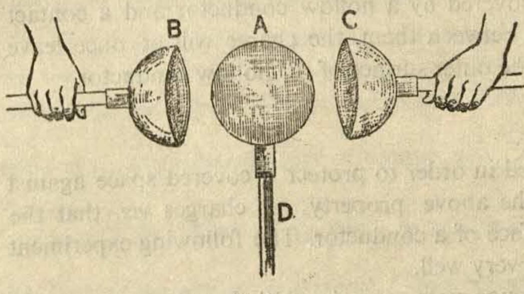


Fig. 1.15 (a)

Again the hemispheres are fitted over the sphere *A* and now *contact* is made between the two. On removing the hemispheres, they are found to be charged

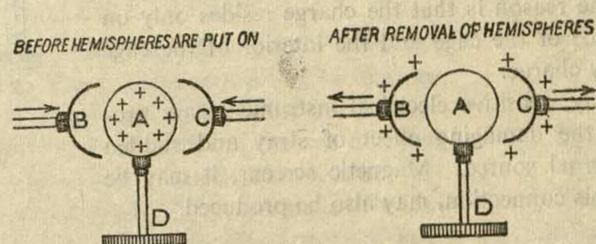


Fig. 1.15 (b)

but no charge, at all, remains on the sphere *A*. All the charge on the sphere *A* must have passed to the hemispheres [Fig. 1.15(b)]. What is the reason of it ?

When the hemispheres fully cover the sphere *A* and contact is made between them, they all constitute a single conductor whose outer surface is the outer surface of the hemispheres. The charge, therefore, leaves the inner sphere *A* and spreads over the outer surface of the hemispheres.

(3) **Experiment with a hollow conductor :** A deep metallic can *A* rests on an insulating stand (Fig. 1.16). The can is strongly charged with positive electricity. A proof-plane is now made to touch the inside surface of the can and is then presented before the disc of an electroscope. The proof-plane shows no sign of electrification proving that here is no charge on the inside surface of the can. The outside surface of the can is now tested similarly by the same proof-plane. The leaves of the electroscope will be found to diverge, showing that charges have passed on to the outer surface.

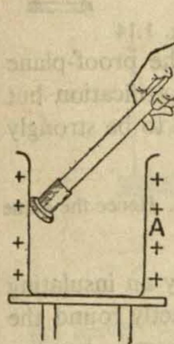


Fig. 1.16

From the above experiments, it may be said that *the charge of a conductor resides only on the outer surface of it.* This fact is very significant because it gives a simple way to transfer charge from one body to another completely. If a charged body be completely covered by a hollow conductor and a contact is made between them, the charge will at once leave the inner body and will spread over the outer surface of the hollow conductor.

1.21. Electric screen :

Electric screens can be produced in order to protect a covered space against the influence of electric charge by the above property of charges *viz.* that the charges reside only on the outer surface of a conductor. The following experiment will illustrate the electrical screening very well.

A cage, made of copper wire-gauge rests on an insulating stand *A* (Fig. 1.17). A gold-leaf electroscope (uncharged) is kept inside the case. If now, a charged rod be brought near the cage, the leaves of the electroscope will exhibit no divergence. Even if the cage itself is charged, the electroscope will remain unaffected. The reason is that the charge resides only on the outer surface of the cage and the interior of the cage is free from any charge.

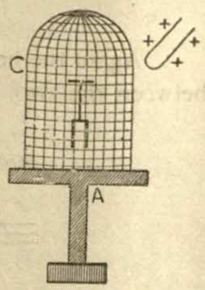


Fig. 1.17

In this way, sensitive electrical instruments are protected against the damaging effect of stray and sudden charges of external source. Magnetic screens, it may be mentioned in this connection, may also be produced.

1.22. Distribution of charge on a conductor and surface density of charge :

When static charges are acquired by a non-conductor like hard rubber, glass or amber, they remain where they were first located. When a conductor like copper, acquires a charge, the charge quickly spreads over the entire surface.

With a metallic sphere, whether solid or hollow, the charge spreads uniformly over the surface. In the case of conductors of other shape, the charge distributes itself according to surface curvature, concentrating more at points and less where the walls are more nearly straight.

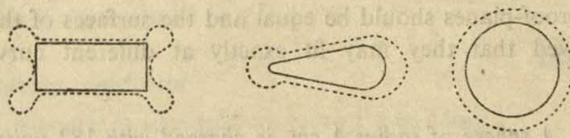


Fig. 1.18

Fig. 1.18 shows qualitatively how charges will distribute over the surfaces of different shapes. The distance of the dotted lines from any point on the surface of each conductor represents the concentration of charge at that point. The last conductor being spherical, the distribution of charge over the surface is uniform but in other two cases, it is not so.

Definition : *When a conductor is charged with electricity, the amount of charge per unit area of the surface of the conductor surrounding a point measures the electric surface-density at that point.*

In the case of a spherical conductor, the distribution of charge over its surface, as we have seen, is uniform and therefore its surface-density of charge is everywhere the same. If r be the radius of the sphere and Q be the total charge given to it, its surface-density of charge $\sigma=\frac{Q}{4\pi r^2}$.

From the above expression, we see that $\sigma \propto \frac{1}{r^2}$. Now $\frac{1}{r}$ is called the *curvature of the surface* at a point. This we see that σ is directly proportional to the square of the curvature. This result holds good for conductors of any shape.

The following experiment shows that the surface-density of charge at different points on the surface of an irregularly shaped body is different.

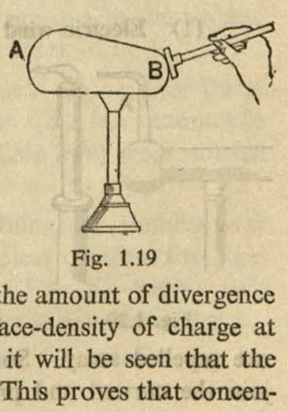


Fig. 1.19

Experiment : AB is a pear-shaped conductor charged with electricity. The curvature of the surface near the point B is greater than that near the point A (Fig. 1.19). If a proof-plane be touched at different points of the conductor, it will collect charges according to the surface-density of charge there. If the proof-plane be now held near the disc of an electroscope, the amount of divergence of the leaves will give an approximate idea of the surface-density of charge at those points. If experiment is carried out in this way, it will be seen that the divergence is less in the case of A than in the case of B. This proves that concentration of charge is more at curved points than at straight points.

In performing the above experiment care should be taken in following respects :—(i) Every time, the proof-plane should be discharged before it is made to touch a new point on the surface of the conductor, (ii) Every time, the proof-plane should be held at equal distance from the disc of the electroscope. (iii) Preferably, a number of proof-planes be used, instead of a single one. (iv) The areas of all the proof-planes should be equal and the surfaces of the proof-planes should be so curved that they may fit exactly at different curvatures of the surface.

Example : *A sphere of radius* 4 *cm. is charged with* 182 *units of electricity. What is its surface density of charge* ?

Ans. Surface density of charge $\sigma=\frac{Q}{4\pi r^2}$; Here $Q=182$ units and $r=4$ cm.

So $\sigma=\frac{182}{4\times\frac{22}{7}\times(4)^2}=\frac{182\times 7}{4\times 22\times 16}=0{\cdot}9$ units/cm^2

1.23. Action of points :

We have seen in the preceding article that electric charge is more concentrated at places on the surface which are more curved. If the place is very pointed and sharp, high concentration of electricity will take place at the point. It will then charge the molecules of air in the vicinity of the pointed end with opposite kind of charge due to induction and will attract the air molecules towards it. Due to this attraction, the air molecules will fall upon the pointed end with the result that the charge on the point is reduced. In this way, the pointed end of a charged conductor loses charge. This process is known as **electric discharge.** So *a conductor which is required to retain tis charge for a long time should be rounded and without any sharp point.*

The attracted air molecules, on contact with the point, acquire the same kind of charge as the point and are then repelled. The following experiments very well illustrate the above action of points.

(1) **Electric wind :** The conductor A has a sharp point in front of which stands a candle flame. When the conductor is uncharged, the flame is straight. If the conductor is now charged strongly by an electric machine, the flame will be deflected [Fig. 1.20]. What is the reason of it ?

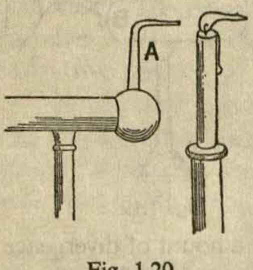


Fig. 1.20

Electric discharge takes place from the pointed end of the conductor A and the surrounding air molecules, falling on the pointed end, are charged with the same kind of charge as the conductor and are repelled away. Some of the uncharged air molecules, in the vicinity, also join the current and produce a strong draught which deflects the candle flame. This sort of air current is known as *electric wind.*

(2) **Electric whirl** : A conductor *P* (Fig. 1.21) consists of several wires arranged as the spoke of a wheel having their ends bent at right angles. When connected to an electric machine, an electric wind streams out from the ends of the wires and the resulting reaction on the wires causes the wheel to rotate in the opposite direction.

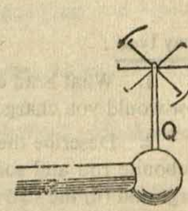


Fig. 1.21

1.24. Lightning conductor :

Lightning conductors are used to protect a building from lightning damage. It consists of a pointed conductor (*R*) placed at the top of a structure and connected by a heavy wire to a metal plate buried in moist earth (Fig. 1.22). The top end of the conductor may contain a number of pointed ends. It is also called *lightning arrester*.

Suppose a positively charged cloud passes over a house. Its attraction makes electrons flow up to the rod from the earth. On reaching the pointed end, the electrons leak off and quietly neutralise the charge of the cloud before it can cause damage by suddenly finding path to earth through the structure itself.

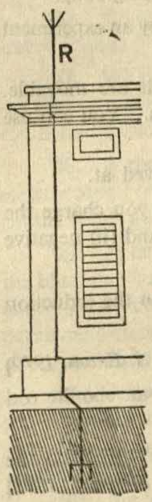


Fig. 1.22

A good lightning conductor should have the following requisites :

(i) The rod must not melt due to heat resulting from electric discharge.

(ii) The top end of the rod should consist of one or more than one pointed end.

(iii) From the pointed end to the earth, the rod should be continuous. It should be buried deep into the earth.

Without the protection of a lightning arrester, the lightning usually strikes the highest point of a building and the current passes to the earth through the path of least resistance. Considerable heat is produced by the passage of the current and masonary tends to split open due to the sudden expansion of steam from the moisture contained in it.

In a lightning storm, the following are the safe places : (i) a steel frame building (ii) a building provided with lightning arrester (iii) an automobile or a shed with an earthed metal roof. One should always keep away from isolated trees, wall, telegraph or telephone posts, wire fencing etc.

It may be pointed out in this connection that lightning and thunder take place simultaneously. But as the sound velocity is much less compared to light velocity, we hear the sound long after the flash is seen. For this reason, it is commonly said that *if one hears the thunder, one is saved from being thunder-struck.*

Exercises

Essay type :

1. What is an electroscope ? Describe and explain the action of a gold-leaf electroscope. How would you charge it by conduction ?

2. Describe the construction of a gold-leaf electroscope. You are given an electroscope, an ebonite rod and some flannel. How could you detect with these things (i) the existence of charge and (ii) the sign of the charge in a body? [*H. S. Exam. 1978*]

3. How could you prove that equal charges of opposite sign are produced by friction ? [*H. S. Exam. 1978*]

4. What is an electron ? Explain the electronic theory of electrification. Explain the process of electrification by rubbing with the help of this theory.

5. What is electrostatic induction ? "Induction precedes attraction"—explain the statement. How has the phenomenon been explained with the help of electronic theory ? [*H. S. Exam. 1982, '84*]

6. What is electrostatic induction ? Prove, by experiment, that the nearer end of the induced body gets opposite charge and the furthest end similar charge as the inducing body.

7. What are bound and free charges ? Why are they so called ? Prove by an experiment that induction develops simultaneously both kinds of electricity in equal amounts.

8. There are two metallic spheres at the end of two insulated rods. The rods are movable. Describe a process by which the spheres may be given equal but opposite charges. You can use a glass rod rubbed with silk but you cannot touch the spheres with the rod.

9. Describe Faraday's ice-pail experiment. Mention the conclusions arrived at.

10. There is a conductor *A* placed on an insulating stand. How would you charge the conductor *A* by means of a negatively charged body *B* with (i) positive charge and (ii) negative charge ?

11. Show that the induced charge and the inducing charge are equal when the induction is complete.

12. How would you charge a gold-leaf electroscope by induction ? [*H. S. Exam. 1979*]

13. Can a gold-leaf electroscope be charged positively and negatively by an ebonite rod and a piece of flannel ?

14. "Charge of a charged conductor always resides on the outer surface"—Explain the statement with suitable experiment. [*H. S. Exam. 1978*]

15. A deep metallic can is charged with electricity. A proof-plane is touched on the inner surface of the can and is then brought before an uncharged electroscope. Explain, with reason, what will happen.

16. Discuss how electric charge is distributed on the surface of a conductor of any arbitrary shape. How can this be studied experimentally ? Describe an experiment to show the discharging effects of the pointed ends of conductors. [*H. S. Exam. 1981*]

17. A conductor which is required to retain charge for a long time should be rounded. Why ? Describe some experiments to illustrate the 'action of points'.

Short answer type :

18. What is electrification ? What is static electricity ?

19. "Repulsion is a surer test of electrification than attraction." Explain the statement. [*H. S. Exam. 1979, '82*]

20. Answer the following questions :—(i) Electrostatic experiments usually work poorly on days when humidity is very high. Explain why ? (ii) Why is a chain kept dangling on the

road from the body of a petrol carrying truck ? (iii) What is the difference between a conductor and an insulator ? (iv) A positively charged glass rod attracts a suspended object. Can we conclude that the object is negatively charged ? A positively charged glass rod repels a suspended object. Can we conclude that the object is positively charged ?

21. How would you explain the two types of electrification according to the electronic theory ?

22. Why cannot a metallic rod be charged by rubbing if it is held in hand ? What should you do to charge the rod ?

23. Standing on an insulating stand a man touched an insulated but charged conductor. Will the conductor lose charge completely ?

24. An uncharged metallic box with a hole on its upper surface is placed on the disc of a gold leaf electroscope. An insulated positively charged conductor is allowed to go inside the box through the hole. Explain the behaviour of the leaves of the electroscope in the following cases :

(i) the box is momentarily earth-connected and the conductor is removed (ii) the box is momentarily earth-connected and the conductor is removed after making a contact with the box.

25. A strongly electrified body can attract another feebly electrified body charged with the same kind of electricity. Explain how it is possible ?

26. A charged ebonite rod is made to touch the disc of a gold leaf electroscope. The leaves diverge. But when the rod is removed, the divergence decreases a little. Explain it.

27. What will happen in the following cases : (*a*) A positively charged rod is held near the disc of a gold leaf electroscope (*b*) the disc is touched momentarily with hand (*c*) the charged rod is now removed.

28. A rod strongly charged with negative electricity is brought near the disc of a gold leaf electroscope. How would the leaves diverge if (i) the rod is slowly brought down from afar and (ii) the rod is quickly brought down from afar.

29. A needle is mounted vertically, point upwards, on the disc of a gold leaf electroscope, the blunt end being in metallic contact with the plate. When a negatively charged body is brought close to the needle point, without touching it, and is then withdrawn, the gold leaf is left with a permanent divergence. What is the sign of the charge causing this divergence and how was this charge produced ?

30. A gold leaf electroscope is so constructed that for a few degrees divergence the leaves touch the case and are thereby earthed. Describe and explain the behaviour of the leaves when :

(i) a positively charged, insulated body is brought towards the disc of the electroscope until the leaves touch the case.

(ii) the positively charged body is then moved slowly closer to the disc.

31. Will a solid metal sphere hold a larger quantity of electric charge than a hollow sphere of the same diameter ? Where does the charge reside in each case ?

32. A hollow metallic body of irregular shape has a hole on its surface. The body is charged and placed on an insulating stand. You are testing the distribution of charge over the surface of the body with the help of the proof plane and a gold-leaf electroscope. What change in the divergence of the leaves would you notice when the proof-plane collects charge from (i) the flatter part of the surface (ii) more curved part of the surface (iii) inside the body.

33. When does a lightning occur ? How can buildings be protected from lightning ?

34. What kind of shelter is safe and what kind of shelter is unsafe during a thunderstorm ?

Objective type :

35. Mark the correct and incorrect statements in the following :

(*a*) The difference between a conductor and an insulator, according to electronic theory,

is that the electrons are firmly attached to the atoms of an insulator and they cannot move about freely whereas the electrons in a conductor can freely move from one atom to another.

(*b*) The end of the conductor nearer a charged body acquires charge same as the charged body and the end remote from the charged body acquires charge opposite to the charged body, while induction takes place between the charged and the uncharged body.

(*c*) If a charged body be brought in contact with the inside surface of hollow conductor, the whole charge of the body is transferred to the conductor.

(*d*) A conductor which is required to retain charges for a long time, may have a shape without very sharp bends.

Numerical problems :

36. A hollow metallic sphere of radius 2 cm. is charged with 20 units of charge. Find the surface density of charge (i) on the outer surface of the sphere (ii) on the inner surface of the sphere.
[Ans. (i) 0·4 units/sq. cm. (nearly) (ii) 0]

37. A hollow spherical conductor of radius 1 cm. is given 6·28 units of charge. Calculate the surface density of charge (i) on the outer surface and (ii) on the inner surface.
[Ans. (i) 0·5 units/sq. cm. ; (ii) 0]

38. How much charge should be given to a sphere of radius 25 cm so as to make the surface density charge equal to $5/\pi$?
[Ans. 12500 units]

2

ELECTRIC FIELD AND ELECTRIC POTENTIAL

2.1. Force of attraction or repulsion between two electric charges : Coulomb's law :

It has already been demonstrated that like charges repel and unlike charges attract. Nothing that has thus far been said, has indicated just how strong the repulsion or attraction might be or how it depends on the magnitude of the charges and the distance between them. The first quantitative measurement of the force between two charged bodies was made by Charles Augustine Coulomb in 1780. He proved experimentally that :

The force acting between two charges is directly proportional to the product of the two charges and inversely proportional to the square of the distance between them.

Thus, if q_1 and q_2 are two charges kept at a distance r from each other, [Fig. 2.1] then, the force F between them according to Coulomb's law, is $F \propto q_1 \cdot q_2$ and $F \propto \frac{1}{r^2}$

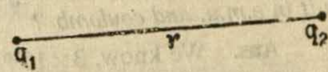


Fig. 2.1

Hence $F \propto \frac{q_1 \cdot q_2}{r^2}$ or, $F = \frac{1}{K} \frac{q_1 \cdot q_2}{r^2}$ where $\frac{1}{K}$ is the constant of proportionality. The value of the constant K depends upon the medium concerned and it is called the *permittivity* of the medium.

In air (truly speaking in vacuum) K is supposed to be equal to 1. So, in air two charges q_1 and q_2 kept separated by a distance r will exert a force on each other which is given by $F = \frac{q_1 \cdot q_2}{r^2}$. .. (i)

K being 1 for air, we can unite $F_{air} = \frac{q_1 q_2}{r^2}$ and $F_{medium} = \frac{q_1 q_2}{K r^2}$ $\therefore$ $\frac{F_{air}}{F_{medium}} = K$

Thus, *the permittivity of a medium may be defined as the ratio of the force of attraction or repulsion between two charges placed in air to the force between the same two charges kept at the same distance in the medium.*

[**Note** : According to the c.g.s. system, the permittivity of vacuum is taken as 1. The permittivity of air is 1·000517—almost equal to 1. This is why the permittivity of air, in general, is taken as 1. But according to *S.I.* or *M.K.S.* units, the permittivity of vacuum is not equal to 1. If K_0 be the permittivity of vacuum in *S. I.* system then $K_0 = \frac{1}{9} \times 10^{-9}$; Further in the c.g.s. system, permittivity is a pure number without any unit but in *S.I.* or *M.K.S.* system it has a definite unit. This difference arises due to the fact that in *S.I.* system, the unit of charge is not derived from Coulomb's law but from the unit of electric current. The 'Coulomb' in this system is defined as follows :

The charge passing through any section of a conductor per second when 1 amp. of current flows through it is defined as 1 Coulomb.

$$\text{Now } K_0 = \frac{q_1 \cdot q_2}{F \cdot r^2}$$

In *S.I.* system, q_1 and q_2 are expressed coulomb, the force F in newton and the distance r in metre. Hence unit of K in vacuum will be (coulomb)^2/(newton × metre^2) or $C^2 N^{-1} m^{-2}$]

2.2. Electrostatic unit of charge :

If $F=1$ dyne, $r=1$ cm., and $q_1=q_2=q$ (say), then from equation (i), we get $q^2=1$ or $q=\pm 1$

Definition : *An electric charge which repels an identical charge at a distance of one centimeter from it in air with a force of one dyne is said to have a magnitude of one electrostatic unit.* This is abbreviated as e.s.u. It is also known as *Statcoulomb.*

Besides e.s.u., there is another unit known as **electromagnetic unit** (abbreviated as *e.m.u.*) based on the magnetic effect of electric current (*vide* current electricity). There is also a practical unit of charge known as **coulomb** which is again the unit of charge in M.K.S. system. The following relations should be remembered :

1 e.m.u. of charge $=3\times 10^{10}$ e.s.u. of charge.

1 e.m.u. of charge $=10$ coulombs.

or, 1 coulomb $=3\times 10^{9}$ e.s.u. of charge.

Example 1 : *The charge of an electron is* $4{\cdot}65\times 10^{-10}$ *e.s.u. How much is it in e.m.u. and coulomb* ?

Ans. We know, 3×10^{10} e.s.u. $=1$ e.m.u. of charge.

$$\therefore\ 4{\cdot}65\times 10^{-10}\ \text{e.s.u.}=\frac{4{\cdot}65\times 10^{-10}}{3\times 10^{10}}=1{\cdot}55\times 10^{-20}\ \text{e.m.u.}$$

i.e. charge of an electron $=1{\cdot}55\times 10^{-20}$ e.m.u.

Again, 3×10^{9} e.s.u. $=1$ coulomb.

$\therefore\ 4{\cdot}65\times 10^{-10}$ e.s.u. $=4{\cdot}65\times 10^{-10}\div 3\times 10^{9}=1{\cdot}55\times 10^{-19}$ coulomb

i.e. charge of an electron $=1{\cdot}55\times 10^{-19}$ coulomb.

Example 2 : *Two point charges of magnitude* 32 *units and* 36 *units are placed* 12 *cm. apart in air. What force will act between them* ?

Ans. We know, $F=\dfrac{q_1 \cdot q_2}{r^2}$

Here, $q_1=32$ units ; $q_2=36$ units ; $r=12$ cm. $\therefore\ F=\dfrac{32\times 36}{(12)^2}=8$ dynes.

Example 3 : *Two electric charges, one* 20 *times as strong as the other, exert a force of* 250 *mg. wt. on each other when they are placed* 10 *cm. apart in air. Find the magnitude of each charge.*

Ans. Here, $F=250$ mg. wt. $=\dfrac{250}{1000}\times 980$ dynes $=\dfrac{25\times 98}{10}$ dynes.

Let q be the magnitude of one charge ; the magnitude of the other charge $=20q$. Now, we know, $F=\dfrac{q_1 q_2}{r^2}$

Here, $F=\dfrac{25\times 98}{10}$ dynes : $q_1=q$; $q_2=20q$; $r=10$ cm.

$$\therefore \quad \frac{25\times98}{10}=\frac{20q\times q}{10\times10} \text{ or, } q^2=25\times49$$

or, $q=35$ units ; other charge$=35\times20=700$ units.

Example 4 : *Suppose you have a large number of identical charges. Any two of them at* 10 *cm. seperation repel with a force of* 3×10^{-5} *dyne. If one of them is repelled by a group of charges which are at a distance of* 10 *cm. and the force of repulsion is* 6×10^{-1} *dyne, how many charges are there in the group ?*

Ans. Suppose, the magnitude of each charge is q. From the first part of the question, we may write, $3\times10^{-5}=\dfrac{q^2}{(10)^2}$ $\therefore$ $q^2=3\times10^{-5}\times(10)^2=3\times10^{-3}$

Regarding the second part of the question, let there be 'n' number of charges in the group. So, $6\times10^{-1}=\dfrac{nq^2}{(10)^2}$

$$\text{or, } n=\frac{6\times10^{-1}\times(10)^2}{q^2}=\frac{6\times10^{-1}\times(10)^2}{3\times10^{-3}}=20{,}000.$$

Example 5 : *Two pithballs each weighing* 0·1 *gm. and suspended from a point by silk threads* 40 *cm. long are equally charged and repel each other to a distance of* 10 *cm., between them. What is the charge on each ball ?*

Ans. O is the point of suspension. A and B are the pith balls [Fig. 2.2]. Resolving tension T along and perpendicular to AC we have, $T\cos AOC=mg$ and $T\sin AOC=F$

Dividing we get $\dfrac{F}{mg}=\tan AOC=\dfrac{AC}{OC}$

Now $AC=\frac{10}{2}=5$ cm. and

$OC=\sqrt{(40)^2-(5)^2}=\sqrt{1575}=39{\cdot}69$ cm.

$$\therefore \quad F=0{\cdot}1\times980\times\frac{5}{39{\cdot}69}=12{\cdot}34 \text{ dynes}$$

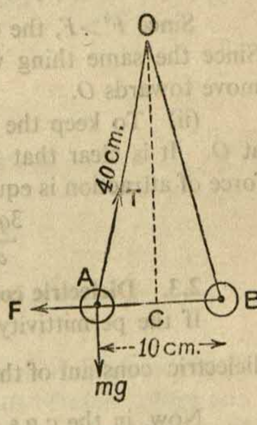


Fig. 2.2

If q be the charge on each pith ball, then $F=\dfrac{q^2}{(10)^2}$

$$\therefore \quad 12{\cdot}34=\frac{q^2}{(10)^2} \text{ or } q^2=(10)^2\times12{\cdot}34 \text{ or } q=35{\cdot}1 \text{ units.}$$

Example 6 : *Three charges, each of value q, are placed at the corners of an equilateral triangle. A fourth charge Q is placed at the centre of the triangle. (i) If $Q=-q$, will the charges at the corners move towards the centre or fly away from it ? (ii) For what value of Q will the charge remain stationary ?* [*I.I. T. 1978*]

Ans. Let us first consider the force experienced by the charge q at A. The charge q at B will exert a force of repulsion F_1 on it along $\overrightarrow{BA}$ such that $F_1=\frac{q^2}{a^2}$ where a=length of each arm of the triangle ABC. Similarly, the charge q at C exerts a force of repulsion $F_2=\frac{q^2}{a^2}$ along $\overrightarrow{CA}$. If F be the resultant of these two forces, then,

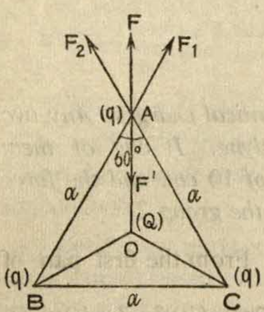


Fig. 2.3

$$F=(F_1{}^2+F_2{}^2+2F_1F_2\cos 60°)^{\frac{1}{2}}$$
$$=[(q^2/a^2)^2+(q^2/a^2)^2+2.\frac{q^2}{a^2}\times\frac{q^2}{a^2}\times\tfrac{1}{2}]^2$$
$$=\left(\frac{3q^4}{a^4}\right)^{\frac{1}{2}}=\sqrt{3}.\frac{q^2}{a^2}\text{ along }\overrightarrow{OA}$$

(i) If $Q=-q$., it will exert a force of attraction F' on q at A along $\overrightarrow{AO}$ such that $F'=q^2/(OA)^2=\frac{q^2}{(a/\sqrt{3})^2}=\frac{3q^2}{a^2}$ $[OA=OB=OC=a/\sqrt{3}]$

Since $F'>F$, the charge at the corner q will move towards the centre A. Since the same thing will happen to all the corner charges, all the charges will move towards O.

(ii) To keep the charges at the corners stationary, let a charge Q be kept at O. It is clear that this charge should be negative and of such value that its force of attraction is equal to the resultant force F. Hence,

$$\frac{3q.Q}{a^2}=\sqrt{3}.\frac{q^2}{a^2}\text{ or }Q=\frac{q}{\sqrt{3}}$$

2.3. Dielectric constant of a medium :

If the permittivity of a medium be K and that of vacuum be K_0, then the dielectric constant of the medium ϵ is given by $\epsilon=\frac{K}{K_0}=\frac{\text{permittivity of the medium}}{\text{,, ,, ,, vacuum}}$

Now, in the c.g.s. system $K_0=1$; so $\epsilon=K$ *i.e.* in the c.g.s. system, the permittivity of a medium is numerically equal to its dielectric constant. Hence there is practically no difference between these two terms in the c.g.s system. For example, the permittivity of mica in this system is 5·7 ; so the dielectric constant of mica is also 5·7. But such is not the case in *S. I.* or *M. K. S.* system. In this system, $K=\epsilon.K_0$ *i.e.* the permittivity of a medium is the product of the permittivity of vacuum and the dielectric constant of the medium. For example, if the dielectric constant of mica be 5·7, its permittivity$=5{\cdot}7\times\frac{1}{9}\times10^{-9}=6{\cdot}3\times10^{-10}$ $C^2N^{-1}m^{-2}$.

Since the dielectric constant of a medium is the ratio of two permittivities, it is sometimes referred to as *relative permittivity*.

Example 1 : *A charge of 80 e.s.u. is kept at a distance of 25 cm. from another charge of −70 e.s.u. in a medium. If the force of attraction between them is 4 dynes, find the dielectric constant of the medium.*

Ans. If K be the permittivity of the medium, then

$$F=\frac{1}{K}\cdot\frac{q_1q_2}{r^2} \text{ or } 4=\frac{1}{K}\cdot\frac{80\times70}{(25)^2}$$

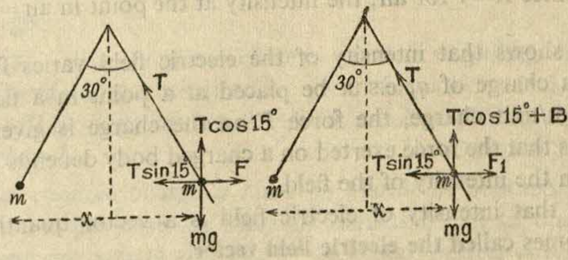
$$\therefore\quad K=\frac{80\times70}{4\times(25)^2}=2{\cdot}25 \text{ (nearly)}$$

Since in the c.g.s. system the permittivity of a medium is numerically equal to its dielectric constant, the dielectric constant of the given medium is 2·25 (nearly)

Example 2 : *Two identical charged spheres are suspended by strings of equal length. The strings make an angle of 30° with each other. When suspended in a liquid of density 0·8 gm/cm³, the angle remains the same. What is the dielectric constant of the liquid ? The density of the material of the spheres is 1·6 gm/cm³.* [*I. I. T. 1976*]

Ans. When in air [Fig. 2.4(i)], the following forces are acting on any of the two spheres ; (a) weight mg of the sphere veritically downward (b) force of repulsion $F=q^2/x^2$ where x is the distance between the spheres and (c) the tension T.

Since the sphere is at rest, $T\cos 15°=mg$ and $T\sin 15°=F$ or $\tan 15°=\dfrac{F}{mg}$ . . . (i)



(i) Fig. 2.4 (ii)

When immersed in the liquid [Fig. 2.4 (ii)], the upthrust B of displaced liquid will act on the sphere vertically upwards. If r be the radius of each sphere and σ the density of liquid, then $B=\frac{4}{3}\pi r^3\sigma g$. In this case, $T\cos 15°+B=mg$ and

$T\sin 15°=F_1$ or $\tan 15°=\dfrac{F_1}{mg-B}$ . . . (ii)

From (i) and (ii) we get,

$$\frac{F_1}{F}=\frac{mg-B}{mg}=1-\frac{B}{mg}=1-\frac{\frac{4}{3}\pi r^3\sigma g}{\frac{4}{3}\pi r^3\rho g}=1-\frac{\sigma}{\rho}=1-\frac{0{\cdot}8}{1{\cdot}6}=\frac{1}{2}$$

[ρ=density of the material of the spheres]

But $\dfrac{F_1}{F}=\dfrac{1}{K}$ [ref. art 2.1] $\therefore\ \dfrac{1}{K}=\frac{1}{2}$ or $K=2$.

2.4. Electric field :

We know that bodies with like kinds of electric charge repel each other and those with unlike kinds of charge attract even though there appears to be nothing in the intervening space. In order to describe these electrostatic forces acting across free space, an imaginary invisible medium, called an electric field, has been invented.

Definition : *The region surrounding an electric charge where its influence is felt is called an electric field of the charge.*

Theoretically, the field extends upto infinity but practically it is found that a charge exerts its influence (*i.e.* a force of attraction or repulsion) over a limited region.

Intensity of the field : *Intensity or strength of the electric field at any point is defined as the force which would act upon a unit positive charge if placed at that point.* The direction of this force gives the direction of the electric field at that point. In the C.G.S. system the unit of intensity is 'dyne' and in M.K.S. system, 'Newton'.

Suppose, we take a point distant r cm. from a charge q e.s.u. in a medium whose di-electric constant is K. If a unit positive charge be placed at that point, the force felt by the unit charge, according to Coulomb's law is given by $F=\frac{q\times 1}{K.r^2}=\frac{q}{K.r^2}$ dynes. So the intensity of the electric field at that point

$E=\frac{q}{K.r^2}$ dyne. Since $K=1$ for air, the intensity at the point in air$=\frac{q}{r^2}$.

This clearly shows that intensity of the electric field varies from point to point. If, again a charge of q_1 e.s.u. be placed at a point in a field where the intensity is E dynes/unit charge, the force F on the charge is given by, $F=Eq_1$ dynes. This shows that the force exerted on a charged body depends on the charge of the body and on the intensity of the field.

It is evident that intensity of electric field is a vector quantity. For this reason, it is sometimes called the **electric field vector.**

Example 1 : *What will be the intensity of the field exactly midway between two charges* $+30$ *units and* $+60$ *units placed* 12 *cm. apart in air* ?

Ans. The point exactly midway between the charges is evidently 6 cm. away from each charge. Now, the intensity due to $+30$ units of charge at 6 cm. away$=\frac{30}{(6)^2}$ and its direction is towards $+60$ unit charge.

Again, the intensity due to $+60$ units of charge at 6 cm. away$=\frac{60}{(6)^2}$ and its direction is towards $+30$ units charge.

These two intensities act along the same line but in the opposite directions. Hence, the resultant intensity at the point$=\frac{60}{(6)^2}-\frac{30}{(6)^2}=\frac{30}{(6)^2}=0{\cdot}83$ dyne in a direction towards $+30$ unit charge.

Example 2 : *Two point charges of magnitude* $+4$ *and* $+9$ *are placed* 10 *cm. apart. Find out the points on the straight line joining them where the force experienced by a unit positive charge will be* (*i*) *equal and opposite and* (*ii*) *equal and in the same direction.*

Ans. (i) In order that the force be equal and opposite, the point in question, should be situated between the charges because both the charges are positive.

Suppose x cm.=the distance of the point from $+4$ unit charge.

So, $(10-x)$ cm.= " " " $+9$ " "

According to the question, $\frac{4}{x^2}=\frac{9}{(10-x)^2}$ or, $\frac{2}{x}=\frac{3}{(10-x)}$

or, $20-2x=3x$ $\therefore$ $x=4$ cm.

i.e. the point is 4 cm. from $+4$ unit charge or 6 cm. from $+9$ unit charge.

(ii) In order that the force may be equal and in the same direction, the point in question should lie on the same side of the charges and nearer to $+4$ unit charge.

Suppose x cm.=the distance of the point from $+4$ unit charge.

Then $(10+x)$ cm. = " " " " " $+9$ " "

According to the question, $\frac{4}{x^2}=\frac{9}{(10+x)^2}$ or, $\frac{2}{x}=\frac{3}{10+x}$

or, $20+2x=3x$ $\therefore$ $x=20$ cm.

i.e. the point is 20 cm. on the left of $+4$ unit charge or 30 cm. on the left of $+9$ unit charge.

Example 3 : *Two negative charges of unit magnitude each and a positive charge q are placed along a straight line. At what position and for what value of q, the system will be in equlibrium ? Check whether if it is a stable, unstable or neutral equilibrium ?* [*Jt. Entrance 1985, I. I. T. 1973*]

Ans. The positive charge q must be placed in between the two negative charges because in that case the force of attraction due to one negative charge may be equal and opposite to that due to the other negative charge and the system will be in equilibrium. Let the distance between the negative charges be r and the $+q$ charge is placed at a distance x from left hand negative charge [Fig. 2.5].

Force of attraction on $+q$ due to the negative charge at $A=\frac{q\times 1}{x^2}$ and that due to the negative charge at $B=\frac{q\times 1}{(r-x)^2}$

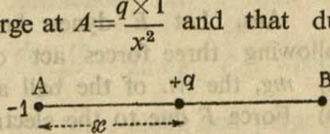


Fig. 2.5

For the equilibrium of the charge $+q$,

$$\frac{q\times 1}{x^2}=\frac{q\times 1}{(r-x)^2} \quad \text{or} \quad x=r-x \quad \text{or} \quad x=\frac{r}{2}$$

For the equilibrium of the charges at A and B, the force of attraction on any one of them due to $+q$ will be equal and opposite to the force of repulsion between them. Hence, $\frac{q\times 1}{(r/2)^2}=\frac{1\times 1}{r^2}$ or $q=\frac{1}{4}$ unit.

The equilibrium is unstable because if the charges are sightly displaced, they will not return to their previous positions.

Example 4 : *Four charges, $+q$, $+q$, $-q$ and $-q$ are placed respectively at A, B, C and D of a square $ABCD$. The length of each arm of the square is a. Find the intensity of the field at O, the centre of the square.*

Ans. Intensity at O due to $+q$ at $A=\frac{q\times 1}{(AO)^2}=\frac{q}{(a/\sqrt{2})^2}=\frac{2q}{a^2}$ along $\overrightarrow{OC}$

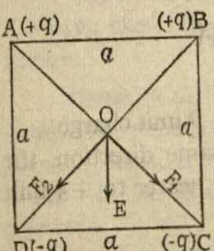


Fig. 2.6

[Fig. 2.6]. Intensity at O due to $-q$ at $C=\frac{q}{(OC)^2}=\frac{q}{(a/\sqrt{2})^2}$

$=\frac{2q}{a^2}$ also along $\overrightarrow{OC}$.

So the total intensity at O due to these two charges is given by $F_1=\frac{2q}{a^2}+\frac{2q}{a^2}=\frac{4q}{a^2}$ along $\overrightarrow{OC}$

Similarly, the total intensity at O due to the charge $+q$ at B and $-q$ at D is $F_2=\frac{4q}{a^2}$ along $\overrightarrow{OD}$.

Now the forces F_1 and F_2 are equal and perpendicular to each other. Hence the resultant intensity at $O=\sqrt{\left(\frac{4q}{a^2}\right)^2+\left(\frac{4q}{a^2}\right)^2}=\frac{4\sqrt{2}q}{a^2}$ and its acts along $\overrightarrow{OE}$, the bisector of the $\angle\ COD$.

Example 5 : *A pith ball carrying a charge of 1 e.s.u. is suspended by an insulated thread of length 50 cm. When a uniform electric field is applied in a horizontal direction, the ball is found to deflect by 2 cm. from the vertical. If the mass of the ball is 0·5 gm, what is the magnitude and direction of the electric field ?* [*I. I. T. 1973*]

Ans. Let E dynes be the field intensity. The following three forces act on the pith ball at rest : (a) mg, the *wt.* of the ball acting vertically downwards (b) Force F due to the electric field acting horizontally given by $F=E\times e=E\times 1$ dyne and (iii) Tension T along the thread

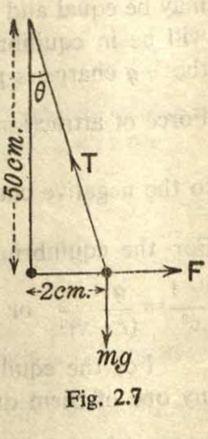


Fig. 2.7

If θ be the angle made by the thread with the vertical, then, $T\cos\theta=mg=0{\cdot}5\times 980=490$

and $T\sin\theta=F=E$.

$\therefore\ \tan\theta=\frac{E}{490}$ or $\frac{2}{50}=\frac{E}{490}$ or $E=19{\cdot}6$ dynes

2.5. Electric intensity at a point near a charged conductor :

Consider a sphere of radius r charged with $+Q$ units of electricity [Fig. 2.8]. The intensity of the electric field at P infinitely close to the charged conductor is the force between the charge Q, considered as if concentrated at the centre O of the sphere and unit positive charge placed at P.

The distance between these two charges $+Q$ and $+1$ may be taken approximately as r and the intensity F is given by

$$F=\frac{Q\times 1}{r^2}=\frac{Q}{r^2} \quad \text{.. (i)}$$

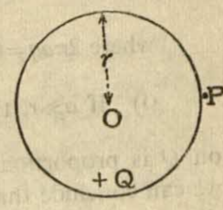


Fig.2.8

Now, let σ be the surface-density of charge on the sphere. We have then $\sigma=\frac{Q}{4\pi r^2}$ or, $Q=4\pi r^2\sigma$.

Substituting this value in eqn. (i) we get

$$F=\frac{4\pi r^2\sigma}{r^2}=4\pi\sigma$$

The expression for intensity does not involve radius of the sphere but involves surface-density of charge. Hence, the above expression for intensity is equally true for any charged conductor. Thus, the intensity at the point in air near a charged conductor is equal to 4 π times the charge-density adjoining the point. This is known as **Coulomb's law.**

[**N.B.** For rigorous proof of the law, any advanced book may be consulted.]

2.6. Force on a point charge placed on the axis of a uniformly charged thin ring :

Let O be the centre of a fixed thin ring of radius a charged uniformly and let Q e.s.u. be a negative point charge placed at P on the axis of the ring at a distance r from the centre of the ring [Fig. 2.9]. Let $+q$ e.s.u. be the charge per unit length of the ring. Consider a small length of the ring at A containing a small charge dq. The force of attraction on Q due to the charge at $A=\frac{dq\times Q}{x^2}$ along $\overrightarrow{PA}$, where

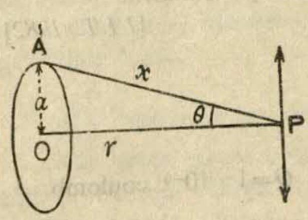


Fig. 2.9

x is the distance between A and P. Resolving this force along and perpendicular to the axis OP, we get the component along the axis$=\frac{Q.dq}{x^2}$. cos θ directed along $\overrightarrow{PO}$.

The component perpendicular to the axis will be cancelled by an equal and opposite component derived from the force of attraction on Q due to a small length at a diametrically opposite point on the ring. Considering the whole

ring, therefore, the total force that acts on Q is along the axis directed towards the centre O and is given by.

$$F=\int\frac{Q.dq.}{x^2}\cos\theta=\int\frac{Q.dq}{x^2}\cdot\frac{r}{x}=\int\frac{Q.rdq}{(r^2+a^2)^{\frac{3}{2}}}$$

$$=\frac{Q.r}{(r^2+a^2)^{\frac{3}{2}}}\int dq=\frac{2\pi aq.Qr}{(r^2+a^2)^{\frac{3}{2}}}$$

where $2\pi aq$=total charge on the ring$=\int dq$

(i) If $a \gg r$, the total force $F=\dfrac{2\pi aq.Qr}{a^3}$ or $F \propto r$. This shows that force on Q is proportional to r and is directed along the centre of the ring. Hence, we can conclude that the charge Q moves along the axis simple harmonically with the centre O as the mean position.

The time-period of the motion is given by,

$$T=2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}=2\pi\sqrt{\frac{r}{(2\pi aq\,Qr)/ma^3}}=2\pi\sqrt{\frac{r.ma^3}{Q.r.q_1}}=2\pi\sqrt{\frac{ma^3}{Q.q_1}}$$

Where $q_1=2\pi aq$=total charge on the ring and m=the mass of the charge Q.

(ii) If the charge on the ring and the point charge at P be of the same nature *i.e.* either positive or negative, the force will be a force of repulsion and the point charge will move away from the centre along the axis.

Example : *A thin fixed ring of radius* 1 *metre has a positive charge* 1×10^{-5} *coulomb uniformly distributed over it. A particle of mass* 0·9 *gm. and having a negative charge of* 1×10^{-6} *coulomb is placed on the axis at a distance of* 1 *cm. from the centre of the ring. Show that the motion of the negatively charged particle is approximately simple harmonic. Calculate the time-period of oscillation.*

[*I.I.T. 1982*]

Ans. For 1st part see art. 2.6

Time period of oscillation $T=2\pi\sqrt{\dfrac{ma^3}{Q.q_1}}$

In the present case, m=0·9 gm ; a=100 cm ; $Q=1\times10^{-6}$ coulomb = $3\times10^9\times10^{-6}$ e.s.u. and $q_1=1\times10^{-5}\times3\times10^9$ e.s.u.

$$\therefore\quad T=2\pi\sqrt{\frac{0{\cdot}9\times(100)^3}{9\times10^{-6}\times10^{-5}\times10^{18}}}=2\pi\sqrt{\frac{1}{100}}=0{\cdot}63 \text{ sec (nearly).}$$

2.7. Electric potential of a conductor :

It is known from experiments that if a conductor A is charged with $+ve$ electricity and if an uncharged insulated conductor B be brought in contact with the charged conductor, the uncharged conductor B receives a certain amount of charge from the conductor A.

Again if we take two *identical* spherical conductors, one having greater amount of positive charge than the other and make contact between them, then

the conductor having greater charge parts with certain amount of its charge which is received by the other conductor and the two conductors acquire the same amount of charge and potential. It is never seen that the conductor having less charge gives up some of its charge to the other conductor.

Thus, when a body is charged it acquires a condition which determines whether the body can deliver charge to or receive charge from another body. This electrical condition is called the potential of the conductor.

In the first case just discussed, the charged body A acquires a higher potential than the uncharged body and hence it parts with some of its $+ve$ charge till their potentials are equal. In the second case, the two conductors being identical, the one having greater amount of $+ve$ charge acquires higher potential than the other. So after contact, the first conductor parts with charge which is received by the other till there is equalisation of potential.

Hydrostatic and thermal analogy of potential : Without stretching the analogy too far, we may consider temperature in heat and level in hydrostatics analogous to potential in electrostatic.

(i) We know that a body at a higher temperature gives up heat to a body at lower temperature when the bodies are in contact with each other and that the transference of heat continues till there is equalisation of temperature of the two bodies. So potential of a conductor may be considered analogous to temperature of a body.

(ii) If we take two vessels containing water at different levels and connect them by a pipe, water will flow from the vessel whose water-level is higher to the vessel whose water-level is lower. The difference in hydrostatic level of water, therefore, determines the flow of water through the pipe just as potential determines the flow of charge from one body to another. The flow of water ceases as soon as the level becomes the same just as the flow of charge ceases as soon as the potentials of the conductors become equal. So potential may be considered analogous to hydrostatic level.

2.8. The earth has zero potential :

The earth is a conductor of electricity. Every moment the earth receives electric charge from different sources and at the same time it supplies electric charge to different sources too. These exchanges are almost equal ; further the earth is so big that its potential does not appreciably change due to small gain or loss of charge. In this respect, the level of sea may be cited as an analogue. Sea water is so huge in quantity, that some water added to or taken out from the sea will not cause any change in its level. For this reason, sea level is considered to be of zero height and all other heights are measured from the sea-level. Similarly, the potential of the earth is considered to be zero because it always remains the same and all other potentials are measured with reference to the earth potential. A body with positive potential is regarded to be higher in potential than the earth and a body with negative potential lower than the earth. But whatever may be the potential, as soon as the body comes in contact with the earth, its potential becomes zero.

This can be explained according to the modern electron theory in the following way :

A positively charged body means a body which has a deficit of electrons. When such a body is connected to the earth, electrons from the earth flow to the body to make up the deficiency [Fig. 2.10(a)] and the body becomes neutral. Its potential then becomes same as the potential of the earth which is zero. Similarly a negatively charged body means a body which has a surplus of electrons. When such a body is connected to the earth, the surplus electrons, at once, flow to the earth, leaving the body neutral. The potential of the body also becomes same as that of the earth [Fig. 2.10(b)].

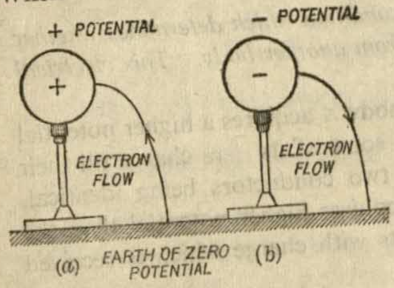


Fig. 2.10

In general it is difficult to calculate potential of a point relative to earth. This is because the electric field due to a charged body near a conducting surface is complicated. In theoretical calculation, therefore, we often find it convenient to consider charges so far from the earth that the effect of the earth on the field is negligible ; we call these '*isolated charges*'.

2.9. Potential at a point :

In the gravitational case, a body when free to move will always move from higher to lower level ; in the same way, a positive charge of electricity will, if free to move, travel from a point of higher potential to a point of lower potential. Potential then, is analogous to level and determines the direction in which a charge will travel when free to move. Consider a region initially free from all electrical influence. Imagine a small charge $+q_1$ placed anywhere in the region. It may be moved about in the region without any work being done, since there is no electrical force acting on it.

But let a charge $+q$ be placed at any point A in this region before the charge $+q_1$ is brought in. To move $+q_1$ from one point to another in the region, some work will have to be done now, for the charge $+q_1$ will be repelled by the charge $+q$. To move $+q_1$ nearer to $+q$, some external source will have to do work against the repulsive force, while if $+q_1$ moves away from the charge $+q$, the repulsive force does the work. Work is also done when either or both of q and q_1 are negative.

We thus see that the presence of the charge $+q$ endows the region around it a property by virtue of which work must be done in moving another charge from one point to another in the region or from a far away point to a point in the region. This property of the point is called its **electric potential.**

Definition : *The electric potential at any point in an electric field may be defined as the work done in bringing a unit positive charge from the infinity up to the point.*

Suppose, V is the potential at a point P in an electric field and W work is done in bringing $+q$ unit of charge from infinity upto the point P.

Then $V=\frac{W}{q}$ or, $W=V\times q$. *i.e.* **Work done = Potential × Charge**

If the potential and charge are expressed in e.s.u., the work done will be expressed in erg.

According to the above definition, the **potential difference** between two points A and B is, therefore, the work done in bringing a unit positive charge from B to A. It is to be remembered that the work done in taking the charge from B to A does not depend on the path along which we carry it, just as the work done in climbing a hill does not depend on the route we take. If this were not true, we could devise a perpetual motion machine, in which we do less work in carrying a charge from B to A *via* P than it does for us in returning from A to B *via* Q [Fig. 2.11].

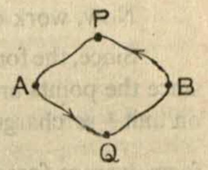


Fig. 2.11

The fact that the potential difference between two points is independent of the path chosen between the points is the most important property of potential in general. This property can be conveniently expressed by saying that *the work done in carrying a charge round a closed path in an electrostatic field such as BPAQB is zero.*

2.10. Units of potential :

If 1 e.s.u. of positive charge is brought from infinity up to a point in an electric field, accomplishing an work of 1 erg, the potential of the point will be 1 e.s.u.

If 1 e.m.u. of positive charge is brought from infinity up to a point in an electric field, accomplishing an work of 1 erg, the potential of the point will be 1 e.m.u.

Since 1 e.m.u. of charge$=3\times10^{10}$ e.s.u. of charge and equal amount of work is done in both the cases, we have,

$$1 \text{ e.m.u. of potential} = \frac{1}{3\times10^{10}} \text{ e.s.u. of potential.}$$

The practical unit of potential is, however, **volt**. The potential at a point in an electric field is 1 volt if 1 joule of work is done in bringing a coulomb of positive charge from infinity up to the point. It is also the M.K.S. unit of potential.

Remember, 1 volt$=\frac{1}{300}$ e.s.u. of potential.

2.11. Calculation of potential at a point in an electric field due to a point charge :

Suppose a point charge $+q$ is placed at A. Its potential at P distant r from A is to be calculated[Fig. 2.12]. Take several points P_1, P_2, P_3....etc. very near to each other at distances r_1, r_2, r_3....etc. from A respectively. Suppose, the potentials at P, P_1, P_2....etc., V, V_1, V_2....etc. respectively. Now, according to the definition,

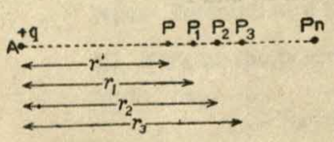


Fig. 2.12

$V-V_1$=work done in bringing a unit$+ve$ charge against the repulsive force from P_1 to $P=W_1$ (say).

Similarly, V_1-V_2=work done in bringing a unit$+ve$ charge from P_2 to $P_1=W_2$ (say) etc.

Now, work done=force$\times$distance.

Since, the forces acting on unit$+ve$ charge at P, P_1, P_2 etc. are all different and since the points are all very near to one another, we can assume that average force on unit$+ve$ charge is uniform between P and P_1 ; between P_1 and P_2 etc. The uniform average force on unit$+ve$ charge between P and $P_1=\dfrac{q}{r\times r_1}$ and that between P_1 and $P_2=\dfrac{q}{r_1\times r_2}$ etc. (for proof see the end of the article).

$$\therefore \text{ the work done } W_1=\frac{q}{r.r_1}(r_1-r)=\frac{q}{r}-\frac{q}{r_1}$$

$$W_2=\frac{q}{r_1.r_2}(r_2-r_1)=\frac{q}{r_1}-\frac{q}{r_2}\,;\; W_3=\frac{q}{r_2.r_3}(r_3-r_2)=\frac{q}{r_2}-\frac{q}{r_3} \text{ etc.}$$

$$\text{Hence,}\quad V-V_1=\frac{q}{r}-\frac{q}{r_1}\,;V_1-V_2=\frac{q}{r_1}-\frac{q}{r_2}$$

$$\vdots \qquad \vdots$$

$$V_{n-1}-V_n=\frac{q}{r_{n-1}}-\frac{q}{r_n}$$

Hence r_n is the distance of P_n from A and V_n its potential.

Adding we get, $V-V_n=\dfrac{q}{r}-\dfrac{q}{r_n}$.

Now, when the point P_n is situated at infinity, $r_n=\infty$

So, $\dfrac{q}{r_n}=0$ and $V_n=0 \quad \therefore \quad V=\dfrac{q}{r}=\dfrac{\text{charge}}{\text{distance}}$

[Calculation of average forces]

force on unit$+ve$ charge placed at $P=\dfrac{q}{r^2}$

" " " " " " $P_1=\dfrac{q}{r_1^2}$

Hence, the average force on unit$+$ve charge between P_1 and P

$$=\tfrac{1}{2}\left(\frac{q}{r_1^2}+\frac{q}{r^2}\right)=\frac{q}{2}\left(\frac{r^2+r_1^2}{r_1^2r^2}\right)=\frac{q}{2}\left[\frac{r^2+(r+\delta)^2}{r_1.^2r^2}\right]$$

where $r_1=r+\delta$, δ being a very small distance.

$$=\frac{q}{2}\left[\frac{r^2+r^2+2r.\delta+\delta^2}{r_1^2.r^2}\right]=\frac{q}{2}\left[\frac{2r^2+2.r.\delta}{r_1^2.r^2}\right]$$

[δ, being a very small quantity δ^2 may be neglected].

$$=\frac{q}{2}\left[\frac{2r(r+\delta)}{r_1^2 r^2}\right]=\frac{q}{2}\left[\frac{2.r.r_1}{r_1^2.r^2}\right]=\frac{q}{r_1.r}$$

In this way, the average forces between other pairs of points may be calculated.]

Potential is a scalar quantity :

It may be pointed out here that potential is **a scalar** quantity and not a vector one. So, potential at a point in a field due to several point charges—positive and negative—is the algebraic sum of the potentials at the point due to individual charges. In other words, *the potential is additive.*

Suppose, several point charges $+q_1$, $+q_2$, $-q_3$ etc. are situated at distances r_1, r_2, r_3..etc. respectively from a point A. Then, the potential V at A due to the charges is given by.

$$V=\frac{q_1}{r_1}+\frac{q_2}{r_2}-\frac{q_3}{r_3}\ldots=\sum\frac{q}{r}$$

With the help of calculus :

The intensity at P due to the charge $+q=\frac{q}{r^2}$ [Fig. 2.5] and its direction is along AP. If the difference of potential between P and P_1 be dV, then according to the definition, dV=work done in bringing a unit$+ve$ charge from P_1 to P=force on unit charge$\times$its displacement$=\frac{-q}{r^2}\times dr$, where dr is the distance between P and P_1.

[Negative sign comes because the force and the displacement are opposite]. Now, by integrating the above expression between limits $r=\infty$ and $r=r$, we get the potential at P. Hence,

V=Potential at P due to the charge $+q$

$$=-\int_{\alpha}^{r}\frac{q}{r^2}dr=-q\int_{\alpha}^{r}\frac{1}{r^2}dr=-q\left[-\frac{1}{r}\right]_{\alpha}^{r}=\frac{q}{r}\Bigg]$$

Example 1 : *A charge* $-$ 10 *e.s.u. is kept at a point A at a distance of* 10 *cm., from a charge q of* 80 *e.s.u. What work is necessary to shift the charge q to a point B at distance* 20 *cm. from q* ?

Ans. Potential at a point distant r from the charge q is $V=\frac{q}{r}=\frac{\text{charge}}{\text{distance}}$

So, if V_A be the potential at A, then $V_A=\frac{80}{10}=8$ e.s.u.

and „ V_B „ „ „ „ B, „ $V_B=\frac{80}{20}=4$ „

$\therefore$ $V_A-B_B=(8-4)=4$ e.s.u.

Hence, the work done in transferring the charge -10 e.s.u. from A to B= $(V_A-V_B)\times10=4\times10=40$ ergs.

Example 2 : *Two positive point charges of* 12 *and* 8 *micro-coulomb respectively are* 10 *cm. apart. Find the work done in bringing them* 4 *cm. closer.*

Ans. Suppose $12\mu c$ charge is kept fixed in position and $8\mu c$ charge is moved 4 cm. closer. The distance of the new position from $12\mu c$ charge is evidently 6 cm.

Now, 1 coulomb$=3\times10^9$ e.s.u.,
hence $12\mu c=12\times10^{-6}c=3\times10^9\times12\times10^{-6}=36\times10^3$ e.s.u.

Similarly, $8\mu c=3\times10^9\times8\times10^{-6}=24\times10^3$ e.s.u.

Now, the p.d. between these two positions,

$$V=Q\left(\frac{1}{r_1}-\frac{1}{r_2}\right)=36\times10^3(\tfrac{1}{6}-\tfrac{1}{10})=\tfrac{36}{15}\times10^3 \text{ e.s.u.}$$

The work done in moving the $8\mu c$ (*i.e.* 24×10^3 e.s.u.) charge, is therefore,

$$W=24\times10^3\times\text{potential difference}$$
$$=24\times10^3\times\tfrac{36}{15}\times10^3 \text{ ergs}$$
$$=5{\cdot}72 \text{ joule}$$

2.12. Electric potential energy of a system consisting of a number of charges :

We have seen in the chapter 'Work, power and energy' (A Text Book of Physics, Vol I) that if we raise a body from the earth's surface, we do some work against the gravitational attraction of the earth and the work done is stored as the potential energy of the system *i.e.* earth+the body. If we release the body, the stored potential energy gradually changes into kinetic energy of the body as it drops. After the body comes to rest on the earth, this K. E. equal in magnitude just before the body strikes the ground to the original P. E. is transformed into heat energy in the same system *i.e.* earth+body.

The same thing happens in electrostatics. Consider two charges q_1 and q_2 at a distance r apart [Fig. 2.12(a)]. If the charges are opposite in sign, some external agent must do some work in trying to increase the separation. This work is regarded as positive. If the charges are, however, of the same sign, they themselves will do the work by virtue of their repulsive force and the work in that case is regarded as negative. The energy represented by this work can be thought of as stored in the system (q_1+q_2) as electric potential energy. This energy, like all other forms of potential energy can be transformed into other forms. If q_1 and q_2, for example, are of opposite nature, and we release them, they will accelerate towards each other, transforming the stored up potential energy into the knietic energies of the accelerating charges. The analogy to the earth+body system is exact, save for the fact that electric forces may be attractive or repulsive whereas gravitational forces are always attractive.

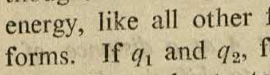


Fig. 2.12 (a)

The electric potential energy of a system of point charges may, therefore, be defined in the following way :

The electrical potential energy of a system of point charges is the work required to assemble the system of charges by bringing them from infinity to the point in question.

Let us suppose that the charge q_2 is shifted to infinity from its position [Fig. 2.12(a)] and is at rest. The potential at the original position of q_2 caused by q_1 is $V=\frac{q_1}{k.r}$. If q_2 is shifted from infinity to the original distance r, the work needed, from the definition of potential, is $W=V.q_2$

It is clear that the above work is stored up in the system (q_1+q_2) as its potential energy. So, electric potential energy $U=W=V.q_2=\frac{q_1q_2}{kr}$

For a system containing more than two charges, the procedure is to calculate the P.E. for every pair of charges separately and then to add the results algebraically. The following example will clarify the procedure.

Example 1 : *Three charges $-4q$, $+q$ and $+2q$ are arranged on the vertices A, B and C of the equilateral triangle ABC in air. Find their mutual potential energy if $q=3\times10^2$ e.s.u. and $a=10$ cm.*

Ans. Total potential energy is the algebraic sum of the energies of each pair of charges. For air $K=1$. So $U=\frac{(+q)\,(-4q)}{a}+\frac{(+q)\,(+2q)}{a}+$

$$\frac{(-4q)(+2q)}{a}=-\frac{4q^2}{a}+\frac{2q^2}{a}-\frac{8q^2}{a}=-\frac{10q^2}{a}$$

$$=-\frac{10\times(3\times10^2)^2}{10}=-9\times10^4 \text{ ergs}$$

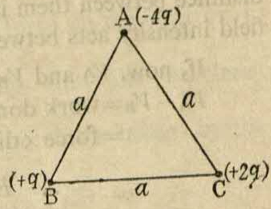


Fig. 2.13

Negative sign indicates that 9×10^4 ergs of work is to be done in dismantling the structure and to remove the charges to infinite separation from one another.

Example 2: *A particle of mass 40 mg and carrying a charge 5×10^{-9} coulomb is moving directly towards a fixed positive point charge of magnitude 10^{-8} coulomb. When it is at a distance of 10 cm. from the fixed positive point charge it has a velocity of 50 cm/s. At what distance from the fixed point charge will the particle come momentarily at rest? Is the acceleration constant during the motion?* [*I.I.T. 1975*]

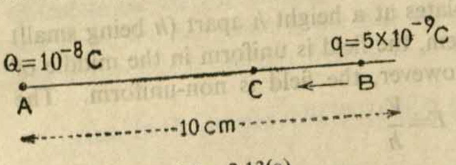


Fig. 2.13(a)

Ans. Let the fixed positive charge Q (10^{-8} coulomb) is located at A and the particle with charge q (5×10^{-9} coulomb) approaching from right arrives at B when its velocity is 50 cm/s. According to the problem $AB=$ 10 cm. [Fig. 2.13(a)]

Now, *K.E.* of the particle at $B=\frac{1}{2}\,m.v^2=\frac{1}{2}\times\frac{40}{1000}\times(50)^2=50$ erg.

And *P.E.* of " " " " $=$Potential at $B\times q.=(Q\times q)/10$

$$=\frac{10^{-8}\times3\times10^9}{10}\times5\times10^{-9}\times3\times10^9$$

$=45$ erg.

[1 coulomb$=3\times10^9$ e.s.u. of charge]

So, total energy of the particle at $B=45+50=95$ erg.

Suppose, the particle advancing through a distance x cm towards A arrives at C and comes momentarily to rest.

K.E. of the particle at $C=0$; P.E. $=$Potential at $C\times q=(Q\times q)/x$.

$$=\frac{10^{-8}\times3\times10^{9}}{x}\times5\times10^{-9}\times3\times10^{9}=\frac{450}{x}\text{ erg.}$$

$\therefore$ Total energy of the particle at $C=\frac{450}{x}$ erg.

As the energy is conserved, $\frac{450}{x}=95 \;\therefore\; x=\frac{450}{95}=4{\cdot}74$ cm.

As the particle advances towards A, the repulsive force increases. Hence its motion becomes gradually retarded until at C, it comes to rest. So its retardation is not constant during the motion.

2.13. Relation between intensity and potential at a point in an electric field :

Suppose two very near points A and B are taken in an electric field and the distance between them is x. If x is very small, it may be assumed that a constant field intensity acts between the points. Let the field intensity be E.

If, now, V_A and V_B be the potentials at A and B respectively ($V_A>V_B$) then,

$V_A-V_B=$work done in bringing a unit$+ve$ charge from B to A.

$=$force$\times$distance between A and $B=E\times x$.

$$\therefore\quad E=\frac{V_A-V_B}{x}$$

If dV be the small difference of potential between A and B and dx the distance between them, then according to the calculus notation, $E=-\frac{dV}{dx}$

The quantity dV/dx is the rate at which the potential changes with distance and is known as the *potential gradient*. The above equation shows that the strength of the electric field is equal to the negative of the potential gradient.

If there be two plane and parallel plates at a height h apart (h being small) and a p.d. of V be maintained between them, the field is uniform in the middle of the plates. At the edge of the plates, however, the field is non-uniform. The intensity of the uniform field is given by, $E=\frac{V}{h}$

Example 1 : *A conducting plate is charged to a potential of* 4000 *volts. A second metal plate, charged to a potential of* 1000 *volts is brought near the first to a distance of* 10 *cm. What is the field intensity at any point between the plates ?*

Ans. The potential of the first plate $V_A=4000$ volts.

$=\frac{4000}{300}$ e.s.u.$=\frac{40}{3}$ e.s.u.

" " " " second $V_B=1000$ volts

$=\frac{1000}{300}$ e.s.u.$=\frac{10}{3}$ e.s.u,

We know, $E=\frac{V_A-V_B}{x}=\frac{\frac{40}{3}-\frac{10}{3}}{10}=\frac{(40-10)}{3\times 10}=1$ e.s.u.

Example 2 : *Charges* $+10$, $+20$ *and* -30 *e.s.u. are respectively placed at distance* 5*cm.*, 10 *cm. and* 15 *cm. from a given point. What is the potential of the point ?*

Ans. We know $V=\frac{q_1}{r_1}+\frac{q_2}{r_2}-\frac{q_3}{r_3}$

Here, $V=\frac{10}{5}+\frac{20}{10}-\frac{30}{15}=2+2-2=2$ e.s.u.

Example 3 : *An electron is liberated from the lower of two large plates separated by a distance of* 50 *m.m. The upper plate has a potential of* 250 *volts relative to the lower. How long does the electron take to reach it ? Mass of electron* $=9{\cdot}1\times 10^{-28}$ gm and charge$=1{\cdot}6\times 10^{-19}$ *coulomb.*

Ans. Between large parallel plates, close together, the electric field is uniform except near the edges of the plates. The potential gradient is, therefore, uniform in the central part of the plates and its magnitude is V/h, where $V=250$ volts and $h=50$ mm$=5$ cm. $V=250$ volts$=\frac{250}{300}=\frac{5}{6}$ e.s.u.

$\therefore$ Intensity $E=\frac{V}{h}=\frac{5}{6\times 5}=\frac{1}{6}$ e.s.u./cm. Now, the force on the electron= Intensity$\times$charge$=1{\cdot}6\times 10^{-19}\times 3\times 10^{9}\times\frac{1}{6}=8\times 10^{-11}$ dynes and the acceleration $f=\frac{\text{force}}{\text{mass}}=\frac{8\times 10^{-11}}{9{\cdot}1\times 10^{-28}}=\frac{8}{91}\times 10^{18}$ cm/s^2 Hence, the time taken by the electron to reach the upper plate is given by $S=\frac{1}{2}ft^2$ or $t=\sqrt{\frac{2s}{f}}$

$$\therefore \quad t=\sqrt{\frac{2\times 5\times 91}{8\times 10^{18}}}=1{\cdot}06\times 10^{-8} \text{ sec.}$$

Example 4 : *Four charges* $+q, +q, -q$ *and* $-q$ *are placed respectively at the corners A, B, C, and D of a square with side a arranged in the given order. Calculate the electric potential and intensity at O, the centre of the square. If E and F are the mid-points of the sides BC and CD respectively, what will be the work done in carrying a charge e from O to E and from O to F ?* [*I.I.T.* 1977]

Ans. In the square *ABCD*, [Fig. 2.6, Page 250] $AO=BO=CO=DO=\frac{a}{\sqrt{2}}$.

Now potential at O is given by,

$$V_0=\frac{q}{AO}+\frac{q}{BO}-\frac{q}{CO}-\frac{q}{DO}=\frac{2\sqrt{2}.q}{a}-\frac{2\sqrt{2}.q}{a}=0$$

Again potential at E, the mid point of BC is given by

$$V_E=\frac{q}{AE}+\frac{q}{BE}-\frac{q}{CE}-\frac{q}{DE}=\left(\frac{q}{AE}-\frac{q}{DE}\right)+\left(\frac{q}{BE}-\frac{q}{CE}\right)=0$$

[$\because$ $AE=DE$ and $BE=CE$]

Also, potential at F, the mid-point of CD is

$$V_F=\frac{q}{AF}+\frac{q}{BF}-\frac{q}{CF}-\frac{q}{DF}=\frac{2q}{\sqrt{5}a}+\frac{2q}{\sqrt{5}a}-\frac{2q}{a}-\frac{2q}{a}$$

$$\left[AF=BF=\frac{\sqrt{5}a}{2} \text{ and } CF=DF=\frac{a}{2}\right]$$

or $$V_F=\frac{4q}{\sqrt{5}a}-\frac{4q}{a}=\frac{4q}{a}\left(\frac{1}{\sqrt{5}}-1\right)$$

$\therefore$ Work done in carrying a charge e from O to E $=e(V_E-V_0)$

$=e\times 0=0$

" " " " " " , O to F $=e(V_F-V_0)$

$$=e\times\frac{4q}{a}\left(\frac{1}{\sqrt{5}}-1\right)$$

2.14. Potential difference and the principle of action of a gold-leaf electroscope :

In discussing the use of a gold-leaf electroscope, it was pointed out in art 1.3 that the existence of charge in a body as well as the sign of the charge may be determined by the electroscope. It was also mentioned that the leaves of the electroscope, acquiring similar charge, repel each other and diverge. It may, however, be proved that the divergence of the leaves is due to the existence of a *potential difference* between the leaves and the tin plates—and not due to the repulsive force between the like charges of the leaves.

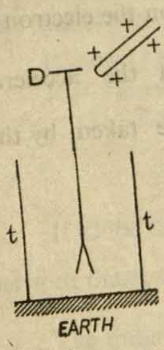


Fig. 2.14

Fig 2.14 shows how the presence of charge in a rod is detected by a gold-leaf electroscope. When the positively charged rod is brought in the vicinity of the disc (D) of the electroscope, the potentials of the disc and the leaves increase. But the tin-plates (t,t) being connected to the ground, have zero potential. So a difference of potential is created between the plates and the leaves which causes the leaves to diverge.

In fig. 2.15, the electroscope has been shown to be placed on an insulating stand. The tin-plates (t,t) are therefore not connected to the earth now but the disc D of the electroscope is grounded. As a result the potentials of the disc and the leaves will be zero. If, now, a negatively charged rod be brought *near the tin-plates*, the potentials of the plates will fall *i.e.* there will again be a difference of potential between the plates and the leaves and consequently the leaves will diverge.

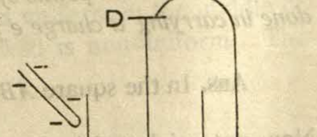


Fig. 2.15

In Fig. 2.16. the electroscope has been shown to be placed on an insulating stand but the disc D of the electroscope is connected by a wire to the tin-plates. In this case, if a charged rod be brought near the disc or if the electroscope be given a charge by any other means, the leaves will not diverge because, the leaves and the tin-plates being connected together have the same potential. So from these experiments we may conclude that a potential difference between the leaves and the tin-plates is necessary for a divergence of the leaves.

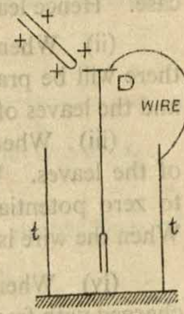


Fig. 2.16

We sum up these observations by saying that the electroscope indicates the potential difference between its leaves and the tin-plates.

2.15. Some problems involving induction and potential :

(*a*) *An insulated gold-leaf electroscope with its disc and tin-foils connected by a wire is charged with positive electricity by induction. The connecting wire is then removed by means of an insulator. The disc of the electroscope is then touched with a finger. Explain what will happen.*

When the electroscope is charged with $+ve$ electricity, the disc, the leaves and the tin-foils being connected together, attain same potential and hence *the leaves although charged with $+ve$ electricity do not diverge.*

When the connecting wire is removed, there is no change in the potential between the leaves and the tin-foils and hence no divergence of the leaves will be observed.

Next when the disc is touched with a finger, the disc and the leaves attain zero potential but the tin-foils retain their previous $+ve$ potential since the electroscope is insulated. Due to this difference of potential between the leaves and the tin-foils, the leaves will now diverge with $-ve$ charge.

So this experiment shows that the divergence of leaves of electroscope depends not only on the charge acquired by the electroscope but also on the potential difference between the leaves and the tin-foils.

(*b*) *Two uncharged electroscopes have their discs connected by a copper wire. Explain what happens when* (*i*) *a charged body is brought near one disc* (*ii*) *with the charged body still in position, the wire is removed by an insulator* (*iii*) *with the charged body still in position, the wire is replaced, touched by the hand and removed* (*iv*) *The charged body is removed.*

Two electroscopes with their leaves and the connecting wire may be considered as a composite conductor.

(i) When a charged body (say, positively charged) is brought near any one of the discs, induction takes place. As a result, there will be flow of electrons from the leaves of both the electroscopes and also from the disc of remote electroscope to the disc of the electroscope near the charged body. This flow ceases

when the composite conductor attains a common potential which is $+ve$ in this case. Hence leaves of both the electroscopes diverge with $+ve$ electricity.

(ii) When the wire is removed, keeping the inducing body still in position, there will be practically no change in the condition of the potential of the system and the leaves of both the electroscopes remain diverged as before.

(iii) When the wire is replaced, there will be no change in the divergence of the leaves. When touched momentarily, the composite conductor is reduced to zero potential. Consequently, the leaves of both the electroscopes collapse. When the wire is removed the leaves remain still collapsed.

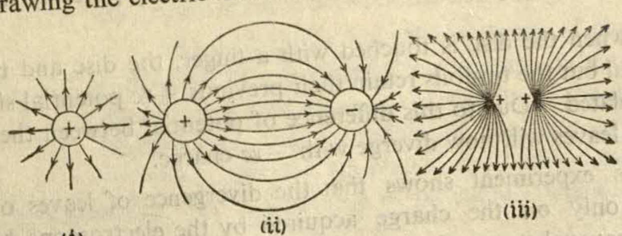
(iv) When the inducing body is removed, the first electroscope becomes charged with free $-ve$ electricity and acquires $-ve$ potential ; the leaves accordingly diverge with $-ve$ electricity. The leaves of the other electroscope which is now disconnected from the first and which is remote from the inducing body, behave like those of an uncharged electroscope and remain practically collapsed.

2.16. Electric lines of force :

Just as a magnet is surrounded by a magnetic field, so an electric charge has an electric field around it. The lines of force of the field may also be drawn.

Definition : *An electric line of force is a line drawn in an electric field such that a tangent drawn at any point on the line gives the direction of the electric field at that point.*

The method adopted for drawing the magnetic lines of force may also be adopted for drawing the electric lines of force. Fig. 2.17 shows the electric lines



(i) (ii) (iii)

Fig. 2.17

of force in some typical cases. In Fig 2.17 (i) lines of force due to a charged sphere have been shown. The lines of force are all radial and start from the centre of the sphere. In figs. 2.17 (ii) and 2.17 (iii), lines of force due to two equal but opposite charges facing each other and two equal but like charges facing each other have been shown. From fig. (iii), it is seen that like magnetic neutral point, the electric field can also have neutral points.

Characteristics of electric lines of force :

The following are the characteristics of electrical lines of force :

(i) Lines of force start from positively charged conductor and end on negatively charged conductor.

(ii) Two lines of force never intersect each other ; for, if they intersect two tangents can be drawn to the lines at the point of intersection, giving two directions of the intensity at a point which is absurd.

(iii) The lines of force tend to contract lengthwise.

(iv) The lines of the force touch a conductor normally.

(v) Each line of force carries equal but opposite charges at its two ends.

(vi) Lines of force repel each other laterally.

(vii) Lines of force do not exist inside a conductor. So, electric lines of force are not closed curves.

2.17. Explanation of electrostatic induction by lines of force :

Electrostatic induction may be explained by considering the above characteristic properties of lines of force. When a positively charged glass rod is held in a room, the lines of force from the charged rod spread through the surrounding medium and terminate on the earth-connected walls of the room. These lines carry $+ve$ charge at the end where they originate (in this case positively charged glass rod) and an equal amount of $-ve$ charge at the end where they terminate (in this case the walls of the room).

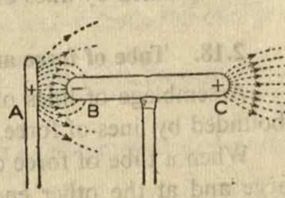


Fig. 2.18(a)

When an insulated uncharged conductor BC is held in the intervening space somewhere near A [Fig. 2.18(a)], some of these lines are intercepted by the insulated conductor BC and the result is that the intercepted lines ending on the conductor find their corresponding negative charge. Now, due to the tendency of the lines of force to contract lengthwise, their end terminating on the conductor BC are pulled more towards the end B and we see negative charge develop at the end B and an equal amount of $+ve$ charge at the remote end C.

The $+ve$ charge at C, in turn, sets up lines which proceed on as usal as shown in fig. 2.18(a).

A finger held near C is equivalent to an earth-connected conductor brought near C. The lines of force from the end C will naturally select shorter path to find their equivalent negative charge and hence these lines will terminate on the finger [Fig. 2.18 (b)]. Now with the approach of the finger towards the conductor the ends of the lines carrying equal amounts of $+ve$ charge and $-ve$ charge approach each other and ultimately coalesce when the finger touches the conductor and thus the lines of force at the end C vanish when the conductor is momentarily earthed by touching it with finger.

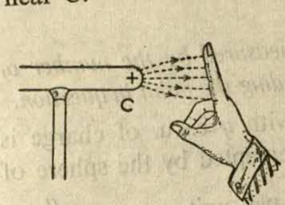


Fig. 2.18(b)

Further, with the earth connection of BC, some more lines from the rod A choose to terminate on the conductor in preference to distant earth-connected

walls of the room [Fig. 2.19 (i)]. This accounts for the slight increase of $-ve$ charge on BC when the latter is grounded.

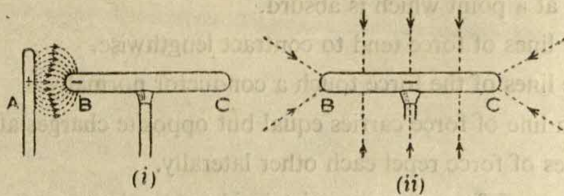


Fig. 2.19

The removal of the rod A enables the lines of force terminating on BC to be distributed over the entire surface of the conductor as shown in fig. 2.19 (ii). The conductor is now freely charged with $-ve$ electricity.

In the same way, the charging of gold-leaf electroscope by induction can also be explained by lines of force.

2.18. Tube of force and Faraday tube :

Assemblage of lines of force constitutes a tube of force *i.e.* a tube of force is bounded by lines of force.

When a tube of force carries at its end where it originates, one e.s.u. of $+ve$ charge and at the other end where it terminates, one e.s.u. of $-ve$ charge, the tube of force is called a unit tube of force or *Faraday tube.*

According to Maxwell, q e.s.u. of $+ve$ charge sends out $4\pi q$ lines in air. Hence, a Faraday tube in air is made of 4π lines of force. If the tube be placed in a medium of permittivity K, a Faraday tube consists of $4\pi/K$ lines of force.

Let us now consider a point charge q placed in a medium of permittivity K. Imagine a sphere of radius r with q as centre. According to Maxwell, the total number of lines of force originating from q and spreading over the surrounding space is $\frac{4\pi q}{K}$. These lines are intercepted by the imaginary sphere whose area is $4\pi r^2$.

$\therefore$ Number of lines of force passing normally through unit area of the sphere $=\frac{4\pi q}{K\times 4\pi r^2}=\frac{q}{Kr^2}$. Let us know consider a point P on the surface of the sphere. The intensity at P due to the charge $q=q/Kr^2$.

Thus we see that *electric intensity at a point is measured by the number of lines of force that passes normally across unit area surrounding the point in question.*

Again, the number of Faraday tubes associated with q e.s.u. of charge is numerically equal to q/K. These Faraday tubes are intercepted by the sphere of area $4\pi r^2$. Hence, the number of Faraday tubes passing per unit area $=\frac{q}{K.\times 4\pi r^2}$

$=\frac{1}{4\pi}\cdot\frac{q}{Kr^2}=\frac{1}{4\pi}\times$ intensity at a point on the sphere.

Thus, *intensity* $=4\pi\times$ *number of Faraday tubes passing per unit area normally.*

2.19. Surface of a charged conductor is equipotential :

We have already seen that the earth has the same potential (zero) all over its surface, because it is a conductor. In a conductor there can be no difference of potential, because a potential difference or a gradient would set up an electric field ; electrons of the conductor would then redistribute themselves throughout the conductor, under the influence of the field until the field vanishes. This is true whether a conductor has a net charge, positive or negative or whether it is uncharged ; it is true whatever be the actual potential of the conductor, relative to another body.

Any surface or volume over which the potential is constant is called an *equipotential.* The volume or the surface may be that of a material body or simply a surface or a volume is space. For example, as we shall see in art 2.20., the space inside a hollow charged conductor is an equipotential volume. Equipotential surfaces may be drawn throughout any space in which there is an electric field.

Consider an isolated point charge q. At a distance r from the charge the potential is $\frac{q}{r}$. A sphere of radius r with centre at q is therefore an equipotential surface of value $\frac{q}{r}$. In fact, all spheres with centre at q are equipotential surfaces whose potentials are inversely proportional to their radii. An equipotential surface has the property that along any direction lying on its surface, there is no electric field ; for there is no potential gradient. *Equipotential surfaces are, therefore, always at right angles to lines of force.*

Experimentally it may be shown in the following way that a charged conductor has same potential all over its surface.

A pear shaped insulated conductor A is charged with positive electricity. A proof-plane is connected to the disc of an uncharged gold-leaf electroscope by a wire and the electroscope is kept at a sufficient distance away so that the charged conductor cannot produce any induction on the electroscope.

Now place the proof-plane on the conductor and slide the proof-plane all along the surface. (Fig. 2.20). It will be seen that the divergence of the leaves remains the same at every point. This proves that the surface of a charged conductor is equipotential, The reader should compare this experiment carefully with the experiment to investigate the distribution of charge over the surface of such a conductor described in art 1.18. The two experiments make it clear that *although the charge is unequally distributed the potential is uniform over the surface.*

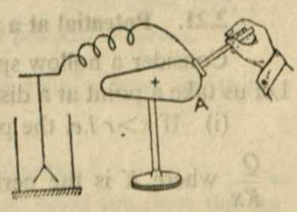


Fig. 2.20

If the conductor is charged with positive and negative electricity simultaneously by induction, the same result will follow. Although, induction develops simultaneously opposite charges in equal amount in a conductor, the surface of

the conductor will behave as an equipotential surface. The sign of the potential will, however depend on the inducing charge. If the inducing charge is $+ve$ the potential of the conductor is also $+ve$. If the inducing charge is $-ve$, the potential of the conductor is $-ve$ too.

2.20. Potential inside a hollow conductor :

If a hollow conductor contains no charged body, then whatever may be the charge on its outside surface there is none on its inside. Inside it, therefore, there is no electric field ; the space within the conductor is an equipotential volume. If the conductor has an open end, like a can, then most of the space inside it is equipotential ; near its mouth there is a weak field.

By the following experiment it may be proved that the potential inside a hollow conductor is everywhere the same and it is equal to the potential on the outer surface of the conductor.

A hollow, deep and insulated metal can C is charged positively and a proof-plane P is inserted well within the can [Fig. 2.21]. The proof-plane is connected to the disc D of a gold-leaf electroscope by a wire. If the proof-plane is moved here and there inside the can, the divergence of the leaves will remain unchanged. The potential inside the can is, therefore, same everywhere because we know that the divergence of the leaves depends upon the potential.

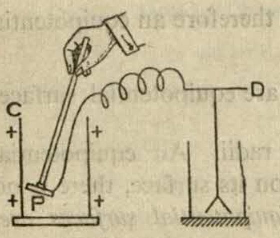


Fig. 2.21

Now, touch the proof-plane on the outside surface of the can. The divergence will remain unchanged. This shows that the potential inside is same as the potential on the outside surface of the conductor.

It is important to note that since the potential inside the can is same everywhere and since the intensity of the field is zero [see art 2.4] there will be no lines of force inside a hollow conductor, except at the edge, if the conductor is an open one.

2.21. Potential at a point due to a hollow charged sphere :

Consider a hollow sphere of radius r charged with Q units of electric charge. Let us take a point at a distance x from the centre of the sphere.

(i) If $x > r$ i.e. the point lies outside, the sphere the potential at the point $= \frac{Q}{Kx}$ where K is the permittivity of the medium. The sphere behaves as if the whole charge is concentrated at its centre.

(ii) If $x = r$ i.e. the point is on the outside surface of the sphere, the potential at the point $= \frac{Q}{Kr}$. Here, also, the sphere behaves as if the whole charge is concentrated at its centre. Further since r is same for all points on the outside

surface of the sphere, potential at every point on the surface is the same. In other words ; the outside surface is an equipotential surface.

(iii) If $x<r$ *i.e.* the point is within the hollow sphere, the potential at that point$=\frac{Q}{kr}$. This shows that the potential inside the sphere is everywhere constant and is equal to the potential on the surface of the sphere.

Since the space inside a hollow sphere is equipotential, intensity there is zero.

Example : *A charge Q is distributed over two concentric hollow spheres of radii r and R (R>r) such that the surface densities are equal. Find the potential at the common centre.* [*I.I.T. 1981*]

Ans. Let the small sphere contain Q_1 and the larger one Q_2 charges. Therefore, $Q=Q_1+Q_2$. Since the surface densities of charge are equal, $\frac{Q_1}{4\pi r^2}=\frac{Q_2}{4\pi R^2}$

$$\text{or } \frac{Q_1}{Q_2}=\frac{r^2}{R^2} \quad \text{or } \frac{Q_1+Q_2}{Q_2}=\frac{r^2+R^2}{R^2}$$

$$\therefore \quad Q_2=\frac{R^2}{r^2+R^2}(Q_1+Q_2)=\frac{R^2.Q}{r^2+R^2}\ ; \quad \text{Similarly, } Q_1=\frac{r^2Q}{r^2+R^2}$$

At the common centre, the potential due to the spheres

$$V=\frac{Q_1}{r}+\frac{Q_2}{R} \quad \text{[Sec art 2.21(iii)]}$$

$$=\frac{r^2Q}{(r^2+R^2)r}+\frac{R^2.Q}{(r^2+R^2)R}=\frac{Q(r+R)}{r^2+R^2}$$

Exercises

Essay type :

1. State Coulomb's law in connection with the mutual force between two point charges and hence define electrostatic unit of charge. [*H. S. Exam. 1978, '81*]

2. Explain the action of a gold-leaf electroscope in terms of potential difference.

3. Prove that the potential at a distance r from a point charge $+q$ is $\frac{q}{r}$. What will be the potential at the point due to multiple charges ?

4. What are electric lines of force ? What are their characteristics ?

5. What do you mean by an equipotential surface ? How would you prove that the surface of a charged conductor is equipotential ?

6. Show how (i) the surface density (ii) the intensity of electric field (iii) potential varies over the surface of a pear-shaped conductor charged with electricity. Describe experiments you would perform to support your answer in cases (i) and (iii).

7. Show that the electric intensity at a point near a charged spherical conductor is independent of the radius of the sphere but is depended on its surface-density of charge. Will it be true in the case of a non-spherical conductor ?

8. A point charge $-q$ is placed on the axis of a thin ring which is uniformly charged with positive electricity. Show that the point charge moves along the axis simple harmonically.

Short answer type :

9. What is an electric field ? Define the intensity at a point in an electric field. What do you mean by the direction of the field at a point ?

10. What is meant by the terms (i) potential and (ii) field strength in electrostatics ? State whether each quantity is a scalar or a vector ?

11. What is meant by the terms (i) potential of a charged body (ii) potential of a point in an electric field (iii) the potential difference between two points ?

12. Potentials of two points A and B are respectively -50 e.s.u. and -10 e.s.u. Which point has higher potential ? What is their potential difference ?

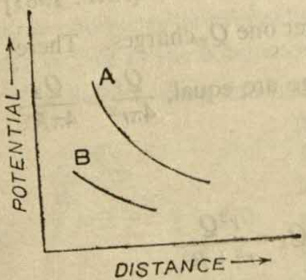


Fig. 2.22

13. What is the relation between intensity and potential difference ? What do you mean by potential gradient ?

14. The potential gradients in two electric fields have been shown in Fig. 2.22. The field represented by the curve A is stronger than that represented by B. Do you agree ?

15. The earth has zero potential—explain the statement. Two bodies, charged negatively and positively are separately connected to the earth. What will be their potentials before and after the connection ?

16. Can two lines of force intersect each other in an electrostatic field ?

17. A point charge $+Q$ is kept at a point. What work will be done in taking a unit positive charge along the circumference of a circle of radius r with the charge $+Q$ at its centre ? [*I.I.T. 1974, '85*]

[**Hints** : No work is done. The sphere drawn with radius r and centre at Q is an equi-potential. Since the potential difference between two points on the sphere (or the circle) is zero, no work is done when the unit charge is taken round the circle.]

18. Suppose that the earth has a net charge which is not zero. Under this circumstances, can earth be taken as standard reference and its potential as zero for the purpose of measurement of potential ?

19. If the intensity at a point in an electric field be zero, will the potential there be also zero ? If the potential at the point be zero, will the intensity be zero ?

20. If the intensity at a point in an electric field be known, can you calculate potential at that point ? If not, what further information is needed ?

21. If the potential is constant throughout a given region of space, what information do you get about the intensity in that region ?

22. q_1 and q_2 are two point charges kept d apart. There is no point in their field where the intensity is zero. What inference can you derive from it ? [*Jt. Entrance 1982*]

[**Hints** : Charges are equal in magnitude but opposite in nature]

23. Two point charges of unknown magnitude and sign are at a distance d apart. The electric field strength is zero at one point between them on the line joining them. What is your conclusion about the charges ?

24. The potential difference between two conductors is very high. What will happen in the following cases : (i) the conductors are joined by a wire (ii) both positive and negative ions are present in the space between the conductors (iii) the space between the conductors is vacuum. [*Jt. Entrance 1982*]

25. The work done in carrying a point charge from one point to another in an electrostatic

field depends on the path along which the point charge is carried. Comment on this statement. [*I.I.T. 1981*]

26. Two point charges $+q$ and $-q$ are placed at d distance apart. What are the points at which the resultant electric field is parallel to the line joining the two charges. [*I.I.T. 1978*]

[**Hints** : All points lying on the perpendicular bisector of the straight line joining the charges].

27. State Coulomb's law ? What is a statcoulomb ?

28. A gold-leaf electroscope is placed on an insulating stand and the disc is connected to its tin-plates. A positively charged rod is now brought close to the disc of the electroscope. How will the leaves behave ?

29. Two concentric metal spheres are insulated from the earth and also from one another and a charge of $+q$ is given to the inner sphere. What will be the electrical condition of the outer sphere. How will it be changed (*a*) by connecting the outer sphere to earth momentarily and (*b*) by afterwards connecting the inner sphere to earth ?

Objective type :

30. (a) Select the correct answer in the following cases :—(*a*) How much is 1 coulomb equal to e.s.u. of charge ? Ans. 3×10^{10} ; 10 ; 3×10^{9} (*b*) How much is 1 volt p.d. equal to e.s.u. ? Ans. $1/3\times10^{10}$; 1/300 ; 3×10^{9} (*c*) A hollow conductor is charged with electricity and a proof-plane is moved well within the conductor. A gold-leaf electroscope connected with the proof-plane shows no deflection. Why ? Ans. no charge is within the conductor ; field-intensity is same everywhere inside it ; potential is same everywhere inside it. (*d*) A battery has 12 volt p.d. between its electrodes. If the negative terminal of the battery is earthed, what will be the new p.d. ? Ans. —12 volts ; 0 ; +12 volts (*e*) What will be the direction of electron flow when a negatively charged body is earthed ? Ans. No flow ; From the earth to the body ; From the body to the earth. (*f*) A hollow metal sphere of radius 5 cm is charged such that the potential on its surface is 10 volts. What is the potential at the centre of the sphere ? Ans. zero ; 10 volts ; 5 volts. [*I.I.T. 1983*] (*g*) Two identical metal spheres of exactly equal masses are taken. One is given a positive charge Q coulombs and the other an equal negative charge. Their masses after charging (i) are different (ii) are the same (iii) vanish.

(b) Fig. 2.23 shows lines of constant potential in a region at which an electric field is present. The values of the potential are written in brackets. Of the points A, B and C, the magnitude of electric field is greatest at the point..... [*I.I.T. 1984*]

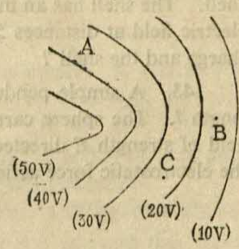


Fig 2.23

Numerical problems :

31. The charges of two small spherical metallic conductors are +12 and +8 c.s.u. respectively. What will be the force between them if they are 8 cm. apart in air ? [Ans. 1·5 dynes]

32. A point charge A of +25 e.s.u. is placed between two charges B and C on the same line with them. The amount of charges in B and C are +5 e.s.u. and —30 e.s.u. respectively. The distance between the charges A and B is 2·5 cm. and that between A and C is 5 cm. What force will act on the charge A ? [Ans. 50 dynes along AC]

33. Two equal and like charges repel each other with a force of 50 mg. wt. when they are 19 cm. apart in air. What is the magnitude of each charge ? [Ans. 70 e.s.u.]

34. Two point charges, +30 units and +60 units, are kept 12 cm. apart. Find the point where the intensities due to the charges are equal and opposite. [Ans. 7·02 cm. away from +60 unit charge]

35. Two small spheres A and B are charged with +9 units and +16 units of electricity.

They are separated by a distance 28 cm. How far from A along the line AB, will the intensities due to the spheres be equal ? [Ans. (i) 12 cm. towards B (ii) 84 cm. away from B]

36. (i) A pith ball weighing 0·05 gm. is charged with 100 e.s.u. of charge. What must be the charge on a ball placed 10 cm. directly above the pith-ball, which will hold the pith-ball in equilibrium ? [Ans. 49 e.s.u.]

37. A metal ball carrying 20 e.s.u. positive charge weighs 2 gm. It lies at rest just below another small ball carrying a negative charge of 128 e.s.u. and suspended by a thread. What is the distance between the centres of the balls ? ($g=980$ cm/s^2) [*H. S. Exam. 1984*] [Ans. 1·14 cm. (nearly)]

38. Equal positive charges are placed at the corners of an equilateral triangle. What is the electric field intensity at a point equidistant from each corner ? [Ans. 0]

39. Two point charges, each $+10^3$ e.s.u., are kept at two points A and B 20 cm. apart. From the middle point of AB, a particle charged with -10^3 e.s.u. is thrown along the perpendicular bisector of the line with an energy 10^4 erg. How far will the point ascend ? [Ans. 199·7 cm. (nearly)]

[Hints : The potential energy at the highest point$=10^4$ ergs.]

40. Two small spheres, each of mass 100 mg. are suspended from a point by threads 50 cm. long. They are equally charged and they repel each other to a distance of 20 cm. What is the charge on each sphere. $g=980$ cm/s^2. [Ans. 89·44 units]

41. Two small metallic spheres, each of mass 5 gm. are suspended by two weightless threads from a point, each thread being 50 cm. long. When the spheres are charged with equal amounts of charge, the threads stand apart making an angle of 30° between them. Calculate the charge on each sphere. [Ans. 567 e.s.u. (nearly)]

42. A point charge of 20 e.s.u. is placed at the centre of a thick insulated metallic spherical shell. The shell has an internal and external radii of 10 and 12 cm and no net charge. Find the electric field at distances 5, 11 and 15 cm from the centre. What is the force between the point charge and the shell ? [*Jt. Entrance 1982*] [Ans. 0·8 dyne ; 0 ; 0·88 dynes ; 0.]

43. A simple pendulum consists of a small sphere of mass m suspended by a thread of length l. The sphere carries a $+ve$ charge q. The pendulum is placed in an uniform electric field of strength E directed vertically upwards. With what period will the pendulum oscillate if the electrostatic force acting on the sphere is less than the gravitational force ?

[*I.I.T. 1977*] [Ans. $T=2\pi\sqrt{\dfrac{ml}{mg-Eq}}$]

[Hints : The effective force on the bob$=mg-Eq$. ; so its acceleration $f=\dfrac{mg-Eq}{m}$]

44. Two spheres of different radii are connected by a very long fine wire. The entire assembly is then raised to a potential V. Show that surface charge density of the spheres are inversely proportional to their radii.

[Hints : $q_1=\dfrac{V}{r_1}$ and $q_2=\dfrac{V}{r_2}$]

Harder problems :

45. Three charges -1, 2 and 3 micro-coulombs are placed respectively at the corners A, B and C of an equilateral triangle of side 200 cm. Calculate (i) the potential and (ii) the field intensity at a point P which is half-way along BC.

[Ans. (i) 40×10^4 volt (ii) $9{\cdot}5\times10^{-3}$ volt/cm. making 18°25′ with CB]

46. A certain charge Q is to be divided into two parts q and $(Q-q)$. What is the relationship between Q and q if the two parts placed at a given distance apart are to have a maximum force of repulsion ? [Ans. $q=Q/2$]

47. The points A, B and C form an equilateral triangle of side a. Point charges of equal magnitude q are placed at A and B. Find the electric field strength and potential at C due to these charges when (i) both the charges are positive and (ii) the charge at A is positive and the charge at B is negative. [Ans. (i) $E=\frac{\sqrt{3}q}{a^2}$; $V=\frac{2q}{a}$ (ii) $E=\frac{q}{a^2}$ paralllel to AB ; $V=0$]

48. Two plane parallel conducting plates are held horizontal, one above the other, in vacuum. Electrons having a speed of 6×10^8 cm./s and moving normally to the plates enter the region between them through a hole in the lower plate which is earthed. What p.d. must be applied to the other plate so that the electrons just fail to reach it ? e/m for electron$=1{\cdot}8\times10^8$ coulomb/gm. [Ans. 10^5 volt]

[**Hints** : $\frac{1}{2}mv^2=V.e.$ where V is the p.d. between the plates].

49. Charges of $+16$, -16, and $+32$ units are placed at the corners A, C and D of a square $ABCD$ of which the side is 4 cm. Find the electric intensity at B. [Ans $\sqrt{3}$ units making 9°54′ with AB]

50. Two charges of magnitude $+100$ and $+100$ e s.u. are kept at the corners A and B of an equilateral triangle ABC of side 10 cm. Calculate the magnitude and direction of electric intensity at C. [Ans. $\sqrt{3}$ dynes, 30° with BC]

51. Two point charges are located on the x-axis, a charge $+1$ e.s.u. at $x=3$ cm. and a charge -4 e.s.u. at $x=-3$ cm. Calculate the electric field on the y-axis at a distance of 4 cm. from the origin. Calculate the location of a point on the x-axis (beyond the unit charge) where the field is zero. [Ans. (i) 0·1537 e.s.u. ; 14°34′ (ii) 6 cm. from $+1$ charge]

52. At the eight corners of a cube of side a cm are situated equal charges of q units each. What is the (i) potential and (ii) intensity at the centre of the cube ? [Ans. (i) $\frac{16q}{\sqrt{3}a}$ (ii) 0]

[**Hints** : The distance of the centre from any corner$=\sqrt{3}a/2$.]

53. A charge Q is placed at each of the two opposite corners of a square. A charge q is placed at each of the other two corners. If the resultant electrical force on Q is zero, how are Q and q related ? [Ans. $Q=-2\sqrt{2}.q$]

54. A charge P of $+250$ e.s.u. is placed between two other charges Q and R of $+50$ and -300 e.s.u. respectively. If $QP=5$ cm and $RP=10$ cm, find the result ant force on P. [Ans. 1250 dynes along R]

55. The distance between two negatively charged specks of dust is 1 mm and they repel each other with a force of 4 dynes. If the charge of one is four times the charge on the other, calculate the number of excess electrons on each speck. Charge on an electron$=5\times10^{-10}$ e.s.u. [Ans. 2×10^8 ; 8×10^8]

56. Two small equal conducting pith balls hang from the same point by silk threads 70 cm long. A charge is given to one of them and then they are allowed to touch. If they come to rest 20 cm. apart and each ball weighs 3 decigrams, find the original charge. $g=980$ cm/s^2 [*Jt. Entrance 1981*] [Ans. $\pm130{\cdot}5$ e.s.u.]

57. Two indentical spheres, each of mass m are suspended by a silk thread of length l from a point. The spheres are charged, each with q amount of electricity. If the angle between the threads is very small, show that in equilibrium, the distance between the spheres$=\left(\frac{2q^2l}{mg}\right)^{\frac{1}{3}}$

58. Three identical small spheres weighing 0·1 gm are suspended from a point with silk threads each having length 20 cm. What charges should be imparted to the spheres so that each thread makes an angle 30° to the vertical ? Assume the charges to be equal. [*Jt. Entrance. 1984*] [Ans. 100 e.s.u. (nearly)]

[**Hints** : The spheres will be at the corners of an equilateral triange of length $\sqrt{\frac{3}{4}}l$.]

59. An infinite number of charges, each equal to q are placed along the x-axis at $x=1$, $x=2$, $x=4$, $x=8$ .... and so on. Find the potential and the electric field at $x=0$ due to the set of charges. What will be the potential and electric field if in the above set up, the consecutive charges have opposite sign ? [*I.I.T. 1974*] [Ans. (i) $V=2q$; $E=4q/3$ (ii) $V=2q/3$; $E=4q/5$]

60. Charge q is uniformly distributed over a thin circular ring of radius a. Show that the electric intensity at a point on the axis of the ring distant r from the centre is given by $E=\dfrac{q.r}{(a^2+r^2)^{\frac{3}{2}}}$

What is the potential there ? [Ans. $\dfrac{q}{(a^2+r^2)^{\frac{1}{2}}}$]

61. An electron of charge e and mass m is constrained to move along the axis of the above ring of charge. Show that the electron performs simple harmonic oscillations with a time-period

$$T=2\pi\sqrt{\frac{ma^3}{qe}}.$$

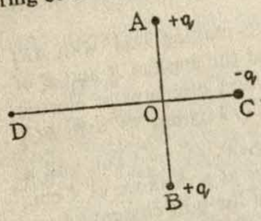


Fig. 2.24

62. Two fixed, equal positive charges, each of magnitude 5×10^{-5} coulombs are located at points A and B separated by a distance of 6 metre cm [Fig. 2.24]. An equal but opposite charge moves towards them along the line COD, the perpendicular bisector of the line AB. The moving charge, when it reaches the point C, at a distance of 4 meter from O has a kinetic energy of 4 joules. Calculate the distance of the furthest point D which the negative charge will reach before returning towards C. [*I.I.T. 1985*] [Ans. 1·72 metre (nearly)]

3

CAPACITANCE AND CAPACITOR

3.1. Capacitance or Capacity :

We know that if we pour equal quantities of water in several vessels of different size, water will come up to different levels in them. Similarly, when conductors of different size are charged with equal quantities of electricity, the potentials of the conductors will be different. Place two metallic cans of unequal size on the discs of two indentical gold leaf electroscopes [Fig. 3.1]. The cans are now given equal charges Q. It will be seen that the divergence of the leaves will be greater for the smaller can, proving that it has acquired a higher potential that the other. Under the circumstances, the larger can is said to have a bigger capacitance than the smaller one.

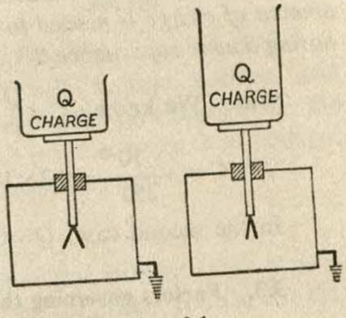


Fig. 3.1

If the cans are now connected by a wire, charge will flow from the smaller can to the larger until their potentials are equal. This is analogous to the flow of water from a vessel of higher level to another of lower level when connected by a pipe. When the cans in the above experiment acquire a common potential, the leaves of the two electroscopes diverge equally.

It has been seen that for a given conductor, the potential is proportional to its charge. If Q amount of charge given to a conductor raises its potential by V, then $Q \propto V$ or $Q=C.V.$ where C is a constant known as the **capacitance** or capacity of the conductor. Hence,

$$\text{Capacitance } (C)=\frac{\text{Charge } (Q)}{\text{Potential } (V)}$$

Now, if $V=1$ $Q=C$ *i.e.* the charge necessary to raise the potential of a conductor by unity, is numerically equal to the capacitance of the conductor.

3.2. Units of capacitance :

If in the above relation, $Q=1$ e.s.u. and $V=1$ e.s.u. then $C=1$ e.s.u. *i.e.* if 1 e.s.u. of charge raises the potential of a conductor by 1 e.s.u. the capacitance of the conductor is regarded as 1 e.s.u. But if the charge and the potential are expressed in practical units (or, M.K.S. units), *viz. coulomb* and *volt* respectively, the capacitance will be expressed in practical unit *i.e.* in **Farad.** A conductor is said to have 1 farad capacitance if 1 coulomb charge raises its potential by 1 volt.

A capacitance of 1 farad, abbreviated as F, is very large and for practical purposes is seldom used. Smaller units are usually used. They are as follows :

1 micro-farad (μF)$=10^{-6}$ farad

1 micro-micro-farad ($\mu\mu F$) or picofarad (pF)$=10^{-12}$ farad.

We know, 1 coulomb $=3\times10^{9}$ e.s.u. of charge

and 300 volts $=1$ e.s.u. of potential.

$$\text{Hence, 1 farad} = \frac{1 \text{ coulomb}}{1 \text{ volt}} = \frac{3\times10^{9} \text{ e.s.u. of charge}}{\frac{1}{300} \text{ e.s.u. of potential}}$$
$$= 9\times10^{11} \text{ e.s.u. of capacitance}$$

$$\therefore\ 1\mu F = 10^{-6} \text{ farad} = 9\times10^{5} \text{ e.s.u. of capacitance}$$

Example : *To increase the potential of a conductor by 250 volts* 5×10^{-7} *coulombs of charge are needed. What is the capacitance of the conductor ? What amount of charge is needed to increase the potential twice as before of a conductor having double capacitance ?*

Ans. We know, $C=\frac{Q}{V}$; here $Q=5\times10^{-7}$ coulomb and $V=250$ volt

$$\therefore\ C=\frac{5\times10^{-7}}{250}=0{\cdot}2\times10^{-8} \text{ Farad}$$

In the second case, $Q=C.V=2\times0{\cdot}2\times10^{-8}\times2\times250=2\times10^{-6}$ coulomb.

3.3. Factors governing the capacitance of a conductor :

In art 3.1 we have seen that for a given value of charge Q, $C\propto\frac{1}{V}$ *i.e.* for a given value of Q, the factors which increase the potential of a conductor are responsible for a decrease in its capacitance and vice versa. The factors are as follows :—

(i) **The area of the conductor** : The potential of a conductor decreases with the increase of the area of the conductor ; hence its capacitance increases.

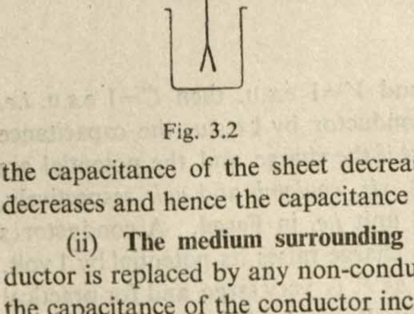


Fig. 3.2

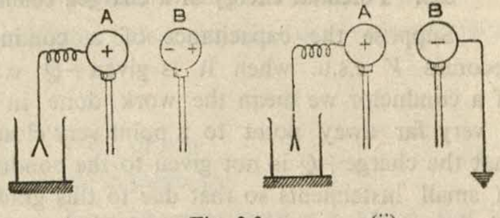
A thin sheet of tin hangs from an ebonite rod and the sheet is connected to the disc of an electroscope (Fig. 3.2). The sheet is given a charge from a source of electricity. The leaves of the electroscope will diverge. If the sheet is rolled a little by the ebonite rod, the divergence of the leaves increases, showing that the potential of the sheet increases. Since the charge remains the same, it implies that the capacitance of the sheet decreases. If the sheet is unrolled, the divergence decreases and hence the capacitance increases.

(ii) **The medium surrounding the conductor** : If air surrounding the conductor is replaced by any non-conducting medium like paraffin, sulphur, glass etc, the capacitance of the conductor increases.

(iii) **Presence of an earth-connected conductor** : If another conductor (uncharged) is present near the charged conductor under test, specially if the

uncharged conductor is earth-connected, capacitance of the charged conductor under test increases remarkably.

A is a positively charged conductor. If it is connected to an electroscope, the leaves of the electroscope will diverge and the divergence will be a measure of the potential of the conductor. If now, an insulated conductor B (uncharged) is brought in its vicinity the divergence will be found to have decreased a little [Fig. 3.3 (i)]. This shows that the potential of the conductor A decreases or its capacitance increases. The reason is that the conductor B will acquire positive and negative charges due to induction which will reduce the potential of A. On connecting the conductor B to the ground, [Fig. 3.3 (ii)] the leaves will collapse appreciably and hence the capacitance of A will increase very much.



(i) Fig. 3.3 (ii)

3.4. Capacitance of a spherical conductor :

Suppose R is the radius of a spherical conductor charged with Q e.s.u. of charge. Now, the potential on the surface of the conductor relative to that at infinity is $V=\frac{Q}{R}$. [The charge of the conductor is supposed to be concentrated at the centre].

Hence, the capacitance of the sphere $C=\frac{Q}{V}=\frac{Q}{Q/R}=R$

i.e. the capacitance in e.s.u. of a sperical conductor placed in air is numerically equal to its radius.

The sphere, whose radius is 1 cm., has capacitance 1 e.s.u. So, in order to make the capacitance of a sphere 1 farad or 9×10^{11} e.s.u. the radius of the sphere should be made 9×10^{11} cm. For this reason capacitance is sometimes expressed in terms of centimetre. For example, the capacitance of a conductor is 5 cm. means the capacitance in air of a spherical conductor of radius 5 cm.

[**N.B.** The radius of the earth is $6{\cdot}4\times10^{8}$ cm ; a sphere of 1 farad capacity has a radius of 9×10^{11} cm *i.e.* one thousand time more than the radius of the earth. This shows how big the unit 'farad' is.]

If the sphere described above is surrounded by a medium other than air, whose di-electric constant is K, then the potential of the conductor is $V=\frac{Q}{K.R}$

Hence its capacitance $C=\frac{Q}{V}=K.R.$

This means that the capacitance of the sphere increases K times, when it is surrounded by the above medium than when it is in air.

Example : *If the earth is supposed to be a perfect sphere of radius* 6400 *kilometres, what would be its capacitance in micro-farads ? 1 microfarad*$=9\times10^{5}$ e.s.u,
[*H.S, Exam. 1981*]

Ans. Radius of the earth=6400 kilometers=6400×10^5 cm.

Hence, capacitance=6400×10^5 e.s.u.=$\frac{6400\times10^5}{9\times10^5}$=711·1 micro-farads.

3.5. Potential energy of a charged conductor :

Suppose the capacitance of a conductor is C e.s.u. and its potential becomes V e.s.u. when it is given $+Q$ e.s.u. of charge. Now, by potential of a conductor we mean the work done in bringing a unit $+ve$ charge from a very far away point to a point very close to the conductor. If we suppose that the charge $+Q$ is not given to the conductor all at a time but is given in small instalments so that due to this gradual supply of charge, the potential of the conductor slowly rises from zero to V, then the total work done in completing the process will remain in the conductor as its potential energy.

To calculate the potential energy in a simple way, we may assume that the potential of the conductor has not undergone a gradual change but has remained steady at the average value $V/2$ e.s.u. during the whole process. Under this circumstances the work done in giving the charge $+Q$ to the conductor

$$W=\text{average potential}\times\text{charge}$$
$$=\tfrac{1}{2}V\times Q=\tfrac{1}{2}V\times CV=\tfrac{1}{2}CV^2 \text{ erg.} \quad (\because \; Q=CV)$$

Hence, the potential energy of the conductor=$\frac{1}{2}CV^2$ erg.

Alternatively,

the potential energy$=\frac{1}{2}CV^2=\frac{1}{2}C\left(\frac{Q}{C}\right)^2=\frac{1}{2}\frac{Q^2}{C}$ erg $\left[\because \; V=\frac{Q}{C}\right]$

[**N.B.** If C is expressed in farads, Q in coulombs and V in volts, the potential energy should be expressed in Joules.]

[*Calculus treatment* : Suppose a conductor of capacity C e.s.u. is charged with $+Q$ e.s.u. of charge and the corresponding potential it acquires is V e.s.u. The charge is given to the conductor not all at once but by small quantities of $+ve$ charge in succession. Now, suppose q is the charge the conductor acquires in this process at any instant and the corresponding potential is v. If at this stage, a small charge $+dq$ is given to the conductor, the corresponding work done $dW=v\times dq=\frac{q}{C}\times dq$ $\left[\because \; v=\frac{q}{C}\right]$

$\therefore$ Total work done *i.e.* the potential energy stored in the conductor, is the integration of dW between limits $q=0$ and $q=Q$ (the final charge)

$$\therefore \quad W=E=\int_0^Q \frac{q}{C}.dq=\frac{1}{C}\int_0^Q q.dq=\frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q=\frac{1}{C}\left[\frac{Q^2}{2}\right]=\frac{1}{2}\frac{Q^2}{C} \text{ ergs} \;\Big]$$

3.6. Sharing of charges between two conductors at different potentials :

Suppose two conductors A and B of capacitances C_1 and C_2 respectively are given charges Q_1 and Q_2 respectively.

The potential acquired by the conductor $A=\frac{Q_1}{C_1}=V_1$ (say)

and „ „ „ „ „ „ $B=\frac{Q_2}{C_2}=V_2$ („)

If the conductors are now joined by a thin and long wire, charge will flow from the conductor of higher potential to the conductor of lower potential till their potentials are equalised. If $V_1>V_2$, charge will flow from the conductor A to the conductor B. Suppose the common potential of the conductors after connection $=V$.

Since, the total charge of the conductors before and after connection remains the same, we have $Q=Q_1+Q_2$

or $V(C_1+C_2)=V_1C_1+V_2C_2$

$$\therefore \quad V=\frac{V_1C_1+V_2C_2}{C_1+C_2}=\frac{Q_1+Q_2}{C_1+C_2}=\frac{\text{Total charge}}{\text{Total capacitance}} \qquad \text{. . . (i)}$$

After connection, if the conductors A and B retain charges q_1 and q_2 due to sharing of charges, then

$$q_1=C_1.V=C_1\frac{(Q_1+Q_2)}{C_1+C_2}=\frac{C_1Q}{C_1+C_2} \qquad \text{. . . (ii)}$$

$$\text{and } q_2=C_2V=C_2\frac{(Q_1+Q_2)}{C_1+C_2}=\frac{C_2Q}{C_1+C_2} \qquad \text{. . . (iii)}$$

If the conductors are spherical, having radii R_1 and R_2 respectively, then we know, $C_1=R_1$ (numerically) and $C_2=R_2$.

In that case, $q_1=\dfrac{R_1Q}{R_1+R_2}$ and $q_2=\dfrac{R_2Q}{R_1+R_2}$

Loss of energy due to sharing of charges :

It may be proved in the following way, that there is always a loss of energy due to sharing of charges between two conductors.

Referring to the case mentioned above, total energy of the conductors A and B before connection $=\frac{1}{2}C_1V_1^2+\frac{1}{2}C_2V_2^2$

Total energy of the conductors after connection and sharing of charges,

$$=\tfrac{1}{2}C_1V^2+\tfrac{1}{2}C_2V^2=\tfrac{1}{2}V^2(C_1+C_2)$$

$$=\tfrac{1}{2}(C_1+C_2)\frac{(C_1V_1+C_2V_2)^2}{(C_1+C_2)^2} \qquad \text{[From eqn. (i)]}$$

$$=\tfrac{1}{2}\frac{(C_1V_1+C_2V_2)^2}{C_1+C_2}$$

$$\text{Loss of energy}=\tfrac{1}{2}(C_1V_1^2+C_2V_2^2)-\tfrac{1}{2}\frac{(C_1V_1+C_2V_2)^2}{C_1+C_2}$$

$$=\tfrac{1}{2}\left[\frac{(C_1+C_2)(C_1V_1^2+C_2V_2^2)-(C_1V_1+C_2V_2)^2}{C_1+C_2}\right]$$

$$=\tfrac{1}{2}\frac{C_1C_2(V_1-V_2)^2}{C_1+C_2}$$

This is a positive quantity irrespective of the values of V_1 and V_2. Thus, whenever two charged conductor are connected by a wire, there is a loss of energy. As total energy is conserved, this energy appears as spark producing heat, sound and light energy.

Example 1. *A conductor of capacitance 15 units is charged to a potential of 40 units and is then made to share its charge with an uncharged sphere, whereby the common potential drops to 30 units. Find the radius of the sphere as well as the charges shared by them. What will be the loss of energy due to sharing of charges ?*

Ans. Here, $C_1=15$ units and $V_1=40$ units.

Hence its charge $Q=C_1V_1=40\times 15=600$ units.

Now, common potential $V=\dfrac{\text{Total charge}}{\text{Total capacitance}}=\dfrac{600}{15+C_2}$

$\therefore\ 30=\dfrac{600}{15+C_2}$ or, $C_2=5$ units. So, the radius of the sphere $=5$ cm.

Again, $q_1=\dfrac{C_1Q}{C_1+C_2}=\dfrac{15\times 600}{15+5}=450$ units

and $q_2=\dfrac{C_2Q}{C_1+C_2}=\dfrac{5\times 600}{15+5}=150$ units.

Also loss of energy$=\frac{1}{2}\dfrac{C_1C_2(V_1-V_2)^2}{C_1+C_2}=\frac{1}{2}\times\dfrac{15\times 5(40-0)^2}{15+5}=3000$ ergs.

Example 2. *A spherical conductor of 10 cm radius is charged with +100 units of charge and it is then connected with another spherical conductor of 5 cm. radius carrying −50 units of charge. What will now be the charge on the spheres ? Find also the potential of each sphere before and after connection.*

Ans. Total charge $Q=100-50=+50$ e.s.u. ; charge on the sphere of radius 10 cm.$=\dfrac{r_1}{r_1+r_2}\cdot Q=\dfrac{50\times 10}{10+5}=33{\cdot}3$ e.s.u. ; So, the charge on the other sphere$=50-33{\cdot}3=16{\cdot}7$ e.s.u.

Potential of the 1st conductor before contact$=\dfrac{\text{charge}}{\text{capacity}}=\dfrac{+100}{10}$
$=+10$ e.s.u.

" " 2nd " " " $=-\frac{50}{5}=-10$ e.s.u.

Potential after contact (common)$=\dfrac{\text{Total charge}}{\text{Total capacity}}=\dfrac{50}{10+5}=\dfrac{50}{15}$
$=+3{\cdot}3.$ e.s.u.

3.7. Capacitor and its principle :

A capacitor is an electrical arrangement for storing up electric charges in much the same way that a reservior is a container for storing up water. The general form of a capacitor is that of two conductors—one insulated and the other earth-connected—placed close to each other, with the intervening space filled up by air or any other non-conducting medium. The capacitors in practical use have simple geometric shape. For example, in a parallel-plate capacitor, two parallel conducting plates are used or in a spherical capacitor, two concentric metallic spheres of different radii are used.

Principle : Suppose A is an insulated metallic plate and it is charged to a potential $+V$. On bringing a similar plate B (uncharged) near it, induction takes place. The inside surface of the plate B acquires $-ve$ charge and the outside surface $+ve$ charge [Fig. 3.4(i)] Now, the negative charge on the inside surface of B will tend to decrease the positive potential of A but the positive charge on the outside surface of B will, however, try ot increase the positive potential of A. Since the negative charge of B is nearer to the plate A, its influence will predominate and the potential of A, on the whole, will decrease a little. Consequently, the capacitance of the plate A will increase because $C \propto \frac{1}{V}$. The plate A will, therefore, take some more charge in order to restore its potential to V.

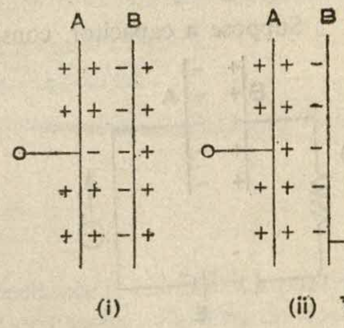


Fig. 3.4

If the plate B is now earth-connected, the free positive charge on it at once flows to the earth [Fig. 3.4 (ii)]. Its influence on the potential of the plate A will now be absent. As a result, the potential of the plate A will decrease still further so that it will be able to hold more charge to make its potential V; in other words, the capacitance of the plate A increases very much.

This arrangement of artificially increasing the capacitance of an insulated conductor by bringing an earth-connected conductor in its vicinity, is called a capacitor.

Significance of a capacitor : Question may arise as to what is the significance of artificially increasing the capacitance of an insulated conductor.

Electric charges need be stored for some practical purposes and electric capacitors have been devised in order to store a large quantity in comparatively smaller space.

If a conductor be given charges gradually, its potential rises until at a very high potential, charges begin to leak away from the conductor in much the same way as water spills out of a vessel when the vessel is full to the brim. Ordinarily, a conductor has a limit to hold charge, which depends upon the shape, size and the surrounding medium of the conductor. If, in that condition, the potential of the conductor be somehow decreased, the leakage of charge will stop and the conductor will be in a position to hold more charge. The function of a capacitor is to enable its insulated conductor to hold more charge, by keeping its potential low. The plate A of the capacitor shown in fig 3.4, would hold more charge in presence of the earth-connected plate B than it would in absence of the plate B.

Charging and discharging of a capacitor :

When a capacitor is connected to a battery it becomes charged. The process is known as *charging*. When the battery is disconnected and the plates of the capacitor are joined together by a wire, the capacitor loses its charge. The

process is known as *discharging* of a capacitor. These processes take place in the following way.

Suppose a capacitor, consisting of two plates A and B, is connected to a battery of e.m.f.E. Electrons flow from the negative terminal of the battery on to the plate A of the capacitor [Fig. 3.4 (a)] and at the same rate, electrons flow from the other plate B of the capacitor towards the positive terminal of the battery. Positive and negative charges thus appear on the plates and oppose the flow of electrons which causes them. As the capacitor is charged due to accumulation of charge in the plates, the potential difference between them increases and the *charging current* falls to *zero* when the potential difference becomes equal to the battery e.m.f. E. If a suitable current meter (say a micro-ammeter) is included in the circuit, the meter will record maximum current when the circuit is switched on but the current slowly decreases to zero when the p.d. between the plates A and B equals the battery e.m.f. E. The capacitor is then said to be fully charged. The process of charging, it is to be remembered, is not instantaneous. It takes some time for the capacitor to be fully charged. The current at the commencement of charging is maximum and it gradually falls to zero value as the charging continues.

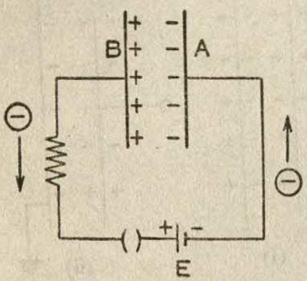


Fig. 3.4(a)

When the battery is disconnected and the plates A and B are joined together by a wire, electrons flow back from the plate A to plate B until the positive charge on B is completely neutralised. A current, known as *discharge current* flows for a time in the wire and at the end of the time, the charges on the plates disappear. The meter included in the circuit will show the same maximum current as before but in the opposite direction when the plates A and B are joined. Then the current slowly decreases to zero. The capacitor is, then said to be completely discharged. The process of discharge is also not instantaneous. It takes some time for the capacitor to lose its charge completely. Discharge current at the commencement of discharge is also maximum and it gradually falls to zero value as the discharging process continues but the charging current and discharge current are oppositely directed [Fig 3.4(b)].

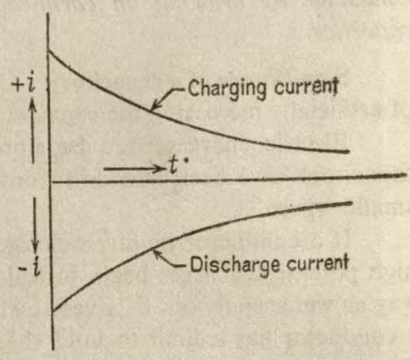


Fig. 3.4(b)

3.8. Capacitance and potential of a capacitor :

A capacitor has two plates—one insulated and the other earth-connected. The capacitance of a capacitor means the capacitance of its insulated plate.

Definition : *The capacitance of a capacitor is defined as the ratio of the charge given to its insulated plate to the difference of potential between the plates.*

$$\text{Capacitance of a capacitor} = \frac{\text{Charge given to the insulated plate}}{\text{P. D. between the plates}}$$

If C be the capacitance, q the charge given to the insulated plate and V the $p.d$ between the plates of the capacitor, then

$$C=\frac{q}{V} \text{ or } q=C.V.$$

It is to be noted in this connection that capacitance is essentially a positive quantity. It has no negative value like potential or charge. Further, the capacitance of a conductor and that of a capacitor are expressed in the same unit. In the same way, the potential of a capacitor means the potential of its insulated plate, the potential of the other plate being zero.

An analogy can be made between a capacitor carrying a charge q and a rigid container of volume v containing n moles of a perfect gas. The pressure of the gas P is directly proportional to n when the temperature remains constant, according to the ideal gas equation $n=\left(\frac{v}{R_0T}\right).P$. where R_0 is the universal gas constant.

For the capacitor, $q=(C).V$

Comparison shows that the capacitance C of the capacitor is analogous to the volume v of the container, provided the temperature remains fixed.

It is to be noted that any amount of charge can be put on the capacitor and any mass of gas can be put in the container, upto certain limits. The limits correspond to electrical rupture or breakdown for the capacitor and to mechanical rupture of the walls for the container.

3.9. Specific inductive capacity :

The medium filling up the space between the plates of a capacitor is called the *di-electric.* It has been found experimentally that if non-conductors like paraffin, sulphur, glass, ebonite, mica etc. be used instead of air as dielectric in a capacitor, its capacitance is greatly increased. Michael Faraday, in 1837, first investigated the effect of filling the space between the plates of a capacitor with a non-conducting material. He constructed two identical capacitors, in one of which he placed a dielectric, the other containing air at normal pressure. When both capacitors were charged to the *same potential difference*, Faraday found that the charge on the one containing the dielectric was greater than that on the other. Since q is larger, for the same V, if a dielectric is present, it follows from the relation $C=q/V$ that the capacitance of a capacitor increases if a dielectric is used between the plates. For these reasons, the above non-conductors are said to have greater specific inductive capacity than air.

The specific inductive capacity (abbreviated as S.I.C.) *of a medium is defined as the ratio of the capacitance of a capacitor with the medium as dielectric to the capacitance when air is used as a dielectric.* In other words, *S.I.C.* of a dielectric,

$$K = \frac{\text{Capacitance of the capacitor with the medium as dielectric}}{\text{Capacitance of the capacitor with air as dielectric.}}$$

For example, when glass is used as dielectric in a capacitor, the capacitance is found to be 8·5 times its capacitance when air is used as the dielectric. Hence, the S.I.C. of glass is 8·5. It is to be noted that specific inductive capacity is also known as *dielectric constant.*

Action of a dielectric :

We know that a molecule is a collection of atomic nuclei, positively charged surrounded by a cloud of negatively charged electrons. Ordinarily the centre of gravity of the electrons of an atom coincides with those of the protons (or, the c.g. of negative charges coincides with that of positive charges) [Fig. 3.5(i)] so

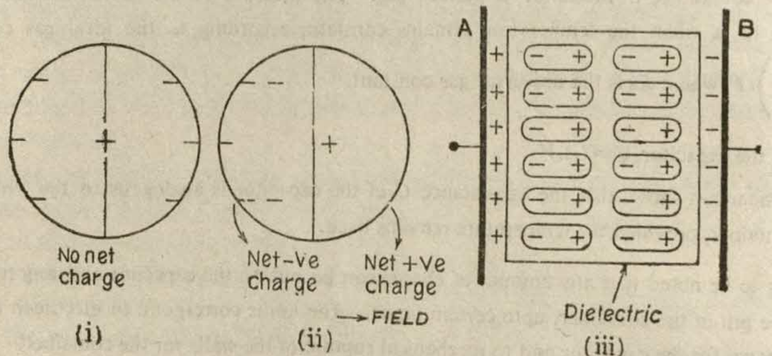


Fig. 3.5

that the atom has no net charge. On applying an electric field, the nuclei (*i.e.* +ve charges) are urged in the direction of the field and the electrons (*i.e.* negative charges) in the opposite direction [Fig. 3.5(ii)]. Thus each molecule is distorted, one end having an excess of positive charge and the other an excess of negative charge. Such distorted molecules are called *polar molecules.* When a dielectric is in a charged capacitor, its molecules are in an electric field and are consequently polarised. At the surface of the dielectric, therefore, charges appear as shown in fig. 3.5(iii). These charges are of opposite sign to the charges on the plates *A* and *B* and so reduce the p.d. between the plates. As the capacitance is inversely proportional to the p.d. between the plates, insertion of the dielectric, therefore, increases the capacitance of the capacitor.

Some molecules, it is believed, are permanently polarised. For example the water molecule consists of an oxygen atom with two hydrogen atoms, making roughly a right-angled structure. It is found that the apex of this triangle is negatively charged and its base is positively charged. Dielectric whose molecules

are permanently polarised, increases the capacitance of a capacitor more than those whose molecules are polarised merely by the action of the electric field.

3.10. Different types of capacitors :

(i) **Parallel plate capacitor** : A parallel plate capacitor consists of two flat metal plates of any shape, kept parallel to each other at a little distance away. If a glass plate be inserted between the plates as shown in fig. 3.6(i), the capacitor is known as Epinus's parallel plate capacitor.

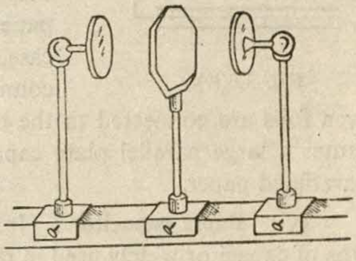


Fig. 3.6(i)

(ii) **Spherical capacitor** : A spherical capacitor consists of two concentric spheres of different radii—one being charged and the other connected to the earth. The space between the spheres may be filled up by air or any other medium.

(iii) **Cylindrical capacitor**: It consists of two co-axial cylinders. The cylinder A is charged and fixed. Leaving a little gap, there is another earth-connected cylinder C kept co-axially with A. [Fig. 3.6(ii)]. The purpose of the cylinder C is to keep the lines of force at the end of the cylinder A undistorted. The cylinder B inside, is co-axial with outer cylinder A and can move along the axis forward or backward. It is earth-connected. A micrometer screw M is provided to measure the displacement of the cylinder B. In order to protect the charged cylinder A from the influence of stray, outside electric field, it is covered by another earth-connected cylinder Q (shown by dotted line).

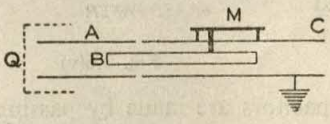


Fig. 3.6(ii)

(iv) **Variable air capacitor** : It is, in fact, a modified parallel plate air capacitor. Its capacitance can be varied at will. These capacitors are commonly used in radio sets. It consists of two sets of aluminium plates—one set fixed and the other movable. When the knob is turned, the movable set of plates rotates between the fixed set, thus increasing or decreasing the effective plate area and hence the capacitance. Between two sets of plates, air acts as a dielectric. The less the thickness of the air film, the more will be the capacitance.

Fig. 3.6 (iii) shows the type of variable air capacitor. The shapes of the plates are such that when the knob K is turned in one direction, the effective plate area increases or decreases uniformly, causing a uniform variation of its capacitance. In the figure, A represents the fixed plates and B the rotating plates. The plates A are joined to the terminal S and the plates B to the terminal S_2.

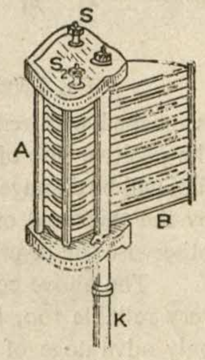


Fig. 3.6(iii)

(v) **Mica capacitor or block capacitor** : This is a type of parallel plate capacitor in the form of a small block and fixed in the sense that no parts in it are movable and the capacitance cannot be varied. This type of capacitor is widely used in radio sets. It consists of alternate sheets of tin-foils shown by thin lines and paraffined paper shown by thick lines [Fig. 3.6 (iv)]. In this case, odd foils are connected together and finally connected to the external terminal A. Similarly, even foils are connected to the terminal B. The arrangement, as a whole, constitutes a large parallel plate capacitor with a core of paraffined paper.

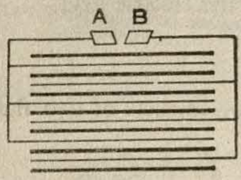


Fig. 3.6(iv)

(vi) **Paper capacitor** : It is the most common type of capacitor widely used in radios and the ignition system of automobiles. Two long strips of tin-foils are glued to the two faces of a thin paper or polyester film. The paper is generally soaked in paraffin or oil and rolled up with another paraffin strip of paper into a small compact unit. Each sheet of tin foil becomes one plate of the capacitor and the paper becomes the dielectric separating them. [Fig. 3.6 (v)].

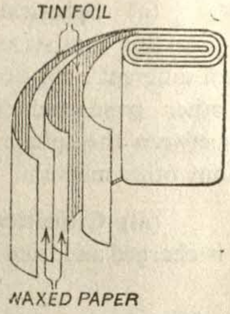


Fig. 3.6(v)

(vii) **Electrolytic capacitors** : Electrolytic capacitors are made by passing a direct current between two sheets of aluminium foil, with a suitable electrolyte or liquid conductor between them [Fig 3.6(vi)]. A very thin film of aluminium oxide is then formed on the anode plate which is on the positive side of the d.c. supply. This film is an insulator. It forms the dielectric between the two plates, the electrolyte being a good conductor. Since the dielectric thickness t is very small and $C \propto \frac{1}{t}$ the capacitance can be very high. Several thousand microfarads may easily be obtained in a capacitor of small volume. To maintain the oxide film, the anode terminal is marked with red colour or with a + sign. This terminal must be connected to the positive side of the circuit in which the capacitor is used, otherwise the oxide film will be damaged. The voltage applied to it must not exceed a value determined by the thickness of the dielectric film and marked on the body of the capacitor ; otherwise the layer of aluminium oxide may break down.

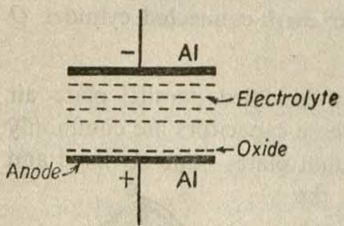


Fig. 3.6(vi)

The above conditions limit the usefulness of these capacitors. They are not very reliable too, because the oxide layer is apt to break down with age. The only advantage of these capacitors is that they provide a high capacitance in a limited space. They are used in domestic radio receivers but in high grade apparatus, they are avoided.

In some variety of electrolytic capacitors, tantalum sheets are used as electrodes. They are kept immersed in an electrolyte of sulphuric acid.

3.11. Calculation of capacitance of some types of capacitors :

(i) **Parallel plate capacitor** : If the charged plate A and the earth-connected plate B of the capacitor are of same shape and size, [Fig. 3.7], and if they are very near to each other, the lines of force between the plates are straight and parallel. Consequently, the intensity of electric field between the plates may be regarded uniform.

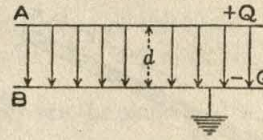


Fig. 3.7

Suppose, area of each plate$=\alpha$ sq. cm.
charge given to the plate $A=+Q$ e.s.u.
distance between the plates$=d$ cm.
dielectric constant of the medium$=K$.

The surface density of charge in the plate A is $\sigma=\frac{Q}{\alpha}$; if the intensity of the field between the plates be E, then it can be proved that $E=\frac{4\pi\sigma}{K}$.

If the potential difference between the plates be V e.s.u. (the plate B being earth-connected has zero potential), then according to the definition, $V=$work done in bringing a unit$+ve$ charge from the plate B to the plate $A=$force on unit charge $\times$ distance$=\frac{4\pi\sigma}{K}\times d$

If C be the capacitance of the capacitor, $C=\frac{Q}{V}=\frac{Q}{\frac{4\pi\sigma d}{K}}=\frac{K.Q}{4\pi\sigma.d}$

Now, charge on the plate A is $Q=\alpha.\sigma$ $\therefore$ $C=\frac{K.\alpha.\sigma.}{4\pi.\sigma.d}=\frac{K.\alpha}{4\pi d}$ e.s.u.

If the medium be air, then $K=1$, and in that case, $C=\frac{\alpha}{4\pi d}$ e.s.u.

Example 1 : *Calculate the capacitance of a parallel plate capacitor, the potential plate of which has an area* 200 *sq. cm. The distance between the plates is* 1 *cm. and it is filled up by an ebonite plate of S.I.C.*$=3$.

Ans. The capacitance of a parallel plate capacitor is given by,

$C=\frac{K.\alpha}{4\pi d}$ e.s.u. ; Here, $K=3$; $\alpha=200$ sq. cm ; $d=1$ cm ; so,

$$C=\frac{3\times200}{4\times3{\cdot}14\times1}=47{\cdot}7 \text{ e.s.u.}$$

Example 2 : *The distance between the plates of a parallel plate air capacitor is* 1 *mm. What should be the area of the plate so as to make the capacitance equal to* 1 *micro-farad* ?

Ans. For a parallel plate air capacitor $C=\frac{\alpha}{4\pi d}$ e.s.u.$=\frac{\alpha}{4\pi d}\times\frac{10^{-11}}{9}$ farad.

Here $C=1$ micro-farad $=10^{-6}$ farad ; $d=1$ mm. $=0{\cdot}1$ cm.

$$\therefore \quad 10^{-6}=\frac{\alpha}{4\pi\times 0{\cdot}1}\times\frac{1}{9}\times 10^{-11}$$

or, $\alpha=4\pi\times 0{\cdot}1\times 9\times 10^5$ sq. cm. $=1{\cdot}13\times 10^6$ sq. cm.

Example 3 : *A capacitor has square plates, each of side a, making an angle* θ *with each other as shown in Fig. 3.7(a). Show that for small* θ, *the capacitance is given by* $C=\dfrac{Ka^2}{4\pi d}\left(1-\dfrac{a\theta}{2d}\right)$ *where K is the dielectric constant.*

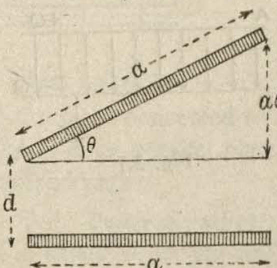


Fig. 3.7(a)

Ans. The distance between the plates at left end $=d$ and that at right end $=d+a\sin\theta=d+a\theta$ [$\because$ θ is small]

$\therefore$ Average distance between the plates $=\frac{1}{2}[d+d+a\theta]=d+\frac{1}{2}.a\theta$.

We can now consider the arrangement as a parallel plate capacitor whose plates are separated by a distance $=d+\frac{1}{2}.a\theta$.

Now, for parallel plate capacitor,

$$C=\frac{KA}{4\pi\times\text{distance}}=\frac{K.a^2}{4\pi(d+\frac{1}{2}a\theta)}=\frac{Ka^2}{4\pi d}\left(1+\frac{1}{2}\frac{a\theta}{d}\right)^{-1}=\frac{Ka^2}{4\pi d}\left(1-\frac{a\theta}{2d}\right)$$

(ii) **Capacitance of a parallel plate capacitor with a compound dielectric :**

The two plates A and B of a parallel plate capacitor are separated by a distance d and the space between them is filled up by a medium of dielectric constant K_1. In this space another medium of thickness t and dielectric constant K_2 is introduced. [Fig. 3.8]. Such arrangement is called *a capacitor with a compound dielectric.* It is clear from the figure that the thickness of the dielectric K_1, in this case, is $(d-t)$. The introduction of a second medium does not, however, disturb the uniformity of the field in the capacitor.

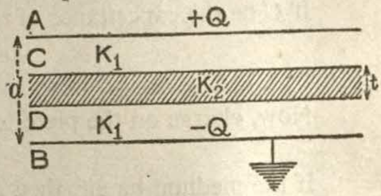


Fig. 3.8

Let α be the surface area of the charged plate A and, therefore, its surface-density of charge $\sigma=\dfrac{Q}{\alpha}$, where Q is the charge given to the plate. If E_1 be the intensity of the uniform field in the dielectric K_1, then $E_1=\dfrac{4\pi\sigma}{K_1}$ and that in the dielectric K_2 is $E_2=\dfrac{4\pi\sigma}{K_2}$. If V be the *p.d.* between the plates,

$$V=\frac{4\pi\sigma}{K_1}(d-t)+\frac{4\pi\sigma}{K_2}.\ t=4\pi\sigma\left(\frac{d-t}{K_1}+\frac{t}{K_2}\right)=\frac{4\pi Q}{\alpha}\left(\frac{d-t}{K_1}+\frac{t}{K_2}\right)$$

If C be the capacitance of the capacitor, $C=\frac{Q}{V}=\frac{\alpha}{4\pi\left(\frac{d-t}{K_1}+\frac{t}{K_2}\right)}$

If $K_1=1$ (air) and $K_2=K$, then, $C=\frac{\alpha}{4\pi\left(d-t+\frac{t}{K}\right)}$

[*Note* : It is to be noted that the introduction of a dielectric of thickness t in the air medium of the capacitor increases the capacitance. To get back the previous value of the capacitance $\left(=\frac{\alpha}{4\pi d}\right)$, the distance between the plates is to be increased. If the increase of distance be x, then it is easy to see that $x-t+\frac{t}{K}=0$ or $x=t\left(1-\frac{1}{K}\right)$cm.]

Example : *On introducing an insulating material,* 4 *mm. thick, into the space between the plates of a parallel plate capacitor, it was found that in order to restore the capacitance to its original value, the distance between the plates had to be increased by* 3·5 *mm. Calculate the dielectric constant of the material.*

Ans. To restore the capacitance to the original value, the increase of distance is given by, $x=t\left(1-\frac{1}{K}\right)$ Here, $x=3{\cdot}5$ mm ; $t=4$ mm ; hence $3{\cdot}5=4\left(1-\frac{1}{K}\right)$

$$\text{or} \quad \frac{3.5}{4}=1-\frac{1}{K} \quad \therefore \quad K=8$$

So, the dielectric constant of the material $=8$.

(ii) **Spherical capacitor** : Suppose, the radius of the inner sphere is r_1 and that of the outer sphere r_2. The inner sphere is given a charge of $+Q$ units and the outer sphere is earthed (Fig. 3.9).

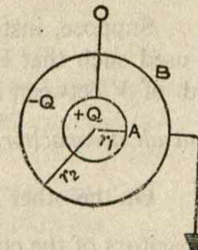


Fig. 3.9

If V be the potential of the inner sphere, then,

$V=$potential due to its own charge $(+Q)$ $+$potential due to the charge $(-Q)$ induced on the inner surface of the sphere B.

$$=\frac{Q}{r_1}-\frac{Q}{r_2}=Q\left(\frac{1}{r_1}-\frac{1}{r_2}\right)=Q\left(\frac{r_2-r_1}{r_1r_2}\right)$$

If C be the capacitance of the capacitor,

$$C=\frac{Q}{V}=\frac{Q}{Q\left(\frac{r_2-r_1}{r_1.r_2}\right)}=\frac{r_2.r_1}{r_2-r_1} \text{ e.s.u.}$$

If the medium between the spheres be something else than air, having dielectric constant K, then, $C=\frac{K.r_1r_2}{r_2-r_1}$ e.s.u.

[N.B. (i) Capacitance of the inner sphere alone is its radius r_1. The effect of surrounding the sphere by an earth-connected concentric sphere of radius r_2, is to increase its capacity from r_1 to $r_1r_2/(r_2-r_1)$

(ii) Capacitance $C=\frac{r_1 r_2}{r_2-r_1}=\frac{r_1}{1-r_1/r_2}$ which is very nearly equal to r_1 if r_2 is very great ; but as r_2 becomes smaller the capacity increases until when r_2 is very nearly equal to r_1 the capacity becomes very great.

(iii) If the inner sphere is earthed and the outer sphere charged, it may be shown that the capacitance $C=\frac{K r_2^2}{r_2-r_1}$.]

3.12. Combination of capacitors :

(i) **Series combination :** In series combination of several capacitors, the second plate of the first capacitor is connected to the first plate of the second capacitor ; the second plate of the second capacitor to the first plate of the third capacitor and so on. All the plates of the capacitors except the last plate of the last capacitor, are insulated. The last plate of the combination is earth-connected (Fig. 3.10). If the first plate A of the first capacitor be given a charge $+Q$, it will induce $-Q$ charge on the inside surface of the plate B and $+Q$ on the first plate C of the second capacitor. In this way induction will take place from capacitor to capacitor due to which the first plate of each capacitor will get $+Q$ charge and the second plate $-Q$ charge on its inner surface. If the p.d.'s between the plates of the successive capacitors be V_1, V_2, V_3 etc. and that between the plates A and G of the whole combination be V, then,

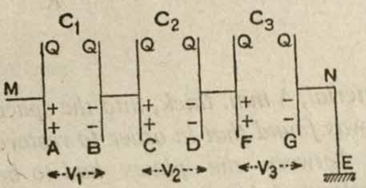


Fig. 3.10

$$V=V_1+V_2+V_3+\quad \cdots \quad \text{(i)}$$

Suppose, instead of the combination, a single capacitor of capacitance C_s is used such that the same charge $+Q$ given to the single capacitor produces a p.d. of V between its plates. The single capacitor, in this case, will be called an *equivalent capacitor*. For equivalent capacitor, $V=\frac{Q}{C_s}$.

On the other hand, if C_1, C_2, C_3 .. etc. be the capacitances of the individual capacitors of the combination, then we have, $V_1=\frac{Q}{C_1}$; $V_2=\frac{Q}{C_2}$ etc.

So, according to the *eqn*. (i) we have,

$$\frac{Q}{C_s}=\frac{Q}{C_1}+\frac{Q}{C_2}+\frac{Q}{C_3}+\ldots \quad \text{or,} \quad \frac{1}{C_s}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\ldots$$

i.e. the reciprocal of the equivalent capacitance is equal to the sum of the reciprocals of the individual capacitances. It is to be noted that the equivalent capacitance in series combination is less than the capacitance of any individual capacitor of the combination but the series combination produces a high potential difference.

(ii) **Parallel combination** : In this combination, the insulated plates of a number of capacitors are connected to one point M while all the earth-connected plates are joined to another point N. A source of electric charge is connected to the point M and the point N is grounded [Fig. 3.11]. In this arrangement, each capacitor has a potential difference V, but the charges will be distributed among them according to their capacitances. If the charges in the successive capacitors be Q_1, Q_2, Q_3, etc, then the total charge $Q = Q_1 + Q_2 + Q_3 + \ldots$ (i).

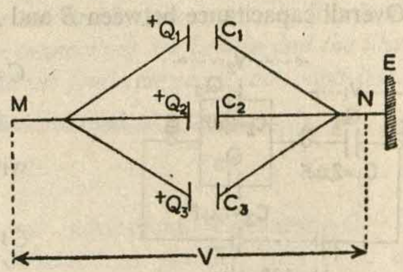


Fig. 3.11

Suppose, instead of the combination, a single capacitor of capacitance C_p be used such that same charge $+Q$ given to the single capacitor produces a p.d. of V between its plates. The single capacitor, in this case, will be called an equivalent capacitor. For equivalent capacitor, therefore, $Q = C_p V$.

On the other hand, if C_1, C_2, C_3.. etc be the capacitances of the individual capacitors of the combination, then we have,

$$Q_1 = C_1 V \; ; \; Q_2 = C_2 V \ldots \text{ etc.}$$

So, according to the eqn. (i) we have,

$$C_p V = C_1 V + C_2 V + C_3 V + \ldots\ldots$$

$$\text{or } C_p = C_1 + C_2 + C_3 + \ldots$$

i.e. the equivalent capacitance is the sum of the individual capacitances. It is to be noted that due to parallel combination, the equivalent capacitance is greatly enlarged.

[N.B. It should be noted with care that these two formulae are just the reverse of those for series and parallel resistors.]

Example 1 : *Two condensers have capacitances of* 10 *and* 15 *units respectively. The first one is charged to* 10 *and the second one to* 5 *units of potential. If the condensers are connected in parallel, what would be their common potential* ? [*H. S. Exam. 1978*]

Ans. The charge in the first condenser, q_1 = capacitance × potential
$= 10 \times 10 = 100$ units

" " " " second condenser q_2 = capacitance × potential
$= 15 \times 5 = 75$ units

Total charge $= q_1 + q_2 = 100 + 75 = 175$ units.

Since the condensers are in parallel, their total capacitance
$= 10 + 15 = 25$ units

$$\text{Common potential} = \frac{\text{Total charge}}{\text{,, capacitance}} = \frac{175}{25} = 7 \text{ units}$$

Example 2 : *Find the charges on the capacitors in fig.* 3.12 *and the potential differences across them.*

Ans. Capacitance between A and B, $C'=C_2+C_3=3\mu F$ (connected parallel). Overall capacitance between B and D,

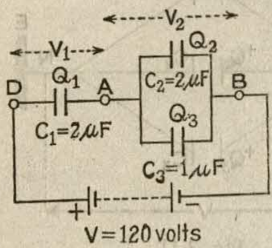


Fig. 3.12

$$C=\frac{C_1C'}{C_1+C'}=\frac{2\times 3}{2+3}=1{\cdot}2\mu F$$

Here, C_1 may be supposed to be in series with C'.

Charge stored in the capacitance $C=Q_1=CV=1{\cdot}2\times 10^{-6}\times 120=144\times 10^{-6}$ coulomb

$$\therefore\quad V_1=\frac{Q_1}{C_1}=\frac{144\times 10^{-6}}{2\times 10^{-6}}=72 \text{ volt.}$$

Also, $V_2=V-V_1=120-72=48$ volts.

$Q_2=C_2V_2=2\times 10^{-6}\times 48=96\times 10^{-6}$ coulomb.

$Q_3=C_3V_2=1\times 10^{-6}\times 48=48\times 10^{-6}$ coulomb.

Example 3 : *Find the equivalent capacitance between A and B of the arrangement shown in fig.* 3.13. *Suppose* $C_1=10\mu f$; $C_2=5\mu f$ *and* $C_3=4\mu f$.

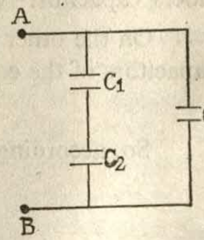


Fig. 3.13

Ans. As C_1 and C_2 are in series, their equivalent capacitance is given by $C'=\dfrac{C_1\times C_2}{C_1+C_2}=\dfrac{10\times 5}{10+5}=\dfrac{10}{3}\ \mu f$

Again C_3 and C' are in parallel ; the equivalent capacitance between A and $B=C_3+C'=\dfrac{10}{3}+4=\dfrac{22}{3}=7{\cdot}33\ \mu f$.

Example 4 : *From the fig.* 3.14, *find the value of the capacitance C if the equivalent capacitance between points A and B is to be* 1 *microfarad. All the capacitances are in microfarads.* [*I.I.T. 1977*]

Ans. As $2\mu F$ and $2\mu F$ capacitors are in parallel their equivalent capacitance $C_1=2+2=4\mu F$. Again $8\mu F$ and C_1 being in series, their equivalent capacitance

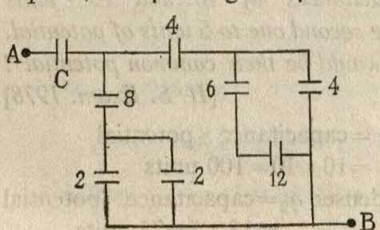


Fig. 3.14

$C_2=\dfrac{8\times C_1}{8+C_1}=\dfrac{8\times 4}{8+4}=\dfrac{8}{3}\ \mu F$. Again $12\mu F$ and $6\mu F$ being connected in series, they give an equivalent capacitance $C_3=\dfrac{12\times 6}{12+6}=4\mu F$. Again C_3 and $4\mu F$ are in parallel combination. So, their equivalent capacitance $C_4=C_3+4=4+4=8\mu F$. Now, $8\mu F$ and $4\mu F$ being in series, they give an equivalent capacitance $=\dfrac{8\times 4}{8+4}=\dfrac{8}{3}\mu F$. This $\frac{8}{3}\mu F$ and C_2 (found earlier) are in parallel connection. Their equivalent capacitance $C_5=\frac{8}{3}+C_2=\frac{8}{3}+\frac{8}{3}=\frac{16}{3}\mu F$. Finally C_5 and C are in series combination. Their equivalent capacitance $C_6=\dfrac{C_5\times C}{C_5+C}=\dfrac{\frac{16}{3}\times C}{\frac{16}{3}+C}\mu F=\dfrac{16C}{16+3C}\mu F.$

According to the question $C_6=1\mu F$.

$\therefore \quad 1=\frac{16C}{16+3C}$ or $C=\frac{16}{13}=1\frac{3}{13}\,\mu F.$

Example 5 : *Two capacitors of capacitance* $4\mu F$ *and* $2\mu F$ *are respectively joined in series with a battery of e.m.f.* 100 *volts. The connections are broken and the like terminals of the capacitors are then joined. Find the final charge on each capacitor.*

Ans. When connected in series, the equivalent capacitance C is $\frac{1}{C}=\frac{1}{4}+\frac{1}{2}=\frac{3}{4} \quad \therefore \quad C=\frac{4}{3}\ \mu F.$

Charge on each capacitor=charge on equivalent capacitor$=CV=\frac{4}{3}\times10^{-6}\times100=\frac{400}{3}\times10^{-6}$ coulomb. When connected in parallel, they have a common potential, say V_1. Since total charge remains same, the initial total charge $=\left(\frac{400}{3}+\frac{400}{3}\right)\times10^{-6}$ coulomb$=(4+2)\ V_1\times10^{-6} \quad \therefore \quad V_1=\frac{400}{9}$ volts.

$\therefore$ Charge on larger capacitor$=4\times\frac{400}{9}=\frac{1600}{9}\ \mu C.$

charge on smaller „ $=2\times\frac{400}{9}=\frac{800}{9}\ \mu C.$

Example 6 : *A parallel plate capacitor consists of two plates of area* 500 *sq. cm. each separated by a sheet of mica* 0·075 *mm. thick. Find its capacity in micro-farads, if dielectric constant for mica is* 6·5.

Ans. For a parallel plate capacitor, we know $C=\frac{K.\alpha}{4\pi d}$ e.s.u.

Hence, $C=\frac{6{\cdot}5\times500}{4\times3{\cdot}14\times0{\cdot}0075}$ e.s.u.$=\frac{6{\cdot}5\times500}{4\times3{\cdot}14\times0{\cdot}0075\times9\times10^5}\mu F=0{\cdot}038\mu F.$

Example 7 : *The thickness of air layer between the two coatings of a spherical air capacitor is* 2 *cm. The capacitor has the same capacitance as that of a sphere of* 120 *cm. diameter. Find the radii of the surfaces of the air capacitor.*

Ans. For a spherical air capacitor, we know $C=\frac{r_2r_1}{r_2-r_1}$

According to the problem, $C=60$ e.s.u. and $r_2-r_1=2$ cm.

$\therefore \quad 60=\frac{r_2r_1}{2}$ or, $r_2r_1=120$

Now, we have, $r_2-r_1=2$ and $r_1.r_2.=120$
$(r_2+r_1)^2=(r_2-r_1)^2+4r_1r_2=4+480=484 \quad \therefore \quad r_2+r_1=22$ cm.
Solving, we get $r_2=12$ cm. and $r_1=10$ cm.

Example 8 : *Each of* 64 *identical water droplets is charged with a p.d. of* 220 *volts. What will be the p.d. if all the droplets coalesce to form one single large drop ? What will be the change of energy of the system.*

Ans. Let r be the radius of each droplet and R that of the single large drop. Since the total volume of all the 64 droplets is equal to the volume of the single drop, we have $\frac{4}{3}\pi R^3=64\times\frac{4}{3}\pi r^3$ or $R^3=64r^3$ or $R=4r.$

Now, the capacitance of each small droplet, C_1=its radius=r e.s.u.
Similarly ,, ,, large drop, C_2=its radius=R e.s.u.
Charge on each droplet, $q_1=C_1V_1=r\times\frac{220}{300}$ e.s.u.$=\frac{11}{15}\times r$ e.s.u.
,, ,, large drop, $q_2=64\times q_1=64\times\frac{11}{15}\times r$ e.s.u.

$\therefore$ *P-D.* of the large drop$=\dfrac{\text{its charge}}{\text{,, capacitance}}=\dfrac{q_2}{C_2}=\dfrac{64\times 11\times r}{15\times R}$

$=\dfrac{64\times 11\times r}{15\times 4\times r}=\dfrac{176}{15}$ e.s.u.$=\dfrac{176}{15}\times 300$ volt$=3520$ volts.

Again energy of the 64 droplets $E_1=64\times\frac{1}{2}C_1V_1^2=32\times r\times\left(\frac{11}{15}\right)^2$ ergs.

And energy of the large drop $E_2=\frac{1}{2}\times C_2V_2^2=\frac{1}{2}\times 4r\times\left(\frac{176}{15}\right)^2$ ergs.

$$\therefore\ \frac{E_2}{E_1}=\frac{2r\times 176\times 176\times 15\times 15}{32\times r\times 11\times 11\times 15\times 15}=16 \text{ or, } E_2=16E_1$$

3.13. Energy stored in a charged capacitor :

The work done in charging a capacitor remains stored up in the capacitor as its potential energy. It may be proved in the same way as was done in art 3.5 that if Q be the charge of a capacitor, C its capacitance and V the p.d. between the plates of the capacitor, its potential energy $W=\frac{1}{2}\frac{Q^2}{C}=\frac{1}{2}QV=\frac{1}{2}C.V.^2$

(i) If Q, V and C are expressed in e.s.u., W will be expressed in ergs.

In the case of a parallel-plate capacitor, $C=\dfrac{KA}{4\pi d}$ e.s.u. If the surface-density of charge on the plate of the capacitor be σ e.s.u./cm^2, then $Q=A\sigma$ e.s.u. Hence the potential energy of a parallel-plate capacitor in e.s.u.

$$W=\frac{1}{2}\frac{Q^2}{C}=\frac{1}{2}\frac{A^2\sigma^2\times 4\pi d}{KA}=\frac{2\pi\sigma^2 A.d}{K} \text{ ergs.}$$

(ii) If Q, V and C are expressed in coulomb, volts and farads respectively, W the energy will be expressed in joules.

In the case of a parallel-plate capacitor, $\dfrac{1}{C}=\dfrac{4\pi d}{K.A}$ or $C=\dfrac{KA}{4\pi d}$ e.s.u.

$=\dfrac{KA}{4\pi d}\times\dfrac{1}{9\times 10^{11}}F.$ If the surface-density of charge of the charged plate of the capacitor be σ coulomb/cm^2 then $Q=A.\sigma.$ coulomb

Hence, the potential energy of a parallel-plate capacitor, in practical units,

$$W=\frac{1}{2}\frac{Q^2}{C}=\frac{1}{2}\frac{A^2\sigma^2\times 4\pi d}{K.A.}\times 9\times 10^{11}=\frac{2\pi\sigma^2 A.d.}{K}\times 9\times 10^{11} \text{ joule.}$$

[*Alternative calculus proof* :

Suppose during charging, the capacitor has at any instant q amount of charge. If v be the potential difference between the plates of the capacitor at the instant, then $q=C.v$ where C is the

capacitance of the capacitor. If, now, a small amount be charge dq be given to the capacitor, the work done $dW=v\times dq$. The charge dq is so small that the $p.d$ of the capacitor remains unchanged. So, total work done in giving the capacitor the full charge Q is

$$W=\int dW=\int_0^Q v\times dq=\int_0^Q \frac{q}{C}dq=\left[\frac{q^2}{2C}\right]_0^Q=\frac{Q^2}{2C} \quad \therefore\ W=\frac{1}{2}\frac{Q^2}{C}=\frac{1}{2}\frac{V^2C^2}{C}=\frac{1}{2}CV^2\Big]$$

Heat produced in charging a capacitor :

If a high resistance R is included in the capacitor circuit, the rate of charging is slow. When the charging current ceases to flow, the final charge Q on the capacitor is same as if negligible resistance was present in the circuit, because the whole of the applied p.d. V of the battery which is used for charging the capacitor, appears across the plates of the capacitor when the charging is full. Thus, the energy stored in the charged capacitor$=\frac{1}{2}Q.V$. The source of this energy is evidently, the battery. But the battery, in supplying a charge Q at a p.d. V, spends an energy $Q.V$ during the charging process. Half of it is stored in the capacitor. What about the other half ? The other half appears as heat in the circuit resistance R. If the resistance is high, the rate of production of heat is quick ; if it is low, the rate is also slow. In both cases, however, the total amount of heat produced is the same *viz.* $\frac{1}{2}\,Q.V$. So, we can conclude that when a capacitor is charged by a battery *half the energy delivered by the battery is spent in charging the capacitor and half is dissipated as heat produced in the resistance.*

Example 1 : *Two capacitors of* 2μ*f and* 4μ*f capacitances are connected in parallel and a p.d. of* 300 *volt is applied at their terminals. Find the total energy stored in the combination.*

Ans. Total capacitance of the combination $C=2+4=6\mu f=6\times10^{-6}$ farad.

So, the energy stored $W=\frac{1}{2}CV^2=\frac{1}{2}\times6\times10^{-6}\times(300)^2=0{\cdot}27$ joule

Example 2 : *A parallel-plate capacitor has been formed with the help of two brass plates, each of area* 1 *sq. metre placed parallel to each other with a separation of* 10 *cm. If the intervening space be filled up with a glass block and if the surface-density of charged plate be* 0·01 *coulomb/sq. cm. find the potential energy of the capacitor. Specific inductive capacity of glass*=8.

Ans. Potential energy of a parallel-plate capacitor in practical units

$$W=\frac{2\pi\sigma^2 A.d.}{K}\times9\times10^{11}\ \text{joules.}\quad \text{[ref. art 3.13]}$$

Here, $\sigma=0{\cdot}01$ coulomb/sq. cm ; A=1 sq. metre=100×100 sq. cm ; $d=10$ cm ; $K=8$.

$$\text{Hence,}\quad W=\frac{2\times3{\cdot}14\times(0{\cdot}01)^2\times(100)^2\times10\times9\times10^{11}}{8}=\frac{2\times3{\cdot}14\times9\times10^{12}}{8}$$

$$=70{\cdot}6\times10^{11}\ \text{joules.}$$

Example 3 : *The area of the plates of a parallel-plate capacitor is A and they are separated by a distance d. A battery charged the plates to a potential difference of* V_1. *Then the battery is disconnected and a slab of thickness d is intro-*

duced into the plates. Calculate the energy of the capacitor before and after the introduction of the slab. Explain if there be any difference of energy.

Ans. Before the introduction of the slab, the energy of the capacitor $W_1=\frac{1}{2}C_1V_1^2$. After the introduction of the slab the capacitance $C_2=KC_1$ and the potential $V_2=V_1/K$. where $K=S.I.C.$ of the slab.

So, the energy of the capacitor after the slab is introduced $W_2=\frac{1}{2}C_2V_2^2=$

$$\frac{1}{2}KC_1\left(\frac{V_1}{K}\right)^2=\frac{W_1}{K}$$

Since $K>1$, the energy W_2 is less by a factor $1/K$. The missing energy would be apparent to the person who introduced the slab. He would feel a "pull" on the slab and would have to restrain it if he wanted to introduce the slab without acceleration. This means that the capacitor+slab system would do some work on the person and the work$=W_1-W_2=\frac{1}{2}C_1V_1^2(1-1/K)$.

3.14. The Leyden jar :

It is one of the earliest types of capacitors used in the laboratories for demonstration purposes. It is named so to commemorate the first attempt at storing up electric charge at Leyden in Holland in 1745. So far as the construction is concerned, it is a parallel plate capacitor.

The apparatus in the usual form consists of a fairly tall thin-walled cylindrical glass jar (V) with tin-foil coatings both inside and outside running up to about two-thirds of its length as shown in the fig. 3.15. The tin-plates (M_1 and M_2) may be regarded as the plates of a capacitor, the glass wall being the dielectric medium. There is a lid S made of some non-conducting material tightly fitting in the mouth of the jar. Through the central hole of the lid is admitted a glass rod R. The lower end of the rod is in electrical contact with the inside tin foil (M_1) and the upper end projects a little out of the jar and carries a small brass ball K known as the charging knob. Considering the Leyden jar as a parallel plate capacitor, its capacitance may be calculated in the following way :

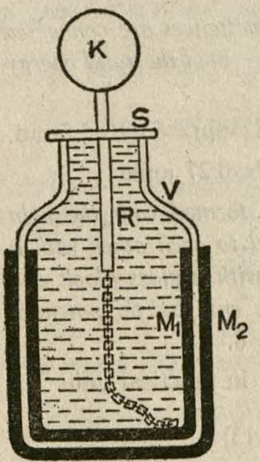


Fig 3.15

Suppose, r=radius of the cylindrical jar
h=height of the tin plates
d=thickness of glass wall
K=dielectric constant of glass.

Area of each tin plate A=area of the vertical portion+area of the round bottom$=2\pi rh+\pi r^2$

Now, the capacitance of a parallel plate capacitor is

$$C=\frac{KA}{4\pi d}=\frac{K(2\pi rh+\pi r^2)}{4\pi d}=\frac{Kr(2h+r)}{4d}$$

This expression shows that the capacitance increases when d is less and r is large.

To charge a Leyden jar, hold the outside coating (M_2) of the jar with hand (*i.e.* earth-connected) and bring the knob K in contact with the prime conductor of an electric machine. If the knob K is given negative electricity the inner tin-foil M_1 gets the negative electricity which induces positive charge on the inside surface of the tin-foil M_2 and negative charge on the outside surface. Since the outside surface is earth-connected, the free induced negative charges flows to the earth. To charge the Leyden jar positively, the charging knob should be given positive charge.

To discharge a charged Leyden jar, a discharging tong as shown in fig. 3.16 is to be used. It is made of brass and is provided with an insulating handle. The discharging process consists of bringing one of the knobs of the tong in contact with the outside tin-foil M_2 and the other knob slowly near the charging knob K of the Leyden jar. A spark will pass between the two and the jar will be discharged partially. If the tong be again brought in the same position as before, another spark, though comparatively feeble, will pass. This shows that a charged Leyden jar cannot be discharged by a single attempt. For complete discharge, several successive discharge processes are to be adopted.

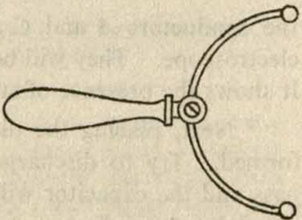


Fig. 3.16

The appearance of *residual charge* after each discharge is explained in terms of energy being stored up due to straining of the intervening dielectric medium during charging. A series of discharge is necessary before the dielectric can recover itself fully from its electrical straining. Had the dielectric been a gaseous substance like air, the capacitor could have been discharged all at once.

Example : *The diameter of a Leyden jar is* 15 *cm. and its tin plates are* 18 *cm. high. The thickness of the glass is* 0·25 *cm. If the dielectric constant of glass be* 6·4, *find the capacitance of the Leyden jar.*

Ans. For a Leyden jar, $C=\dfrac{K.\alpha}{4\pi d}$ e.s.u. where K=dielectric constant of the medium; α=the area overlapped by the inside and the outside tin-plates and d=thickness of glass.

Here, $K=6{\cdot}4$; $\alpha=\pi r^2+2\pi.r.h=[\pi\times(7{\cdot}5)^2+2\pi\times7{\cdot}5\times18]$ sq. cm. ; $d=0{\cdot}25$ cm.

$$\therefore \quad C=\frac{6{\cdot}4\times[2\pi\times7{\cdot}5\times18+\pi(7{\cdot}5)^2]}{4\pi\times0{\cdot}25} \text{ e.s.u} =2088 \text{ e.s.u.}$$

3.15. Seat of charge in a Leyden jar :

Where does the charge of a Leyden jar reside ? In the tin-plates, or on the glass wall ? If experiments are done with a Leyden jar whose parts are detachable, it will be seen that the charge resides on the glass wall *i.e.* on the dielectric medium. For this reason the dielectric of a capacitor is very important. The tin-plates only act as the conductors of the capacitor.

A and *C* are two separate conductors and *G* is a glass vessel [Fig. 3.17]. After charging the capacitor, the inner conductor *A* and the glass vessel *G* are removed by means of insulating handles and placed on a glass plate *P*. Now,

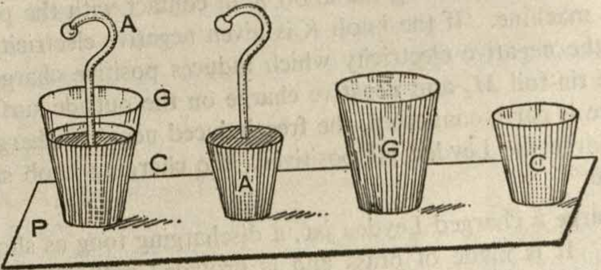


Fig. 3.17

the conductors *A* and *C* are examined with the help of an uncharged gold-leaf electroscope. They will be found to have no charge. The glass jar is next tested. It shows the presence of strong charge.

Next, placing the different parts in their proper positions the capacitor is formed. Try to discharge the capacitor with a discharging tong. Sparks will pass and the capacitor will be discharged. This proves that *the seat of charge is the dielectric medium*.

Exercises

Essay type :

1. What do you mean by capacitance of a conductor ? What are the factors on which it depends ? Explain with suitable experiments.

2. Prove that the capacitance of a spherical conductor is numerically equal to its radius. What are the e.s.u. and practical units of capacitance ? What is the relation between them ?

3. Obtain an expression for the potential energy of a charged conductor.

4. "An arrangement which artificially increases the capacitance of an insulated conductor is called a capacitor"—explain the meaning of the statement.

5. What is a condenser ? Explain the principle of a condenser. Define the capacitance of a condenser. State the factors on which it depends. [*H. S. Exam. 1980*]

6. Describe a parallel plate air condenser and obtain an expression for its capacitance. What change in its capacitance will take place if an ebonite plate is inserted between the parallel plates ? Explain, in brief, the action of the ebonite plate in this respect.

7. Show that the capacitance of a parallel plate capacitor $C=\frac{A}{4\pi d}$ where A=area of any one plate of the capacitor and d=the distance between the plates.

8. Deduce an expression for the capacitance of a spherical capacitor. How will its capacity change if the inner sphere is earth-connected instead of the outer one ?

9. Find the equivalent capacitance of a number of capacitors connected in (i) series and (ii) parallel.

10. Describe the construction of a Leyden jar. How can you charge and discharge a Leyden jar ? Describe an experiment to study the seat of charge in a Leyden jar.

Short answer type :

11. "The capacitance of a conductor is 5 cm"—explain the statement.

12. The capacitance of a capacitor is 12 Farad—explain the statement. [*H.S. Exam. 1982*]

13. A charged spherical conductor may be considered as a capacitor. What is the other plate of the capacitor ?

14. What are the potential and capacitance of a capacitor ? What is specific inductive capacity ?

15. The expression for the capacitance of a parallel plate capacitor is only approximately correct. What is the reason of it ?

16. Two copper spheres of same radius—one hollow and the other solid—are charged to the same potential. Which one will contain larger charge ?

[*Hints* : Both the spheres have equal charge ; charge always resides on the outer surface of the conductor and $Q=C.V.$ Radii being same, the spheres have equal capacitance. Since potentials are same, Q will also be same.]

17. A capacitor is connected across a battery. Why does each plate receive a charge of exactly same magnitude ? Is this true even if the plates are not of same size ?

18. 'The dielectric constant of glass is 8·5'—what does this statement mean ?

19. A liquid dielectric filled parallel plate capacitor has been designed to have a capacitance C so that it may be operated safely at or below a certain maximum p.d. of V_m without breakdown. However, the designer did not do a good job and the capacitor occasionally suffered breakdown. What can be done to redesign the capacitor, keeping C and V_m unchanged and using the same dielectric ?

20. A very thin (thickness may be ignored) aluminium foil is introduced into the space between the plates of a parallel plate capacitor. What effect will it produce on the capacitance of the capacitor when (i) the foil is insulated (ii) the foil is connected with the upper plate ?

21. (*a*) Will a capacitor hold more or less charge at a given potential difference when (i) there is a dielectric in the capacitor (ii) there is no dielectric ?

(*b*) Two conductors carry like charges of the same magnitude. Can there be a p.d. between the conductors ?

[*Hints* : Yes; if the conductors have different capacitance] [*Jt. Entrance 1985*]

22. A dielectric slab is inserted in one end of a charged paralled plate capacitor (the plates are horizontal and the charging battery is disconnected) and then released. Describe what happens. Neglect friction.

23. A capacitor is charged by a battery which is then disconnected. A dielectric slab is then inserted between the plates. Describe qualitatively what happens to the charge, the capacitance, the potential difference and the stored energy.

Objective type :

24. Select the correct answer :

(a) What is the practical unit of capacitance ? Ans. Microfarad, Farad, Volt

(b) If C and V be the capacitance and potential difference of a capacitor, what is its potential energy ? Ans. $\frac{1}{2}C.V$; $\frac{1}{2}C.V^2$. ; $C.V.$

(c) What happens to the potential of a capacitor when one of its plates is charged and the other earthed ? Ans. Potential increases ; Potential decreases ; Potential becomes zero.

(d) Two condensers of capacities $2\mu F$ and $4\mu F$ are joined in parallel. What is the total capacity ? Ans. $6\mu F$; $\frac{4}{3}\,\mu F$; $8\mu F$.

(e) You are given a parallel plate air capacitor of capacity $4\mu F$. The distance between the plates is doubled. What is the new capacity ? Ans. $8\mu F$. ; $2\mu F$. ; $4\mu F$.

(f) Will the equivalent capacitance increase or decrease when several capacitors are joined in parallel ? Ans. Increases ; Decreases ; Remains unchanged.

(g) If the distance between the plates of a parallel plate air capacitor is increased, what happens to its capacitance ? Ans. Increases ; Decreases ; Remains unchanged.

(h) A parallel plate capacitor is joined to a battery. The charge in the capacitor is Q_0, its potential and energy are V_0 and U_0 respectively. Keeping the battery connected, if a slab of some insulating material is inserted in the space between the plates of the capacitor, which of the following conditions will be applicable ? $Q_0 > Q$; $V_0 < V$; $U_0 > U$.

Numarical problems :

25. A conductor of capacitance 4 units is charged with 100 units of +ve charge and is connected to another conductor of capacitance 2 units charged with 20 units of −ve charge. Calculate the change of potentials and the charge of each conductor.

[Ans. 1st conductor → +25 to +13·3 units ; +53·3 units
second ,, → −10 to +13·3 ,, +26·7 ,,

26. Two conductors of capacitances 20 and 30 units are connected by a thin wire and are, then, given 100 units of charge. Find their potentials and charge.

[Ans. Potential=2 units ; q_1=40 units ; q_2=60 units]

27. A capacitor of 10$\mu\mu F$ capacitance is charged to 50 volts of potential difference. Then the charging battery is disconnected and the capacitor is connected to another capacitor. As a result, the potential difference is reduced to 35 volts. What is the capacitance of the second capacitor ? [Ans. 43 $\mu\mu f$]

28. Two tin-plates, each having size 20 cm×25 cm are pasted on two sides of a mica-sheet, 0.1 mm thick. Find the capacitances, in microfarad, of the capacitor so formed. Di-electric constant of mica=5 [Ans. 0·022μF]

29. Three capacitors of 4, 6 and 12μF capacitance are put in series and the combination is connected to a 500 volt battery. Find (i) the equivalent capacitance (ii) charge in each capacitor (iii) voltage across each capacitor. [Ans. (i) 2μF (ii) 0·001 coulomb (iii) 250v. 167v. and 83v.]

30. Two circular plates, each of radius 8 cm, form a parallel plate capacitor with a gap of 1 mm between them. If the capacitor is charged to 100 volt potential difference, find the amount of charge stored in the plates of the capacitor. [Ans. $1{\cdot}8\times10^{-8}$ coulomb (nearly)

31. A capacitor consists of 200 circular sheets of tinfoils separated by mica of S.I.C., 6 and thickness 0·5 mm., alternate plates being connected together. If the capacitance of the capacitor be 0·4 microfarad, find the radius of the tinfoils. [Ans. 7·7 cm.]

32. The capacitance of a parallel-plate capacitor is 100$\mu\mu f$ and the area of its plates is 100 cm². If the capacitor has a *p.d.* of 50 volt, calculate the surface density of charge in the capacitor plate. [Ans. $0{\cdot}05\times10^{-9}$ coulomb/cm²]

33. The capacitances of two capacitors are 4 and 9 e.s.u. respectively. They are charged to potentials of 1 and $\frac{2}{3}$ e.s.u. respectively. If they are now connected in parallel, what will be their charges ? [Ans. $q_1=3\frac{1}{13}$; $q_2=6\frac{12}{13}$ e.s.u.]

34. Two spheres of radii 2 and 4 cm. respectively are each charged with 24 units of electricity. If they are now connected by a fine wire, find the charge on each sphere.

[Ans. 16 units ; 32 units]

35. Show that when two equal capacitors are connected in parallel, the system has four times the capacitance of that obtained when the capacitors are joined in series.

36. Calculate the capacitance of a spherical capacitor in microfarad, if the diameter of the outer sphere is 30 cm. and that of the inner sphere is 20 cm. the space between them being filled with a liquid of S.I.C.=2. [Ans. $6{\cdot}6\times10^{-5}\mu$F]

37. Two capacitors of capacitances 5 and 10 units are charged to potentials 16 and 10 units respectively. Find their common potential when they are connected in (i) parallel and (ii) series. [Ans. (i) 12 units (ii) 54 units.]

38. Two metal plates, each having area 500 sq. cm. are separated by a distance 0·075 cm. If the intervening space be filled up by mica of dielectric constant 6·28 and the surface-density of the charged plate be 0·1 coulomb/sq cm., find the potential energy of the capacitor.
[Ans. $3 \cdot 37 \times 10^{11}$ Joule]

39. The plates of a parallel-plate capacitor are 2 cm. apart. A dielectric medium of thickness 1 cm. and dielectric constant 5 is introduced between the plates. The distance between the plates is then so adjusted that the capacitance remained unaltered. Find the present distance between the plates. [Ans. 2·8 cm]

40. Two circular plates A and B of a parallel plate capacitor have each an effective diameter of 10 cm and are 2 mm. apart. Two other plates C and D of a similar capacitor have each an effective diameter 12 cm and are 3 mm apart. A is earthed. B and C are connected together and D is connected to the positive pole of a 120 volt battery whose negative pole is earthed. Calculate (i) the combined capacity (ii) the energy stored in it (iii) the energy stored in each capacitor. [Ans. (i) $1 \cdot 7 \times 10^{-11} F$ (ii) $1 \cdot 22 \times 10^{-7} J$ (iii) $CD \rightarrow 6 \cdot 2 \times 10^{-8} J$; $AB \rightarrow 6 \times 10^{-8} J$]

Harder Problems

41. A slab of dielectric is introduced between the plates of a parallel plate capacitor. Show that the capacitance per unit area of the capacitor will be doubled if the dielectric constant $K = \dfrac{2x}{2x-d}$ where x=thickness of air layer before the introduction of the slab and d=the thickness of the slab.

42. Calculate the total capacitance between A and B of the arrangement shown in fig 3.18 $C_2 = 10\mu f$ and $C_1 = C_3 = C_4 = C_5 = 4\mu f$ [Ans. $4\mu f$]

43. The plates of a parallel plate air capacitor consisting of two circular plates, each of 10 cm. radius, placed 2mm. apart, are connected to the terminals of an electrostatic voltmeter. The system is charged to give a reading of 100 in the voltmeter scale. The space between the plates is then filled with an oil of dielectric constant 4·7 and the voltmeter reading falls to 25. Calculate the capacitance of the voltmeter. [Ans. $3 \cdot 3 \times 10^{-11} F$ (nearly)]

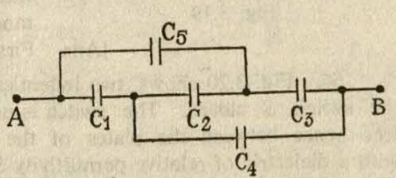


Fig. 3.18

44. A capacitor C_1 is charged to a p.d. of V_0. The charging battery is then removed and the capacitor is connected to an uncharged capacitor C_2. Find the final p.d. across the combination. Also calculate the energy stored before and after the second capacitor is joined.

$$\left[\text{Ans. } \frac{V_0 C_1}{C_1 + C_2} ;\ E_0 = \tfrac{1}{2} C_1 V_0^2 ;\ E = \left(\frac{C_1}{C_1 + C_2}\right) E_0 \right]$$

45. For making a capacitor, you are given two plates of copper, a sheet of mica (thickness =·1 mm ; $K=6$), a sheet of glass (thickness=2 mm ; $K=7$) and a slab of paraffin (thickness =1 cm ; $K=2$). To obtain the largest capacitance, which sheet (or sheets) should you place between the copper plates ? [Ans. Mica]

[*Hints* : $C \propto \dfrac{K}{d}$, where d=thickness]

46. Three capacitors of 1, 2 and $3\mu f$ capacitances are connected in series and a *p.d.* of 110 volt is applied at the end of the combination. Find the terminal *p.d.* and the charge in each capacitor. [Ans. $V_1 = 600V$; $V_2 = 300V$; $V_3 = 200V$; $Q = 6 \times 10^{-4}$ coulomb]

47. Calculate the energy stored in a capacitor of $4\mu F$ capacitance charged to 1000 volts. If another uncharged capacitor of $2\mu F$ capacitance be connected in parallel with the first, what will be the potential of the combination ? [Ans. 0·002 joule ; $666 \cdot 6V$]

48. Two drops of water, radii 1 mm and 2 mm. respectively are charged with 15000 and

and 7500 volts respectively. If the drops coalesce, what will be the change of energy in ergs ? [Ans. 52·5 ergs (nearly)]
1 volt=$\frac{1}{300}$ e.s.u.

49. Three condensers of capacities 1, 3 and 4 units are connected respectively in (i) series and (ii) parallel. Compare the equivalent capacities in the two cases. [Ans. 4/39]

50. The ratio of the radii of two insulated metal spheres is 4 : 1. They are given equal amounts of charge and are then connected by a thin wire. Assuming that the wire takes no charge, find the ratio of surface density of charge of the spheres when the connection is cut off. [Ans. 1 :4]

51. The ratio of the capacities of three condensers is 1 : 2 : 3. The equivalent capacity of the condensers when connected in parallel is greater than that when the condensers are connected in series by $5\frac{5}{11}\mu F$. Find the capacities of the condensers. [Ans. 1, 2 and 3μf]

52. Find the potential differences between A and B and B and C of the arrangement shown in fig. 3.19 [Ans. 7·69 V ; 92·3 V]

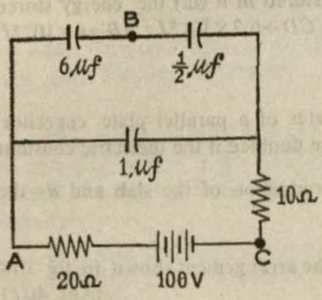


Fig. 3.19

53. A leyden jar has a diameter of 14 cm and the height of the tin foils is 20 cm. Thickness of glass=0·4 cm. Find the capacity of the jar if K for glass is 6·5. [Ans. 1336·56 e.s.u]

54. Twenty seven identical drops of mercury are charged simultaneously to the same potential of 10 volts. What will be the potential if all the charged drops coalesce to form one single drop ? [Ans. 90 volts]

55. How would you combine four capacitors, each having capacitance of 1μf, so as to produce a capacitance of 0·75 μf.
[Ans. First three in parallel and the fourth in series with them]

56. Fig. 3.20 shows two indentical parallel plate capacitors connected to a battery with the switch S closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of relative permittivity 3. Find the ratio of the total electrostatic energy stored in both the capacitors before and after the introduction of the dielectric.
[*I.I.T. 1983*] [Ans. 3 : 5]

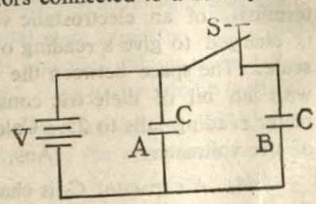


Fig. 3.20

[*Hints* : Before introduction, total energy $=\frac{1}{2}CV^2+\frac{1}{2}CV^2=CV^2$; After introduction, the energy of $A=\frac{1}{2}C_1V^2=\frac{1}{2}\times 3C.V^2$ and that of $B=\frac{1}{2}C_2V_2^2=\frac{1}{2}\times 3C\times\left(\frac{V}{3}\right)^2=\frac{CV^2}{6}$

$$\therefore \text{ Total energy}=\frac{3}{2}CV^2+\frac{CV^2}{6}=\frac{5}{3}CV^2 \text{]}$$

57. Two parallel plate capacitors are so arranged in series that the second plate of the first capacitor is rigidly fixed to the first plate of the second and this portion of the combination is movable. Show that for all positions of the movable part, the equivalent capacity remains constant.

58. A condenser consists of 11 rectangular pieces of tin foils each measuring 15 cm × 20 cm all joined together with 10 similar pieces of tin foil joined together and alternating with the first set. If the tin foils are separated by sheets of mica of thickness 0·2 mm, whose specific inductive capacity is 6·28, what is the capacity of the condenser ? [Ans. 15×10^4 e.s.u.]

4

ELECTRIC MACHINES

4.1. Introduction :

A machine which can produce, in a very short time, sufficient amount of positive or negative electricity, is called an electric machine. To be exact, these machines do not produce any new electric charge, they simply separate the positive charges from the negative ones. Electric machines are generally of two kinds : (i) *Frictional machine* and (ii) *Induction machine*. Frictional machines are almost obsolete now. Of the induction machines, however, (i) Electrophorus and (ii) Van-de-Graff generator are very important.

4.2. The Electrophorus :

The different parts of the machine are as follows (Fig. 4.1).

(*a*) **Collector or cover A** : It is a metallic disc provided with an insulating glass handle *H*. It is called collector because it collects charge.

(*b*) **Cake B** : It is a plate made of insulating material like ebonite, resin etc. Its size is slightly greater than that of the collector. The upper surface of the cake is rough. When placed on the cake, the collector touches the surface of the cake only at few points.

(*c*) **Sole C** : It is a shallow metallic dish upon which the cake is placed.

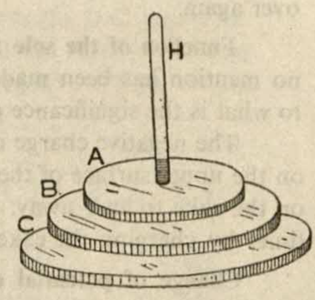
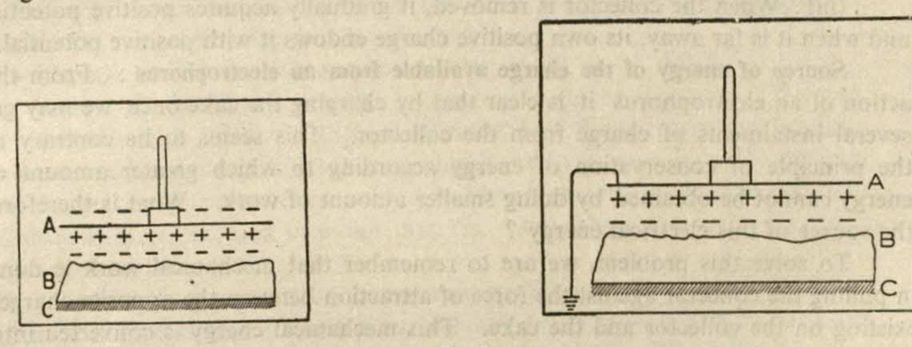


Fig. 4.1

Action : In order to use the instrument, it is first well dried. It is better to warm it a little in sun shine. Having placed the ebonite cake *B* on the sole, the upper surface of the cake is rubbed with fur or flannel so as to charge the surface with negative electricity. The collector *A* is now placed on the cake. The collector



(i) Fig. 4.2 (ii)

is charged with negative electricity by conduction at the points of contact. This charge is insignificantly small in comparison with the charge produced by induction

because the two plates are separated from each other by a thin layer of air. The collector is thus charged at its lower face with $+ve$ electricity and at its upper face $-ve$ electricity by induction. [Fig. 4.2(i)].

The collector is then momentarily touched with finger when the free negative charge on the upper surface of the collector escapes to the earth [Fig. 4.2(ii)].

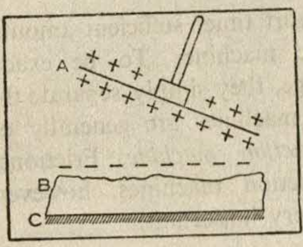


Fig. 4.3

The collector with the positive charge is then removed by the insulating glass handle. The positive charge gradually spreads over the entire surface of the collector as it is taken further away (Fig. 4.3). This charge may be used for any other purpose. Clearly, more the volume of the cake, more is the charge taken away by the collector each time.

After using the charge when the collector becomes discharged, it may be charged again by repeating the above process. In this way, by charging the cake once, the collector may be charged over and over again.

Function of the sole : In the action of the electrophorus, described above, no mention has been made about the sole *C*. Question may, therefore, arise as to what is the significance of the metallic sole ? The answer is as follows :

The negative charge on the free surface of the cake induces a positive charge on the upper surface of the sole which reduces the tendency of the negative charge on the cake to leak away. For this reason, the collector may be charged several times by charging the cake once.

Change of potential of the collector at different stages of operation : The potential of the collector *A* changes sign several times at different stages of operation. These changes are as follows :

(i) When the collector is placed on the negatively charged cake, the collector enters a field of negative potential and hence acquires a negative potential.

(ii) When the collector is touched with fingers, it is earthed and then its potential becomes zero.

(iii) When the collector is removed, it gradually acquires positive potential and when it is far away, its own positive charge endows it with positive potential.

Source of energy of the charge available from an electrophorus : From the action of an electrophorus it is clear that by charging the cake once, we may get several instalments of charge from the collector. This seems to be contrary to the principle of conservation of energy according to which greater amount of energy cannot be obtained by doing smaller amount of work. What is therefore, the source of this electrical energy ?

To solve this problem, we are to remember that mechanical work is done in pulling the collector against the force of attraction between the opposite charges existing on the collector and the cake. This mechanical energy is converted into electrical energy which finally appears in the collector. So repeated collection of charge by the collector does not go against the principle of conservation of energy.

4.3. Van-de-Graff generator :

This instrument, first devised by Robert Van-de-Graff of Princeton University U.S.A. in 1931, is capable of producing a small current at a very high voltage.

Description : Fig. 4.4 shows a schematic diagram of the instrument. Two hollow metallic spheres each of diameter of about 50 cm. are placed on two tall columns (X, X), made of some insulating meterial like glass. The spheres serve the purpose of positive and negative terminals of a D.C. generator. In each column, there is a pair of pulleys (P_1, P_2)—the pulley P_1 being connected to an electric motor and the pulley P_2 remaining inside the hollow sphere. They are coupled to each other by a flat belting made of an insulating material like silk. The beltings can rotate in the direction of the arrow. The belting enters the hollow sphere through the aperture S_1 and emerges from another aperture S_2.

Action : The working of this machine depends on the charging action and the collecting action of points and the collecting action of hollow conductors. Let us consider the action of right hand belting first. The small sphere opposite to the point D near the belting acquires a small $+ve$ charge from the $D.C.$ generator. On account of induction, the point D gets $-ve$ charge and the free $+ve$ charge escapes to the earth. Due to the action of point, the negative charge on the pointed end D leaks to the belt which carries the charge into the hollow sphere above. When the charge comes near the point G, it induces $+ve$ charge on the pointed end G and $-ve$ charge on the sphere. The opposite charges on G and the belt very soon neutralise each other, leaving the sphere negatively charged. In the same time the pointed end C on the left column, receives $+ve$ charge from its nearest small sphere and discharges the charge to the belt. The belt, in its turn, deposits the charge to the hollow sphere above.

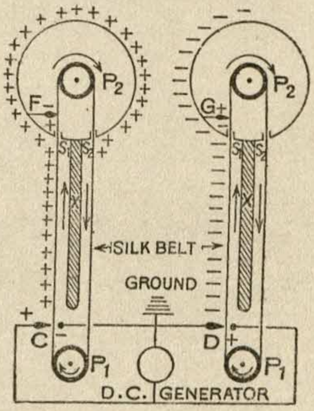


Fig. 4.4

The action of the two belting being reciprocal, the opposite charges on the two spheres increase at an enormous rate as the beltings rotate, till the final potential is reached. As the potential difference between the terminals increases, there is likelihood of discharge in air, because air, under normal pressure, cannot bear too much electrical straining. To prevent such discharge, the spheres and the belt are made as smooth as posible and the whole instrument is enclosed in a big metallic vessel where the air is kept at a very high pressure.

It is needless to mention that since invention, the instrument has undergone a number of modifications. Now-a-days, we see giant Van-de-Graff generators in operation in the Carnegie Institute of Washington and in the Wisconsin University, U.S.A. The machine can generate a p.d. of nearly 5 million volts.

Exercises

Essay type :

1. Describe and explain the action of an electrophorous.

2. Explain how an electrophorous produces almost any amount of charge, when once excited.

3. Describe and explain the mode of operation of a Van-de-Graff generator. [*H. S. Exam. 1979*]

Short answer type :

4. What is the use of an electrophorous ? What is the function of its sole ?

5. What are the changes of potential of the collector of an electrophorous at different stages of its working ?

6. 'Electrophorous does not violet the laws of conservation of energy'—Explain.

7. Why are the sphere and the belt of a Van-de-Graff generator made smooth ?

8. Why is the metallic vessel in which the generator is enclosed earth-connected ?

CURRENT ELECTRICTTY

1

ELECTRIC CURRENT & ELECTRIC CELLS

Introduction :

The age we are passing through, may very well be called the age of electricity because at every step of our life in this age we take the help of electric power. It is the electricity that lights up our houses, factories, offices, etc, and operates the communication media like telegraph, telephone, radio etc. Theatres, cinema, television and other amusement sources owe their existence to electric power. Transports like electric train, tram etc. are driven by electricity. Various machines in factories and workshops are dependent on electric power. Electricity has made our life easy and comfortable and is thus very intimately associated with our life.

1.1. Electric current :

In connection with electric potential in electrostatics, it has been said that when two charged bodies are connected by a wire, charge flows from the body of higher potential to the body of lower potential until the bodies acquire a common potential. Also when an uncharged body is brought in contact with a charged body, a flow of charge takes place from the charged body to the uncharged body in very much the same way as the flow of water takes place from a vessel full of water to an empty vessel (kept in the same level).

So, it should be remembered that as level difference produces a *hydrostatic pressure* causing a *flow* of water from higher to lower level, similarly a potential difference produces an *electric pressure* causing a *flow* of electricity from higher to lower potential.

This flow of electricity from one point to another is called electric current. If this flow takes place in a particular direction, the current is called *direct current*, abbreviated as *D.C.* On the other hand, if the flow changes its direction alternately right and left after a specified interval of time, the current is called *alternating current*, abbreviated as *A.C.*

In general, two charged bodies, when connected by a wire, produce a transient flow of charge because their potentials are equalised in a moment. In order to make the flow of charge steady, the potential difference between the charged bodies should be maintained constant.

1.2. Conventional direction of current :

An electric current has already been described as electric charge in motion. But charge may be of two kinds—positive and negative. So the question is which of the two charges flowing from one point to another constitutes an electric current ?

The conventional rule, in this regard, is that a current is produced when positive charges flow through a conductor. Suppose *A* and *B* are two points, the potential of *A* being higher than that of *B*. If the points are, now joined by a conducting wire *positive*

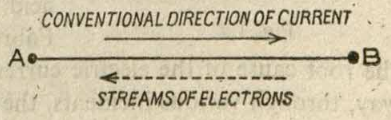


Fig. 1.1

charge will flow from A to B (Fig. 1.1). This is the conventional direction of current. This convention has been followed in this book.

According to the modern electronic theory, however, the direction of current is indicated in a different way. In a conductor, there are copious free electrons. When a P.D. is created at the ends of a conductor, these negatively charged *electrons move from the end of lower potential to the end of higher potential.* So according to electron theory, an electric current is a flow of negative electrons in a direction opposite to the conventional direction.

1.3. History of the discovery of the electric cells :

Electric cells were first devised by Alessandro Volta, an Italian scientist. But before that, a casual discovery by Galvani of the existence of electricity in the convulsive movement of the muscles of a frog is actually responsible for the discovery of the cells.

In 1786, Luigi Galvani, a Professor of Anatomy at Bologna University in Italy was experimenting with freshly cut frogs. One day, the body of a freshly cut frog, soaked in saline water, was hanging from a copper hook. Galvani suddenly noticed that each time the leg of the frog swung by air, touched the iron railing of the corridor, the leg was thrown into muscular convulsions. It had been known from a long time that muscles of dead animals could be caused to contract by means of electric shock. From this observation, Galvani came to the conclusion that electricity was inherent in the frog.

Professor Alessandro Volta, however, could not accept the explanation offered by Galvani. He was convinced that the contact between the two dissimilar metals involved in the incident was responsible for the generation of electricity and not the body of the frog itself. The frog's body was a conductor of electricity and a current was produced as soon as the metals came in contact through the conducting body of the frog.

He then tried to establish his theory by preparing the famous *pile*, known as Voltaic pile in 1800. The pile consists of pairs of copper and zinc discs separated by pieces of cotton soaked in dilute sulphuric acid. On connecting the first copper disc and the last zinc disc by a wire, a current is produced (Fig. 1.2).

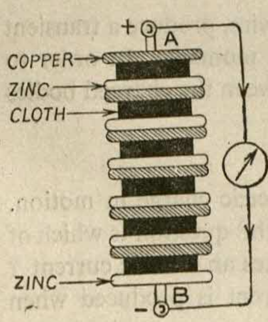


Fig. 1.2

According to Volta, a potential difference and hence a current will be produced if two dissimilar metals are brought in contact. But in respect of the above experiment of Volta's pile, his theory was found to be inconsistent because on examination of the pile it was observed that some chemical action took place between zinc and sulphuric acid. From these observations, Davy, De la Rive, Fabroni and others came to the final conclusion that the root cause of the electric current in the pile was the chemical action. In this way, through various incidents, the scientists realised the basic principle of electric cells.

1.4. Simple Voltaic cells :

The arrangement which converts chemical energy into electrical energy in the form of continuous flow of electric current is called an electric cell. A simple cell is called a voltaic cell because Volta first constructed it.

Description : Fig. 1.3 shows a schematic diagram of a simple voltaic cell. It consists of a zinc plate (Zn) and a copper plate (Cu) dipped in the dilute sulphuric acid kept in a glass vessel. Two binding screws are provided with the plates. When a wire is connected between the screws, chemical action between zinc and sulphuric acid starts and bubbles of hydrogen are found to rise along the copper plate. An electric current also flows along the wire from the copper plate to the zinc plate.

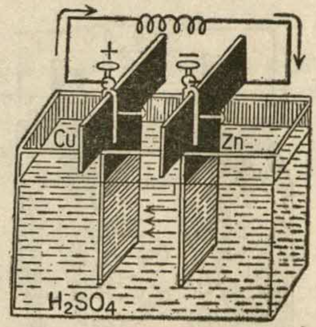


Fig. 1.3

The current stops if the wire is taken out of the screw terminals but the potential difference between the zinc and the copper plates exists. The copper plate is said to have higher or positive potential and the zinc plate lower or negative potential. They are also known as positive and negative poles respectively. When a wire connects the terminals of the plates, a current flows to the zinc plate from the copper plate along the wire which tends to reduce the potential difference between the plates. But more chemical action takes place between zinc and sulphuric acid which maintains the potential difference and hence a continuous current.

Action of the simple voltaic cell : To follow the action of a simple voltaic cell, some idea of ions and ionic dissociation is necessary.

We know that atoms of a substance are made up of a number of fundamental particles of which electrons are very significant. Ordinarily atoms and molecules are neutral in character but if one or more electrons be driven out somehow, from an atom or a molecule, they become electrically charged. Electrons themselves being negatively charged particles, a deficit of electrons in an atom endows the atom with positive charge while the expelled electrons combining with a neutral atom make it negatively charged. Such electrically charged atoms or molecules are called *ions* and the process is known as *ionisation.* Ions are represented by putting + or − sign on the chemical symbol of an atom. When an ion has a deficit or surplus of a number of electrons, equal number of (+) or (−) signs, as the case may be, is put on the chemical symbol of the atom. Thus, if a single electron is detached from helium atom, the ion is represented by He^+.

It has been found that when a solution is made of a compound in a solvent, the molecules of the compound are dissociated into ions which remain in the solution. For example, when common salt ($NaCl$) is dissolved in water, each molecule of $NaCl$ dissociates into a positively charged (Na^+) ion and a negatively charged (Cl^-) ion. These ions move in a random way in the solution. This process is called ionisation.

Let us now come back to the action of a simple cell. In order to make dilute sulphuric acid, when acid is mixed with water, each molecule of the acid due to ionic dissociation, is split up into positive H^+ ion and negative SO_4^- ion according to the following scheme : $H_2SO_4 \rightarrow (H^+ + H^+) + SO_4^{--}$.

These ions move in a haphazard way in the solution.

Now, when the zinc plate is dipped in the acid, Zn^{++} ions carrying $+ve$ charge go into the acid and attract the negatively charged $(SO^4)^{--}$ ions. They form neutral $ZnSO_4$ molecules according to the equation $Zn^{++} + SO_4^{--} = ZnSO_4$. As positive zinc ions leave the zinc plate, the plate acquires a surplus of electrons and the plate becomes negatively charged (Fig. 1.4). For this reason, the zinc plate is referred to as the 'fuel' of the cell.

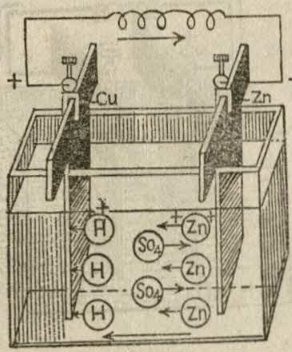


Fig. 1.4

The solution near the zinc plate acquires a positive charge due to the presence of positively charged zinc ions and repels the positively charged H^+ ions present there towards the copper plate. H^+ ions, on the arrival at the copper plate, attract one electron each from the plate and form neutral H_2 molecules which bubble out of the acid as hydrogen gas. The copper plate losing electrons, becomes positively charged and attracts electrons from the Zn-plate, to make up the deficit. The electrons, travelling along the wire from zinc plate to the copper plate give rise to the current.

Thus, we see that due to chemical action in the cell, the copper plate acquires positive charge and hence a positive potential and the zinc plate negative charge and hence a negative potential. When the plates are not joined by a wire, the p.d. existing between the plates is called the *electromotive force* (abbreviated as e.m.f.) of the cell. This electromotive force is responsible for driving the current in a circuit. The practical unit used to express e.m.f. is 'volt'. The e.m.f. of a simple voltaic cell is about 1·08 volts.

It is to be noted that as there is a flow of current in the wire outside the cell, there is also a flow of current in the liquid inside the cell. External flow of current takes place from the copper to the zinc plate (conventional direction), but the internal flow takes place in the opposite direction *i.e.* from the zinc to the copper plate. (Fig. 1.4).

Whenever current flows through a conductor, it has to overcome some opposition, known as the '*resistance*' of the conductor. Liquid conductors also offer resistance to the flow of current. Such resistance offered by the liquid in a cell is called the '*internal resistance*' of the cell while the resistance offered by the external circuit is called the '*external resistance*'. Detailed discussion about resistance will be taken up later on.

Calculation of E.M.F. of a voltaic cell :

According to Faraday, the energy required for maintaining the current in a voltaic cell is derived from the chemical reaction that takes place inside the cell. So, the e.m.f. of a voltaic

cell may be found out if the heat equivalent of chemical energy spent in the cell is converted into electrical energy.

Suppose, H_1 and H_2 are the heat, in calories, produced (heat will be absorbed at one electrode and evolved at the other) at the electrodes as a result of chemical action when a unit quantity of electricity flows through the cell. So, when q units of electricity pass through the cell, the amount of heat evolved at one electrode is H_1q and that absorbed at the other is H_2q. calories.

The amount of heat available$=(H_1q-H_2q)$ cal. The corresponding energy liberated $=J(H_1q-H_2q)$ joules where $J=$mechanical equivalent of heat.

Again, for q units of electricity flowing through the cell, the amount of energy consumed $=E.q$ where E is the *e.m.f.* of the cell.

$$\therefore \quad J(H_1q-H_2q)=E.q \qquad \ldots(i)$$

Now, the amount of heat liberated when one *gm-equivalent* of zinc goes into solution is 54,600 calories and the amount of heat absorbed for the evolution of 1 *gm-equivalent* of hydrogen at the copper plate is 29,700 cal. The quantity of electricity that passes through the cell in the circumstances stated above is called *Faraday* and it is equal to 96,500 coulomb (See Chapter V).

Hence, in this case $H_1q=54{,}600$ cal ; $H_2q=29{,}700$ cal. ; $q=96{,}500$ coulomb and $J=4{\cdot}18$ joule. So, $4{\cdot}18(54{,}600-29{,}700)=E\times 96{,}500$ [From eqn, (i)]

$$\text{or, } 4{\cdot}18\times 24{,}900=E\times 96{,}500$$

$$\text{or } E=\frac{4.18\times 24{,}900}{96{,}500}=1{\cdot}08 \text{ volts (nearly)}$$

This is the e.m.f. of a voltaic cell.

1.5. Defects of simple voltaic cell :

The simple cell described above has two principal defects *viz,* (i) **Local action** and (ii) **Polarisation.** For these defects, the current given by a simple cell gradually diminishes and finally stops altogether.

(i) **Local action** : Ordinarily, zinc plate available in market is not pure. It contains several impurities like iron, carbon, lead, arsenic etc. When such a zinc plate is dipped in sulphuric acid, the impurities together with the acid set up tiny local cells at the zinc surface. The small currents produced by these local cells do not join the mainstream and serve no useful purpose. Whether the plates of the cell are joined by a wire or not, these local cells always produce currents which waste the zinc plate and heat up the cell. The cell, consequently is damaged very soon.

Means of removal : To prevent local action, pure zinc plate may be used instead of the plates usually available in markets. But there are two difficulties in using pure zinc. First, there is hardly any chemical action between pure zinc and sulphuric acid. Second, pure zinc is very costly. For these reasons, commercial zinc plates are used in the construction of simple cells.

Fortunately, local action can be prevented easily by giving the commercial zinc plate a coating of mercury. Zinc dissolves in mercury and a bright coating of zinc amalgam is formed all over the surface of the plate. The amalgam covers up the impurities and prevents them from coming in contact with the acid ; only zinc comes in contact with the acid. The cell, therefore, operates without any local action. The impurities, in course of time, get loose and drop on the bottom of the vessel as the zinc plate is consumed due to chemical action with the acid.

(ii) **Polarisation** : If the plates of a simple voltaic cell are joined by a wire the current given by it, is found to weaken gradually and at last, the cell stops

delivering any current. This gradual weakening of the current is due to *polarisation.*

Experiment : Connect a calling bell to the terminals of a simple cell. The bell rings with usual sound at first ; but after sometime, the sound begins to fall and finally the bell stops ringing. If the copper plate of the cell be now taken out and brushed vigorously by a small paint brush, and replaced, the bell will again ring with usual sound. This is due to the polarisation of the cell.

Explanation : In describing the action of a simple cell, it has been said earlier that positively charged H^+ ions move towards the copper plate and attracting electrons from the plate, form neutral H_2 gas which bubbles out of the cell. But the rate of arrival of ions is greater than the rate at which hydrogen gas escapes from the cell. As a result, some of the neutral H_2 molecules stick to the plate. After some time, a layer of hydrogen gas covers up the entire surface of the plate. When this happens, the newly arrived H^+ ions cannot reach the copper plate and the current delivered by the cell begins to fall. After some time the H^+ ions form a layer on the neutral gas and repel the fresh H^+ ions which happen to come near by. These H^+ ions then start moving toward the zinc plate and an electromotive force starts acting in the opposite direction. This e.m.f. is known as *back electromotive force*. At this time, the cell is said to be completely polarised and it fails to supply any more current in the external circuit.

Means of removal : (i) **Mechanical means** : Polarisation can be removed by occasionally taking the copper plate out and cleaning the bubbles of hydrogen gas by a paint brush. If such a clean copper plate is replaced in the cell, current in its full strength will be available. This is known as mechanical means. Polarisation can also be removed to some extent by using a copper plate having rough surface because bubbles cannot conveniently gather on rough surface. But these mechanical means are not very effective.

(ii) **Chemical means** : In this method, an oxidising agent like MnO_2 or HNO_3 is used in the cell which oxidises hydrogen into water by secondary chemical reaction. Consequently, hydrogen gas cannot accumulate on the copper plate and polarisation cannot occur. This oxidising agent is known as **depolariser**. In Leclanche's cell, MnO_2 (manganese dioxide) is used as a depolariser (See Leclanche's cell, page 315).

(iii) **Electro-chemical means** : In this method, two liquids are used in the cell, such that hydrogen molecules, produced by the first liquid (which is the electrolyte of the cell) react with the second liquid and produce molecules of the element of which the positive plate is made. As there is no free hydrogen molecule, there is no polarisation too. In Daniel cell, for example, solution of copper sulphate ($CuSO_4$) in water is used as a depolariser (See Daniel cell, page 317).

1.6. Types of primary cells :

A cell, in which electric current is produced by the chemical reaction between several chemical substances, is called a primary cell. Primary cells contain

three principal elements *viz*, (i) positive and negative poles or electrodes (ii) active liquid and (iii) a depolariser. Different types of primary cells are divided broadly into two classes *viz*. (i) *Single fluid* and (ii) *Double* or *two fluid*.

1.7. Different single fluid cells :

(i) **Leclanche's cell** : This cell was invented by Georges Leclanche in about 1865. Fig. 1.5 shows a schematic diagram of the cell. In a glass vessel is kept a solution of ammonium chloride (NH_4Cl) in water in which is partially dipped an amalgamated zinc rod. There is a porous pot at the middle of the glass vessel dipped in ammonium chloride. The pot is filled up by a mixture of

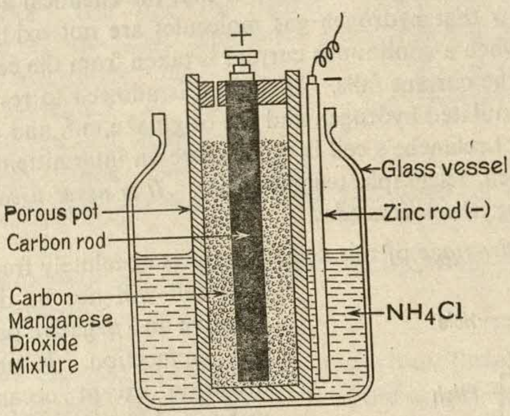


Fig. 1.5

powdered carbon and manganese dioxide in which is inserted a gas carbon rod. There is a small opening at the top of the porous pot through which gas escapes. Its negative pole or electrode is the zinc rod and positive pole or electrode is the carbon rod. Ammonium chloride solution is the active electrolyte and manganese dioxide is the depolariser. The e.m.f. of the cell is about 1·5 volts.

As the solution of ammonium chloride evaporates, it tends to form small crystals which 'creep' up the sides of the vessel. To prevent such creeping, the upper part of the vessel is painted with a special back wax paint. The crystals do not adhere to that paint.

Action : Due to chemical action between Zn and NH_4Cl, positively charged H^+ ions are liberated and the zinc rod itself becomes negatively charged :

$$Zn^{++}+2NH_4Cl=ZnCl_2+2NH_3+(H^++H^+)^*$$

[*$Zn \rightarrow Zn^{++}+2e$

$2NH_4Cl \rightarrow 2NH_3+2H^++2Cl^-$

$Zn^{++}+2Cl^- \rightarrow ZnCl_2$

$Zn+2NH_4Cl=ZnCl_2+2NH_3+2H^++2e$]

The free ammonia gas (NH_3) dissolves in water. When the solution becomes saturated, the gas instead of going into solution, escapes through the opening of the porous pot. The H^+ ions, in their bid to proceed towards the carbon rod enter into the porous pot through the pores and hand over the charge to the carbon rod and are finally converted into neutral hydrogen molecules. Manganese dioxide present in the porous pot, then reacts with the hydrogen molecules and oxidises them into water molecules :

$$H_2+2MnO_2=Mn_2O_3+H_2O.$$

So, hydrogen gas cannot accumulate on the carbon rod and hence cannot produce polarisation in the cell.

The main *disadvantage* of this cell is that the chemical action between MnO_2 and H_2 is so slow that hydrogen gas molecules are not oxidised as soon as they are formed. So when a continuous current is taken from the cell, some polarisation takes place and the current falls. If the cell is allowed to rest for a while, MnO_2 oxidises the accumulated hydrogen and the original e.m.f. and current are restored. For this reason, a Leclanche's cell is used where an intermittent current is required such as electric bell, telegraph, telephone etc. *It is never used where a continuous current over a long period is needed.*

The main *advantage* of this cell is that it is absolutely free from local action. The cell will not be damaged even if its positive and negative poles are left without any connection. It practically needs no attention except occasional addition of water and powdered ammonium chloride.

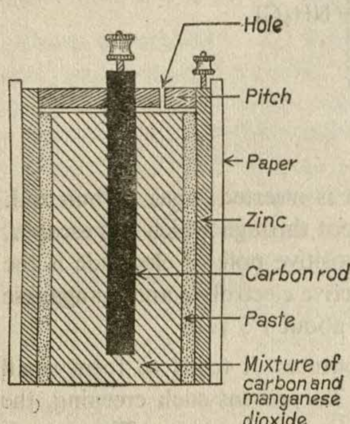


Fig. 1.6

(ii) **Dry cell** : The dry cell is a form of Leclanche cell in which the liquid is replaced by a paste. For this reason it is called a dry cell although it is not perfectly dry. These cells are widely used in torch lights, radio sets and transistor sets, for sending current. Fig. 1.6 shows the sketch of a dry cell.

In this cell, a zinc cylinder is used as a container as well as the negative pole of the cell. A carbon rod is placed in the vessel, which forms the positive pole of the cell. It is surrounded by a mixture of manganese dioxide and powdered carbon. This mixture is placed in a calico bag and the intervening space between the calico bag and the zinc cylinder is packed with a paste made of NH_4Cl solution, coke and a little water. Due to chemical action between NH_4Cl and Zn, positive hydrogen ions are liberated which travel towards the carbon rod through the pores of the calico bag. The upper part of the cell is closed by sand, pitch etc, leaving a small hole in the pitch for the outlet of the gas. The whole thing, after being wrapped up in paper, is sent to the market.

1.8. Two fluid cell : Daniel cell :

John Daniel, a Professor of chemistry of King's College, London invented this cell in 1836. It consists of a copper vessel in which a solution of $CuSO_4$ (copper sulphate) in water is kept (Fig. 1.7). The vessel itself acts as the positive pole of the cell. A few pieces of copper sulphate crystals are kept in two perforated shelves on the upper part of the vessel to keep the solution in a saturated condition. A porous earthenware pot containing dilute sulphuric acid and an amalgamated zinc rod is placed inside the copper can. The zinc rod acts as the negative pole of the cell. Sulphuric acid is the active electrolyte and $CuSO_4$ solution is the depolariser of the cell. Its e.m.f. is 1.1 volt which remains fairly constant for a long while. So this cell is useful when a very constant but feeble current is required for a great length of time.

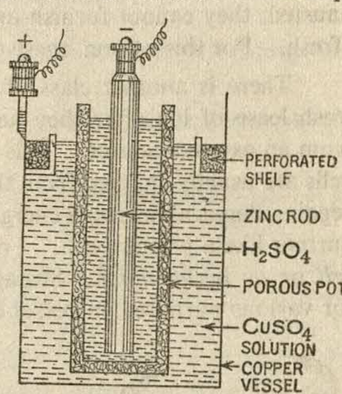


Fig. 1.7

Action : Zn being acted on by H_2SO_4 liberates positively charged hydrogen ions according to the equation :

$$Zn^{++}+H_2SO_4=ZnSO_4+(H^++H^+)$$

These positively charged hydrogen ions come out through the pores of the earthenware pot and travel towards the walls of the copper vessel. But before reaching the wall, they react with $CuSO_4$ and produce positively charged Cu^{++} ions according to the equation :

$$(Cu)^{++}(SO_4)^{--}+(H^++H^+)=Cu^{++}+H_2SO_4.$$

The copper ions hand over charge to the copper vessel and are deposited on the wall. Hence, the copper vessel becomes the positive pole of the cell.

Polarisation does not occur in this cell for the simple reason that copper and not hydrogen is deposited on the positive pole. The cell can, therefore, maintain a steady e.m.f. for long and can furnish a steady current. As the action of the cell goes on, the copper sulphate solution tends to become weaker but the crystals of $CuSO_4$ kept in the shelves keep the solution saturated.

Only *disadvantage* of this cell is that if the cell be allowed to stand idle copper sulphate molecules diffuse through the porous pot and damage the zinc rod. For this reason, the parts of the cell are kept dismantled when the cell is not in use.

Calculation of E.M.F. :

When 1 gm-equivalent of zinc goes into solution $5{\cdot}461\times10^4$ calories of hear are liberated and when 1 gm-equivalent of copper is deposited, the amount of heat absorbed is $2{\cdot}892\times10^4$ calories.

Here, $H_1q=5{\cdot}461\times10^4$ cal ; $H_2q=2{\cdot}892\times10^4$ cal ; $q=96{,}500$ columb and $J=4{\cdot}18$ joules.

$\therefore\ J(H_1q-H_2q)=E.q$

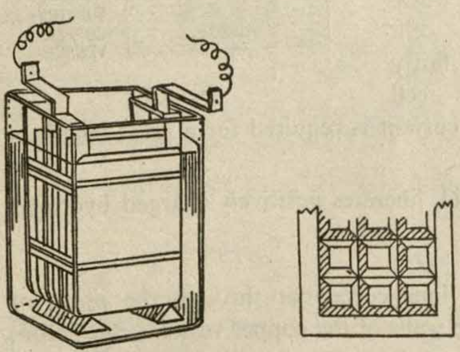
or $4{\cdot}18(5{\cdot}461\times10^4-2{\cdot}892\times10^4)=E\times96500$

or $E=\dfrac{4{\cdot}18\times2{\cdot}569\times10^4}{96500}=1{\cdot}12$ volts (nearly)

1.9. Accumulator or Storage cell or Secondary cell :

In primary cells like Leclanche's cell, dry cell etc, current is produced by the chemical action between its various components. When the chemicals are exhausted, they cannot furnish any more current. The cell has got to be prepared afresh. For this reason, these cells are known as **primary cells.**

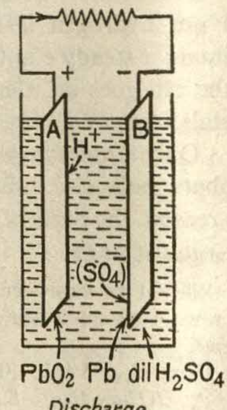
There is another class of cells called *secondary cells*, which can be given a fresh lease of life after they have run down, by passing a current through them from an external source. This is known as '*charging*' of the cell. The secondary cells are usually charged from the mains supply. As a result of charging, the cell regains chemical potential energy and at the expense of this energy, it again delivers current like a primary cell. For this reason, the cell is sometimes called a *storage cell* or *an accumulator*. Storage cells are widely used in ships, trains and cars for various purposes, in petrol engines as well as in laboratories.



(a) Fig. 1.8 (b)

Description : The cell was devised by Plante' in 1859. It consists of a thick glass vessel in which dil. H_2SO_4 is taken. Several lead plates are held parallel to each other in the acid and the plates are alternately connected to the positive and negative electrodes of the cell [Fig. 1.8(*a*)]. In modern form of accumulator, instead of lead plates, lead grids as shown in the fig. 1.8(*b*) are used. In the spaces of the grids, a mixture of litharge (PbO) and H_2SO_4 in the form of a paste is used for the cathode plate and a paste of red lead and H_2SO_4 is used for the anode plate. The e.m.f. of the cell is 2·1 volts.

Principle of action : The action of the cell consists of two processes : (i) *discharge of cell i.e.* when the cell delivers current in the external circuit and (ii) *charging of the cell i.e.* when the cell is fully discharged, it is revitalised by passing current through it from some external source.



Discharge

Fig. 1.8(c)

(i) **Discharge of cell** : When used to produce current, the oxide coated plate *A* forms the positive pole and the metal plate *B*, the negative pole [Fig. 1.8(*c*)]. When the poles are connected by a conductor, current flows from *A* to *B* through the external circuit and from *B* to *A* through the acid. The following chemical action takes place during discharge.

In the acid electrolyte, sulphate ions migrate to the lead plate and hydrogen ions to the oxide coated plate. On deposi-

tion, hydrogen ions reduce lead peroxide to monoxide which forms lead sulphate as follows :

$$PbO_2+H_2 \rightarrow PbO+H_2O$$
$$PbO+H_2SO_4 \rightarrow PbSO_4+H_2O$$

The lead sulphate is insoluble in the electrolyte and it forms a coating on the plate.

At the negative plate, $(SO_4)^{--}$ions also form lead sulphate according to : $Pb+SO_4 \rightarrow PbSO_4$.

When both the plates are coated with lead sulphate, the accumulator is completely discharged and no more electrical energy should be drawn from it.

During discharge process, sulphuric acid is effectively removed from the solution to form lead sulphate with the result that *the specific gravity of acid solution is reduced.* In a freshly charged accumulator, the specific gravity is about 1·25, whilst in the fully discharged state it is about 1·18. The specific gravity is, however, used as a guide to determine the charged or the discharged state of a cell.

(ii) **Charging of cell** : When an accumulator is completely discharged, it is restored to the original state by passing current through the cell in the opposite direction. This is achieved by applying a p.d. by the D.C. mains so that the positive plate A becomes the anode and the negative plate the cathode [Fig. 1.8(d)].

During charging process, H^+ ions migrate to the negative plate B and reduce the lead sulphate formed during discharge of the cell to metallic lead :

$$PbSO_4+H_2=H_2SO_4+Pb.$$

At the positive plate A, the $(SO_4)^{--}$ ions produce lead peroxide (PbO_2) and sulphuric acid : $PbSO_4+SO_4+2H_2O \rightarrow PbO_2+2H_2SO_4$

The net result is the formation of a layer of lead peroxide on the positive plate A, a layer of spongy lead on the negative plate B and an increase in the concentration of sulphuric acid. The charging process is continued till the specific gravity of the acid solution rises to a specific value (1·25).

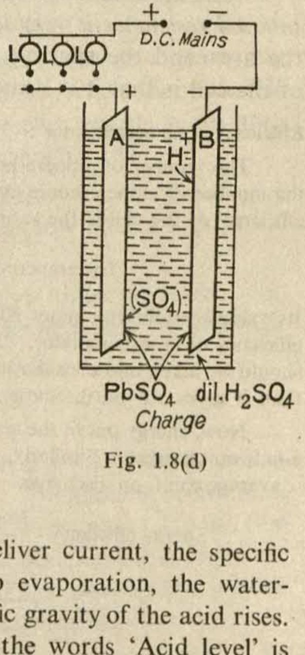


Fig. 1.8(d)

Notes on accumulator :

When the cell is fully charged and is ready to deliver current, the specific gravity of the acid in the cell becomes 1·25. Due to evaporation, the water-content of the liquid in the cell diminishes and the specific gravity of the acid rises. To prevent such rise of specific gravity, a mark with the words 'Acid level' is given on the outside surface of the glass vessel. Whenever the level of the acid falls below the mark, distilled water is added to the acid in order to bring the level back to the specified mark. This keeps the specific gravity of the acid at the desired value.

As the cell delivers current, the chemical action that takes place inside the cell, lowers the specific gravity of the acid and the voltage of the cell also falls slowly from its full value 2·1 volts. When the specific gravity falls to about 1·18 and the voltage to about 1·8, the cell becomes fully 'discharged' and it is not capable of delivering any more current. A fresh charging is needed in order to bring the cell to its original strength. It is to be remembered that *the state of the cell (whether charged or discharged) is best ascertained by testing the specific gravity of the acid* because the voltage may remain almost unchanged when the cell is being discharged.

Capacity of an accumulator :

It goes without saying that the cell cannot deliver more energy than what is stored in it during charging. In other words, the principle of conservation of energy is applicable to the storage cell. The capacity of delivering energy of an accumulator is expressed in terms of **Ampere-hour** which is the product of the current in ampere that the accumulator can supply and the time in hour during which such supply of current is available without discharging the cell. For example, an accumulator of 60 ampere-hour capacity can supply, when fully charged, a current of 1 ampere for 60 hours or a current of 2 amp. for 30 hours.

The accumulator can supply a steady but strong current for a long time. *Its internal resistance is very low* of the order of $\frac{1}{10}$ to $\frac{1}{100}$ ohm. It depends on the the area and the spacings of the plates of the accumulator. The disadvantage of the cell is that it is heavy and requires careful handling and regular attention.

Efficiency of an accumulator :

The number of ampere-hours supplied to the accumulator during charging is greater than the number of ampere-hours available from it without discharging it too far. The ampere-hour efficiency of the cell is the ratio of these two numbers *i.e.*

$$\text{The ampere-hour efficiency} = \frac{\text{ampere-hours available during discharge}}{\text{,, \quad ,, \quad supplied during charging}}$$

Its value is ordinarily about 90%. However, ampere-hour effiiency is not a true picture of the efficiency of an accumulator. To judge the performance of an accumulator, its energy efficiency should be taken into consideration because it takes not only more ampere-hours during charging than it gives out on discharge, but takes them in at a higher voltage also.

Now, energy put in the accumulator during charging = quantity of charge supplied × average e.m.f. on charging. Similarly, energy delivered during discharge = quantity of charge given out × average e.m.f. on discharge.

$$\therefore \text{ Energy efficiency} = \frac{\text{Energy available during discharge}}{\text{,, \quad supplied during charging}}$$

$$= \frac{\text{amp-hour} \times \text{average e.m.f. on discharge}}{\text{,, \quad ,, } \times \text{ ,, \quad ,, \quad ,, charging}}$$

$$= \text{amp-hour efficiency} \times \frac{\text{average e.m.f. on discharge}}{\text{,, \quad ,, \quad ,, charging}}$$

$$\simeq \text{amp-hour efficiency} \times \frac{1{\cdot}8}{2}$$

The energy efficiency of an accumulator is about 80%.

Caution : The two terminals of an accumulator should never be *short-circuited i. e.* directly connected by a connecting wire. This will cause permanent damage to the cell.

1.10. Some important facts in connection with electric cells :

The following important facts in connection with electric cells should always be borne in mind :

(i) The e.m.f. of a cell does not depend upon the size of the cell; it depends upon the materials of the cells. Cells made of same materials but of different sizes have same e.m.f.

(ii) If the plates of the cell be large and are close to each other, the internal resistance of the cell becomes low and the current given by it is high.

(iii) The total amount of charge delivered by a cell depends upon the quantity of material used in the cell.

(iv) The seat of e.m.f. of a cell is the surface of contact between the plates and the active electrolyte of the cell.

1.11. Distinction between a primary cell and a secondary cell :

(i) In a primary cell, a non-reversible chemical reaction takes place in the materials of the cell and thereby an electric current is produced. When the chemicals of the primary cell are exhausted, they can not be activated to the original condition by passing a charging current through them. The action of a secondary cell is exactly opposite. The cell produces a current as a result of a reversible chemical reaction taking place in the materials of the cell. When the materials are exhausted, they can be activised by charging. For this reason, a primary cell is called an irreversible cell and a secondary cell a reversible cell.

(ii) The internal resistance of a primary cell is high ; so it cannot give intense current. A secondary cell, on the other hand, has low internal resistance and gives a strong current.

(iii) When a primary cell is once exhausted, a new cell is to be prepared. So, the use of primary cell is costly. A secondary cell can be activated by charging several times.

1.12. Standard cell :

A cell whose e.m.f. remains constant and which is used as the standard of e.m.f. for determining the e.m.f. of other cells by comparison, is called a standard cell. A standard cell is never used to supply current. It is free from local action and polarisation. The e.m.f. of a standard cell, however, changes a little with temperature. *Weston Cadmium cell* has been recommended internationally as a standard cell for the purpose of standardisation.

1.13. Effect of electric current :

When current flows in a closed circuit, three effects, mentioned below, are seen. Each of these effects may be used to determine the current strength.

(1) **Heating effect** : When current flows through a wire, the wire becomes heated. We become acquianted with the heating effect of electric current through various happenings of our daily life. When, for example, current flows through the filament of an electric lamp, the filament becomes so hot that it emits light. Many useful appliances have been made by utilising this heating effect of current.

(2) **Magnetic effect** : When a current flows through a wire a magnetic field is created around the wire. A magnetic needle brought near the wire will be deflected which proves the existence of a magnetic field. This is known as the the magnetic effect of current.

(3) **Chemical effect** : When current flows through a liquid conductor, like acidified water, copper sulphate solution, silver nitrate solution etc, a chemical action takes place in the liquid, as a result of which, the molecules of the liquid are found to dissociate. This is known as the chemical effect of current or *electrolysis.*

All the three effects of current can be simultaneously demonstrated by an arrangement shown in fig. 1.9.

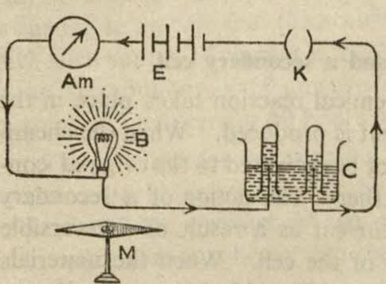


Fig. 1.9

The arrangement consists of a torch bulb *B*, a magnetic needle *M* and a vessel containing some water. Two test tubes, filled with water, are placed in an inverted position inside water. *E* is a battery of three storage cells connected in series and *K* is a plug key. All the instruments are placed in series in the same circuit so that same current flows through each one of them, which is recorded by an ammeter *Am.* When a plug is inserted in the hole of the key *K*, current flows through the circuit. At the same time, the bulb glows, the needle *M* is deflected and gas starts collecting in the tubes. In this way, the heating, the magnetic and the chemical effects can be demonstrated simulatenously.

1.14. Physiological effects of electric current :

Human body is a conductor of electricity. Whenever opportunity is given, current will flow through human body, producing various reactions which are, in general, harmful. Those of you who have worked with electric appliances and instruments, must have felt electric 'shock' sometime or other. 'Shocks' are likely to occur when defective fan, heater, iron, switch etc. are handled.

Due to electric shock, the nerves of the body become temporarily paralysed and sensation of pain, involuntary movement of mu scles, twitchings etc are caused. Strong shocks, due to large voltage, may cause severe burning and even death.

If the hands and feet are moist, the current passing through the body will be stronger and hence the shock is severe. High voltage also causes very severe shock. For these reasons, precautions are to be taken while working with elec-

trical equipments requiring high voltage—specially those connected to the 'mains'. A wooden stand or a wooden board and a pair of rubber gloves should be used as a measure of protection against shocks.

Electrical shocks sometimes produce good results in the treatment of some diseases like rheumatism, paralysis, loss of memory, insanity etc. The process of curing such diseases by repeated electric shock is known as '*shock therapy*'.

Exercises

Essay type :

1. What is a simple voltaic cell ? Explain its action. Mention the chief defects of a simple cell and describe them.

2. How is potential difference developed between copper and zinc electrodes when they are dipped in dilute H_2SO_4 ? [*H. S. Exam. 1982*]

3. What do you mean by local action and polarisation in a voltaic cell ?

4. Describe a Leclanche's cell. What steps are taken in this cell to obviate the chief defects of the cell ?

5. Describe and explain the action of a Daniel cell. What methods have been adopted to remove the cheif defects of the cell ? [*H. S. Exam. 1982*]

6. What is the construction of a dry cell ? For what purpose is this cell used ?

7. Why are simple cells not used now-a-days to obtain a current ? Describe any other cell and explain the methods adopted for the removal of its defects.

8. What is a storage cell ? What is its difference with a Daniel or a Leclanche's cell ? Describe a storage cell.

9. Describe a secondary cell. Explain the principle of its action. What is its difference with a simple voltaic cell ? [*H. S. Exam. 1979*]

10. What is a secondary cell ? Why is it so called ? Explain the action of any type of secondary cell. [*H. S. Exam. 1985*]

11. Describe an experiment to demonstrate (i) the heating effect (ii) the magnetic effect and (iii) the chemical effect of a current simultaneously. Draw a diagram of the circuit.

12. What do you mean by efficiency of an accumulator ? Why isn't the ampere-hour efficiency a true picture of the performance of an accumulator ?

Short answer type :

13. Answer the following questions : (*a*) Does the e.m.f. of a cell depend on its size ? (*b*) What is the advantage of taking large plates and keeping them close together in a cell ? (*c*) What is the seat of e.m.f. in a cell ? (*d*) On what factors does the e.m.f. of a cell depend ? (*e*) Why is the zinc plate of a simple voltaic cell called the fuel of the cell ? (*f*) Why a coating of mercury is given to the zinc plate of a Leclanche's cell ?

14. Why should you not short-circuit the terminals of a storage cell ? What does a storage cell store ?

15. What do you mean by 'ampere-hour' ? [*H. S. Exam. 1979*]

16. Should you draw currents from a standard cell for ordinary use ?

17. What is the harm if current is drawn continuously for a long time from a Leclanche cell ?

18. What type of cell should you use for the following purposes : (i) for lighting a bi-cycle lamp (ii) to put on the bulb of a room (iii) to ring an electric bell ? Explain your answer.

19. What are the differences between a primary cell and a secondary cell ?

20. What is the best of way of knowing whether a secondary cell is charged or discharged ?

21. Fill up the blanks with suitable words from the parenthesis :

(a) The arrangement which converts chemical energy into electrical energy in the form of continuous flow of electric current is called an——. (Volta's pile, electric resistance, electric cell)

(b) The gradual weakening of current due to a deposition of other material on the electrodes of a simple voltaic cell is referred to as the —— of the cell. (Local action, potential, polarisation)

(c) The seat of — of a cell is the surface of contact between the electrode and the electrolyte. (Potential difference, internal resistance, electromotive force)

(d) The capacity of an accumulator of delivering charge is expressed in —— units. (ampere-hour, volt, ampere)

Numerical Problems :

22. The heat liberated when 1 gm of zinc is dissolved in sulphuric acid is 1622 cal and the heat absorbed when 1 gm of copper is deposited from copper sulphate solution is 881 cal. The atomic weights of zinc and copper being 65·4 and 63·6 respectively and valency being 2, calculate the e.m.f. of the cell. [Ans. 1·1 volts]

23. How much total charge can an accumulator of 2 ampere-hour capacity deliver ? [Ans. 7200 coulmbs]

24. The capacity of a secondary cell is 30 ampere-hour. How much electric charge can be drawn from it without damaging it? [*H. S. Exam. 1985*] [Ans. 108×10^3 coulombs]

———

2

OHM'S LAW AND RESISTANCE

2.1. Current strength :

A current flows through a wire when it is connected to the terminals of a cell. There is a similarity between such a current flowing in a wire and the flow of a liquid through pipes. The rate at which the liquid flows past any point in a system of piping may be measured by the amount passing in each unit of time—for instance in 'grammes per second' (Fig. 2.1). In the electrical case,

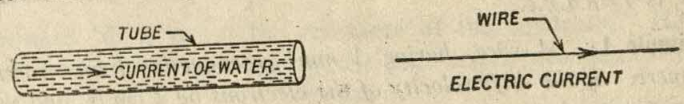


Fig. 2.1

the strength of the current (usually called the 'current') is similarly measured by *the amount of charge passing any section of the wire per unit time.* If Q amount of charge passes any section of the wire in 't' seconds, the current-strength or simply the current in the wire $I=\dfrac{Q}{t}$

If $Q=1$ e.s.u. and $t=1$ sec, then $I=1$ e.s.u. *i.e.* if 1 e.s.u. of charge passes any section of a conductor in 1 sec., then the current flowing through the conductor 1 e.s.u.

Current flows in a closed circuit. If there be any break anywhere in the circuit, the current will cease to flow. If we take continuous piping system of uniform circular section and allow water to flow through it, then at every section of the pipe, the rate of flow will be equal. Similarly, when current flows through a wire, its strength at every point of the wire remains same.

The water system consisting of a series of pipes joined to a circulating pump corresponds to a simple electric circuit made up of a series of wires connected to a battery. The purpose of the pump is to maintain a pressure difference between its inlet and outlet in order to keep the water circulating. Similarly, the function of the battery is to maintain an electrical potential difference (P.D.) between its two terminals. It is the P.D. which keeps the current flowing in the circuit.

2.2. Conduction of electricity in metals :

Metals, in general, are good conductors of electricty as they contain copious free electrons. The conduction of electricity in metals is due to these free electrons. Free electrons have thermal energy which depends on the temperature of the metal. They wander freely from atom to atom in the metal. When a battery is connected across the ends of the metal, an electric field is set up which produces a drift of the electrons towards the higher potential end (see art 1.2). This drift constitutes an 'electric current'.

Consider a metallic wire of cross-sectional area α through which a current I is flowing. Let us suppose that there are n electrons per unit volume of the wire. Now, in 1 second all those electrons within a distance v (v being the drift velocity of electrons) to the right of the plane B *i.e.* in a volume $\alpha.v$ will flow through the plane B as shown in fig. 2.2. This volume evidently contains $n.\alpha.v.$ electrons and hence a charge $n\,\alpha.v.e.$ passes across the plane B in 1 second, where e is the charge carried by each electron. Hence, the current I in the wire, according to the definition, is $I=n.\alpha.v.e.$

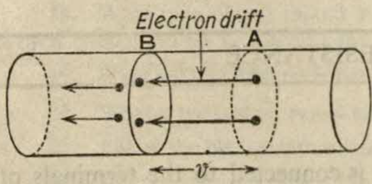


Fig. 2.2

Example 1 : *A wire, having* 1 *mm*^{2} *cross-sectional area carries a current of* 3·2 *ampere. If the drift velocity of the electrons be* 2 *cm./s. find the number of electrons per unit volume of the wire. Charge of an electron*$=1{\cdot}6\times10^{-19}$ *coulomb.*

Ans. We know, $I=n.\alpha.v.e.$ or $n=\dfrac{I}{\alpha.v.e.}$

Here $\alpha=1$ mm$^2=10^{-2}$ cm^2 ; $v=2$ cm/s ; $e.=1{\cdot}6\times10^{-19}$ coulomb.

Hence $n=\dfrac{3{\cdot}2}{10^{-2}\times2\times1{\cdot}6\times10^{-19}}=10^{21}$

So, the number of electrons per unit volume (c.c.) of the wire$=10^{21}$

Example 2 : *The diameter of an aluminium wire is* 0·25 *cm. It is joined, end to end, with a copper wire of diameter* 0·16 *cm. If a current of* 10 *amp. flows through the combination find the current density of each wire.*

Ans. Current will be distributed uniformly across the cross-sections of the wires. Only at the junction, the distribution will be somewhat non-uniform. So, the distribution of current along each wire may be regarded uniform.

Now, the area of the cross-section of aluminimum wire$=\pi\left(\dfrac{0{\cdot}25}{2}\right)^2=0{\cdot}05$ sq. cm. So, current density in aluminium wire, $J_{AL}=\dfrac{10}{0{\cdot}05}=200$ amp per cm^2

Again, area of the cross-section of copper wire$=\pi\left(\dfrac{0{\cdot}16}{2}\right)^2=0{\cdot}02$ sq. cm.

So, current-density in copper wire, $J_{CU}=\dfrac{10}{0{\cdot}02}=500$ amp/cm^2

2.3. Ohm's law :

G.S. Ohm first established a relationship between the current and potential difference in 1826. This relationship is called Ohm's law. The law is as follows :

For a given conductor, the strength of the current (I) *that passes through it, is proportional to the potential difference* (V) *maintained between the ends of the conductor provided its temperature and other physical conditions remain constant.*

For example, a current I flows through a conductor AB whose terminal potentials are respectively V_a and V_b [Fig. 2.3(a)]. According to Ohm's law, $(V_a - V_b) \propto I$.

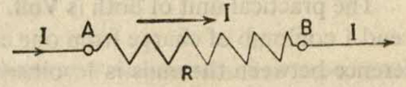


Fig. 2.3(a)

So, keepingt he temperature and other physical conditions (*i.e.* length, cross-section etc.) unchanged, if the terminal p.d. $(V_a - V_b)$ is increased, the current (I) increases and vice versa.

Now, we can write, $(V_a - V_b) = R.I.$ [R=a constant]

$$\therefore \quad \frac{V_a - V_b}{I} = R.$$

This constant 'R' is called the **resistance** of the conductor. Hence, *the resistance of a conductor is the ratio of the p.d. across it to the current flowing through it.*

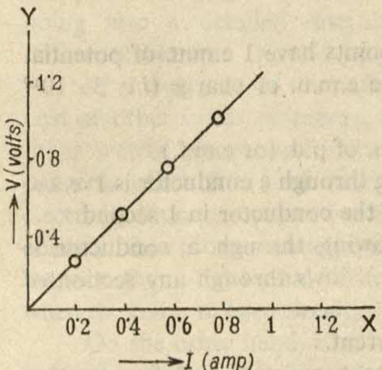


Fig. 2.3(b)

Most of the metallic conductors like copper, aluminium etc. obey Ohm's law *i.e.* the terminal *p.d.* (V) at their ends is proportional to the current (I) flowing through them and the *V-I* graph for them is a straight line passing through the origin [Fig 2.3(b)]. The resistance of these conductors is constant and the current becomes reversed when the *p.d.* is reversed but the strength of the current remains the same. They are known as *Ohmic conductors*. But there are some conductors which do not obey Ohm's Law. The *V-I* graph for these conductors is neither a straight line nor does it pass through the origin. Consequently, their resistance is not constant. If *p.d.* is reversed in these conductors, a feeble current is obtained in some of them while no current is obtained in the other. These conductors are called *non-ohmic conductors*. Vacuum valves like diode, triode etc, semi-conductors, some electrolytes are the examples of non-ohmic conductors.

2.4. Practical and absolute units of different electrical quantities :

A. In studying current electricity we shall come across various terms like current, resistance etc. The practical units of these terms are very important.

(i) **Quantity of electricity** : The practical unit of a quantity of electricity is **Coulomb.** It is the amount of electricity which passing through a solution of silver nitrate, will deposit 0·001118 gm. of silver on the cathode plate.

(ii) **Current strength** : The practical unit of current strength is *ampere*. If one coulomb of charge (*i.e.* electricity) passes across any section of a wire in one second, the current strength of the wire will be called 1 ampere.

That is, $I \text{ (ampere)} = \dfrac{Q \text{ (coulomb)}}{t \text{ (seconds)}}$ or, $Q = I\,t$.

There are two smaller units of current. They are milliampere and micro-ampere.

10^3 milliamperes $(mA) = 1$ ampere (A)
10^6 microamperes $(\mu A) = 1$ ampere.

(iii) **Potential difference and Electromotive force :**

The practical unit of both is **Volt.** If 10^7 ergs or 1 joule of work are necessary to send 1 coulomb of charge from one end of a conductor to the other, the potential difference between the ends is 1 volt.

(iv) **Resistance :** The practical unit of resistance is **Ohm.**

$$1 \text{ Ohm} = \frac{1 \text{ volt}}{1 \text{ ampere}}$$

Besides, 10^6 ohms = 1 meg-ohm.
and 10^{-6} ohm = 1 micro-ohm.

B. Absolute Units :

(i) *E.S.U. of potential difference :* Two points have 1 e.s.u. of potential difference if one erg of work is done to move 1 e.s.u. of charge from one point to the other.

(ii) *E.M.U. of potential difference :* Two points have 1 e.m.u. of potential difference if one erg of work is done to move one e.m.u. of charge (*i.e.* 3×10^{10} e.s.u.) from one point to the other.

$\therefore$ 3×10^{10} e.m.u. of p.d. (or e.m.f.) = 1 e.s.u. of p.d. (or e.m.f.)

(iii) *E.S.U. of current :* The current flowing through a conductor is 1 e.s.u. if a charge of 1 e.s.u. flows through any section of the conductor in 1 second.

(iv) *E.M.U. of current :* The current flowing through a conductor is 1 e.m.u. if a charge of 1 e.m.u. (*i.e.* 3×10^{10} e.s.u.) flows through any section of the conductor in 1 second.

$\therefore$ 1 e.m.u. of current = 3×10^{10} e.s.u. of current.

(v) *E.S.U. of resistance :* A conductor has a resistance of 1 e.s.u. if a current of 1 e.s.u. flows through it under a p.d. of 1 e.s.u.

(vi) *E.M.U. of resistance :* A conductor has a resistance of 1 e.m.u. if a current of 1 e.m.u. flows through it under a p.d. of 1 e.m.u.

C. Relation between practical unit and the absolute electromagnetic unit : It is to be remembered that,

1 volt = 10^8 e.m.u. of potential
and 1 ampere = 10^{-1} e.m.u. of current

$$\therefore \quad 1 \text{ ohm} = \frac{1 \text{ volt}}{1 \text{ ampere}} = \frac{10^8}{10^{-1}} = 10^9 \text{ e.m.u. of resistance.}$$

2.5. International ampere, volt and ohm :

Ampere, volt and ohm, being the most popular units of current, potential difference and resistance respectively, they are defined by an international agreement in terms of easily realisable quantities, in the following way :

(i) **International ampere :** According to international agreement, 1 ampere is that steady current which passing through a silver nitrate solution, desposits ·001118 gm. of silver in 1 second.

International ampere is slightly less than *true ampere* which is $\frac{1}{10}$th of e.m.u. of current.

(ii) **International volt** : It is $\frac{1}{1{\cdot}0183}$ of the e.m.f. of a standard Weston Cadmium cell at 10°C.

International volt is slightly bigger than *true volt* which has been defined earlier.

(iii) **International ohm** : It is the resistance of a column of mercury of length 106·3 cm., sectional area 1 sq. mm. and mass 14·4521 gm. at 0°*C*. It is a little bigger than the *true ohm* which is equal to 10 e.m.u. of resistance.

2.6. Difference between electromotive force and potential difference :

In discussing electrical circuits, we shall very often use the terms 'potential difference' and 'e.m.f.' In art 2.4, it has been mentioned that they have the same unit. But it is to be borne in mind that they are not identical things. Before going into a detailed discussion of Ohm's law, we should realise clearly the difference between the two terms.

If at any part of a circuit, electrical energy is found to be produced at the cost of other forms of energy, then we say that an e.m.f. exists in that part. In other words, electromotive force may be supposed to be such a source which converts other forms of energy into electrical energy. Positive and negative electricity become separated as a result of the electromotive force and the separated charges, then possess some potential energy which creates a difference of potential between them. Potential difference between the terminals of a cell, for example, when the cell is in **open circuit**, is called the electromotive force of the cell.

On the other hand, if at any part of a circuit, electrical energy is converted into other forms of energy, then we say that a potential difference exists in that part. In passing through the potential difference, the electrical potential energy of the charges disappears giving rise to heat energy, mechanical energy, chemical energy and so on.

In brief, it may be said that potential difference created between the plates of a cell as a result of the chemical action in the cell, is called the e.m.f. of the cell. But as soon as the cell drives a current through a circuit, the potential difference existing between its plates, diminishes a little due to the current passing through the resistance of the electrolyte of the cell (this is known as the "internal resistance" of the cell). This potential difference between the plates is usually called the *p.d.* of the cell. Hence it is clear that *p.d.* of a cell is slightly less than its e.m.f. It is true not only in the case of cells but in all other cases involving e.m.f. and p.d.

Further, if e.m.f. be regarded as the cause, the p.d. is its effect.

2.7. Resistance and laws of resistance :

The term 'resistance' is very important in current electricity. Its significance will be clear if we take the example of water flowing through a pipe as mentioned in art 2.1.

We have seen that if there be a pressure difference at the two ends of the pipe, water will flow continually through it. If, now, keeping the pressure-

difference same, the pipe be made wider or narrower or longer or shorter, will the rate of flow of water remain same ? It is easy to realise that the rate of flow of water depends upon the cross-section and the length of the pipe. If the pipe be wider (*i.e.* cross-section bigger), the rate of flow increases and if the pipe be longer, the rate of flow decreases. In other words, it may be said that the flow of water encounters less resistance in a wider pipe but greater resistance in a longer pipe.

Same thing happens when electric current flows through a wire. The strength of the current obviously depends on the cross-section and the length of the wire. Current encounters less resistance in a thicker wire but greater resistance in a longer wire.

Laws of resistance : If a wire has length l, cross-sectional area A and resistance R then,

(i) $R \propto l$ when the cross-section A is kept unaltered. Wires of same material and equal cross-section but of different length, have, therefore, different resistance which is proportional to the lengths of the wires.

(ii) $R \propto \frac{1}{A}$ when the length l is kept unaltered. Wires of same material and equal length but of different cross-section, have different resistance which is inversely proportional to the cross-section of the wire.

(iii) Wires of equal length and cross-section but made of different materials have different resistances.

$$\text{Hence, } R \propto \frac{l}{A} \quad \text{or,} \quad R = \rho . \frac{l}{A} \ [\rho = \text{a constant}]$$

The constant 'ρ' is called the *specific resistance* or the *resistivity*. It depends upon the material of the conductor. The resistivity of a material is increased by even a small amount of impurity ; and alloys, such as constantan, may have resistivities far greater than any of their constituents.

Definition of specific resistance : In the above formula, if we put $l=1$ and $A=1$, then $R=\rho$ *i.e.* the specific resistance of a material is the resistance of 1 cm. length of the material when the area of cross-section is 1 sq. cm. In other words, it is the resistance between the opposite faces of a cube of the material, having each side equal to one centimetre. For example, the specific resistance of copper is $1{\cdot}62 \times 10^{-6}$—this means that a cube of copper having length, breadth and height each equal to 1 cm. has a resistance $1{\cdot}62 \times 10^{-6}$ ohm between its opposite faces.

Unit of specific resistance : We can find the unit of specific resistance in the following way :

We know $R = \rho \times \frac{l}{A}$; Putting the units of different quantities we have,

$$R \ \text{(ohm)} = \rho \times \frac{l \text{ (cm.)}}{A. \text{(sq. cm.)}} \quad \therefore \rho = \frac{R \text{ (ohm)} \times A \text{ (sq. cm.)}}{l \text{ (cm.)}} = \frac{RA}{l} \text{ ohm. cm.}$$

Example 1 : (1) *A metal wire of radius* 3 *mm. and length* 31·4 *cm. has a resistance of* $0{\cdot}2 \times 10^{-3}$ *ohm. Determine the resistivity of the metal.*

[*H. S. Exam. 1978*]

Ans. We know $R = \rho . \frac{l}{A}$ $\therefore$ $\rho = \frac{R.A}{l}$

Here, $R = 0{\cdot}2 \times 10^{-3}$ ohm ; $l = 31{\cdot}4$ cm. ; $A = \pi r^2 = \pi(0{\cdot}3)^2$ sq. cm.

$$\therefore \quad \rho = \frac{0{\cdot}2 \times 10^{-3} \times \pi(0{\cdot}3)^2}{31{\cdot}4} = \frac{0{\cdot}2 \times 10^{-3} \times 3{\cdot}14 \times (0{\cdot}3)^2}{31{\cdot}4} = 18 \times 10^{-7} \text{ ohm. cm.}$$

Example 2 : *The ratio of the resistances of two wires A and B was* 1·2. *The wire A was* 1·2 *metre long and had a resistivity* 100×10^{-6} *ohm.-cm. Its diameter was* 1·2 *mm. The wire B had* 0·8 *mm. diameter and a resistivity* 28×10^{-6} *ohm-cm. What was the length of the wire B* ?

Ans. With usual notation, for the wire A, we have $R_1 = \rho_1 \frac{l_1}{A_1}$ and for the wire B, $R_2 = \frac{\rho_2 l_2}{A_2}$

$\therefore$ $\frac{R_1}{R_2} = \frac{\rho_1}{\rho_2} . \frac{l_1}{l_2} . \frac{A_2}{A_1} = \frac{\rho_1}{\rho_2} . \frac{l_1}{l_2} . \frac{d_2^2}{d_1^2}$ [d_1 and d_2 are the diameters of A and B respectively]

$$\therefore \quad 1{\cdot}2 = \frac{100}{28} \times \frac{1{\cdot}2}{l_2} \times \left(\frac{0{\cdot}8}{1{\cdot}2}\right)^2 \quad \therefore \quad l_2 = \frac{100 \times 1{\cdot}2}{28 \times 1{\cdot}2} \times \frac{64}{144} = 1{\cdot}59 \text{ metres}$$

Example 3 : *Each of length, diameter and sp. resistance of two wires are in the ratio* 1 : 3. *If the resistance of the thinner wire is* 20 *ohms., find the resistance of the other wire.* [*Jt. Entrance 1982*]

Ans. The resistance R_1 of the thinner wire $R_1 = \frac{\rho_1 l_1}{\pi r_1^2}$ and that of the thicker wire $R_2 = \frac{\rho_2 l_2}{\pi r_2^2}$ $\therefore$ $\frac{R_1}{R_2} = \frac{\rho_1}{\rho_2} . \frac{l_1}{l_2} . \left(\frac{r_2}{r_1}\right)^2 = \frac{1}{3} \times \frac{1}{3} \times \left(\frac{3}{1}\right)^2 = 1$ $\therefore$ $R_1 = R_2$

So, the resistance of the thicker wire = 20 ohms.

2.8. Resistor. conductor and conductance :

A substance which allows electric current to flow through it easily is called a conductor. No substance is, however, a perfect conductor. Every substance has an ability to resist the flow of electricity through it. If the resistance of a conductor is utilised to diminish the current in a circuit, it is better to call it a *resistor* than a conductor.

For metallic wires of given dimensions (*i.e.* length and cross-section) silver offers least resistance to the current. Silver, being too costly for ordinary use, copper which comes next to silver as a conductor, is widely used for preparing connecting wires etc. In order to reduce the current in a circuit appreciably, high resistances are sometimes required. To prepare such high resistances, special alloys like eureka (60% copper and 40% nickel), manganin (84% copper, 12% manganese and 4% nickel) and nichrome (80% nickel and 20% chromium) are used. Eureka and manganin have a resistance about 25 times, and nichrome about 60 times that of copper.

Conductance : Current flows very easily through a substance whose resistance is very low. For this reason the quality of a material opposite to its resistance is called *conductance.*

Resistance of a wire is defined as $R=\frac{V}{I}$ where V is the p.d. across the wire and I the current in it. Hence conductance of the wire is defined as $S=\frac{I}{V}$. The unit of conductance is *Siemens*, symbol S.

A quantity opposite to the specific resistance is called the *specific conductivity.* The unit used to denote specific conductivity is *mho* (ohm spelled in a reverse direction). For example, if a copper wire has resistance of ·01 ohm, its conductivity is $\frac{1}{\cdot 01}=100$ siemens. If, again, the specific resistance of copper be 2×10^{-6} ohm. cm., its specific conductivity is $\frac{1}{2\times10^{-6}}=0{\cdot}5\times10^{6}$ mho-cm.

Since conductance is a property opposite to the resistance, the factors which increase the resistance of a body, evidently decrease the conductance of it.

2.9. Effect of different factors on resistance :

In general, the resistance of a conductor is little affected by surrounding conditions other than temperature. There are, however, some special cases which are stated below :—

(i) **Effect of temperature** : The resistance of a conductor, in general, increases with the increase of temperature, the relation being $R_t=R_0\ (1+\alpha t)$ where R_t=the resistance of the conductor at $t°C$, R_0=resistance of the conductor at $0°C$ and α=a constant, known as the *temperature co-efficient of resistance.*

The temperature co-efficient of resistance of a resistor is defined as the increase of resistance per unit resistance for $1°C$ *rise in temperature.*

The resistance of some substances like carbon, vulcanised India rubber (V.I.R.) etc., decreases with the increase of temperature. The resistance of a carbon filament lamp in cold condition becomes almost halved when brought to incandescence. The resistance of V.I.R. at $0°C$ is almost four times that at 24°C. For this reason, the temperature co-efficient of resistance is regarded positive for metals but negative for carbon, V.I.R. and electrolytes.

The resistance of semi-conductors also decreases with the increase of temperature.

Example 1 : *Ratio of two resistances—one made of nichrome and the other of german silver—is* 2·06 *at* 0°C. *If the temperature coefficient of nichrome is* $4\times10^{-4}\ C^{-1}$ *and of german silver* $3\times10^{-4}C^{-1}$, *what would the ratio of the resistances become if the temperature were raised by* 100°C ?

Ans. When the temperature is raised the resistance increases according to the equation : $R_t=R_0\ (1+\alpha t)$. Thus, the new resistance of nichrome wire at

$100°C$, is $R_A = R_0^1(1+4\times10^{-4}\times100) = R_0^1\times1{\cdot}04$. Similarly, the new resistance of german silver wire is $R_B = R_0^2(1+3\times10^{-4}\times100) = R_0^2\times1{\cdot}03$

$$\therefore \quad \frac{R_A}{R_B} = \frac{R_0^1}{R_0^2}\times\frac{1{\cdot}04}{1{\cdot}03} = \frac{2{\cdot}06\times1{\cdot}04}{1{\cdot}03} = 2{\cdot}08$$

Example 2 : *The resistance of a wire measured by a Wheatstone bridge was found to be* 5 *ohms in melting ice. When the coil was heated to* $100°C$, *a* 100 *ohm resistor had to be connected prallel to it in order to keep the bridge balanced at the same point. Calculate the temperature coefficient of resistance of the coil.*

[*H. S. Exam. 1982*]

Ans. Here, $R_0=5$ ohm ; suppose R_{100}=resistance of the coil at $100°C$. Then $R_{100}=R_0(1+\alpha.t)$ or $R_{100}=5(1+\alpha t)$.

When R_{100} ohm and 100 ohm are in parallel, their equivalent resistance $=\dfrac{100\times R_{100}}{R_{100}+100}$. As the null point remains unchanged, we may write

$$\frac{100\times R_{100}}{R_{100}+100}=5 \quad \text{or} \quad R_{100}=\frac{500}{95} \text{ ohm}$$

$$\therefore \quad \frac{500}{95}=5\,(1+\alpha.t) \quad \text{or} \quad 1+\alpha.t=\frac{100}{95} \quad \text{or} \quad 1+100.\alpha=\frac{100}{95}$$

$$\therefore \quad \alpha=\frac{5}{95}\times10^{-2}=5{\cdot}3\times10^{-4}\ \text{ohm}/°C.$$

(ii) **Effects of light, magnetic field and pressure :**

Light, magnetic field and pressure have some effects on the resistance of a conductor and all these effects have important practical applications.

The resistance of selenium, a semi-matallic element, is found to diminish when light falls on it. The more the intensity of light the more is the diminution of the resistance. A piece of selenium has, therefore, the highest resistance in darkness. The circuit current can be regulated conveniently by using a selenium and controlling the intensity of light incident on it. For this reason, selenium cell is used for automatic street-light, alarm signals etc. Now-a-days, of course, photo-electric cells are found to be more effective than selenium cells.

In bismuth, the resistance is changed by a magnetic field. The stronger the magnetic field, the higher is the resistance. Utilising this effect, a method has been devised for the determination of the intensity of magnetic field.

The resistance of carbon granules decreases when pressure is applied on them. For this reason, carbon granules are used in carbon microphone.

Example : 1 *Find the resistance of a cubic centimetre of copper* (*a*) *when drawn into a wire of* 0·32 *mm. diameter and* (*b*) *when hammered into a flat sheet of thickness* 1·2 *mm. the current flowing perpendicularly through the sheet from one face to the other. Sp. resistance of copper*$=1{\cdot}59\times10^{-6}$ *ohm. cm.*

Ans. (i) we know, $R=\rho.\dfrac{l}{A}$ Here, $A=\pi r^2$; again, $\pi r^2\times l=1$ c.c. $\therefore\ l=\dfrac{1}{\pi r^2}$

Hence, $R=\rho.\dfrac{1}{(\pi r^2)^2}=\dfrac{1{\cdot}59\times10^{-6}}{\{3{\cdot}14\times({\cdot}016)^2\}^2}=2{\cdot}458$ ohms.

(ii) Since current is flowing perpendicularly from one face to another, $l=0{\cdot}12$ cm ; Also, $A\times l=1$ c.c. $\therefore\ A=\frac{1}{l}$

Hence, $R=\rho.\frac{l}{\frac{1}{l}}=\rho.l^2=1{\cdot}59\times10^{-6}\times(0{\cdot}12)^2=2{\cdot}3\times10^{-8}$ ohm.

Example 2 : *A wire has a resistance of* $1{\cdot}2$ *ohms per metre and a specific resistance of* $47{\cdot}5\times10^{-6}$ *ohm. cm. What is its diameter ?*

Ans. We know, $R=\rho.\times\frac{l}{A}$

If 'd' be the diameter, $A=\frac{\pi d^2}{4}$; Hence $R=\rho\times\frac{4l}{\pi d^2}$

Here, $R=1{\cdot}2$ ohms ; $\rho=47{\cdot}5\times10^{-6}$; $l=1$ metre$=100$ cm.

$$\therefore\quad 1{\cdot}2=\frac{47{\cdot}5\times10^{-6}\times4\times100}{3{\cdot}14\times d^2}$$

or $d^2=\frac{47{\cdot}5\times10^{-6}\times4\times100}{3{\cdot}14\times1{\cdot}2}$ or, $d=0{\cdot}071$ cm.

Specific rsesistance of some materials

(In ohm.-cm.)

Materials	Sp. resistance	Materials	Sp. resistance
Aluminium	$3{\cdot}2\times10^{-6}$	Brass	$7-9\times10^{-6}$
Copper	$1{\cdot}8\times10^{-6}$	German silver	$15-40\times10^{-6}$
Platinum	$10{\cdot}8\times10^{-6}$	Eureka	$49-52\times10^{-6}$
Silver	$1{\cdot}6\times10^{-6}$	Manganin	$0{\cdot}42\times10^{-6}$
Mercury	$94{\cdot}1\times10^{-6}$	Tungsten	$5{\cdot}5\times10^{-6}$
Iron	$9{\cdot}8\times10^{-6}$	Nichrome	100×10^{-6}

2.10. Internal resistance of a cell and lost volt :

When a cell sends a current through a *closed* circuit, the current flows through the external circuit as well as through the active liquid of the cell. Inside the cell, the current flows from the negative pole to the positive pole and in doing so, the current has to overcome the resistance of the liquid between the plates. This resistance is known as the *internal resistance* of the cell. The strength of the current which is available from a cell, depends not only on its e.m.f. but also on the internal resistance. In order to obtain a large current, the internal resistance of the cell must be low.

Suppose A is the positive pole and B the negative pole of a cell [fig. 2.4]. The cell is sending current I through an external resistor R. The current in the external circuit flows from the positive pole A to the negative pole B but inside the cell, it flows through the active liquid from the negative pole B to the positive pole A. Consequently a drop of potential $I.r.$ will take place across the internal resistance of the cell which will reduce the external e.m.f. E by an equal amount. So, the terminal p.d. across the external resistor $= E-I.r.$ This relation shows that greater the internal resistance of the cell, less is the driving force or the potential differnce which drives current through the external circuit.

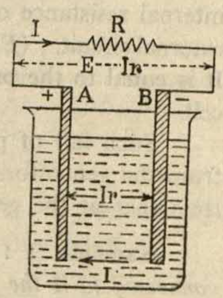


Fig. 2.4

The internal resistance of a cell depends upon the distance between the plates, the sizes of the plates, the nature of the active liquid and a few other factors. If the plates of the cell are large in size and are placed close to each other, the internal resistance becomes low. The following are the values of the internal resistance of some widely used cells :

Leclanche's cell→between 1 ohm and 5 ohms.
Dry cell→between 0·1 ohm and 0·5 ohm.
Daniel cell→between 1 ohm and 6 ohms.
Storage cell→nearly 0·01 ohm.

Lost volt : The internal resistance of a cell behaves as if it is connected in series with the cell. Suppose, a cell, of e.m.f. E and internal resistance r is connected to a resistor of resistance R (Fig. 2.5). In this case, this total resistance of the circuit may be supposed to be made up of the external resistance R and the internal resistance r of the cell connected in series. If I be the current in the circuit, then from Ohm's law, we have,

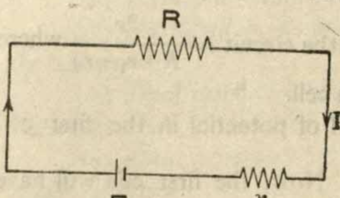


Fig. 2.5

$I=\dfrac{E}{R+r}$ because the circuit resistance$=R+r$.

$\therefore$ $E=I.R+I.r=V+I.r.$, where V is the terminal $p.d.$ of the cell. Since the e.m.f. (E) of a cell and its internal resistance (r) remain fairly constant, the terminal $p.d.$ (V) of the cell depends upon the current (I) and in no case, can exceed the e.m.f. (E) of the cell.

When $I=0$, $V=E$ *i.e.* when there is no current in the circuit the terminal *p.d.* becomes equal to the e.m.f. of the cell. For this reason, the e.m.f. of a cell *is generally referred to as the p.d. of the cell when the cell is in open circuit.*

From the equation mentioned above we also have,

$$I.r=E-V \text{ or } r=\frac{E-V}{I}.$$

From the equation we can find the internal resistance of a cell.

Now, E is the terminal *p.d.* of the cell in open circuit and V is the *p.d.* of the

cell in closed circuit. The *e.m.f.* $(E-V)$, therefore sends current through the internal resistance of the cell. This current does not do any useful work in the external circuit. $(E-V)$ is referred to as **the internal fall of potential** or **lost volt**. It is equal to the product of the current (I) and the internal resistance (r) of the cell.

Such fall of potential is also found along a power transmission line away from the generator. For this reason, the *p.d.* at the receiving end is always less than that at the generating end.

Example 1 : *The e.m.f. of a cell is 2 volt ; but when a resistor of 10 ohms is connected to it the p.d. between the terminals of the cell becomes 1·6 volts. Calculate the internal resistance of the cell and the lost volt.*

Ans. Suppose, the current in the circuit is I.

So, $I=\frac{1\cdot6}{10}=0\cdot16$ amperes. Now, we know, $r=\frac{E-V}{I}$;

Here, $E=2$ volts ; $V=1\cdot6$ volts and $I=0\cdot16$ amp.

Hence, $r=\frac{2-1\cdot6}{0\cdot16}=\frac{0\cdot4}{0\cdot16}=2\cdot5$ ohms.

Also, lost volt$=I.r=0\cdot16\times2\cdot5=0\cdot4$ volt.

Example 2 : *Two cells each of same e.m.f. but of internal resistances r_1 and r_2 are connected in series through an external resistance R. Find the value of R in terms of r_1 and r_2 for which the first cell will have zero p.d. across it.*

[*Jt. Entrance 1984*]

Ans. According to Fig. 2.5(i), the current in the circuit $i=\frac{2e}{R+r_1+r_2}$ where e is the e.m.f. of each cell.

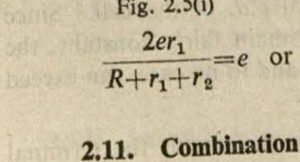


Fig. 2.5(i)

The internal fall of potential in the first cell $=i\times r_1=\frac{2er_1}{R+r_1+r_2}$. Now, the first cell will have zero p.d. across it, when the internal fall of potential is equal to its e.m.f. Hence for zero p.d.

$$\frac{2er_1}{R+r_1+r_2}=e \quad \text{or} \quad R+r_1+r_2=2r_1 \quad \text{or} \quad R=r_1-r_2$$

2.11. Combination of resistors :

Occasion arises when a number of resistors are to be used together in a combination, known as the combination of resistors. All these resistors combined together behave like a single resistor, called the *equivalent resistor*, which maintains the same current or the same potential difference of the circuit as before. Resistors can be combined in two ways : (i) **Series combination** and (ii) **Parallel combination.**

(i) **Series combination** : A number of resistors are said to be connected

in series if they are connected end to end consecutively so that the same current flows through each.

Fig. 2.6(i) shows three resistors r_1, r_2, r_3 connected in series. The same current I passes through each. Their equivalent resistance can be found out in the following way :

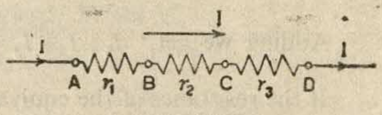


Fig. 2.6(i)

Suppose, the potentials at A, B, C etc. be V_A, V_B, V_C etc. respectively. Considering the points A and B, we can write, according to Ohm's law,

$$V_A - V_B = I.r_1$$

Similarly, $V_B - V_C = I.r_2$

and $V_C - V_D = I.r_3$

Adding, $V_A - V_D = I\,(r_1 + r_2 + r_3)$

If R be the resistance of the equivalent resistor, then, $V_A - V_D = I.R.$,

So, $I.R. = I.(r_1 + r_2 + r_3)$ $\therefore$ $R = r_1 + r_2 + r_3$

For a large number of resistors connected in series, we can write, in general, $R = r_1 + r_2 + r_3 + r_4 + \ldots\ldots\ldots\ldots$

Hence, by adding individual resistances, we get the equivalent resistance.

In connection with the series combination of resistors, the following points are to the noted :

(i) *Current is same through all the resistors.*

(ii) *Total p.d.=sum of individual p. d's across the individual resistors.*

(iii) *Individual p. d. is directly proportional to the individual resistance of the resistors.*

(iv) *Total resistance of the combination is greater than the greatest individual resistance.*

(v) *Total resistance of the combination=sum of the resistances of the individual resistors.*

(ii) **Parallel combination** : A number of resistors are said to be connected in parallel when they are placed side by side and their corresponding ends joined together so that the main current is distributed among them.

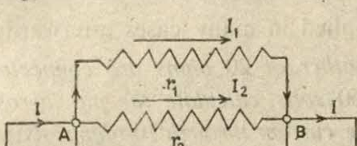


Fig. 2.6(ii)

Figl 2.6 (ii) shows three resistors r_1, r_2, r_3 connected in parallel. Corresponding ends of the resistors are connected to the points A and B. The main current I, arriving at A divides into I_1, I_2 and I_3 through the resistors r_1, r_2, and r_3 respectively and combine again into the main current I at the point B. The equivalent resistance may be calculated in the following way.

Suppose the potentials at A and B are respectively Va and Vb. Since the ends of each resistor are connected to A and B the *p.d.* at the ends of each resistor is $(Va - Vb)$.

So, according to Ohm's law, $I_1=\dfrac{Va-Vb}{r_1}$; $I_2=\dfrac{Va-Vb}{r_2}$; $I_3=\dfrac{Va-Vb}{r_3}$

Adding we get, $I_1+I_2+I_3=I=(Va-Vb)\left(\dfrac{1}{r_1}+\dfrac{1}{r_2}+\dfrac{1}{r_3}\right)$

If the resistance of the equivalent resistor is R and if it is connected between A and B, the circuit current will remain unchanged. Hence, for the equivalent resistor, $I=\dfrac{Va-Vb}{R}$

$$\therefore \quad \frac{V_a-V_b}{R}=(V_a-V_b)\left(\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}\right) \quad \text{or,} \quad \frac{1}{R}=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}$$

For any number of resistors, it may be written in general that

$$\frac{1}{R}=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}+\ldots\ldots$$

i.e. sum of the reciprocals of individual resistances is equal to the reciprocal of the equivalent resistance.

In connection with parallel combination of resistors, the following points are to be noted :

(*i*) *P.D. across each resistor is the same.*

(*ii*) *Total current=sum of currents passing through individual resistors.*

(*iii*) *Individual currents are inversely proportional to individual resistances.*

(*iv*) *Equivalent resistance of the combination is less than the smallest individual resistance.*

(*v*) *Reciprocal of the equivalent resistance is the sum of the reciprocals of invidual resistances.*

If only two resistors r_1 and r_2 are connected in parallel, then their equivalent resistance R is given by.

$$\frac{1}{R}=\frac{1}{r_1}+\frac{1}{r_2}=\frac{r_1+r_2}{r_1 r_2} \quad \text{or} \quad R=\frac{r_1.r_2}{r_1+r_2}$$

i.e. equivalent resistance $=\dfrac{\text{product of the two resistances}}{\text{sum}\quad ,,\quad ,,\quad ,,\quad ,,}$

This relation is very useful as it will be applied in many cases afterwards.

Example 1 : *A resistor of* 10 *ohms and another of* 20 *ohms are connected in series. If the p.d. across the combination be* 60 *volts, calculate the p.d. across each and the current through* 10-*ohm resistor.*

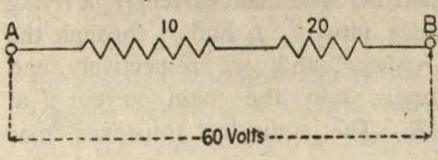


Fig 2.6(iii)

Ans. The resistance between the ends A and $B=(10+20)$ ohms$=30$ ohms [Fig.2.6(iii)]. So according to Ohm's law, current through each resistor, $I=\frac{60}{30}=2$ amp.

Hence, *p.d.* across the 10-ohm resistor$=10\times2=20$ volts.

and .. .. ..20-ohm .. $20\times2=40$ ..

Example 2 : *Between two points X and Y, are connected, in parallel, two resistors* 12 *ohms and* 8 *ohms. The main current in the circuit is* 10 *amperes. Calculate the current in each resistor and the p.d. between X and Y.*

Ans. If the equivalent resistor R be put between X and Y instead of the two resistors, the circuit current will remain 10 amperes. So, the p.d. between X and $Y=10\times R$ [Fig. 2.6(iv)]

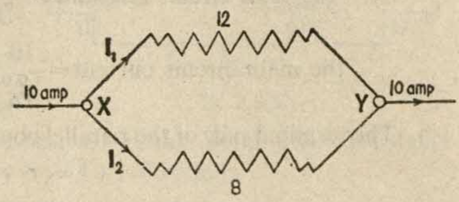


Fig. 2.6(iv)

Now, we know, $\frac{1}{R}=\frac{1}{12}+\frac{1}{8}=\frac{5}{24}$

$\therefore \quad R=\frac{24}{5}$ ohms.

Hence, p.d. between the points X and Y is $=10\times\frac{24}{5}=48$ volts.

Now, current through 12 ohm resistor $=\dfrac{\text{P.D. between } X \text{ and } Y}{12}=\dfrac{48}{12}$

$=4$ amp.

So, current through 8-ohm resistor $=10-4=6$ amp.

Example 3 : *The electrical resistance of a piece of steel wire of diameter* 1 *cm. is reduced to* $\frac{1}{3}$ *of its value by uniformly coating it with copper. What is the thickness of copper coating ? Given specific resistance of copper*$=1{\cdot}8\times10^{-6}$ *ohm-cm. and sp. resistance of steel*$=1{\cdot}98\times10^{-5}$ *ohm-cm.*

Ans. In this case, the resistance of the steel wire is in parallel with the resistance of the copper coating. If R be the resistance of the steel wire and x that of the copper coating, then according to the question,

$\frac{3}{R}=\frac{1}{R}+\frac{1}{x} \quad \therefore \frac{1}{x}=\frac{2}{R}$ or, $x=\frac{R}{2}$ .. ..(i) Now, we know, $R=\rho\times\frac{l}{A}$

For the steel wire, $R=\dfrac{1{\cdot}98\times10^{-5}\times l}{\pi(0{\cdot}5)^2}$

and for the copper coating, $x=\dfrac{1{\cdot}8\times10^{-6}\times l}{2\pi(0{\cdot}5)\times d}$ [d=thickness of the coating]

From eqn. (i) we have, $\frac{R}{2}=x$.

or, $\dfrac{1{\cdot}98\times10^{-5}\times l}{2\pi(0{\cdot}5)^2}=\dfrac{1{\cdot}8\times10^{-6}\times l}{2\pi(0{\cdot}5)\times d}$

or, $d=\dfrac{1{\cdot}8\times0{\cdot}5}{1{\cdot}98\times10}=0{\cdot}045$ cm. (nearly)

Example 4. *A cell of e.m.f.* 10 *volts and internal resistance* 1 *ohm is connected in series with a parallel combination of three resistors* 3, 5 *and* 8 *ohms respectively. What will be current through each of the resistors ?* [*H.S. Exam, 1978*]

Ans. The equivalent resistance of the three resistors connected in parallel is given by, $\frac{1}{R}=\frac{1}{3}+\frac{1}{5}+\frac{1}{8}=\frac{79}{120}$ $\therefore$ $R=\frac{120}{79}$ ohms.

So, the total circuit resistance$=\frac{120}{79}+1=\frac{199}{79}$ ohms

$\therefore$ the main circuit current$=\frac{10}{\frac{199}{79}}=\frac{790}{199}$ amperes.

The terminal p.d. of the parallel combinatiôn

$$=\text{main current}\times\text{equivalent resistance}$$

$$=\frac{790}{199}\times\frac{120}{79}=\frac{1200}{199}\text{ volts.}$$

Hence, current through first resistor$=\frac{1200}{199\times 3}=2$ amp. (approx)

" " second " $=\frac{1200}{199\times 5}=1{\cdot}2$ " "

" " third " $=\frac{1200}{199\times 8}=0{\cdot}75$ " "

(iii) **Equivalent resistance of a mixed circuit** : Sometimes, we are to deal with circuits where the resistors are connected neither in series nor in parallel but in a mixed way. Fig. 2.6(v) shows such a mixed circuit. In the circuit ABCD, there is a resistor r_1 between A and B, two resistors r_2 and r_3 in parallel between B and C and another resister r_4 between C and D. The main current arriving at A is shared between the resistors as shown in the figure.

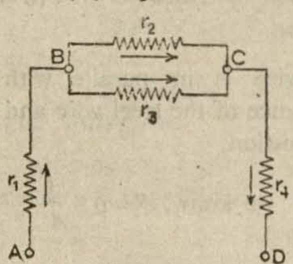


Fig. 2.6(v)

Analysing the circuit, it may be said that it is composed of three resistors *in sreies* and these three resistors are (i) r_1, (ii) r_4 and (iii) the equivalent resistor of r_2 and r_3. Now, the

equivalent resistance of r_2 and $r_3=\frac{\text{Product of the resistances}}{\text{Sum } \text{,, ,, ,,}}=\frac{r_2\times r_3}{r_1+r_3}$

So, the equivalent resistance of the mixed circuit$=r_1+r_4+\frac{r_2\times r_3}{r_2+r_3}$.

Example 1 : *Calculate the total resistance R and the main current I in the mixed circuit shown in fig.* 2.6(vi). *The e.m.f. of the cell E*=6 *volts and its internal resistance r*=1 *ohm.*

Ans. 6 ohm and 12 ohm resistors are connected between A and B in parallel. Their equivalent resistance $R_1=\frac{6\times 12}{6+12}=4$ ohm. So, we can consider

a resistor of 4 ohm existing between A and B. Now between C and D, the 4 ohm resistor and the resistor R_1 (=4 ohm) are in series while 8 ohm resistor is in parallel with them. The combined resistance of the two resistors in series is $R_2=4+4=8$ ohms. So the equivalent resistance between C and D is given by $R_3=\frac{R_2\times 8}{R_2+8}=\frac{8\times 8}{8+8}=4$ ohms. Now we can say that only a resistor R_3 (=4 ohms) is connected between the terminals of the battery. Hence, total circuit resistance $R=R_3+$internal resistance of the battery $(r)=4+1=5$ ohm.

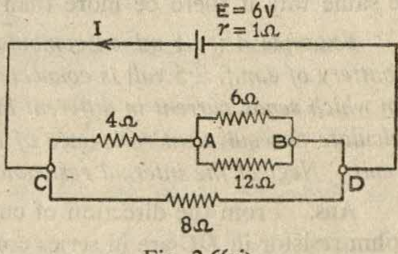


Fig. 2.6(vi)

$\therefore$ Main circuit current $I=\frac{E}{R}=\frac{6}{5}=1{\cdot}2$ amp.

Example 2 : *The values of the resistors in fig.* 2.6 (vii) *are in ohms. Find the equivalent resistance between A and B.* [*I.I.T. 1979*]

Ans. Between A and C, the 3Ω and 3Ω resistors are in series, giving a resistor $R_1=3+3=6$ ohms and the 6 ohm resistor in parallel. If the equivalent resistance be R_2 then $1/R_2=1/R_1+1/6=1/6+1/6=1/3$ $\therefore$ $R_2=3\Omega$. Now we can say that between A and C, there is only one resistor R_2 of value 3 ohms. Considering the points A and D, the aforesaid resistor R_2 and the resistor 3 ohms are in series, giving a total resistor $R_3=R_2+3=3+3=6$ ohms and a 6Ω resistor in parallel. So, if R_4 be their equivalent resistance, then $\frac{1}{R_4}=\frac{1}{R_3}+\frac{1}{6}=\frac{1}{6}+\frac{1}{6}=\frac{1}{3}$ $\therefore$ $R_4=3$ ohms. This means that between A and D, there is only one resistor R_4 of value 3 ohms. Similarly, between A and E, we will have only one resistor R_5 of value 3 ohms. Lastly between A and B, the resistor R_5 (=3 ohms) and EB (=3 ohms) being in series, they give a resistor $R_6=3+3=6$ ohms, which is in parallel with the resistor AB (=3 ohms). Hence the final equivalent resistance R of the circuit is $1/R=1/R_6+\frac{1}{3}=\frac{1}{6}+\frac{1}{3}=\frac{1}{2}$ $\therefore$ $R=2$ ohms.

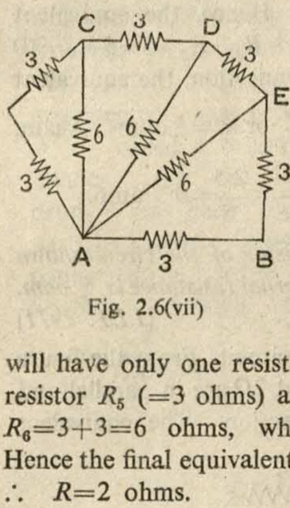


Fig. 2.6(vii)

Example 3 : *If a number of resistances be connected in parallel. show that the equivalent resistance will be less than the smallest resistance of the combination.*

Ans. Suppose r_1, r_2 and r_3 are three resistors in parallel $(r_3<r_2<r_1)$. Now the equivalent resistance of r_1 and r_2 in parallel is $R_1=\frac{r_1r_2}{r_1+r_2}=\frac{r_2}{r_1+r_2}.r_1$. Since r_1 and r_2 are positive it is clear from the above relation that $R_1<r_1$. Again, the equivalent resistance of R_1 and R_3 in parallel is $R_2=\frac{R_1.r_3}{R_1+r_3}=\frac{R_1}{R_1+r_3}.\ r_3$. It shows that $R_2<r_3$. So, $R_2<r_3<r_2<r_1$ *i.e.* the equivalent resistance is less than

r_3, the smallest resistance of the combination. The above fact can be proved in the same way if there be more than three resistors in the combination.

Example 4 : *A mixed combination of resistors has been shown in fig.* 2.6(viii) *A battery of e.m.f.* 2·5 *volt is connected between the points A and B of the combination which sends current in different branches in the directions shown by arrow heads. Calculate the equivalent resistance of the combination and the current in the battery circuit. Neglect the internal resistance of the battery.*

Ans. From the direction of current, it is clear that 3 ohm resistor in AD and 7 ohm resistor in DC are in series connection. So, their total resistance $R_1=3+7=10$ ohms. Again, the resistor R_1 and the resistor 10 ohms in AC, being in parallel connection, give an equivalent resistance R_2 such that $\frac{1}{R_2}=\frac{1}{R_1}+\frac{1}{10}=\frac{1}{10}+\frac{1}{10}=\frac{1}{5}$ or $R_2=5$ ohms. So, it may be considered that there is only one resistor of R_2 (=5 ohms) between A and C. According to the direction of current, this resistor R_2 and the resistor 5Ω in CB are in series combination. Hence, the equivalent resistance of these two resistors $R_3=R_2+5=5+5=10$ ohms. As R_3 and 10 ohms resistor in AB are in parallel connection, the equivalent resistance of the circuit R is $\frac{1}{R}=\frac{1}{R_3}+\frac{1}{10}=\frac{1}{10}+\frac{1}{10}=\frac{1}{5}$ or $R=5$ ohms. Again, the current in the battery circuit $i=\frac{\text{e.m.f.}}{\text{equivalent resistance}}=\frac{2\cdot5}{5}=0\cdot5$ amp.

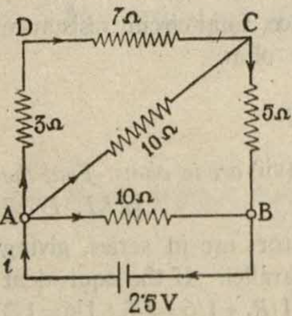


Fig. 2.6(viii)

Example 5 : *Find the current through the* 3 *ohm resistor of the circuit shown in fig* 2.6(ixa). *The e.m.f. of the battery is* 1·8 *volts and internal resistance is* $\frac{2}{3}$ *ohm.* [*I.I.T.* 1971]

Ans. Fig. 2.6(ixb) shows the simplified form of the circuit. From the figure it is clear that between A and B, two resistance of 8Ω and 2Ω are in parallel and between B and C, 4Ω and 6Ω are in parallel connection. The equivalent

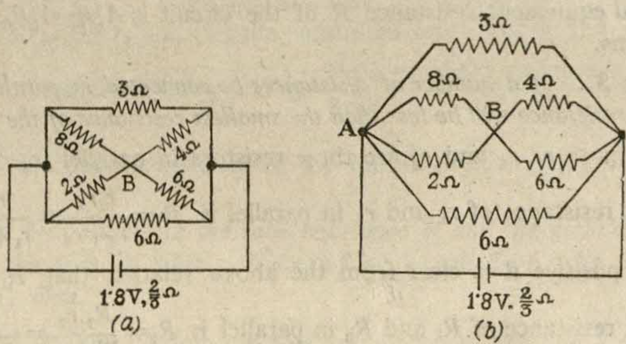


Fig. 2.6(ix)

resistor R_1 of the first is $\frac{1}{R_1}=\frac{1}{8}+\frac{1}{2}=\frac{5}{8}$ or $R_1=\frac{8}{5}=1\cdot6\ \Omega$ and that of the second is $\frac{1}{R_2}=\frac{1}{4}+\frac{1}{6}=\frac{5}{12}$ or $R_2=\frac{12}{5}=2\cdot4\ \Omega$. So, total resistance between A and C is $R_3=1\cdot6+2\cdot4=4\Omega$.

Again since, 3Ω, $R_3(=4\Omega)$ and 6Ω are in parallel, their equivalent resistance R is $\frac{1}{R}=\frac{1}{3}+\frac{1}{R_3}+\frac{1}{6}=\frac{1}{3}+\frac{1}{4}+\frac{1}{6}=\frac{9}{12}$ $\therefore$ $R=\frac{12}{9}=1\frac{1}{3}$ ohm. So, total circuit resistance$=R+$internal resistance of the battery$=\frac{12}{9}+\frac{2}{3}=\frac{18}{9}=2$ ohms.

$$\text{Circuit current } i=\frac{\text{e. m. f.}}{\text{circuit resistance}}=\frac{1\cdot8}{2}=0\cdot9 \text{ ohms.}$$

Again, the total resistance between A and $C=R=\frac{12}{9}$ ohm. and the total circuit current$=0\cdot9$ amp. So, the p.d. between A and $C=R\times i=\frac{12}{9}\times0\cdot9=1\cdot2$ volt. If i_1 be the current in 3Ω resistor, its terminal p.d.$=3\times i_1$. As 3Ω resistor is in parallel connection between A and C, we have $1\cdot2=3\times i_1$ or $i_1=0\cdot4$ amp.

2.12 Potential divider :

In laboratory work, we very often require fractions of large potential difference. We can use two resistors in series to provide such fractions very easily. In [Fig. 2.7(i)] one such arrangement has been shown and it is called a resistance potential divider. Here the large potential drop V has been divided into two parts by the two resistors r_1 and r_2 connected in series. The current flowing through the resistors is $i=\frac{V}{r_1+r_2}$.

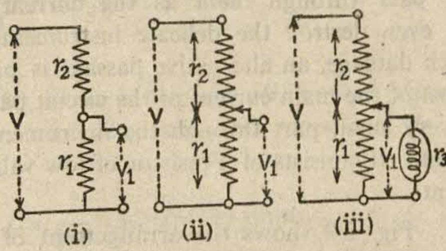


Fig. 2.7

$$\therefore\ V_1=i.r_1=\frac{r_1}{r_1+r_2}.\ V \quad .. \quad .. \quad \text{(i)}$$

$$\text{And}\quad V_2=i.r_2=\frac{r_2}{r_1+r_2}.\ V \quad .. \quad .. \quad \text{(ii)}$$

Regulating the values of r_1 and r_2, the fractions V_1 and V_2 can be made according to convenience.

A resistor with a sliding contact can be similarly used [Fig. 2.7(ii)] to provide a continuously variable potential difference from zero to the full value V. Figure 2.7(ii) shows how the value of V_1 increases from zero to V as the contact point goes from the lowermost point to the top-most. This is evidently a very convenient method of controlling the voltage across a load, for example, an electric bulb [Fig. 2.7 (iii)] The resistance r_3 of the bulb is in parallel with r_1 and the *eqn.* (i) is not applicable in calculating the voltage V_1 across the combination. The voltage can, however be calculated if r_2 is known as in the following example.

Example : *A load of 200 ohms is connected via a potential divider of resistance 400 ohms to a 12 volt supply. What is the p.d. across the load when the slider is one quarter up the divider ?*

Ans. When the slider is one quarter up the divider, the resistance in parallel with the load is 100 ohm [Fig. 2.8]. If R be the equivalent resistance, then

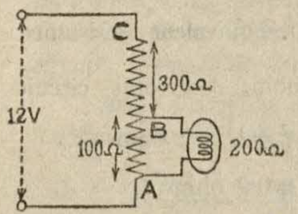


Fig. 2.8

$$\frac{1}{R}=\frac{1}{200}+\frac{1}{100}=\frac{3}{200}. \quad \therefore \quad R=\frac{200}{3}\text{ohms.}$$

Thus, $R_{AB}=\frac{200}{3}$ ohms. ; $R_{AC}=\frac{200}{3}+300=\frac{1100}{3}$ ohms.

$$\therefore \quad V_{AB}=\frac{R_{AB}}{R_{AC.}}V_{AC}=\frac{200/3}{1100/3}\times 12=\frac{2}{11}\times 12$$

$=2{\cdot}2$ volts (nearly). Had there been no load across the resistor AB, its p.d. would have been 3 volts.

2.13. Shunt :

On many occasions, sensitive instruments like galvanometers, ammeters etc, are used in electrical circuits. Intense current should not be allowed to pass through them as the current may be strong enough to damage or even destroy the delicate instruments. To protect the instruments against such damage, an alternative passage is provided with the instruments such that most of the main current of the circuit passes through the alternative passage and a very small part through the instrument. This alternative passage is called a **shunt.** It consists of a resistor of low value connected in parallel with the instrument.

Fig. 2.9 shows the arrangement of a shunt. With a galvanometer G, is connected, in parallel, a resistor S of low resistance. It serves the purpose of a shunt and the galvanometer is called a shunted galvanometer. The main current I, arriving at A, finds two paths—one through the galvanometer and the other through the shunt. Since,the shunt resistance is very low, compared to the galvanometer resistance, most of the current will pass through the shunt S and a small part through the galvanometer G. As a result, the galvanometer will not be damaged. Further by adjusting the shunt resistance, the galvanometer current may be changed according to our convenience. We can find the currents flowing through the two paths in the following way.

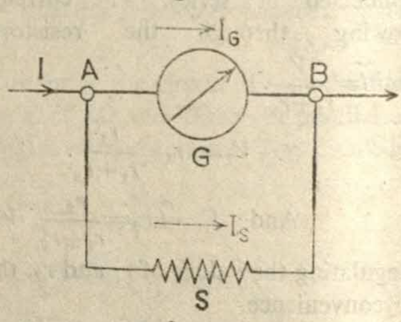


Fig. 2.9

Suppose, the main current$=I$

Current through the galvanometer$=I_g$

.. .. .. shunt $=I_s$

The resistance of the galvanometer$=G$ and the resistance of the shunt$=S$.

Now, $I=I_g+I_s$.

If (V_a-V_b) be the potential difference between A and B, then considering the galvanometer path we have, $V_a-V_b=I_gG$.

Since the shunt is also connected between A and B, $V_a-V_b=I_sS$.

$$\therefore\quad I_gG=I_sS. \quad \text{or,} \quad \frac{I_g}{I_s}=\frac{S}{G}$$

This shows that in two parallel branches of resistances, current divides itself inversely as the ratio of the resistances.

$$\text{Now}\quad \frac{I_g+I_s}{I_s}=\frac{S+G}{G} \quad \text{or,} \quad \frac{I}{I_s}=\frac{G+S}{G} \quad \therefore \quad I_s=\frac{G}{G+S}I.$$

$$\text{Also,}\quad I_g=I-I_s=I\left(1-\frac{G}{G+S}\right)=\frac{S}{S+G}I.$$

Example 1. *A galvanometer of 240-ohm resistance is shunted with a resistance of 10 ohms. If the main current is 5 amp calculate the currents through the galvanometer and the shunt.*

Ans. Here, $G=240$ ohms ; $S=10$ ohms, $I=5$ amp.

We know, $I_g=\frac{S}{S+G}\cdot I=\frac{10}{240+10}\times 5=\frac{50}{250}=0{\cdot}2$ amp.

So, $I_s=5-0{\cdot}2=4{\cdot}8$ amp.

Example 2. *A galvanometer has 199 ohms resistance. How much shunt is to be used with it so that $\frac{1}{200}$th. part of the main current should flow through the galvanometer ?*

Ans. We know, $I_g=\frac{S}{S+G}\cdot I$, here $I_g=\frac{I}{200}$ and $G=199$ ohms.

So, $\frac{I}{200}=\frac{S}{S+199}\times I$ or $S+199=200\ S$ or $S=1$ ohm.

Example 3. *Consider two circuits given in the fig. 2.10. Calculate the resistance R_1 and R_2 such that the current drawn from the batteries is the same as in the two cases but current through R in circuit (b) is reduced to 1/10th of that through R in circuit (a). If R is 11 ohms, find the values of R_1 and R_2* [I.I.T 1970]

Ans. First consider the parallel resistors R and R_1 in the second circuit. Suppose current i is drawn from the battery. According to the question the current through $R=i/10$. So, current through $R_2=i-\frac{i}{10}=9\frac{i}{10}$. Since the resistors are connected parallel, the p.d. across R=p.d. across R_2 or $R_2\times\frac{9i}{10}=R\times\frac{i}{10}$

or $R_2=\frac{R}{9}=\frac{11}{9}=1\frac{2}{9}$ ohms.

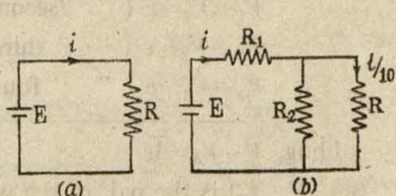


Fig. 2.10

If R_3 be the equivalent resistor of R and R_2 then $\frac{1}{R_3}=\frac{1}{R}+\frac{1}{R_2}=\frac{1}{11}+\frac{9}{11}=\frac{10}{11}$

$\therefore\ R_3=\frac{11}{10}$ ohm. So, the total resistance of the second circuit

$=R_3+R_1=\left(\frac{11}{10}+R_1\right)$ ohms.

Since the first and the second circuits are taking the same current, their resistances are equal. Hence,

$\frac{11}{10}+R_1=R=11$ or $R_1=11-\frac{11}{10}=\frac{99}{10}$ $\therefore$ $R_1=9{\cdot}9$

2.14 Combination of cells :

In order to increase the circuit current or the potential difference, several cells are connected together to form a combination, known as a *battery*. There may be three different types of combination of cells *viz.* (i) *series combination* (ii) *parallel combination and* (iii) *mixed combination.*

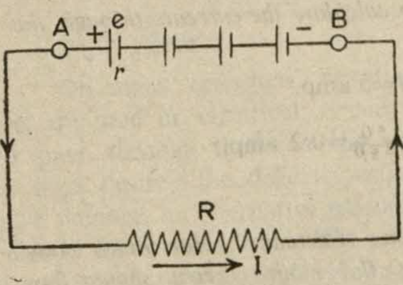


Fig. 2.11

(i) **Series combination** : In series combination of cells, the positive of one is connected to the negative of the next and so on [Fig 2.11]. The adjoining figure shows four cells connected in series. The free positive pole of the first cell and the free nagative pole of the last cell form the two poles of the battery. An external resistor R is connected to the battery. To find the current through the external resistor, we may proceed as follows.

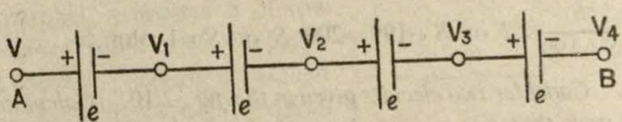


Fig. 2.11(i)

Suppose, the e.m.f. of each cell is 'e' and the internal resistance is 'r'. From fig. 2·11(i) we can write.

$V-V_1=e$ (for the first cell)

$V_1-V_2=e$ (" "second ")

$V_2-V_3=e$ (" " third ")

$V_3-V_4=e$ (" " fourth ")

Adding, $V-V_4=4e$

But $(V-V_4)$ is the p.d. of the whole battery. So, the e.m.f. of the battery $=4e$; if there be 'n' such identical cells, the total e.m.f. of the battery$=ne$.

Since the cells are joined in series, the internal resistances may also be supposed to be joined in series. For 'n' cells, therefore, the total internal resistance$=nr$. The total circuit resistance$=nr+R$ [Fig 2.11 (ii)].

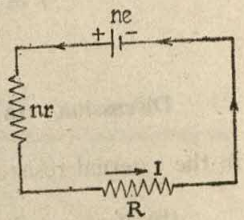


Fig. 2.11(ii)

So, the current through the external resistor,

$$I=\frac{\text{Total e. m. f.}}{\text{Total resistance}}=\frac{ne}{nr+R}.$$

Discussion : (a) When $R>>nr$, we can write,

$I=\frac{ne}{R}$. *i.e.* the current through the external resistor is 'n' times the current supplied by a single cell.

(b) When $nr>>R$, we have, $I=\frac{ne}{nr}=\frac{e}{r}$ *i.e.* the current through the external resistor is equal to the current supplied by a single cell.

So, we see that to get a strong current, cells should be connected in series when the external resistance is large in comparison with the internal resistance of the battery.

(c) If one of the 'n' cells is connected wrongly so that it sends a current in the opposite direction, then $(n-1)$ cells will send current in the same direction, having a total e.m.f.$=(n-1)e$. Hence, the resultant e.m.f. of the battery $=(n-1)e-e=(n-2)e$. But the circuit resistance remains same as before.

So, the current through the external resistor$=\frac{(n-2)e}{nr+R}$.

(ii) **Parallel combination** : When all the positive poles of a number of cells are connected to one point and all the negative poles at another point, the cells are said to be connected in parallel. Fig. 2.12 shows three cells connected in parallel. The point A is the positive terminal of the battery and the point B, the negative terminal. An external resistor R is connected between the points A and B.

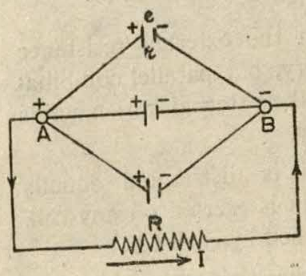


Fig. 2.12

In parallel connection, the resultant e.m.f. of the battery is the same as that of one cell only and in the present case, it is equal to 'e'. Since the cells are connected in parallel, the internal resistance of the battery is to be calculated from the formula for resistors in parallel. For three cells connected in parallel as in fig. 2.12, the internal resistance of the battery$=\frac{r}{3}$. If there be 'n' number of cells, the internal resistance of the battery$=\frac{r}{n}$. The total circuit resistance$=R+\frac{r}{n}$. So, the current in the external resistor, [Fig 2.12 (i)]

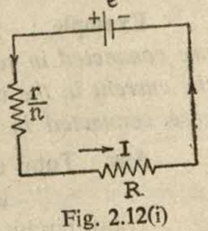


Fig. 2.12(i)

$$I=\frac{\text{total e. m. f.}}{\text{Total resistance}}=\frac{e}{R+\frac{r}{n}}=\frac{ne}{nR+r}.$$

Discussion : (a) When $nR>>r$, we can write $I=\frac{ne}{nR}=\frac{e}{R}$ *i.e.* the current in the external resistor is same as the current supplied by a single cell.

(b) If $r>>nR$, we can write, $I=n.\frac{e}{r}$ *i.e.* the current in the external resistor is 'n' times the current supplied by a single cell.

It is, therefore, clear that to get a strong current, cells should be connected in parallel when the external resistance is small in comparison with the internal resistance of a single cell.

One advantage of parallel combination of cells is that the main circuit current is shared between the cells and hence there is less strain on the cells. But in series combination, the same main current is supplied by each cell. The cells should never be left connected in parallel when not in use because if one of the cells has greater e.m.f. than another, a current will flow through the battery itself and the cells become quickly exhausted. This cannot happen in the case of series combination.

Advantages and disadvantages of series and parallel combinations :

(i) In series combination, the terminal e.m.f. increases; the internal resistance of the battery also increases. In parallel combination, however, the terminal e.m.f. remains same as the e.m.f. of the individual cell but the internal resistance of the battery diminishes.

(ii) Series combination gives strong current when the external resistance is large compared to the internal resistance of the battery, but parallel combination gives strong current in the reverse case *i.e.* when the external resistance is small compared to the internal resistance of the battery.

(iii) In parallel combination, main circuit current is distributed equally among the cells of the battery and hence no undue strain is exerted on any cell. But in series combination, the same main current is supplied by each cell.

(iv) If one of the cells of the battery happens to have greater e.m.f. than another, then a current will flow through the battery itself when the cells are connected in parallel. This may damage the cells quickly. This, however, cannot happen in the case of series combination.

Example : 5 *cells, each of e.m.f.* 1·5 *volts and internal resistance* 3 *ohms are connected in series. A resistor of* 10 *ohms is connected to the battery. Find the current in the circuit. What would be the change in circuit current if the cells were connected in parallel* ?

Ans. Total e.m.f. of 5 cells connected in series$=5\times1{\cdot}5=7{\cdot}5$ volts

" internal resistance of them$=5\times3=15$ ohms.

Total circuit resistance$=15+10=25$ ohms.

$\therefore$ Current in the circuit$=\frac{7\cdot5}{25}=0\cdot3$ ampere.

If the cells were connected in parallel, the battery e.m.f. is same as that of a single cell *i.e.* the battery e.m.f.$=1\cdot5$ volts. If R be the internal resistance of the battery, $\frac{1}{R}=\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=\frac{5}{3}$ $\therefore$ $R=\frac{3}{5}=0\cdot6$ ohm.

So, the total circuit resistance$=10+0\cdot6=10\cdot6$ ohms

$\therefore$ the circuit current$=\frac{1\cdot5}{10\cdot6}=0\cdot14$ ampere.

Hence, the change in circuit current$=(0\cdot3-0\cdot14)=0\cdot16$ ampere.

(iii) **Mixed combination** : When several rows of cells are connected in parallel, each row containing a number of cells in series, the combination is called a mixed combination. In fig. 2.13, 'm' rows of cells are connected in parallel, each row containing 'n' cell in series. The point A is the positive pole and B the negative pole of the battery. A resistor R is connected between these points.

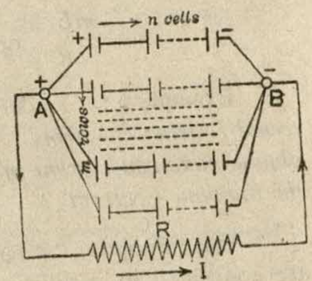


Fig. 2.13

Suppose the cells are all identical. The e.m.f. and the internal resistance of each cell are 'e' and 'r' respectively. From the figure 2.13, it is clear that each row may be regarded as a single cell of e.m.f. 'ne' and internal resistance 'nr'. In that case, such 'm' cells are connected in parallel. Considering the whole battery, its e.m.f.$=ne$ and internal resistance$=\frac{nr}{m}$.

[If R' be the total internal resistance, then, $\frac{1}{R'}=\frac{1}{nr}+\frac{1}{nr}+\ldots m$ times$=\frac{m}{nr}$ $\therefore$ $R'=\frac{nr}{m}$]

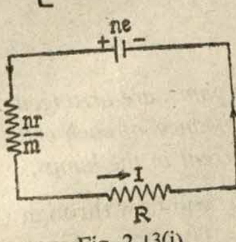


Fig. 2.13(i)

Hence, total circuit resistance$=R+\frac{nr}{m}$ [Fig 2.13 (i)]

So, the current through the external resistor

$$I=\frac{\text{total e. m. f.}}{\text{Total resistance}}=\frac{ne}{R+\frac{nr}{m}}=\frac{mne}{mR+nr}$$

Arrangement for maximum current ;

How can we arrange the cells in a mixed combination so as to obtain the maximum current in the external circuit ?

In the mixed combination, the current in external circuit is given by, $I=\frac{m.n.e.}{mR+nr}=\frac{m.n.e.}{(\sqrt{mR}-\sqrt{nr})^2+2\sqrt{mnR.r}}$

The numerator being a constant, I will be maximum when the denominator is minimum. But the last factor *viz.*, $2\sqrt{mnR.r}$ of the denominator is a constant. Hence, I will be maximum, when $(\sqrt{mR}-\sqrt{nr})^2$ is a minimum. Since a squared quantity cannot be negative, the minimum value of $(\sqrt{mR}-\sqrt{nr})^2$ is zero.

i.e. For I to be maximum, $(\sqrt{mR}-\sqrt{nr})^2=0$.

$$\text{or,}\quad mr=nr \quad \text{or,}\quad R=\frac{n}{m}.r$$

Hence the current in the external circuit will be maximum, when the total internal resistance of the battery $\left(\frac{nr}{m}\right)$ is equal to the total external resistance R.

The maximum value of the current is given by the equation,

$$I_{max}=\frac{nE}{2R} \quad \text{or,} \quad \frac{mE}{2r}\left[\because \quad mR=nr\right]$$

Example 1 : *You are given* 24 *similar cells, each of e.m.f.* 1·5 *volts and internal resistance* 4 *ohms. How you will arrange them in a mixed circuit so as to obtain maximum current in an external resistor of* 6 *ohms ? What is the value of the maximum current ?*

Ans. Suppose, 'n' cells are arranged in series in a row and 'm' such rows are connected in parallel. So, $m\times n=$total no. of cells$=24$ .. ..(i)

Again from the condition of maximum current, we have, $R=\frac{n}{m}\times r$

Here, $R=6$ ohms, and $r=4$ ohms. So, $6=\frac{n}{m}\times 4$ .. ..(ii)

Multiplying eqns. (i) and (ii). $n^2=24\times\frac{6}{4}=36$ $\therefore$ $n=6$. So, $m=4$
i.e. 4 rows are to be made each row containing 6 cells in series.

$$\text{Also,}\quad I_{max}=\frac{nE}{2R}=\frac{6\times 1{\cdot}5}{2\times 6}=0{\cdot}75 \text{ amp.}$$

Miscellaneous example 1. *Two lamps, each of resistance* 50 *ohms, are arranged in series with* 100 *cells, all joined in series. If the internal resistance of each cell is* 1 *ohm and the e.m.f. of each cell is* 1·5 *volts, calculate the current in the lamps.*

Ans. Since the lamps are joined in series, the same current will flow through each lamp. Now, the total resistance of the lamps$=50+50=100$ ohms. Since the cells are also joined in series, their total internal resistance$=100$ ohms. So, the circuit resistance$=100+100=200$ ohms.

Also, the e.m.f. of the battery$=100\times 1{\cdot}5=150$ volts.

$$\therefore \text{ the current}=\frac{\text{Total e.m.f.}}{\text{Total resistance}}=\frac{150}{200}=0{\cdot}75 \text{ amp.}$$

Hence, current through each lamp$=0{\cdot}75$ amp.

Example 2 : *One kilogram of copper is drawn up into a wire (a) 1 mm. diameter and (b) 2 mm. diameter. Compare their resistances at the same temperature.*

Ans. We know, $R=\dfrac{\rho.l}{\pi\left(\dfrac{d}{2}\right)^2}=\dfrac{4\rho l}{\pi d^2}$ where R=resistance of the wire, l= length, d=diameter and ρ=sp. resistance.

If M=mass and D=density of the material of the wire then volume× density=mass, or $\dfrac{\pi d^2}{4}\times l\times D=M$ $\therefore$ $l=\dfrac{4M}{D\pi d^2}$ So, $R=\dfrac{4\rho}{\pi d^2}\times\dfrac{4M}{D.\pi d^2}=\dfrac{16M.\rho}{D.(\pi)^2(d)^4}$

In this case, $R\propto\dfrac{1}{d^4}$, because all other factors are constant.

If the wires have resistances R_1 and R_2 and diameters d_1 and d_2 respectively, then, $\dfrac{R_1}{R_2}=\dfrac{d_2^4}{d_1^4}=\dfrac{(0{\cdot}2)^4}{(0{\cdot}1)^4}=16$ $\therefore$ $R_1 : R_2=16 : 1.$

Example 3 : *ABCD is a square of wire, each side having a resistance of* 8 *ohms. A and B are connected to the terminals of a battery of* 20 *volts. What is the current in the main line ?*

Ans. In fig. 2.14, *ABCD* is the square of wire. The battery is connected to the points *A* and *B*. Here, the main current, arriving at *A*, will be divided into two parts— one flowing along *AD, DC, CB* and the other along *AB* ; they will meet again at *B* and will go back to the battery. From the flow of the current, it is clear that the wires *AD, DC* and *CB* are joined in series and the wire *AB* is in parallel with them.

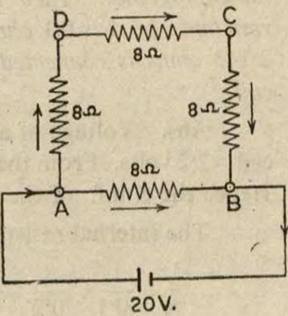


Fig. 2.14

Now, the total resistance of the wires *AD, DC*, and *CB* joined in series=8+8+8=24 ohms. We can now suppose that a single wire of 24 ohms resistance is there instead of the three wires. If *R* be the equivalent resistance of the single wire and the wire *AB*, then $\dfrac{1}{R}=\dfrac{1}{24}+\dfrac{1}{8}=\dfrac{4}{24}$ $\therefore$ R=6 ohms.

So, the total circuit resistance=6 ohms.

$\therefore$ the main current$=\frac{20}{6}=3{\cdot}33$ amp.

Example 4 : *The ends of a uniform wire* 1 *metre long are connected to the terminals of a battery (e.m.f.*=2·1 *volts ; internal resistance*=1·5 *ohms). Find in millivolts, the fall of potential per unit length of the wire, if the resistance of the wire be* 2 *ohms.*

Ans. Total circuit resistance=2+1·5=3·5 ohms.

If I be the circuit current, $I=\dfrac{\text{Total e.m.f.}}{\text{Total resistance}}=\dfrac{2{\cdot}1}{3{\cdot}5}=0{\cdot}6$ amp.

Now, terminal p.d. of the wire = resistance of the wire $\times$ current

$$=2\times 0{\cdot}6=1{\cdot}2 \text{ volts.}$$

$$\therefore \text{ fall of potential per unit length}=\frac{P.D. \text{ of the wire}}{\text{length ,, ,, ,,}}$$

$$=\frac{1{\cdot}2}{100}\text{ volt/cm.}=\frac{1{\cdot}2\times 1000}{100}\text{ millivolt/cm.}=12 \text{ millivolt/cm.}$$

Example 5 : *A circuit contains an ammeter which reads* 1·3 *amp. When a voltmeter is connected to two points A and B of the circuit, it reads* 3·9 *volts. What is the resistance of the portion of the circuit between A and B ?*

Ans. The reading of the ammeter included in the circuit gives the current in all parts of the circuit. Hence, the current flowing in the circuit between A and $B=1{\cdot}3$ amp.

The p.d. of the portion of the circuit is obtained from the voltmeter reading, which is 3·9 volts. So the p.d. between A and $B=3{\cdot}9$ volts. From Ohm's law, therefore, we can write,

$$\text{the resistance between } A \text{ and } B=\frac{P.D. \text{ between } A \text{ and } B}{\text{Current through it}}=\frac{3{\cdot}9}{1{\cdot}3}=3 \text{ ohms.}$$

Example 6 : *Three storage cells A, B and C each having an open circuit voltage of* 2·2 *volts are connected in such a way that their positive terminals are all joined together. Their negative terminals are also joined together. The internal resistance of A is* 0·1 *ohm., of B* 0·2 *ohm. and of C* 0·4 *ohm. If a coil of resistance* 2·143 *ohms is connected across the battery terminals, find the current through the coil.*

Ans. Voltage of a cell in an open circuit is its e.m.f. So, the e.m.f. of each cell = 2·2 volts. From the question, it is clear that the cells are connected in parallel. Hence the e.m.f., of the battery = 2·2 volts.

The internal resistance of the battery R is given by,

$$\frac{1}{R}=\frac{1}{0{\cdot}1}+\frac{1}{0{\cdot}2}+\frac{1}{0{\cdot}4}=\frac{7}{0{\cdot}4} \quad \therefore \quad R=\frac{0{\cdot}4}{7}=0{\cdot}057 \text{ ohm.}$$

Total circuit resistance = 2·143 + 0·057 = 2.2 ohms.

$$\therefore \text{ Circuit current}=\frac{\text{e.m.f. of the circuit}}{\text{resistance of the ,,}}=\frac{2{\cdot}2}{2{\cdot}2}=1 \text{ amp.}$$

Example 7 : *Two identical cells, when connected in series, send a current of* 0·25 *amp through a resistor of* 8 *ohms. When the cells are joined in parallel, a current of* 0·16 *amp flows through the same resistor. Find the e.m.f. and the internal resistance of each cell.*

Ans. Suppose, the e.m.f. of each cell $=e$ and internal resistance $=r$. In the first case, the total e.m.f. $=2e$ and total internal resistance $=2r$. Hence we can write,

$$\frac{2e}{2r+8}=0{\cdot}25 \quad \ldots \quad \ldots \text{(i)}$$

In the second case, the total e.m.f.$=e$ and total internal resistance $=\frac{r}{2}$. So, we have, $\frac{e}{\frac{r}{2}+8}=0{\cdot}16$ .. .. (ii)

From eqn. (i), we get, $\frac{e}{r+4}=0{\cdot}25$ or $e=0{\cdot}25\times r+1$.

" " (ii) we get, $e=0{\cdot}08\times r+1{\cdot}28$

$\therefore\ 0{\cdot}25\times r+1=0{\cdot}08\times r+1{\cdot}28$ or, $0{\cdot}17\times r=0{\cdot}28$

$\therefore\ r=\frac{0{\cdot}28}{0{\cdot}17}=1{\cdot}6$ ohms (nearly) Also, $e=0{\cdot}08\times1{\cdot}6+1{\cdot}28=1{\cdot}408$ volts.

Example 9 : *12 cells, each having the same e.m.f. are connected in series and are kept in a closed box. Some of the cells are wrongly connected. This battery is connected in series with an ammeter and two cells which are also in series. The current is 3 amp when the cells and the battery aid each other and is 2 amp when the cells and the battery oppose each other. How many cells in the battery are wrongly connected ?* [*Jt. Entrance 1985*]

Ans. Let the number of cells wrongly connected be x. If e be the e.m.f. and r the internal resistance of each cell, then the total e.m.f. of the battery$=(12-2x)\,e$* and the total internal resistance$=12r$. When the additional two cells aid the battery, total circuit e.m.f.$=(12-2x)e+2e$ and total circuit resistance $=14r+R$ where R is the resistance of the ammeter.

$$\therefore\ \frac{(12-2x)e+2e}{14r+R}=3 \quad .. \quad \text{(i)}$$

When the additional cells oppose the battery, the total circuit e.m.f.$=(12-2x)e-2e$ and the total circuit resistance$=14r+R$

$$\therefore\ \frac{(12-2x)e-2e}{14r+R}=2 \quad .. \quad \text{(ii)}$$

Solving equations (i) and (ii), we get $x=1$ *i.e.* one cell was connected wrongly in the battery.

Example 10 : *A distant telephone whose resistance is 300 ohms is connected to the exchange 10 miles away by a pair of telephone wires whose resistance singly is 5 ohms per mile. A twig falls across the wire at a certain point, producing the effect of a resistance R connected between the wires at that point. Measurements made at the exchange show a resistance between the terminals of the line as 130 ohms which rises to 160 ohms when the distant telephone is disconnected. How far from the exchange is the fault to be found and what is the value of R ?* [*I.I.T 1971*]

[*Note that the total e.m.f. is not $(12-x)e$ but $(12-2x)e$ because x wrongly connected cells remain in the circuit and their e.m.f.'s act opposite to the e.m.f.'s of the remaining cells of the battery. See art 2.14(c)].

Ans. See fig. 2.15. Let AB be the twig at a distance x miles from the exchange. The distance of the twig from the telephone $=(10-x)$ miles. From the fig. 2.15 it is clear that the wire of resistance $2\times5(10-x)$ ohm and the telephone are in series but the twig of resistance R is in parallel with them. If R_1 be the equivalent resistance of this combination then

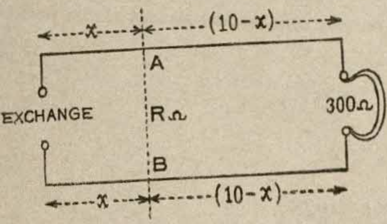


Fig. 2.15

$$\frac{1}{R_1}=\frac{1}{2\times5(10-x)+300}+\frac{1}{R}$$

$$=\frac{1}{400-10x}+\frac{1}{R}=\frac{400-10x+R}{R(400-10x)} \qquad \therefore\ R_1=\frac{R(400-10x)}{400-10x+R}$$

Total circuit resistance$=5x+\dfrac{R(400-10x)}{400-10x+R}+5x=10x+\dfrac{R(400-10x)}{400-10x+R}$

According to the question, $10x+\dfrac{R(400-10x)}{400-10x+R}=130 \quad \ldots. \quad \text{(i)}$

When the distant telephone is disconnected, the circuit resistance

$=5x+R+5x=10x+R$

According to question, $10x+R=160 \quad \ldots. \quad \text{(ii)}$

Subtracting eqn. (i) from eqn (ii) we get $R-\dfrac{R(400-10x)}{400-10x+R}=30$

or, $R^2=30(400-10x+R)=30(400-160+2R)$ [from eqn. (ii)]

$=30(240+2R)$

or, $R^2-60R-30\times240=0$ or $R=120$ ohms.

Putting this value of R in eqn (ii), $x=4$ miles.

Example 11 : *Calculate the steady state current in the 2 ohm resistor shown in the fig. 2.16. The internal resistance of the battery is negligible and the capacitance of the condenser C is 0·2 microfarad.* [*I.I.T 1982*]

Ans. In d.c. circuit, a capacitor has no effect at the steady state. It has some effect only at the start and at the termination of the current. So, at the steady state no current flows through the condenser branch. Now, the equivalent resistance of 2Ω and 3Ω in parallel$=\dfrac{2\times3}{3+2}=\dfrac{6}{5}=1{\cdot}2$ ohm.

So, total circuit resistance$=1{\cdot}2+2{\cdot}8=4$ ohm.

$\therefore$ Circuit current$=\frac{6}{4}=1{\cdot}5$ amp. Now, from the principle of shunt, the current through 2 ohm resistor$=\dfrac{3}{2+3}\times1{\cdot}5=0{\cdot}9$ amp.

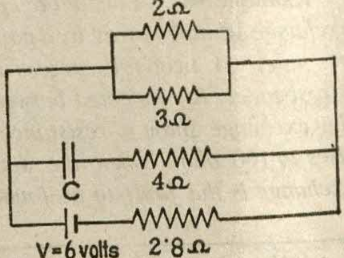


Fig. 2.16.

Exercises

Essay type :

1. Explain the practical units of electric current, potential difference and resistance. State Ohm's law.

2. What is equivalent resistance ? Find the equivalent resistance of two resistors r_1 and r_2 connected in parallel.

3. How will you join two resistors (i) in series and (ii) in parallel ? Find the equivalent resistance in each case.

4. What is a shunt ? Why is it used ? What is the current passing through a shunted galvanometer ?

5. What is a mixed combination of cells ? Under what condition does a mixed combination of cells send maximum current through an external resistor ?

Short answer type :

6. The terminal *p.d.* of a wire of length l and diameter d is V. How will the drift velocity of electrons be modified if (i) V is doubled (ii) l is doubled (iii) d is doubled ?

7. State Ohm's law and explain how the definition of resistance comes from the law. [*H.S. Exam. 1978*]

8. Does the relation $V=iR$ apply to non-ohmic resistors ?

9. What is the difference between the e.m.f. and p.d. of a cell ? Explain why the potential difference between the terminals of a battery is not always the same as its e.m.f. ?

10. The voltage applied to a rheostat is doubled and then trebled. What happens to the resistance of the rheostat ?

[Hints : No change if the temperature is constant.]

11. What will be the effect on the resistance of a wire if (i) the diameter is doubled (ii) the wire is folded back on itself so that the length is halved and the two halves are in parallel ?

[Hints : (i) one fourth (ii) one fourth].

12. Upon what factors does the resistance of a conductor depend ? What is specific resistance ? What is its unit ? [*H. S. Exam. 1982*]

13. What is temperature coefficient of resistance ? Why is the temperature coefficient of resistance of metallic substances considered positive while that of carbon negative ?

14. What is lost volt ? What is its relation with the internal resistance of a cell ?

15. What are the advantages and disadvantages of series and parallel connections of cells ?

16. In connection with parallel combination of resistors, answer the following questions :

(i) Is the p.d. across each resistor same ?

(ii) Is the total current equal to the sum of the currents passing through individual resistors ?

(iii) Is individual current inversely proportional to individual resistor ?

(iv) Is the equivalent resistance less than the least individual resistance ?

(v) Is the reciprocal of the equivalent resistance equal to the sum of the reciprocals of individual resistances ?

17. In connection with the series combination of cells, answer the following questions :

(i) When is this combination helpful in obtaining a large p.d. ?

(ii) Does the internal resistance of the battery become greater than that of any individual cell ?

(iii) Is the combination helpful when the external resistor is less than the internal resistance of the battery ?

18. Is it possible for two cells of e.m.f. e_1 and e_2 and internal resistances r_1 and r_2 respectively to produce a weaker current through an external resistance R connected in series than one of the cells by itself connected to the same resistance ?

[Hints : If $e_2/e_1=r_2/(r_1+R)$]

19. What are the definitions of (i) international ampere (ii) international volt and (iii) international ohm ?

20. What effect does pressure produce on the resistance of a substance ?

21. In a power transmission line, the p.d. at the receiving end is always less than that at the generating end. Why ?

Objective type :

22. (i) If at any part of a circuit, electrical energy is converted into other forms of energy, we say that (i) a p.d. (ii) an e.m.f. (iii) a lost volt exists in that part. Which is correct ?

(ii) An increase in the cross-sectional area of a resistor (i) increases (ii) decreases (iii) makes no difference in the resistance of the resistor. Which is correct ?

(iii) A reduction in the shunt resistance of a galvanometer (i) reduces (ii) increases (iii) makes no difference in the current flowing through the galvanometer. Which is correct ?

(iv) If the external resistance is greater than the total internal resistance of the battery, a series combination of cells gives (i) increased current (ii) reduced current (iii) same current through the external resistance. Which is correct ?

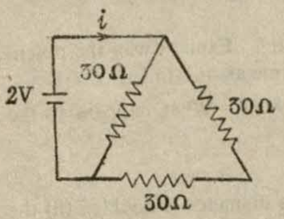


Fig. 2.17

(v) One micro-volt is equal to (i) 10^6 volt (ii) 10^{-6} volt (iii) 10 volt. Which is correct ?

(vi) In some substances, resistance is found to (i) increase (ii) decrease (iii) remain same when light falls on it. Which is correct ?

(vii) The current i in the circuit shown in fig. 2.17 is (i) $\frac{1}{45}$ amp (ii) $\frac{1}{15}$ amp (iii) $\frac{1}{10}$ amp (iv) $\frac{1}{5}$ amp. Which is correct ? [*I. I. T. 1984*]

Numerical Problems :

23. A current of 5 amp flows through a conductor of resistance 10 ohms for 4 minutes. How many (i) coulombs and (ii) electrons pass through any cross-section of the conductor in that time ? $e=1{\cdot}6\times10^{-19}$ coulomb. [Ans. (i) 1200 (ii) $7{\cdot}5\times10^{21}$]

24. A cell of e.m.f. 1·1 volt and internal resistance 1 ohm is connected to the ends of two wires AB and BC joined in series. The positive terminal of the cell is connected to the end A. If the resistance of AB is 4 ohms and that of BC 6 ohms, calculate the p.d. in the following cases (i) between A and B (ii) between B and C. (iii) between A and C. [Ans. 0·4 : 0·6 volt ; 1 volt]

25. A cell of e.m.f. 1·5 volts and internal resistance 2 ohms is connected to two resistances joined in parallel. If the resistance of the wires are 4 ohms and 10 ohms, calculate the current in each wire. [Ans. 0·22 amp ; ·088 amp]

26. A copper wire 100 cm long and of diameter 1 mm is connected in series with another thinner copper wire of length 400 cm. When a certain current passes through them, the terminal p.d. of the two wires are 4 volts and 100 volts respectively. Calculate the diameter of the thinner wire. [*H. S. Exam. 1983*] [Ans. 0·4 mm]

27. A battery of e.m.f. 4 volts when connected to a resistance of 9 ohms is found to have a terminal potential difference of 3 volts. Find the internal resistance of the battery.

[Ans. 3 ohms]

28. Find the resistance of a battery which on open circuit gives an e.m.f. of 6 volts and which, when producing a current of 2 amp., has a p.d. of 4 volts between its poles.

[Ans. 1 ohm]

29. A battery of e.m.f. 6·8 volts and 0·8 ohm internal resistance sends a current of 0·4 amp. through a wire. Calculate the resistance of the wire, its terminal potential difference and internal fall of potential of the battery. [Ans. 16·2 ohms ; 6·48 volts ; 0·32 volt]

30. Two coils have an equivalent resistance of 12 ohms when connected in series and $_3$ ohm when connected in parallel. What is the resistances of each coil ? [Ans. 10 ohms ; 2 ohms]

31. The length of a wire of 60 ohm resistances is stretched to three times the original length. If its density and sp. resistance remain unaltered, what will be its present resistance ? [Ans. 540 ohms]

32. If the specific resistance of platinum at $0°C$ is $8·96 \times 10^{-6}$ ohm-cm and its temperature coefficient $32 \times 10^{-4}/°C$, find the length of a wire of diameter 0·0274 cm which has a resistance of 4 ohms at $50°C$. [Ans. 0·165 cm]

33. Compare the resistance of a wire 1 metre long and 0·01 cm^2 cross-section with that of a one centimetre cube of the same material. [Ans. 10^4]

34. The specific resistance of copper is $1·76 \times 10^{-6}$ ohm-cm. The radius of a copper wire is 1 mm. Calculate the length of a telegraph wire needed for having a resistance of 17·6 ohms. [Ans. 3·14 km]

35. If a copper wire is stretched to make it 0·1% longer, find the percentage change in its resistance. [*I. I. T. 1978*] [Ans. 0·2%]

36. (i) Five electric lamps are connected in parallel. What is the equivalent resistance of the combination, if the lamp has resistance 250 ohms each ? [Ans. 50 ohms]

(ii) Show that if n identical wires are joined in series the combined resistance is n^2 times as great as when they are joined in parallel.

37. A p.d. of 2 milli-volts is applied across a wire 100 cm. long and 2 sq. mm. is cross-section. A current of 0·2 amp. flows through it. What is the specific resistance of the material of the wire ? [Ans. 2×10^{-6} ohm. cm.]

38. Two wires of resistance 10 ohms and 15 ohms are joined in series and three such sets are joined in parallel. What will be the resistance of the whole combination ? [Ans. 8·33 ohms]

39. Two resistors of 300 ohm and 400 ohm are connected in series and the combination is then joined to a source of 60 volt potential difference. A voltmeter connected across 400 ohm resistor gives a reading of 30 volt. What reading will it give when connected across 300 ohm resistor ? [Ans. 22·5 volts]

40. A battery of 12 volts e.m.f. is connected to a group of three resistances joined in series. One resistance is unknown and the others are 3 ohms and 1 ohm. A voltmeter connected across the 3 ohm resistance reads 6 volts. Determine the the value of the unknown resistance. [Ans. 2 ohms]

41. You have several 1000 ohm resistors but you want one of 240 ohms. How would you obtain it ? [Ans. By connecting 4 given resistors in parallel]

42. A galvanometer of 250 ohms resistance, a cell of e.m.f. 1·4 volts and internal resistance 2 ohms and a resistance of 70 ohms are joined in series. Find the current through the galvanometer. If the galvonometer is now shunted by a resistance of 10 ohms, what will be the current through the galvanometer ? [Ans. $4·35 \times 10^{-3}$ amp ; $6·6 \times 10^{-4}$ amp.]

43. A galvanometer of 20 ohms resistance is shunted by a resistance of 5 ohms. Find the current which flows through the galvanometer when a battery of 20 volts e.m.f. and a resistance of 10 ohms is connected in series with it. [Ans. 0·28 amp. (nearly)]

44. What shunt must be used with a galvanometer of resistance 20 ohms so that 1% of the main current passes through the galvanometer ? [Ans. 0·202 ohm]

45. How would you connect two identical cells so as to have (*a*) double voltage (*b*) same voltage as any of them (*c*) no voltage ? How are the internal resistances of the combination related to that of single cell in three cases ?

46. 5 cells, each of e.m.f. 2 volts and internal resistance 0·03 ohm are joined respectively (i) in series and (ii) in parallel. The combination is connected to a wire of resistance 19·85 ohms. Find the current flowing the wire in each case. [Ans. (i) 0·5 amp. (ii) 0·101 amp.]

47. A cell of e.m.f. 1·4 volt and internal resistance 2 ohms is connected with a 100 ohm resistor and an ammeter whose resistance is 4/3 ohm. A voltmeter is also joined across the 100 ohm resistor to read its potential difference. If the ammeter reads 0·02 amp, what will be the resistance of the voltmeter ? If the voltmeter reads 1·1 volt, what is the error in the reading ?
[*I.I.T.* 1975] [Ans. 200 ohms ; —0·23 volt]

48. You are given 48 cells each of which has an internal resistance of 2 ohms. How will you arrange them in order to obtain maximum current in an external resistor of 6 ohms ?
[Ans. 4 rows each containing 12 cells in series]

49. Wires of resistances 1, 2 and 3 ohms are connected in series across a Laclanche's cell of e.m.f. 1·5 volts and internal resistance 3 ohms. Calculate the p.d. across each of the wires and also the drop of potential inside the cell. [Ans. 0·166, 0·33 ; 0·5 ; 0·5 volt]

50. Wires of resistances 1, 2, and 3 ohms are connected in parallel. If the current through the first wire is 0·1 amp, what will be the currents through the other wires ? What is the total current ?
[Ans. 0·05 amp ; 0·03 amp ; 0·18 amp]

51. Two resistors of resistances 20 and 25 ohms are connected in parallel. To this combination is connected in series a resistor of 15 ohms and a battery of e.m.f. 5·4 volts and internal resistance $\frac{8}{9}$ ohm. Determine the current in each resistor and the p.d. across each resistor.
[*H. S. Exam. 1985*] [·11 amp, ·089 amp, 0·15 amp, $\frac{20}{9}V$; $3V$]

52. How many cells each having an e.m.f. of 1·8 volts and internal resistance of 0·07 ohm will be required to send a current of 10 amp through a resistance of 2·2 ohms ? [Ans. 20]

53. If a calling bell is worked with a pair of cells in series, each of e.m.f.=1·5 volts and internal resistance=1·8 ohms, find the resistance of the coil, the current in it being 0·5 amp.
[Ans. 2·4 ohms]

54. Two copper wires, whose lengths are in the ratio 1 : 2 are of the same resistance. Compare the diameters of the wires.
[Ans. 1 : $\sqrt{2}$]

55. A uniform wire, 4 metres long and of resistance 6 ohms/metre is bent into the form of a square *ABCD*. The adjacent corners *A* and *B* are connected to a battery of e.m.f. 3 volts and internal resistance 4 ohms. Find the current along *AB*. [Ans. 0·264 amp.]

56. In an electrification scheme, the dynamo is of negligible resistance but the resistance of the leading wires is one ohm per mile. If the voltage of the power house is 220 volts D.C. what voltage will be available at a station 20 miles off, when a current of 2 amp is drawn from the leads? What can be the maximum strength of the current available there ?
[Ans. 140 volts ; 5·5 amp.]

57. A battery drives a current of 0·325 amp through an external circuit of resistance 19·5 ohms when it is in conjunction with a Leclanche's cell of e.m.f. 1·5 volts and resistance 0·5 ohm. When the Leclanche's cell is in opposition, the current is 0·175 amp. Find the e.m.f. and the internal resistance of the battery.
[Ans. 5 volts ; 0]

58. The ratios of lengths, diameters and sp. resistances of two wires are, each 1 : 2. If the resistance of the thinner wire is 10 ohms, what is the resistance of the other ? [Ans. 10 ohms]

Harder Problems :

59. A current of 1 amp flows through a copper wire. Determine the number of electrons that pass across any section of the wire in one second. Charge of an electron=$1{\cdot}6\times10^{-19}$ coulomb
[*Jt. Entrance 1973*] [Ans. $6{\cdot}25\times10^{16}$]

60. The resistance of a copper wire of 1/12 inch diameter is 8 ohms per mile. What is the resistance of a copper wire whose length is 2 miles and diameter is 1/20 inch ?
[Ans. 44·4 ohms]

61. A copper wire and an iron wire of same length have the same p.d. applied to them. What must be the ratio of their radii if the current is to be the same ? [Ans. $(\rho_{Cu}/\rho_{Fe})^{\frac{1}{2}}$]

62. A resistance is made by joining two wires of the same material. The radii of the two wires are 1 mm and 3 mm respectively while their lengths are 3 cm and 5 cm respectively. A battery of e.m.f. 16 volts and negligible internal resistance is connected across the resistance. What is the p.d. across the shorter wire ? [*I.I.T. 1970*] [Ans. 13·5 volts]

63. Two resistors, when connected in series, have an equivalent resistance of 20 ohms but when connected in parallel, have an equivalent resistance of 4·8 ohms. Find the value of each resistor. [Ans. 12 ohms and 8 ohms]

64. A triangle *ABC* is formed by three resistors. The resistance of *AB* is 40 ohms, of *BC* is 60 ohms and of *CA* is 100 ohms. Find the equivalent resistance between *A* and *B*. [*I.I.T. 1975*] [Ans. 32 ohms]

65. A letter A is constructed of a uniform wire of resistance 10 ohm/cm. The sides of the letter are 20 cm long and the cross piece in the middle is 10 cm long while the apex angle is 60°. Find the resistance of the letter between the two ends of the legs. [Ans. 26·67 ohms]

66. Each of the resistance in the fig 2.18 is *R*. Find the resistance between *A* and *B*. [*I.I.T 1978*] [Ans. $\frac{3}{4}R$]

67. Two cells, each of e.m.f. 4 volt but of internal resistances 3 and 2 ohms respectively are connected in series through an external resistance *R*. Find the value of *R* for which the first cell will have zero p.d. across it. [Ans. 1 ohm]

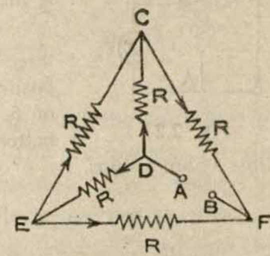


Fig. 2.18

68. A current of 5 amp. is divided into three branches forming a parallel combination. The lengths of the wires in the three branches are in the ratio 2 : 3 : 4 ; their diameters are in the ratio 3 : 4 : 5. Find the currents in each branch if the wires are of same material. [*I.I.T. 1975*]
[Ans. First wire = 1·399 amp ; 2nd wire = 1·658 amp and third wire = 1·943 amp]

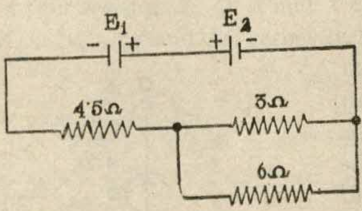


Fig. 2.19

69. In the circuit shown in fig. 2.19, the cells E_1 and E_2 have e.m.f.'s 4 volts and 8 volts and internal resistances 0·5 Ω and 1 Ω respectively. Calculate the current in each resistor and the p.d. across each cell. [*I.I.T. 1973*]

[Ans. 0·5 amp in 4·5 Ω resistor, 0·83 amp in 3Ω resistor, 0·166 amp in 6 ohm resistor ; E_1=4·25 volt ; E_2=7·5 volt]

70. A 36 volt battery is connected to the circuit shown in fig 2·20. Calculate the current through (i) 2 ohm resistance and (ii) 40 ohm resistance. The internal resistance of the battery is 1 ohm. [Ans. (i) 2 amp (ii) 0·6 amp]

71. A cell of e.m.f. 2·4 volt and 0·2 ohm internal resistance is joined in series with a combined resistance of three wires connected in parallel. If the wires have resistances 2, 3 and 6 ohms respectively, find the current in each wire. [Ans. 1 amp ; 0·66 amp ; 0·33 amp]

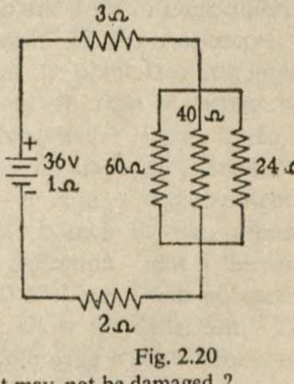


Fig. 2.20

72. The highest current that may be sent through a galvanometer of resistance 20 ohms is 1 milliampere. If a current of 0·2 amp is about to pass through instrument, what shunt will have to be used in order that the instrument may not be damaged ? [*It. Entrance 1973*] [Ans. 0·1 ohm]

73. The terminals of a galvanometer are connected by a wire which has the same resistance as the galvanometer and the lead from the battery is connected to one end of this wire. Find the point on the wire at which the other lead of the battery has to be connected so that exactly one-tenth of the current supplied by the battery will travese the galvanometer.

[*Jt. Entrance 1981*] [Ans. $\frac{1}{5}$]

74. A galvanometer together with an unknown resistance in series is connected across two indentical batteries each of 1·5 volts. When the batteries are connected in series, the galvanometer records a current of 1 amp and when the batteries are connected in parallel, the current is 0·6 amp. What is the internal resistance of the battery ? [*I.I.T. 1973*] [Ans. $\frac{1}{3}$ ohm]

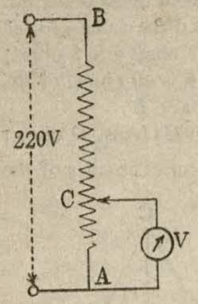


Fig. 2.21

75. A potential difference of 220 volts is maintained across a 12,000 ohm rheostat *AB* (Fig. 2·21). The voltmeter *V* has a resistance of 6000 ohms and a point *C* is at $\frac{1}{4}$th the distance from *A* to *B*. What is the reading in the voltmeter ? [*I.I.T. 1977*] [Ans 40 volt]

76. Three resistors of value 2 ohm, 3 ohm and 4 ohm form the three sides *AB*, *BC* and *CA* respectively of a triangle *ABC*. The positive pole of a battery is connected to the point *B* through a resistor of 5 ohms and the negative pole to the point *C*. The e.m.f. of the battery is 4 volt and the internal resistance is 1 ohm. Find the p.d. across 5 ohm resistor.

[Ans. 2·5 volt]

3

ELECTRICAL MEASUREMENTS

3.1. Measurement of ordinary resistances :

The simplest method of determining a resistance is to connect it in series with an ammeter A and a suitable source of e.m.f. A voltmeter V is connected across the resistance R (Fig. 3.1). If the readings of A and V are I amp and and V volts respectively, $I=\frac{V}{R}$

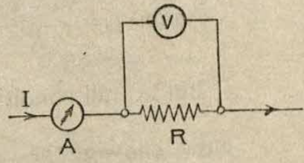


Fig. 3.1

$$\therefore \quad R=\frac{V}{I} \text{ ohms.}$$

Accuracy is limited because a fraction of the current I flows through the voltmeter V. This current is negligible if $R_V >> R$ where R_V is the resistance of the voltmeter. Some rough idea of the value of R is thus obtained. The method is suitable for approximate values, provided meters of suitable ranges are available.

Accurate measurement of ordinary resistance is, however, based on the principle of Wheatstone bridge, first devised by Charles Wheatstone in 1843. Practical applications of this principle are found in meter bridge and P.O. Box, with which ordinary resistances are usually measured.

Principle of Wheatstone bridge : A Wheatstone bridge is a combination of four resistors P, Q, R and X connected in a manner so as to form the four arms AD, DC, AB and BC respectively of a bridge or network (Fig. 3.2). A galavanometer G is connected across DB *i.e.*, across the junctions of the resistors P, Q and the resistors R, X. Similarly, a battery E (as a matter of fact, a single cell) is joined across AC *i.e.*, across the junctions of resistors P, R and the resistors Q, X. The main current from the battery entering the network at A divides up and flows through different resistors of the network. If the resistances of the resistors are so that the point D is at a higher potential than the point B, then a current will pass through the galvanometer from D to B productng a deflection in the galvanometer in one direction. If on the other hand, the point B is at a higher potential than the point D, curent will flow in the galvanometer branch in the opposite direction and the galvanometer will show an opposite deflection. But if the resistances are so adjusted that the potentials at B and D are equal then no current flows along this branch and the galvanometer will show no deflection. This is known as *null condition*. It may be proved in the following way that for a null condition of the galvanometer, $\frac{P}{Q}=\frac{R}{X}$

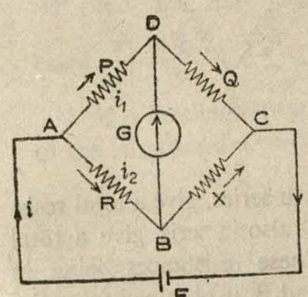


Fig. 3.2

Proof : Suppose the main current entering the network at A divides up into i_1 through P and i_2 through R. If no current flows through the galvanometer *i.e.* at null condition, the current through Q and X must also be equal to i_1 and i_2 respectively. Now from ohm's law, we get,

$$i_1=\frac{V_A-V_D}{P}=\frac{V_D-V_C}{Q} \quad \therefore \quad \frac{P}{Q}=\frac{V_A-V_D}{V_D-V_C}$$

$$\text{Again } i_2=\frac{V_A-V_B}{R}=\frac{V_B-V_C}{X} \quad \therefore \quad \frac{R}{X}=\frac{V_A-V_B}{V_B-V_C}$$

But at null condition, $V_B=V_D$ $\quad \therefore \quad \frac{P}{Q}=\frac{R}{X}$

Now, knowing the values of P, Q and R, the resistance of X may be found out.

Example : *Applying Wheatstone bridge principle to the circuit shown in fig. 3.3 (a), find the equivalent resistance between the points A and B.*

Ans. The circuit shown in fig 3.3(a) when simplified takes the form of Wheatstone bridge as shown in the adjoining figure 3.3(b). Now, from the figure, we see that $\frac{AC}{CB}=\frac{10}{10}=\frac{AD}{DB}$. So, the resistances along the branches AC, CB, AD and BD balance each other and no current flows through the resistance CD. Thus,

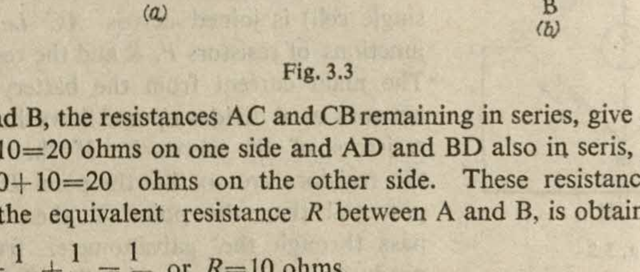


Fig. 3.3

between A and B, the resistances AC and CB remaining in series, give a total resistance $=10+10=20$ ohms on one side and AD and BD also in seris, give a total resistance$=10+10=20$ ohms on the other side. These resistances being in in parallel, the equivalent resistance R between A and B, is obtained from the relation, $\frac{1}{R}=\frac{1}{20}+\frac{1}{20}=\frac{1}{10}$ or $R=10$ ohms.

3.2. Metre bridge :

It consists of three thick copper strips AE, FM and NC of negligible resistance mounted on a wooden board (Fig. 3.4). These copper strips are provided with screw terminals and form two gaps EF and MN between them for the insertion of a resistance box R and an unknown resistance X. A straight wire AC, one metre long, of uniform cross-section is stretched alongside

a metre scale fixed on the same wooden board. The two ends of the wire are soldered to two copper strips at A and C. A brass jockey B can slide parallel to the length of the wire and can make contact at any point with the wire. The jockey terminal is connected to one of the terminals of a sensitive galvanometer

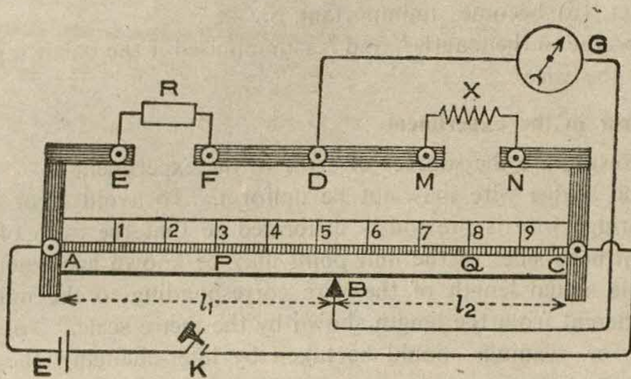


Fig. 3.4

G, the other terminal of the galvanometer being connected to the point D. The instrument as a whole is the practical form of the theoretical Wheatstone bridge circuit shown in fig. 3.2 and the points A, B etc of the fig. 3.2 correspond to the same points A, B etc. of the fig. 3.4.

The jockey B is then moved along the wire AC, right and left, and contact is made at various points along the wire, until a point is found for which the galvanometer gives no deflection. The two lengths l_1 and l_2 of the bridge wire AB and BC are measured from the metre scale. At null condition, we know, $\frac{P}{Q}=\frac{R}{X}$.

If σ be the resistance/unit length of the bridge wire AC, then the resistance of the length $AB=P=l_1\sigma$ and that of the length $BC=Q=l_2\sigma$.

$$\therefore \quad \frac{P}{Q}=\frac{l_1}{l_2}$$

Hence, $\frac{R}{X}=\frac{l_1}{l_2}$ or, $X=R.\ \frac{l_2}{l_1}=R\times\frac{100-l_1}{l_1}$

The experiment is to be repeated by taking different resistances from the resistance box and the mean value of X is to be found out therefrom.

[**N.B.** By this method, the specific resistance of the material of a wire can be easily found out ; for, measuring the diameter 'd' of the wire by a screw-gauge and length l by a metre scale, the specific resistance ρ can be found out from the relation,

$$\rho=X.\ \frac{\pi d^2}{4l}\Big]$$

Accuracy of measurement : The accuracy depends on l_1 and l_2 and the certainty of the balance point. The latter depends on (i) the sensitivity of the

galvanometer and (ii) the width of the contact B. The distance moved on either side of a balance point before the galvanometer detects a current, is a minimum when its resistance is approximately the same as P, Q, R and S *i.e.*, the galvanometer is most sensitive when all the resistances are equal. By using a knife-edge type of contact, (ii) becomes unimportant.

The error due to the length l_1 and l_2 is minimised if the balance point is near the centre of the wire.

Sources of error in the experiment :

The following are the sources of error in the experiment :

(i) The bridge wire may not be uniform. To avoid error due to this reason, the bridge wire is previously calibrated so that the ratio of the lengths of the wire on both sides of the null point may be known accurately.

(ii) The actual length of the wire corresponding to the null point may be slightly different from the length shown by the metre scale. To eliminate the consequent error, readings should be taken by inter-changing the positions of X and R.

(iii) Due to flow of current through the bridge wire as also through various junctions of the bridge, heat may be produced which alters the position of the null point. For this reason, readings should be taken by reversing the direction of the current. Further, a tap key should be included in the battery circuit and current should be allowed to flow as long as it is necessary.

(iv) **End correction** : In the description of the metre bridge, it has been said that the copper strips have negligible resistance. But however thick they may be, they have some resistance. Further the soldering of the bridge wire at A and C also introduces some resistance. Moreover, the wire may not be exactly one metre long. For all these reasons, some extra resistances are supposed to be introduced at the left and the right ends of the bridge wire. If these resistances are not taken into consideration, the results may be inaccurate. This error is '*End error*'. Usually, this end resistances are expressed in terms of the lengths of the bridge wire. They can be found out in the following way. Connect two known resistors P and Q in the two gaps *viz*, EF and MN respectively. Let the end corrections at the left and at the right ends be x cm. and y cm. respectively. If the length of the null point be l_1, then,

$$\frac{P}{Q}=\frac{l_1+x}{(100-l_1)+y} \quad \cdots \quad \cdots \quad \text{(i)}$$

Interchanging P and Q, if the length of the new null point be l_2, then,

$$\frac{Q}{P}=\frac{l_2+x}{(100-l_2)+y} \quad \cdots \quad \cdots \quad \text{(ii)}$$

From equations (i) and (ii), x and y can be found out and these values may be used in subsequent experiments.

(For details, See 'Practical Physics' by the Author)

Example 1 : *If any one of the resistances A and B is connected to the gap of a metre-bridge nearer to the zero mark of the scale and a fixed resistance to the*

other gap, null point is obtained at 4 *cm. mark of the scale. Where will the null point be obtained if under the same condition* (*i*) *the resistances A and B are joined in series and* (*ii*) *joined in parallel* ?

Ans. Since any of the resistances A and B when connected to the gap, gives null point at the same position, it is clear that the resistances are of equal value. If this value be P and the fixed resistance be Q, then

$$\frac{P}{Q}=\frac{40}{60}=\frac{2}{3} \quad \cdots \quad \cdots \quad \text{(i)}$$

(i) When A and B are joined in series the total resistance $=2P$. If the null point is at a distance x cm., then $\frac{2P}{Q}=\frac{x}{100-x} \quad \cdots \quad \cdots \quad$ (ii)

Dividing (i) by (ii) we get $\frac{1}{2}=\frac{2(100-x)}{3x}$ or $x=57{\cdot}14$ cm.

(ii) When A and B are joined in parallel, the total resistance$=\frac{1}{2}P$. If the null point is at a distance y cm then $\frac{P}{2Q}=\frac{y}{(100-y)} \quad \cdots \quad$ (iii)

Dividing (i) by (iii), we get, $2=\frac{3(100-y)}{3y}$ or $y=25$ cm.

Example 2 : *An aluminium wire of resistance* 7·3 *ohms at* 30°*C is placed in the left gap of a metre bridge and the balance is obtained at* 42·6 *cm from the left end of the bridge wire. If the temperature of the aluminium wire be increased to* 100°*C without changing anything else, by how much will the balance point shift? The temperature coefficient of resistance of aluminium*$=3{\cdot}8\times10^{-3}/°C$

[*Jt. Entrance 1983*]

Ans. The resistance of the aluminium wire will increase due to rise of temperature. We know $R=R_0(1+\alpha.t.)$. Hence, $7{\cdot}3=R_0(1+3{\cdot}8\times10^{-3}\times30)$ and $R_{100}=R_0(1+3{\cdot}8\times10^{-3}\times100)$.

So, $\frac{R_{100}}{7{\cdot}3}=\frac{1+3{\cdot}8\times10^{-3}\times100}{1+3{\cdot}8\times10^{-3}\times30}=\frac{1{\cdot}38}{1{\cdot}114} \quad \therefore \quad R_{100}=\frac{7{\cdot}3\times1{\cdot}38}{1{\cdot}114}$ ohms.

From the principle of metre bridge, we get, $\frac{7{\cdot}3}{42{\cdot}6}=\frac{P}{100-42{\cdot}6}$ where P is the resistance in the right gap. $\therefore\ P=\frac{7{\cdot}3\times57{\cdot}4}{42{\cdot}6}$

If x be the distance of the new null point from the left end of the bridge, then,

$$\frac{7{\cdot}3\times1{\cdot}38}{1{\cdot}114\times x}=\frac{P}{100-x}=\frac{73\times57{\cdot}4}{42{\cdot}6(100-x)} \quad \therefore \quad x=47{\cdot}9 \text{ cm.}$$

So, the shift of null-point$=47{\cdot}9-42{\cdot}6=5{\cdot}3$ cm.

3.3. Post office box :

This instrument is also a practical form of Wheatstone bridge principle. It is called Post office box because it was originally used by the Postal

department for the measurement of the resistance of telegraph cables, etc. A section of a P. O. box has been shown in fig. 3.5. The first line of the box

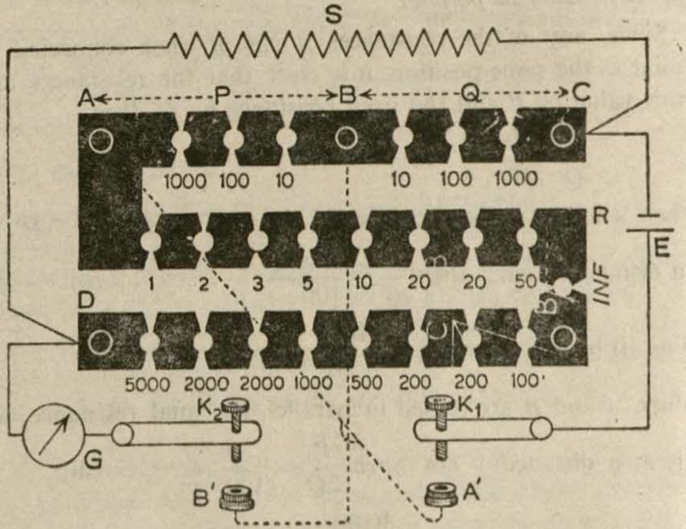


Fig. 3.5

CA is divided into two parts—*CB* and *BA*—each consisting of three resistance coils 10, 100 and 1000 ohms. These two parts act as the resistors *P* and *Q* of the Wheatstone bridge and are known as ratio arms. The third arm *R* of the Wheatstone bridge, extends in this instrument, from *A* to *D* and it consists of different resistance coils of values ranging between 1 to 5000 ohms connected in series. When the plug of any coil is taken out, its resistance will be included in the circuit. As a matter of fact, null condition in the galvanometer is produced by adjusting the resistance in this arm. The unknown resistor *S* is connected between the points *D* and *C*. A galvanometer *G* is inserted between the points *B* and *D* through a tap key K_2 and a battery *E* between the points *A* and *C* through another tap key K_1. That the instrument is a practical form of Wheatstone bridge will be clear from fig. 3.5(a).

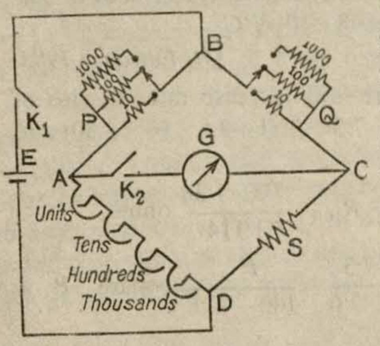


Fig. 3.5(a)

Use of the instrument : The use of the instrument will be clear if we take a specific example. Suppose, a resistor of 5·36 ohms resistance is given to us. We can find its value with the help of the P. O. box in the following way.

(i) Connect the resistor under test across *DC*. Take out 10 ohm plug from each of the ratio arms *P* and *Q*. Now, taking out 1 ohm plug from the third arm *R*, first press the battery-key K_1 and then the galvanometer key K_2. Note the direction of deflection in the galvanometer. Go on putting resistances of gradually

high denominations in the third arm R until for two consecutive resistances, having difference of 1 ohm, the directions of deflections of the galvenometer needle are opposite. For the above mentioned resistor, the deflections will be opposite for 5 ohms and 6 ohms. This shows that the unknown resistor has resistance lying between 5 and 6 ohms.

(ii) Now make the ratio of P and Q 10:1 by taking out 100 ohm plug from the arm P and 10 ohm plug from Q. It will be seen that when 53-ohm resistance is put in the third arm R, the deflection in the galvanometer is opposite to the deflection when 54-ohm resistance is put in it. This shows that the unknown resistor has resistance lying between 5·3 and 5·4 ohms.

(iii) Now take out 1000 ohm plug from the arm P and keep 10 ohm plug in Q as before *i.e.*, the ratio is now 100:1. Following the procedure described before, it will be seen that almost no deflection is produced in the galvanometer when a resistance of 536 ohms is put in the third arm R. In the circumstances, we get from Wheatstone bridge principle,

$$\frac{P}{Q}=\frac{R}{S} \quad \text{or} \quad \frac{1000}{10}=\frac{536}{S} \quad \therefore \quad S=\frac{536}{100}=5{\cdot}36 \text{ ohms.}$$

Example : *Four resistors P, Q, R and S form the four arms of a P. O. Box. Their resistances are* 2, 2, 2 *and* 3 *ohms respectively. What resistance joined in parallel with S gives a null point in the P. O. Box ?*

Ans. Let x ohm be the required resistance. The equivalant resistance of x ohm and S joined in parallel is $S'=\dfrac{S.x}{S+x}=\dfrac{3x}{3+x}$. From the principle of P. O. box. we have, $\dfrac{P}{Q}=\dfrac{R}{S'}$ or, $\dfrac{2}{2}=\dfrac{2(3+x)}{3x}$

or $3x=6+2x$ or $x=6$ ohms.

3.4. Potentiometer :

Description : The potentiometer is a piece of apparatus used for comparing potential differences. It consists of ten pieces of wire of uniform cross-section,

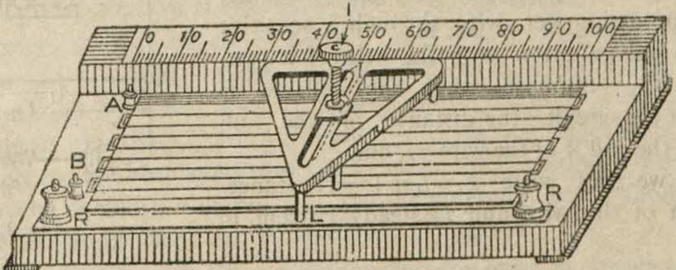


Fig. 3.6

each one metre long and joined in series by thick copper strips at their ends. The wires run parallel to each other and lie on a milk-white glass plate mounted on a wooden board (fig. 3.6). The two ends of the wire are connected to two terminal

screws A and B. The wire is so selected that its material has low temperature coefficient of resistance. Generally manganin or constantan wire is used. A triangular brass jockey slides along a graduated metre scale and during such motion, one of its legs (L) keeps contact with a brass strip $R-R$. One end of a galvanometer is connected to the binding screw provided with the brass strip. By pressing a central key T, the central leg of the jockey may be brought in contact with any wire.

Principle of action : Suppose AB is the potentiometer wire (fig. 3.7). A battery E of constant e.m.f. drives a steady current through the wire. Suppose, the resistance per unit length of the wire $=\sigma$ and the current flowing through the wire $=i$ amp. In this case, the terminal p.d. of the wire $=1000\times\sigma\times i$ [the resistance of the wire $AB=1000\times\sigma$]

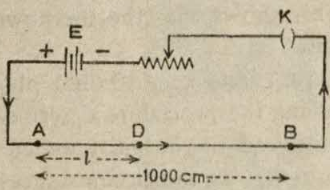


Fig. 3.7

The terminal p.d. of the portion AD of the wire $=l\times\sigma\times i$

$$\therefore \frac{\text{The terminal p.d. of the wire } AB}{\text{,, ,, ,, ,, ,, portion } AD}=\frac{1000\times\sigma\times i}{l\times\sigma\times i}=\frac{1000}{l}$$

$\therefore$ the terminal p.d. of the portion AD

$$=\frac{l}{1000}\times \text{ the terminal p.d. of the wire } AB$$

So, we see that when a steady current flows through the potentiometer wire, the p.d. across any length of the wire is proportional to its length. This is the principle of action of a potentiometer.

3.5. Uses of potentiometer :

(a) **Comparision of e.m.f's of two cells** : Suppose e.m.f's of two cells E_1 and E_2 are to be compared. A battery E (whose e.m.f. should be greater than the e.m.f. of either cells to be compared), a rheostat R_h, a plug key K are connected in series with the potentiometer wire AB (fig. 3.8). The cells E_1 and E_2 may be brought in contact with the potentiometer wire through a sensitive centre-zero galvanometer with the help of a two-way key K_1. Suppose when the cell E_1 is included in the circuit, the distance of the null point from the end A of the wire $=l_1$ cm.

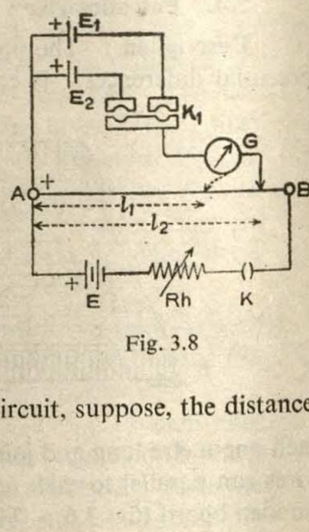


Fig. 3.8

Here we may write $E_1=l_1\sigma i$ [σ=resistance/unit length of the wire and i=steady current in the wire.]

Similarly, when the cell E_2 is included in the circuit, suppose, the distance of the null point $=l_2$ cm. Here, $E_2=l_2.\sigma i$.

$$\text{Dividing, we get } \frac{E_1}{E_2}=\frac{l_1}{l_2}$$

By changing the circuit resistance by the rheostat R_h several observations may be taken and the mean value of the ratio can be found out therefrom.

If one of the two cells compared be a standard cell of known e.m.f. the e.m.f. of the other cell may be determined.

(*b*) **Measurement of e.m.f. of a cell** :

Suppose, the e.m.f., of the cell E_1 is to be measured. A battery E whose e.m.f. is greater than that of the cell E_1, a milli-ammeter (M.A.), a rheostat R_h and a plug key K are connected in series with the potentiometer wire AB. G is a sensitive galvanometer (fig. 3.9)

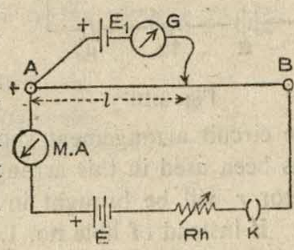


Fig. 3.9

Send a steady current through the wire AB by putting a plug in the plug key K. Next find the position of the null point by sliding the jockey along the wire. Let l cm be the length of the null point. It is clear that, E_1=terminal p.d. of the length l cm.

$$=\frac{l}{1000}\times\text{terminal p.d. of the wire } AB.$$

Now, if the current flowing through the wire AB be i amp. (shown by the milli-ammeter) and the resistance of the wire AB be r, then the terminal p.d. across $AB=i.r.$ $\therefore$ $E_1=\frac{i.r.l.}{1000}$ volt.

It is to be noted that the current shown by the milli-ammeter is to be converted into ampere. By changing the resistance of the circuit by the rheostat, several observations may be taken and from that the mean value of the current can be evaluated.

Accuracy of potentiometer : No current flows through the cells when a balance point is found so that the instrument measures e.m.f.'s accurately. Moving coil voltmeters cannot measure e.m.f. accurately since they take current. Electrostatic voltmeters do not pass current but they are insensitive for small p.d.'s. For these reasons, a potentiometer is widely used.

The precision with which the balance point of a potentiometer can be found depends on the sensitivity of the galvanometer. It is important to realise that accuracy of a potentiometer does not depend on the accuracy of the galvanometer but only on its sensitivity. The galvanometer, it may be pointed out, is used not to measure a current but merely to show one when the potentiometer is off balance. It is said to be used as a null-indicator and the potentiometer method of measurement, like the bridge methods, is called a null method.

The current through the potentiometer wire must be steady—it must not change appreciably between the successive balance points. To realise this, freshly charged or nearly run-down accumulators should be avoided. Non-uniformity of potentiometer wire may also cause some error.

(c) **Measurement of current in a circuit :**

The cell E_2 and a variable resistor R have built up a circuit whose current is to be measured. A standard resistor r of very low value is included in the circuit as shown in the Fig. 3.10. Since the resistor r is of very low value, it will not, by its inclusion in the circuit, disturb the current flowing in the circuit but it will produce a p.d. across its terminals. If p.d. be V, then circuit current $i=\frac{V}{r}$.

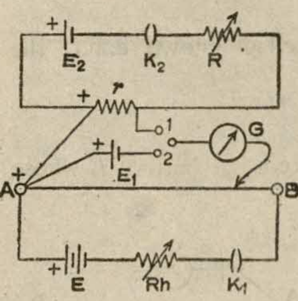


Fig. 3.10

To determine the value of V, the p.d. is to be compared with the e.m.f. of a standard cell E_1 with the help of a potentiometer. For this, the circuit arrangement should be made as shown in fig. 3.10. A two-way key has been used in this arrangement. If the plug be inserted in hole no.1, the resistor r will be brought in the potentiometer circuit through the galvanometer G. If instead of hole no. 1, the plug is inserted in the hole no. 2, the cell E_1 will be included in the circuit.

Suppose when the resistor r is included in the potentiometer circuit, the distance of the null point from $A=l_1$ cm.

We can, therefore, write $V=l_1\sigma.I$. (I=current in the wire AB).

Similarly, when the cell E_1 is included in the circuit, suppose the distance of the null point$=l_2$ cm. In this case, $E_1=l_2.\sigma.I$.

$$\text{Dividing, } \frac{V}{E_1}=\frac{l_1}{l_2} \quad \therefore \quad i=\frac{V}{r}=\frac{E_1.l_1}{r.l_2}$$

All the quantities on the right-hand side of the equation being known, i can be calculated.

It may be pointed out here that for successful performance of the experiment, the e.m.f. of the cell (or the battery) E must be greater than the p.d. across the the resistance r and the *e.m.f.* of the standard cell E_1. If this is not the case, balance point will not be available.

Example 1 : *A potentiometer wire* 600 *cm. long and of* 10*-ohm resistance is connected in serise with a cell of e.m.f.* 4 *volt and internal resistance* 6 *ohm. Find (i) the current in the wire (ii) the fall of potential per cm of the wire (iii) the balance point for a cell of e.m.f.* 1·5 *volt.*

Ans. (i) Current in the wire$=\frac{4}{10+6}=\frac{4}{16}=0{\cdot}25$ amp.

(ii) P.D. at the ends of the wire=current×resistance of the wire

$=0{\cdot}25\times10=2{\cdot}5$ volts.

$\therefore$ Fall of potential per cm.$=\frac{2{\cdot}5}{600}=4{\cdot}17\times10^{-3}$ volt.

(iii) Suppose the length of the balance point $= l$ cm.

$\therefore$ P.D. across the length $= \frac{2\cdot5}{600} \times l$

Hence $\frac{2\cdot5 \times l}{600} = 1\cdot5$ or, $l = \frac{1\cdot5 \times 600}{2\cdot5} = 360$ cm.

Example 2 : *In an experiment for measurement of current by potentiometer, the standard low resistance used was* 0·01 *ohm. The drop of potential per centimetre of the potentiometer wire was* 13×10^{-6} *volt. The balance point was at* 760 *cm of the bridge wire. Calculate the strength of the current flowing through the given line.*

Ans. Potential drop per cm of the wire $= 13 \times 10^{-6}$ volt.

So, p.d. across 760 cm of the wire $= 13 \times 10^{-6} \times 760$ volt.

This p.d. is equal to the p.d. across the low resistance.

Hence, the current in the line $= \frac{\text{p.d.}}{\text{low resistance}}$

$$= \frac{13 \times 10^{-6} \times 760}{0\cdot01} = 0\cdot988 \text{ amp.}$$

Exercises

Essay type :

1. Write the principle of a metre bridge and explain how two resistors can be compared with its help.

2. Discuss the principle of Wheatstone bridge in the measurement of resistance with a diagram. [*H. S. Exam. 1980, '84*]

3. How would you use a metre bridge in measuring the resistivity of the material of a wire ?

4. Describe a Post Office box. Explain how the resistance of a wire can be found out with its help ?

5. How would you proceed to compare e.m.f.'s of two cells by a potentiometer. Explain the basic principle.

6. A cell of e.m.f. E volts is used to drive a current through a resistance R ohm. It is desired to measure the potential drop across the wire accurately by means of a potentiometer. If the current through the potentiometer wire be driven by means of another identical cell, will it be possible to obtain the balance point accurately ? If not, what would you do to perform the experiment successfully ? [*Jt. Entrance 1983*]

7. Describe a potentiometer and explain, in detail, the method you would adopt for measuring the current flowing in a circuit with it.

8. Explain the procedure for comparing the e.m.f.'s of a Leclanche's cell and a Daniel cell.

Short answer type :

9. Will the Wheatstone bridge principle be modified if (i) the positions of the battery and the galvanometer are interchanged (ii) a galvanometer of high resistance is used ? [*H. S. Exam. 1978*]

10. Give the wiring diagram of a metre bridge arrangement.

11. In determining an unknown resistance by a metre bridge we find out of the balance

point (*a*) by taking direct and reverse currents (*b*) by interchanging the positions of known and unknown resistances. Which errors are we able to remove by those processes ?
[*Jt. Entrance 1978*]

12. What type of cell is used in a potentiometer circuit while measuring an unknown e.m.f. or current by a potentiometer ?

13. Potentiometer can measure e.m.f. more accurately than a moving coil voltmeter or an electrostatic voltmeter. Why ?

14. In the potentiometer measurement, should the galvonometer be very accurate or very sensitive ?

Numerical Problems :

15. A 10 ohm resistor is connected to the left gap of a metre bridge and a combination of 10 ohm and 15 ohm in parallel in the other gap. At what point from the left end of the bridge wire will the null point be obtained ? [Ans. 62·5 cm.]

16. A metre bridge is 100 cm long and the standard resistance R_s used is 100 ohms. Null point is obtained at 4.715 cm away from the end where the unknown resistance R_x is fixed. On interchanging the positions of R_s and R_x, the null point is obtained 52·75 cm from the same end. Find R_x. [Ans. 89·38 ohms]

17. To one gap of a metrebridge is fixed a 2-ohm resistor and to other gap a 3-ohm resistor. What will be the position of the null point ? What resistance should be connected in parallel with 3-ohm resistor in order to get the null-point at the middle of the wire ?
[Ans. 40 cm away from 2-ohm resistor ; 6 ohms]

18. The resistance of a 1000 cm long potentiometer wire is 20 ohms. A milli-ammeter included in the potentiometer circuit shows a current of 100 milli-amperes. Find the position of the null point due to a Leclanche's cell of e.m.f. 1·4 volts. [Ans. 700 cm.]

19. A secondary cell of e.m.f. 2 volt and internal resistance 0·1 ohm is connected to the ends of a uniform wire of length 1 metre and resistance 1·5 ohm. A primary cell of e.m.f. 1·5 volt in series with a galvanometer shows no deflection ; find the distance between the points.
[Ans. 80 cm.]

20. A current is passed through an ammeter and a standard coil of resistance 0·1 ohm, connected in series and is adjusted until the ammeter reads 5 amp. The p.d. between the ends of the standard coil is balanced by the p.d. across 128·8 cm of a potentiometer wire. A standard cell of e.m.f. 1·018 volt is found to require 254·5 cm of the same potentiometer wire for a balance. What is the error in the ammeter reading ? [Ans. 0·152 amp. (less)]

[Hints : $\dfrac{\text{P.D. across standard coil}}{\text{,, ,, standard cell}}=\dfrac{128\cdot8}{254\cdot5}$ $\therefore$ P.D. across the coil $=\dfrac{128\cdot8}{254\cdot5}\times1\cdot018$ volt

So current $=\dfrac{128\cdot8\times1\cdot018}{254\cdot5\times0\cdot1}=5\cdot152$ amp. $\therefore$ Error $=5\cdot152-5=0\cdot152$ amp.]

21. The e.m.f. of a cell is balanced by the fall of potential along 150 cm. of a potentiometer wire. When the cell is shunted by a resistance of 14 ohms the length required is 140 cm. What is the internal resistance of the cell ? [Ans. 1 ohm.]

22. The following data were obtained in an experiment to measure the e.m.f. of a Leclanche's cell : the current in the potentiometer=80 milli amp ; resistance of potentiometer wire=20 ohms ; the length of the null point= 950 cm. Find the e.m.f. of the cell. [Ans. 1·52 volts]

Harder Problems :

23. A wire, bent in the form of a square *ABCD*, has each arm of length x cm. The resistance of the wire is r ohm per cm. The corners *A* and *C* as well as *B* and *D* are joined by identical wires of length x cm. Applying the principle of Wheatstone bridge, find the equivalent resistance between *A* and *C*. [Ans. $\frac{1}{2}\,x.r$]

24. P and Q are two fixed resistances, R is a known variable resistance and S is a combination of two equal resistances, each of X ohm. At first, the combination is a series combination and then a parallel combination. In the first case, they balance each other when the known resistance is 27 ohms but when parallel combination is used and P and Q are interchanged, R is changed to 3 ohms to restore balance. Find the value of X and also the ratio P/Q.
[Ans. 9 ohms ; 3/2]

25. Four resistances P, Q, R and S were balanced in a metre bridge in the following way :
(i) P and Q in series in one arm, R and S in series in the other (ii) P and R in series in one arm, Q and S in series in the other. If l_1 and l_2 were the balance lengths in the two cases, show that $l_2 - l_1 = \dfrac{100(R-Q)}{P+Q+R+S}$

26. Find the p.d. between the points X and Y in Fig. 3.11. [Ans. 0·67V]

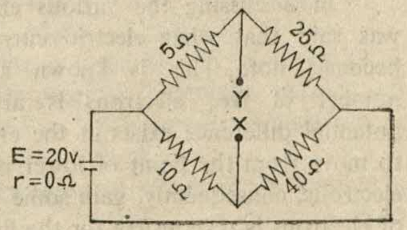


Fig. 3.11

27. A battery of e.m.f. 2 volts and negligible resistance is used across the ends of a metre bridge whose wire has a diameter of 0·1 cm and resistance 1·5 ohms. The metre bridge is used to compare two resistances 1 ohm and 2 ohms respectively. Calculate (i) the current through the battery when the bridge is balanced (ii) resistivity of the bridge wire.
[Ans. (i) 2 amp (ii) $11{\cdot}78\times10^{-5}$ ohm-cm]

28. A wire of resistance 0·1 ohm/cm is bent in the form of a square $ABCD$ of side 10 cm. A similar wire is connected between the corners B and D to form the diagonal BD. Find the effective resistance of this combination between the corners A and C. [*I.I.T. 1971*] [Ans. 1 ohm]

29. The standard low resistance in a potentiometer circuit is 0·1 ohm, the e.m.f. of the standard cell is 1·018 volt. The null point for the standard cell is obtained at 920 cm. while that for the standard low resistance is at 46 cm. What is the current through the low resistance ?
[*Jt. Entrance* 1978] [Ans. 0·5 amp. (nearly)]

30. A potentiometer wire of length 100 cm. has a resistance of 10 ohms. It is connected in series with a resistance and a cell of e.m.f. 2 volts and of negligible internal resistance. A source of e.m.f. 10 millivolts is balanced against a length of 40 cm of the potentiometer wire. What is the value of the external resistance ? [*I.I.T. 1976*] [Ans. 790 ohms]

31. A standard cadmium cell of e.m.f. 1·02 volt gives a null point on a potentiometer wire at a length 70.4 cm. If a Daniel cell in open circuit is used instead of the cadmium cell, the null point is at a distance of 74·5 cm. If a resistance of 10 ohms is now connected with the Daniel cell, the null point shifts to 41·3 cm. Find (*a*) the e.m.f. of Daniel cell (*b*) its internal resistance.
[Ans. (*a*) 1·07 volts ; (*b*) 8 ohms (nearly)]

32. The resistance of 10-wire potentiometer is 20 ohms. With it are connected in series a cell of e.m.f. 2 volts and negligible internal resistance and a resistance box. If each of the 10 wires of the potentiometer is 1 metre long, what resistance should be taken out from the resistance box in order to produce a p.d. of 1 microvolt per 2 mm of the wire ? If an unknown p.d. produces a balance point at 750 cm of the wire, what is its value ? [Ans. 7980 ohm ; 3750 microvolt]

33. A Leclanche's cell and a Daniel cell are connected in series so that their e.m.f.'s act in conjunction. This combination gives a null point at 875 cm on a potentiometer wire. On reversing the Daniel cell, the combination requires only 140 cm. length for balance. Find the e.m.f. of Leclanch's cell, if that of Daniel cell is 1·1 volt. [Ans. 1·52 volt]

4

HEATING EFFECT OF ELECTRIC CURRENT AND THERMO-ELECTRICITY

4.1. Introduction :

In discussing the various effects of electric current, in chapter one, it was said that when electric current flows through a conductor, the conductor becomes hot. This is known as *heating effect* of current. A large number of free electrons is always available in every conductor. When a potential difference exists at the ends of a conductor, these free electrons try to move from the point of lower potential to the point of higher potential. The electrons, consequently, gain some kinetic energy. As a matter of fact, this drift of electrons is responsible for the flow of current through the conductor. Now, these electrons, in their attempt to move from one point to another collide against surrounding molecules of the conductor, which as a result, become possessed with increased kinetic energy. Increased kinetic energy of the molecules means an increase in temperature. For this reason, when current passes through a conductor, heat is produced and the temperature of the conductor increases. As the temperature rises., the radiation of heat from the surface of the conductor increases. Continuing in this manner, a stage is soon reached, when the rate of production of heat equalises the rate of loss of heat due to radiation. Then the conductor attains a steady temperature. This heating effect of current serves many useful purposes. As domestic applications of this effect, we get electric bulb, electric heater, stove, iron, etc. As industrial applications, we get electric arc or arc welding, electric furnace, etc. The law concerning the development of heat by electric current was first established by Dr. Joule in 1841. The law is known as Joule's law.

4.2. Joule's law :

If a current I flows through a conductor of resistance R for time t, Joule's law states that—

(i) the heat (H) produced is proportional to the square of the current (I^2), when the resistance of the conductor and time of flow remain unaltered. *i.e.* $H \propto I^2$ when R and t are constants.

(ii) the heat produced is proportional to the resistance of the conductor when the current and time of flow remain unaltered *i.e.* $H \propto R$ when I and t are constants.

(iii) the heat produced is proportional to the time of flow of the current when the current and the resistance of the conductor remain unaltered *i.e.* $H \propto t$ when I and R are constants.

Mathematically, Joule's law may be expressed in symbols. Thus,

$$H \propto I^2 R.t.$$

4.3. Establishment of Joule's law :

Suppose, the terminal p.d. of a conductor AB is E, and its resistance is R. A current I flows through the conductor for an interval t. (Fig. 4.1)

From the definition of potential difference, it follows that the work done in allowing unit quantity of charge to flow from one end of the conductor to the other$=E$. Since the current I flows through the conductor for time t, the charge transferred is $I.t$. Let the charge be Q. Hence, work done in transferring charge Q from one end of the conductor to the other is given by, $W=E.\ Q=E.I.t$.

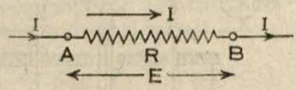


Fig.4.1

So far no units have been mentioned for potential difference, current, etc. According to practical units, if E is expressed in volts, current in amperes and time in seconds, then $W=E.I.t$ Joules$=E.I.t\times 10^7$ ergs.

This work will be converted into heat which will warm up the conductor. If H calories of heat are developed, then

$W=J.H$ (J=mechanical equivalent of heat.)

$$\therefore\ H=\frac{E.I.t\times 10^7}{J}=\frac{E.I.t.\times 10^7}{4{\cdot}2\times 10^7}\ \text{cal}\quad [J=4{\cdot}2\times 10^7\ \text{ergs/cal}]$$

$$=0{\cdot}24\times E\times I\times t\ \text{cal.}\qquad \text{(i)}$$

$$=0{\cdot}24\times I^2\times R\times t\ \text{cal.}\qquad \text{(ii)}\qquad (\because\ E=R\times I)$$

or, $H\propto I^2R.t.$ This is Joule's law.

Further, if $I=1$ amp ; $R=1$ ohm. and $t=1$ sec., then $H=0{\cdot}24$ calorie. This shows, that if 1 amp. of current flows through a conductor of resistance 1 ohm, 0·24 calorie of heat will be produced in the conductor in 1 second.

Important Conclusions :

(a) From eqn. (ii) it is seen that $H\propto I^2$; so if the current is reversed, the quantity of heat produced remains the same. In other words, *the rate of production of heat i.e. Joule effect does not depend on the direction of current.* Hence, it is called irreversible.

(b) $H\propto R$; Also $R\propto l$, the length of the conductor as also $R\propto\frac{1}{A}$ where A is the cross-sectional area of the conductor. This shows that current remaining the same, the quantity of heat produced is directly proportional to the length of the conductor and inversely proportional to the area of the cross-section of the conductor.

(c) From eqn. (i) above, we see that $H=0{\cdot}24\times E\times I\times t$ calories

$$=0{\cdot}24\times\frac{E^2}{R}\times t\ \text{calories}\left[\ \therefore\ I=\frac{E}{R}\right]\ \text{or}\ H\propto\frac{E^2}{R}\times t.$$

Hence we can write, (i) $H\propto E^2$ when R and t are constants

(ii) $H\propto\frac{1}{R}$ when E and t are constants

(iii) $H\propto t$ when E and R are constants.

4.4. Verification of Joule's law :

(i) **To verify** $H \propto I^2$: C is a polished copper calorimeter partly filled with cold water (Fig. 4.2). The calorimeter is provided with an ebonite lid having serveral openings, through which are inserted a coil of eureka wire and a thermometer T. The ends of the coil are connected to two terminals ($S.S$). A battery B, an ammeter A and a rheostat R are connected in series with the coil. Adjust the rheostat to send a suitable current, say I_1 amp. through the coil for given interval (say, 10 minutes) of time. Note the rise of temperature of water from the thermometer T. Let the rise of temperature be $T_1 C$. Now stop the current and allow the water to come back to the room temperature.

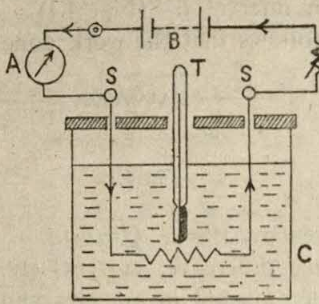


Fig. 4.2

Change the current with the help of the rheostat. Allow the new current, say I_2 amp. to flow through the coil for the same interval of time. Again note the rise of temperature. Let it be T_2°C. If the heat developed in the two occasions be H_1 and H_2 calories then since the thermal capacity of the calorimeter and its centents remains constant, $\dfrac{H_1}{H_2} = \dfrac{T_1}{T_2}$

From the experiment, it will be seen $\dfrac{T_1}{T_2} = \dfrac{I_1^2}{I_2^2}$ $\therefore$ $\dfrac{H_1}{H_2} = \dfrac{I_1^2}{I_2^2}$

i.e. $H \propto I^2$ when R and t are constants.

(ii) **To verify** $H \propto R$: Two identical calorimeters containing equal amounts of cold water are taken. Into the water are inserted two heating coils of different resistances R_1 and R_2. Circuit connections are made as shown in Fig. 4.3. Since the coils are joined in series, same current flows through each. Adjusting the rheostat, send a suitable current for a certain interval of time. Note the rise of temperature of water in the calorimeters with the help of thermometers. Suppose, the rise of temperature is T_1°C in one calorimeter and T_2°C in the other. Since the thermal capacity of the calorimeters and their contents remains constant, the heat developed in the calorimeters will be proportional to the rise of temperature

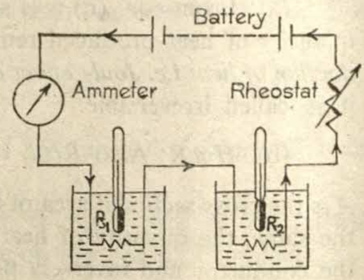


Fig. 4.3

$$i.e. \ \frac{H_1}{H_2} = \frac{T_1}{T_2},$$

From the experiment it will be seen $\dfrac{R_1}{R_2} = \dfrac{T_1}{T_2}$ $\therefore$ $\dfrac{H_1}{H_2} = \dfrac{R_1}{R_2}$

i.e. $H \propto R$ when I and t are constants.

(iii) **To verify** $H \propto t$: The circuit arrangement is same as that shown in Fig. 4.2. Send a suitable current through the heating coil for t_1 sec., say. Suppose, the rise of temperature of water is T_1°C. Stop the current and allow the calorimeter and its contents to return to the room temperature.

Next, allow the same current to flow through the heating coil for different time, say t_2 sec. Again note the rise of temperature. Suppose it is T_2°C.

If H_1 and H_2 be the amounts of heat developed in the two occasions, then $\frac{H_1}{H_2}=\frac{T_1}{T_2}$. From the experiment it will be seen that $\frac{T_1}{T_2}=\frac{t_1}{t_2}$ $\therefore$ $\frac{H_1}{H_2}=\frac{t_1}{t_2}$

i.e. $H \propto t$ when R and I are constants.

Example 1 : *If 20 coulombs of electric charge flow across a potential difference of 220 volts, how much work is done ? If the charge flows through a resistor, how much heat will be generated in it ? J=4·2 joules/cal.*

[*H. S. Exam. 1984*]

Ans. Work done, W=charge × potential difference=20 × 220=4400 joule.

If H be the heat produced, then $W=J.H$ or $H=\frac{W}{J}=\frac{4400}{4\cdot2}$ cal=1047·6 cal.

Example 2 : *How much heat is developed when a current of 0·8 amp flows through a resistor of 10 ohms for 1 minute ?*

Ans. We know, $H=0\cdot24\ I^2R.t$ calories.

Here, $I=0\cdot8$ amp ; $R=10$ ohms; $t=1$ mnt=60 sec.

$\therefore$ $H=0\cdot24(0\cdot8)^2\times10\times60$ calories=92·16 calories.

Example 3 : *A current flows through a 10-ohm coil for 10 minutes and the heat developed is completely supplied to 100 gm of water. If the temparature of water rises from 15°C to 75°C, calculate the current strength.*

Ans. Here, the heat developed, H=mass of water × rise of temp.

$=100(75-15)=100\times60=6000$ cal.

We know, $H=0\cdot24\times I^2\times R\times t$

$\therefore$ $6000=0\cdot24\times I^2\times10\times10\times60$ or, $I^2=\frac{6000}{0\cdot24\times10\times10\times60}=\frac{100}{24}$

or, $I=\sqrt{\frac{100}{24}}=2\cdot04$ amp. (nearly).

Example 4 : *Two resistors of 40 ohms and 60 ohms are connected in series and the combination is then connected to 200 volts main. Calculate the heat developed in each resistor in ½ minute.*

Ans. Total resistance of the resistors=40+60=100 ohms.

So, current through each resistor$=\frac{\text{volts}}{\text{ohms}}=\frac{200}{100}=2$ amp.

Heat developed in 40-ohm resistor

$H_1=0\cdot24\times(2)^2\times40\times30$ cal.=1152 cal.

Heat developed in 60-ohm resistor

$H_2=0\cdot24\times(2)^2\times60\times30$ cal=1728 cal.

Example 5 : *In the circuit shown in fig.* 4.3(i), *the heat produced in the* 5 *ohm resistor due to the current flowing through it is* 10 *calories per second. Find the rate at which heat is produced in the* 4 *ohm resistor.* [*I.I.T 1981*]

Ans. Let V be the p.d. at the ends of the parallel combination. According to Joules law, the rate of heat production in 5 ohm resistor,

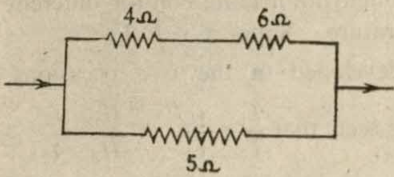


Fig. 4.3(i)

$$H_1 = 0{\cdot}24 \times \frac{V^2}{R} = 0{\cdot}24 \times \frac{V^2}{5} \text{ cal/sec.}$$

From the problem. $H_1 = 10$ cal/s.

So, $10 = 0{\cdot}24 \times \frac{V^2}{5}$ or $V^2 = \frac{50}{0{\cdot}24}$

Again, 4 ohm and 6-ohm resistors being in series having a p.d. of V, the current in them $i = \frac{V}{4+6} = \frac{V}{10}$ amp. So, rate of production of heat in 4-ohm resistor is $H = 0{\cdot}24\ i^2R = 0{\cdot}24 \times \frac{V^2}{100} \times 4 = 0{\cdot}24 \times \frac{50}{0{\cdot}24 \times 100} \times 4 = 2$ cal/s

Example 6 : *Four resistors are put in a mixed circuit as shown in fig.* 4.3 (ii) *and then connected to a battery, which delivers* 5 *amp current to the circuit. What is rate of heat developed in the whole circuit* ?

Ans. If R_1 be the equivalent resistance between A and B, then

$\frac{1}{R_1} = \frac{1}{20} + \frac{1}{20} = \frac{1}{10}$ or $R_1 = 10$ ohms.

Similarly, if R_2 be the equivalent resistance between B and C, then

$\frac{1}{R_2} = \frac{1}{40} + \frac{1}{160} = \frac{5}{160} = \frac{1}{32}$

$\therefore\ R_2 = 32$ ohms.

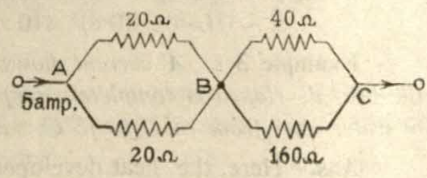


Fig. 4.3(ii)

So, total resistance between A and C is given by $R = R_1 + R_2 = 10 + 32 = 42$ ohms.

If H cal/s be the rate of heat production in the whole circuit, then $H = \frac{i^2R}{4{\cdot}2} = \frac{(5)^2 \times 42}{4{\cdot}2} = 250$ cal/s.

Example 7 : *Two wires have same material and mass but the length of one is double that of the other. Compare the heats developed in them when* (*i*) *their voltage is same* (*ii*) *their current is same.*

Ans. Suppose the length and resistance of one wire are l and R_1 respectively. Then the length of the other wire is $2l$. Its cross-section will be half that of the first because they have equal mass.

Again since their material is the same they have equal specific resistance (ρ). Under this circumstances, $R_1 = \rho\ \frac{l}{A}$ and $R_2 = \rho \times \frac{2l}{\frac{A}{2}} = \rho \times \frac{4l}{A} = 4R_1$

(i) Voltage being the same, the heat produced is inversely proportional to the resistance. Hence, in this case the ratio of heat produced=4 : 1.

(ii) Current being the same the heat produced is proportional to the resistance. Hence, in this case the ratio of heat produced=1 : 4.

Example 8 : *Two wires of resistance r_1 and r_2 are successively connected to a battery of internal resistance r. If in the same time, the heat developed in them be same, prove that $r=\sqrt{r_1.r_2}$.*

Ans. Let e be the e.m.f. of the battery. When the wire of resistance r_1 is joined, the current $i_1=\frac{e}{r+r_1}$. So heat developed in it $=0{\cdot}24\times i_1^2 r_1 t=$

$$0{\cdot}24\left(\frac{e}{r+r_1}\right)^2 . r_1.t \quad .. \quad \text{(i)}$$

When r_2 is connected with the battery, the current $i_2=\frac{e}{r+r_2}$. So heat produced in it$=0{\cdot}24\times i_2^2 r_2.t=0{\cdot}24\left(\frac{e}{r+r_2}\right)^2 r_2.t \quad .. \quad$ (ii)

According to the question eqn^s. (i) and (ii) are equal.

$$\text{So,}\quad \frac{0{\cdot}24\times e^2\times r_1\times t}{(r_1+r)^2}=\frac{0{\cdot}24\times e^2\times r_2\times t}{(r_2+r)^2}$$

or, $r_2(r_1+r)^2=r_1(r_2+r)^2$

or, $r_2.\ r_1^2+r_2.r^2+2r.r_1.r_2.=r_1.r_2^2.+r_1.r^2.+2r_1.r_2.r$

or, $r^2(r_2-r_1)=r_1r_2(r_1-r_2)$ or $r^2=r_1.r_2$ $\therefore$ $r=\sqrt{r_1.r_2}$.

4.5. Determination of J by electrical method :

(i) By Joule's calorimeter.

If E volts be the p.d. across a conductor through which a current of I amp is flowing for t seconds the work done $W=E.I.t$ joules. It will produce some heat in the concuctor. If H calories of heat are produced, then, $W=J.H.$

[J=mechanical equivalent of heat]

So, $E.I.t=J.H$

$$\therefore \quad J=\frac{E.I.t}{H} \text{ Joules/cal.}$$

So knowing E, I, t, and H, the value of J can be found out.

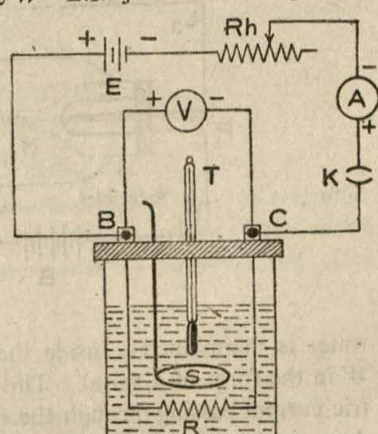


Fig. 4.4

Experiment : The arrangement consists of a nickel-plated copper calorimeter having a stirrer S made of copper (Fig. 4.4). The mouth of the calorimeter can be covered by an ebonite lid. The calorimeter with its stirrer is to be weighed empty. Then some turpentine is poured into the calorimeter and it is reweighed. A heating

coil R is inserted into the oil. Its ends are connected to two screw terminals B and C on the lid of the calorimeter. In series with the coil are connected a battery E, a rheostat R_h, an ammeter A and a key K. A thermometer T is introduced into the oil to record its temperature. A voltmeter V is connected across the heating coil to read the p.d. of the coil.

Allow a current to pass through the heating coil. The heat developed will be taken up by the calorimeter and its contents, whose temperature will, therefore, rise,

Suppose, m=mass of oil taken in gm.
S =sp. heat of the oil.
w =water equivalent of the calorimeter and the stirrer in gm.
θ =rise of temperature in °C.
E =p.d. across the heating coil as shown by the voltmeter in volts.
I =current through the heating coil as shown by the ammeter in ampere.
t =time during which the current flows as recorded by a stop-watch in sec.

Here, the work done $W=E.I.t$ joules, and heat developed $H=(mS+w)\,\theta$ calories.

$$\text{Now,}\quad J=\frac{E.I.t}{H}=\frac{E.I.t}{(mS+w)\theta}\ \text{joules/cal}$$

Experiment is to be repeated by changing the current with the help of the rheostat and the mean value of J to be determined therefrom.

(ii) **By Callender and Barnes' continuous flow calorimeter :**

The essential features of the experimental arrangement have been shown in fig. 4.5. It consists of a narrow glass tube MN through which a steady flow of

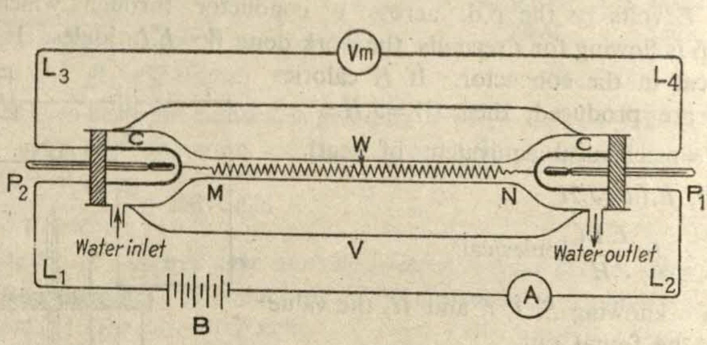


Fig 4.5

water is maintained. Inside the tube is mounted a platinum or a nichrome wire W in the form of a spiral. This produces automatic stirring of water. The electric current passing through the wire W produces heat which warms up the water. As a result, temperature difference between the inflow and outflow of water gradually increases and finally becomes steady. The steady temperature difference is measured by two thermometers P_1 and P_2. The current from the battery B

flows through the heating coil W along the lead L_1 and returns to the battery along the other lead L_2. Two more wires L_3 and L_4 are connected to the heating coil W. To meausre the p.d. across the heating coil, a voltmeter V_m is connected between these wires. The glass tube MN is jacketted by a wider glass tube V from which air is pumped out. This minimises loss of heat due to radiation.

With the apparatus properly set and adjusted, the flow the water through the tube MN is started and the current through the heating coil is allowed to flow. After some time when a stedy state has been reached, the temperature of inflow and outflow liquid are noted respectively from thermometers P_1 and P_2. Suppose the temperatures are θ_1 and θ_2 respectively. The water flowing out in a definite interval of time noted from a stop-watch, is collected and its mass is determined. The ammeter A connected with the heating coil W gives the current (I) flowing through the coil. The p.d.(E) across the heating coil W is noted from the voltmeter V_m.

Calculation : The quantity of heat absorbed by water$=m\ (\theta_2-\theta_1)$.

Heat produced by electric current$=\dfrac{E.I.t}{J}$

$$\therefore \quad \frac{E.I.t}{J}=m(\theta_2-\theta_1)+h \qquad .. \quad \text{(i)}$$

where h represents the heat lost by radiation. To eliminate h, the whole procedure is repeated with a different current I_1 keeping the difference of temperature $(\theta_2-\theta_1)$ unaltered. In the second case, we have,

$$\frac{E_1I_1t}{J}=m_1(\theta_2-\theta_1)+h \qquad .. \qquad \text{(ii)}$$

Subtracting we get, $(\theta_2-\theta_1)\ \ (m_1-m)=\dfrac{(E_1I_1-E.I)t}{J}$

$$\therefore \quad J=\frac{(E_1I_1-E.I)t}{(m_1-m)(\theta_2-\theta_1)}$$

If instead of water, a liquid of specific heat S is taken, then,

$$J=\frac{(E_1I_1-E.I.)t}{S(m_1-m)\ (\theta_2-\theta_1)}$$

Example 1 : *A current of 2 amp is sent through a coil of wire of resistance 3 ohms for 2 minutes. The coil is immersed in 50 gm of water kept in a copper calorimeter of water equivalent 50 gm. The rise of temperature of water is 3·4°C. Calculate the value of J.*

Ans. We know, $J=\dfrac{E.I.t}{(mS+w)\theta}=\dfrac{I^2R.t}{(mS+w)\theta}$ joules/cal.

Here $I=2$ amp, $R=3$ ohms ; $t=2\times60$ sec, $w=50$ gm, $m=50$ gm, $S=1$ and $\theta=3{\cdot}4°C$.

$$\therefore \quad J=\frac{(2)^2\times3\times2\times60}{(50+50)\times3{\cdot}4}=4{\cdot}23 \text{ joules/cal.}$$

Example 2 : *An electric heating coil is connected in series with a resistance X ohms across the* 240 *volts mains, the coil being immersed in a kilogram of water at* 20°*C. The temperature of water rises to boiling point in* 10 *minutes. When a second heating experiment is made with the resistance X short-circuited, the time required to develop the same quantity of heat is reduced to* 6 *minutes. Calculate the value of X.*

Ans. Heat necessary to raise the temperature of a kilogram of water from $20°C$ to $100°C = 1000 \times (100-20) = 1000 \times 80$ cal.

Now, we know, $H = 0{\cdot}24 \times \frac{E^2}{R} . t$ cal. In the first case, the circuit resistance is $R+X$, where R is the resistance of the heating coil. So,

$$1000 \times 80 = 0{\cdot}24 \times \frac{(240)^2}{R+X} \times 10 \times 60 \quad \text{or} \quad R+X = \frac{0{\cdot}24 \times (240)^2 \times 10 \times 60}{1000 \times 80} \quad .. \quad \text{(i)}$$

In the second case, when the resistance X is short-circuited, the circuit resistance$=R$. So,

$$1000 \times 80 = 0{\cdot}24 \times \frac{(240)^2}{R} \times 6 \times 60 \quad \text{or} \quad R = \frac{0{\cdot}24 \times (240)^2 \times 6 \times 60}{1000 \times 80} \quad .. \quad \text{(ii)}$$

Subtracting (ii) from (i), $X = \frac{0{\cdot}24 \times (240)^2 \times 60 \times 4}{1000 \times 80} = 41{\cdot}47$ ohms (nearly)

Example 3 : *In a Joule's electric calorimeter,* 1·5 *kg of oil of sp. heat* 0·6 *is taken and* 3 *amp current is passed through a* 3 *ohm coil immersed in it. For how long is it necessary to pass this current so that the temperature of the oil in the calorimeter rises by* 10°*C ? The water equivalent of the calorimeter and heat loss due to radiation may be assumed to be negligible. Given mechanical equivalent of heat*=4·2 *joules/cal.* [*H. S. Exam. 1980*]

Ans. Let t sec. be the required time.

Heat produced by the current$= \frac{I^2.R.t}{J} = \frac{(3)^2 \times 3 \times t}{4{\cdot}2}$ cal.

Heat absorbed by the oil$=$mass of the oil$\times$sp. heat$\times$rise of temp.
$= 1{\cdot}5 \times 1000 \times 0{\cdot}6 \times 10$ cal.

$$\therefore \quad \frac{(3)^2 \times 3 \times t}{4{\cdot}2} = 1{\cdot}5 \times 1000 \times 0{\cdot}6 \times 10$$

$$\text{or} \quad t = \frac{1{\cdot}5 \times 1000 \times 0{\cdot}6 \times 10 \times 4{\cdot}2}{(3)^2 \times 3} = 1400 \text{ sec} = 23 \text{ mnt. } 20 \text{ sec.}$$

*4.6. Principle of least heat :

In an electric circuit, consisting of several branches, current distribution takes place in such a manner that the heat produced in each branch is a minimum. This is known as the principle of least heat. We shall establish the principle in the case of a simple circuit consisting of two resistances r_1 and r_2 connected in parallel. If i be the main current, then from the principle of shunt [art 2.13], we can write

i_1=current in $r_1=\frac{r_2}{r_1+r_2}.i$ and $i_2=\frac{r_1}{r_1+r_2}.i$.

This distribution can be obtained from the principle of least heat in the following way :

Let the current through the resistance r_1 be i_1. Then the current through the parallel branch of resistance r_2 is $i_2=i-i_1$

Total heat generated per sec in the combination $H=\frac{i_1^2r_1+(i-i_1)^2.r_2}{J}$ cal.

Differentiating, $\frac{dH}{di_1}=\frac{1}{J}[2i_1r_1-2r_2i+2r_2i_1]$ [i is a constant]

Now for minimum heat, $\frac{dH}{di_1}=0$

i.e. $2i_1r_1-2r_2.i+2r_2i_1=0$ [J is not zero]

i.e. $i_1(r_1+r_2)=i.r_2$ or $i_1=\frac{r_2}{r_1+r_2}.i$.

In the same way, it can be shown that for minimum heat the current in the other branch is $\frac{r_1}{r_1+r_2}.i$.

Needless to mention that the principle is valid in the case of complex circuits also.

4.7. Electric energy and power :

Capacity of an electrical machine for doing work is called its energy. For example, if an electrical machine sends Q amount of charge through a conductor whose potential difference is V, then the work done by the machine *i.e.* electrical energy$=Q\times V$.

In practical units, Q is expressed in coulombs, and V in volts.

In that case, the work done or the energy$=Q.V$ joules$=Q.V\times 10^7$ ergs.

Units of power : Power, we know, is defined as the rate of doing work. (See chapter "Work, power and energy" in volume I of this book.) The power of an electric machine is usually expressed in **watts.** The power of 1 watt is defined as the rate of working of 1 joule/sec.

$\therefore$ 1 watt=1 joule/sec$=10^7$ ergs/sec.

We have already seen, work done$=Q.V$ joules, where Q coulombs of charge pass across a p.d. of V volts in t seconds.

$\therefore$ Rate of doing work or power P (in Watts)

$$=\frac{Q\times V}{t}\text{ joules/sec}=I\times V\text{ joules/sec.}\quad\left[\because\ \frac{Q}{t}=I\text{ amp.}\right]$$

It is convenient to memorise the relation, **Watts=Amperes × Volts.**

A bigger unit of energy is generally used to express the energy of a large machine. It is called **Kilowatt (KW)**. 1 K.W.=1000 watts.

Units of energy : Since work done=power×time, we may get the energy of an electric machine which works continuously for time t with power P by multiplying the power with time and from this we can also form an unit of energy.

1 joule is the energy expended when a machine of 1 watt power works for 1 second *i.e.* Joule=watt×second.

If a machine of 1 watt power works continuously for 1 hour, the energy expended is called **watt-hour** *i.e.* Watt-hour=Watt×hour.

Also, 1 watt-hour=1 watt×1 hour=1 watt×3600 seconds=3600 joules.

Electric supply company measures the electricity consumed by the consumers in terms of Kilowatt-hour (abbreviated as KWh) or B.O.T.units. As the name implies the Kilowatt-hour is the energy supplied by a rate of working of 1000 watts for 1 hour.

$$\text{B.O.T. units} = \frac{\text{Watt-hours}}{1000} = \frac{\text{amperes} \times \text{volts} \times \text{hour}}{1000}$$

The electric supply company installs a 'meter' in the house of every consumer, which records the energy consumed in terms of B.O.T. unit.

Very often you will find 'voltage' and 'watt' written on the body of a bulb, such as '220 volt−100 watt' bulb. We can understand the significance of the above writings from the quantities mentioned just now.

'220 volt' means that when the bulb is put on 220 volt p.d. as in the mains supply the bulb will glow the brightest. The filament of the bulb will however, burn out if it is put on a higher voltage line. '100 watt' means that the bulb consumes 100 watt electrical power every second and it takes a current $= \frac{100}{220} = 0{\cdot}45$ amp (nearly).

If the bulb is worked for 10 hours (say), it will consume energy $= 100 \times 10$ $= 1000$ watt hour $= \frac{1000}{1000} = 1$ K.W.H. (or 1 B.O.T unit)

4.8. Fuses :

To safeguard the electrical wiring of a house against damage due to heavy current, fuses are introduced in the circuit. They are nothing but thin wires, capable of carrying a specified current. It is a known fact that when the wire is thin and the current is heavy, the wire may be so hot that it may melt. This is the principle of action of fuses. When for some reason or other, the circuit of a house takes more current from the mains than is safe for the wiring, the fuse melts and blows out, breaking the circuit. It is clear, therefore, that a fuse bears the same relationship to an electric circuit as a safety-valve bears to an engine or a pump.

Usually fuses consist of fine wires made of an alloy of lead and a small amount of tin. They are generally rated by their ampere-capacity. Thus 'a 5-ampere fuse' means such a fuse wire which will melt by the passage of a current more than 5 ampere. So, the circuit containing the above type of fuse is capable of carrying a current upto 5 amperes. It is to be noted that *current-bearing capacity of a fuse wire does not depend upon the length but depends upon its diameter.* It is, however,

important that the length of a fuse wire should be sufficient as otherwise, with the blowing of the fuse wire, an arc might be formed causing a fire. Fuses are generally introduced into an electric circuit mounted on porcelain blocks provided with metal terminals for holding the fuses.

4.9. Output power and efficiency of a circuit :

Consider a circuit consisting of a resistor R connected in series with a battery of e.m.f. E and internal resistance r. The resistor is usually called the 'load' of the circuit, specially when the source of e.m.f. is a dynamo or a generator. Let the current through the circuit be I. If V_{AB} be the p.d. across the load, then $V_{AB}=I.R.=\frac{E}{R+r}.R$ .. (i)

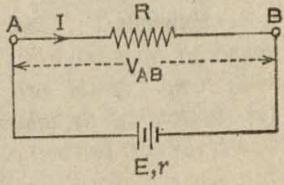


Fig. 4.6

Now, power delivered to the load is called the *output power* P_{out} ; it is given by $P_{out}=I.V_{AB}=I^2R$ .. (ii)

The power generated by the source of e.m.f. is given by $P_{gen}=I.E$ .. (iii)

The difference between the power generated and the power output is the power wasted as heat in the source$=I^2.R$. The ratio of the output power and power generated is known as the *efficiency* η of the circuit as a whole. Thus,

$$\eta=\frac{P_{out}}{P_{gen}}=\frac{I.V_{AB}}{I.E}=\frac{V_{AB}}{E}=\frac{R}{R+r}=\frac{1}{1+r/R}\text{ [From eqn. (i)]}$$

This shows that the efficiency tends to 100% as the load resistance R tends to infinity. For high efficiency, therefore, the load resistance must be several times the internal resistance of the source. When $R=r$ *i.e.* when the load resistance is equal to the internal resistance, the efficiency is 50%.

Maximum power : Let us see how the output power varies with load resistance and when the output power becomes maximum. We have seen that

$$P_{out}=I^2R\text{ ; also } I=\frac{E}{R+r}. \quad \therefore P_{out}=\frac{E^2.R}{(R+r)^2} \quad .. \quad \text{(iv)}$$

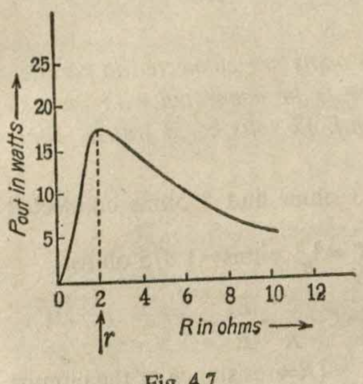


Fig. 4.7

Considering E and r constants, if we plot P_{out} as a function of R, we find that it passes through the peak value [Fig. 4.7] when $R=r$ *i.e.* when the load resistance is equal to the internal resistance of the source, the output power is maximum.

The same result can be shown mathematically by differentiating P_{out} in eqn. (iv) with respect to R and equating the differential co-efficient to zero. Thus,

$$\frac{d}{dR}(P_{out})=\frac{d}{dR}\left[\frac{E^2.R}{(R+r)^2}\right]=\frac{E^2[(R.+r)^2-R.2(R+r)]}{(R+r)^4}$$

for maximum value, $\frac{d}{dR}(P_{out})=0$

i.e. $(R+r)^2=2R(R+r)$ or $R=r$.

Example 1 : *An electric bulb in incandescent state, has a resistance of* 400 *ohms. It is connected to* 200 *volts p.d. for* 10 *hours. If the cost per unit be* 30 *paisa, calculate the total expenditure incurred.*

Ans. Current passing through the bulb$=\frac{200}{400}=\frac{1}{2}$ amp.

$$\text{B.O.T. unit consumed}=\frac{\text{ampere}\times\text{volt}\times\text{hour}}{1000}=\frac{\frac{1}{2}\times200\times10}{1000}=1$$

$\therefore$ Cost=30 paisa.

Example 2 : *Two lamps, connected seperately to a* 100-*volt supply are found to take* 60 *watts and* 75 *watts respectively. Determine the resistance of each lamp. If the two lamps are now joined in series to a* 200-*volt supply, find* (*a*) *the total watts taken by the two lamps* (*b*) *the cost of using the lamps for* 60 *hours, the cost per unit being* 25 *paisa.*

Ans. We know, $\text{watt}=\text{ampere}\times\text{volt}=\frac{(\text{volt})^2}{\text{ohm}}\left[\because\ \text{ampere}=\frac{\text{volt}}{\text{ohm}}\right]$

Now, for the first lamp, $60=\frac{(100)^2}{R_1}\quad\therefore\quad R_1=\frac{(100)^2}{60}=166\frac{2}{3}$ ohms.

For the second lamp, $75=\frac{(100)^2}{R_2}\quad\therefore\quad R_2=\frac{(100)^2}{75}=133\frac{1}{3}$ ohms.

When connected in series their total resistance$=166\frac{2}{3}+133\frac{1}{3}=300$ ohms.

Now, the current passing through the lamps joined in series and connected to 200 volts, $I=\frac{200}{300}=\frac{2}{3}$ amp.

$\therefore$ Watts taken by the first lamp=amp.$\times$volt.=(amp.)$^2\times$ohm

$$=\left(\frac{2}{3}\right)^2\times166\frac{2}{3}$$

Watts taken by the second lamp$=\left(\frac{2}{3}\right)^2\times133\frac{1}{3}$.

$\therefore$ Total watts taken$=\left(\frac{2}{3}\right)^2\times166\frac{2}{3}+\left(\frac{2}{3}\right)^2\times133\frac{1}{3}=133\frac{1}{3}$ watts

$$\text{Hence, total B.O.T. unit consumed}=\frac{\text{Watt}\times\text{hour}}{1000}=\frac{133\frac{1}{3}\times60}{1000}=8.$$

$\therefore$ Cost$=8\times25$ paisa=Rs. 2.

Example 3 : *Two resistors of* 3 *ohms and* 5 *ohms are connected in parallel. Find the value of the third resistor which will have to be connected with them so that the total power consumed from a source of e.m.f.* 12 *volts be* 36 *watts. State how the resistance is to be connected.*

Ans. Suppose the equivalent resistance of 3 ohms and 5 ohms connected in the parallel be R' ; then $\frac{1}{R'}=\frac{1}{3}+\frac{1}{5}=\frac{8}{15}\quad\therefore\quad R'=\frac{15}{8}$ ohms=1·875 ohms.

Now power consumed=p.d.$\times$current$=E\times I=E\times\frac{E}{R}=\frac{E^2}{R}$watt

[R=resistance of the circuit]

$$\therefore \quad 36=\frac{12\times 12}{R} \quad \text{or,} \quad R=\frac{12\times 12}{36}=4 \text{ ohms.}$$

It is clear that the third resistor is to be connected in series with the parallel combination. Suppose, the resistance of the third resistor is x ohm.

$\therefore \quad 4=1{\cdot}875+x \quad$ or $\quad x=4-1{\cdot}875=2{\cdot}12$ ohms (nearly)

Example 4 : *Forty similar electric bulbs are connected in series across a 220 volt d.c. supply. After one bulb is fused, the remaining 39 are connected again in series across the same supply. In which case, will there be more illumination and why?* [*I.I.T. 1969*]

Ans. If each bulb has resistance r, then 40 bulbs in series give a total resistance$=40r$. Current through each bulb $i_1=\dfrac{220}{40r}$ amp. Energy consumed by 40 bulbs per second$=40.i_1^2.r=40\times\left(\dfrac{220}{40r}\right)^2\times r=\dfrac{(220)^2}{40r}$ watt. (P_1 say)

When 39 bulbs are connected, current through each bulb $i_2=\dfrac{200}{39.r}$. Energy consumed by 39 bulbs$=39.i_2^2r=39.\left(\dfrac{220}{39r}\right)^2.r=\dfrac{(220)^2}{39r}$ watt (P_2 say)

Since $P_2>P_1$, more illumination will be produced in the second case.

Example 5 : *Two electric bulbs, each of* 500 *watt and* 220 *volt specification, are joined in series to a line of* 110 *volt. How much power will each bulb take ?*

Ans. We know, power=amp$\times$volt. Current taken by each when connected to 220 volt line$=\dfrac{\text{power}}{\text{volt}}=\dfrac{500}{220}=\dfrac{50}{22}$ amp. So, resistance of each bulb $R=\dfrac{\text{volt}}{\text{amp}}=\dfrac{220}{50/22}=\dfrac{(22)^2}{5}$ ohm. When the two bulbs, connected in series, are joined to 110 volt line, current through them$=\dfrac{110}{2R}$. So, power absorbed by each bulb=volt$\times$current$=110\times\dfrac{110}{2R}=\dfrac{110\times 110\times 5}{2\times(22)^2}=\dfrac{125}{2}=62{\cdot}5$ watts.

Example 6 : *A house-hold uses five* 60-*watt lamps and three* 40-*watt fans for* 5 *hours a day. Find the cost in a month of* 30 *days if electric energy is charged* 25 *paisa per B.O.T. unit.*

Ans. Total watts consumed$=5\times 60+3\times 40=420$

Total energy consumed every day$=420\times 5=2100$ watt-hours.

" " " in a month $=2100\times 30=63000$ watt-hours.

$=63$ KWh (or B.O.T)

$\therefore$ Cost$=63\times 25$ paisa=Rs. 15·75.

Example 7 : *A heater is designed to operate with a power of* 1000 *watts in a* 100 *volt line. It is connected, in combination with a resistance of* 10 *ohms and a resistance R to a* 100 *volt mains. What should be the value of R so that the heater operates with a power of* 62·5 *watts ?* [*I.I.T. 1978*]

Ans. The current taken by the heater when operating in 100 volt line $=\frac{1000}{100}=10$ amp. Its resistance, therefore, is $R_1=\frac{\text{volt}}{\text{current}}=\frac{100}{10}=10$ ohm.

If R' be the total circuit resistance, $R'=\frac{10\times R}{10+R}+10=\frac{20R+100}{10+R}=\frac{20(R+5)}{10+R}$.

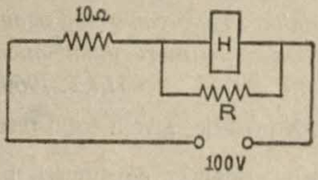


Fig. 4.8

So, circuit current $I=\frac{100}{R'}=\frac{100(10+R)}{20(R+5)}$ amp.

So, the current i_R through the resistance R is

$$i_R=\frac{10}{10+R}\times I=\frac{10}{10+R}\times\frac{100(10+R)}{20(R+5)}=\frac{50}{R+5}$$

Hence, the terminal p.d. of the resistor

$$R=i_R\times R=\frac{50}{R+5}\times R \quad \text{.. (i)}$$

Since the heater takes 62·5 watt, we can write,

$62{\cdot}5=\text{amp}\times\text{volt}=(\text{amp})^2\times\text{resistance}=(\text{amp})^2\times 10$

$\therefore$ Current in heater $i_H=\sqrt{\frac{62{\cdot}5}{10}}=\sqrt{\frac{625}{100}}=2{\cdot}5$ amp.

So, terminal p.d. of the heater $=i_H\times 10=2{\cdot}5\times 10$ .. (ii)

Since the heater and the resistor are in parallel connection, eqns (i) and (ii) are equal. So, $\frac{50\times R}{R+5}=2{\cdot}5\times 10$ or $R+5=2R$ or $R=5$ ohms.

Thermo-electricity

4.10. Seebeck effect :

If two dissimilar metals be joined to their ends so as to form a closed conducting circuit and if a difference of temperature be maintained at two junctions, a feeble current is found to flow through the circuit in a particular direction. This shows that an e.m.f. has been set up in the circuit, the origin of which must be the difference of temperature at the two junctions.

Fig. 4.9 shows a closed circuit where a copper rod and an iron rod have been soldered together at their ends A and B forming two junctions. A sensitive galvanometer G is also included in the circuit. Now, keeping the end A cold, if the end B be heated by a spirit lamp or a burner, the galvanometer will show a deflection. This indicates that a current is flowing in the circuit in a given direction. The current in the circuit becomes reversed if the junction A is heated and the junction B is cooled.

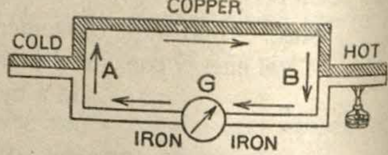


Fig. 4.9

The current produced in this way without the use of a cell or a battery is known as *thermo-electric current* and this branch of electricity is known as *thermo-*

electricity. It was first discovered by Seebeck in 1821 and hence the phenomenon is known as **Seebeck effect.**

In a copper-iron circuit as shown in the fig. 4.9, current flows from iron to copper at cold junction and from copper to iron at the hot junction. A pair of dissimilar metals arranged in this manner is called a **thermo-couple.** The e.m.f. set up in a thermo-couple due to a difference of temperature at the junctions is called **thermo-electric e.m.f.**

It goes without saying that no current flows through a thermo-couple if the junctions are kept at the same temperature.

4.11. Thermo-electric series :

Experiments show that the thermo-e.m.f. set up in a thermo-couple depends upon two factors *viz* (i) the pair of metals forming the couple and (ii) the temperature difference between the junctions. Seebeck studied the above phenomenon with various metals and arranged them in a series known as *thermo-electric series*, from which we can quickly get the direction of current flowing in a couple made of any two metals of the series. If any two metals of the series be taken, current will flow through the *cold junction* from the metal which is higher in the series to the metal which is lower.

The series is as follows :

(i) Antimony	(v) Copper
(ii) Iron	(vi) Nickel
(iii) Zinc	(vii) Constantan
(iv) Lead	(viii) Bismuth.

Thus if antimony and bismuth are taken to form a couple, current flows through the cold junction from antimony (which is higher) to bismuth (which is lower in the series).

4.12. Temperature-E.M.F. relation :

Keeping one junction of a thermo-couple at 0°*C*, if the temperature of the other be gradually increased, the thermo-e.m.f. set up in the circuit also increases. As long as the temperature difference is not very high, the rise of e.m.f. is proportionate to the increase of temperature of the hot junction until at a particular temperature of the hot junction, the e.m.f. becomes maximum. This particular temperature of the hot junction is called the **neutral temperature** of the given couple.

Definition : *The temperature of the hot junction of a thermo-couple at which the thermo-e.m.f., assumes a maximum value while the other junction is kept cold, is a constant for the given couple and is known as the neutral temperature of the couple.*

It is to be remembered that neutral temperature of a given thermocouple is fixed and remains constant whatever may be the temperature of the cold junction. Thus, the neutral temperature of iron-copper couple is 270°*C* whatever may be the temperature of the cold end.

If the temperature of the hot junction be increased beyond the neutral temperature, the thermo-e.m.f., instead of increasing further, decreases until it becomes zero at another given temperature of the hot junction. This particular temperature is called the **temperature of inversion.**

Definition : *The temperature of inversion is the temperature of the hot junction of a thermo-couple at which the thermo-e.m.f. of the circuit becomes zero and tends to be reversed in direction.*

If a graph be plotted between the temperature of the hot junction and the thermo-e.m.f. the cold junction being kept at 0°C, the graph is a parabolic curve (Fig. 4.10). The point A of the graph (270°C) represents the neutral temperature of iron-copper couple and the point B (540°C), the temperature of inversion (when the cold end is at 0°C).

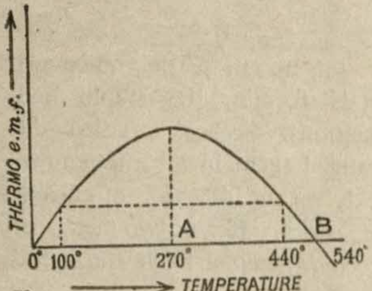


Fig. 4.10

It is important to note that the temperature of inversion of a couple is not a constant quantity. It is as much above the neutral temperature as the cold junction is below the neutral temperature. Thus for a copper-iron couple, the temperature of inversion is 540°C when the cold junction is at 0°C but the temperature of inversion becomes 440°C when the cold junction is kept at 100°C.

In general, suppose the temperature of cold junction $=\theta_1$; the neutral temperature $=\theta_n$ and temperature of inversion $=\theta_2$.

$$\text{Then,}\quad \theta_n-\theta_1=\theta_2-\theta_n \quad \text{or,}\quad \theta_n=\frac{\theta_1+\theta_2}{2}$$

It may be mentioned, further, that if the temperature of hot junction be increased beyond the temperature of inversion, the thermo-e.m.f. again increases but its direction becomes reversed (Fig. 4.10).

For a moderate range of temperature the general relation between the temperature and thermo e.m.f. in a thermo-couple may be stated as follows :

$$E=a\theta+b\theta^2.$$

Where E=e.m.f. set up : a, b,=constants for the given couple : θ=temperature difference between the junctions.

Example : *The temperature of the cold end of a thermo-couple is 2·8°C and the temperature of inversion is 572·2°C. What is its neutral temperature ?*

Ans. If θ_1 be the temperature of the cold end, θ_2 the temperature of inversion, and θ_n the neutral temperature, then, we know,

$\theta_n=\frac{1}{2}(\theta_1+\theta_2)$; Here $\theta_1=2{\cdot}8°C$; $\theta_2=572{\cdot}2°C$.

$$\therefore\quad \theta_n=\tfrac{1}{2}(2{\cdot}8+572{\cdot}2)=287{\cdot}5°C$$

4.13. Peltier effect :

It was discovered by Peltier in 1834 that if a current was sent round the circuit of a thermo couple, heat was evolved at one junction and absorbed

at the other, *i.e.* one junction was heated and the other was cooled. It is known as **Peltier effect** and may be characterised as complementary to Seebeck effect. This feature between the two effects will be clear if we consider both the effects in the same couple. Let us take the copper-iron couple mentioned earlier.

In Seebeck effect, we have seen that when a temperature difference is maintained between the junctions, current flows through the cold junction from iron to copper (Fig. 4.9). If now, keeping the junctions of the couple at same temperature, a current be allowed to circulate with the help of a battery in the same direction, the junction which was heated in Seebeck effect, will now *absorb* heat (Fig. 4.11) and the other junction will *evolve* heat. In other words, the junction through which the current supplied by the battery passes from iron to copper will be heated and the other junction will be cooled. If the direction of the current supplied by the battery is reversed by the reversal of the polarities of the battery, the Peltier effect at the junctions is also reversed.

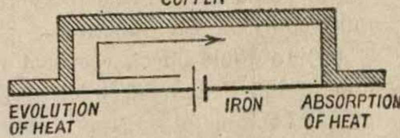


Fig. 4.11

Experimental demonstration of Peltier effect :

At the two ends of a thick iron rod, two pieces of equally thick copper rod are soldered, forming two junctions. The two junctions are introduced in two bulbs of a differential air thermometer (Fig. 4.12). The bulbs are connected by a U-tube which contains some light liquid. With the help of a battery allow a current to pass from copper to iron in one bulb and from iron to copper in the other bulb as shown in the figure. It will be found that a level difference between the liquid columns in the U-tube has been created—the liquid column in the arm *A* having attained a higher level than that in the arm *B*. This shows that the junction corresponding to the arm *B* (where current passes from iron to copper) is heated because the air in that bulb, on being heated tries to expand and pushes the liquid column downwards. The other junction where current passes from copper to iron is evidently cooled.

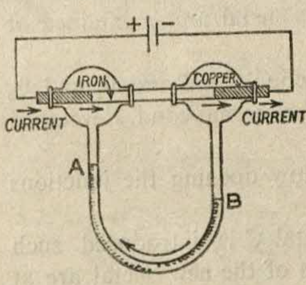


Fig. 4.12

If the battery connections are reversed, the liquid column in the arm *A* will be depressed and the other column in the arm *B* will be elevated. This proves that Peltier effect is a reversible process.

Some heat will, of course, be produced due to Joule effect but it is insignificantly small because the rods are very thick. Moreover, whatever little heat that Joule effect may produce, it will be equal in both the bulbs and will, therefore, not cause any difference in the levels of the liquid columns.

4.14. Distinction between Peltier and Joule effect :

Although both the effects—Peltier and Joule—are concerned with the production of heat by electric current yet there are basic differences between the two. The differences are :

(i) In Peltier effect, heat is evolved as well as absorbed but in Joule effect heat is always evolved.

(ii) Joule effect is not reversible *i.e.* if the direction of the current is reversed, cooling is not produced ; but Pelteir effect is reversible.

(iii) Joule effect takes place throughout the conductor but Peltier effect is confined only at the junctions.

(iv) In Joule effect, the heat produced depends upon the resistance of the conductor but Peltier effect has nothing to do with the resistance.

(v) In Joule effect, the heat produced is proportional to the square of the current passing through the conductor but in Peltier effect, the heat absorbed or evolved is proportional to the current passing through the circuit.

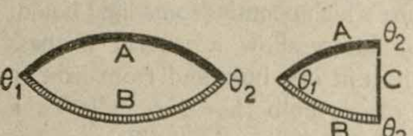
4.15. Laws of thermo-electricity :

There are two well-known laws of thermo-electricity. They are known as (i) *Law of intermediate metals* and (ii) *Law of intermediate temperature.*

(i) Law of intermediate metals :

The law states that the thermo-e.m.f. produced by two different metals having junctions at different temperatures is independent of a third metal introduced in the circuit, provided that the junctions formed by this metal are maintained at the same temperature.

Suppose the junction temperatures of a thermo-couple A/B are θ_1 and θ_2 [Fig. 4.13(i)]. The thermo-e.m.f. developed is $E_{\theta_1}^{\theta_2}(A/B)$. Now opening the junctions at θ_2, a third metal C is introduced such that the junctions of the new metal are at the same temperature θ_2 [Fig. 4.13(ii)]. Under the circumstances, it may be proved that $E_{\theta_1}^{\theta_2}(A/B)=E_{\theta_1}^{\theta_2}(A/C)+E_{\theta_1}^{\theta_2}(C/B)$



(i) Fig. 4.13 (ii)

This law has two main consequences :

(a) A galvanometer and leads may be introduced into a thermo-electric circuit without altering the conditions provided the new junctions are at the same temperature.

(b) Two wires to form a junction may be soldered together even if the solder holds them apart, provided the whole junction is at the required temperature.

(ii) Law of intermediate temperature :

The law states that the thermo-e.m.f. produced by two different metals with junctions at different temperatures θ_1 and θ_3 is given by the sum of the thermo-

e.m.f. 's which exist between the same junctions at θ_1 and θ_2 and θ_2 and θ_3, where θ_2 is some intermediate temperature. Mathematically, the law may be expressed as :

$$E_{\theta_1}^{\theta_3} = E_{\theta_2}^{\theta_3} + E_{\theta_1}^{\theta_2}$$

In general, a given temperature difference may be broken up into a number of successive steps and the law may be applied to such cases of subdivision. Mathematically, it may be represented as

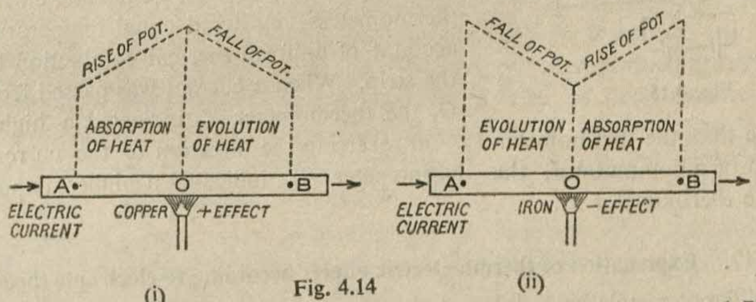
$$E_{0}^{\theta_n} = E_{0}^{\theta_1} + E_{\theta_1}^{\theta_2} + \cdots + E_{\theta_{n-1}}^{\theta_n}$$

where, θ_1 θ_2, θ_3 .. etc. are the successive intermediate temperature between 0° and θ_n.

4.16. Thomson effect :

Thomson in 1856 suggested that when a current flows through unequally heated conductors, heat energy is evolved or absorbed not only at the junctions but also throughout the metals forming the thermo-couple. This is known as **Thomson effect.** He also told that the effect is reversible.

Explanation : Consider the case of thick copper rod AB whose ends are kept at 0°C with the help of ice and the middle point O is heated by a burner [Fig. 4.14(i)]. Ordinarily, heat will be conducted equally along the length of



(i) Fig. 4.14 (ii)

the rod on both sides of the point O and if we take two points (say A and B) equi-distant from the mid-point O on two halves of the rod, the temperatures at those two points will be equal. But when current is passed along the rod in the direction of the arrow, the temperature at the point A will be found to be lower than that at the point B. It means that from A to O heat is absorbed and from O to B, heat is evolved. This is known as *positive Thomson effect.* Similar effect is observed in the case of zinc, silver, antimony, cadmium etc. So, by positive Thomson effect we mean that when current flows in a homogeneous conductor from the point of lower temperature to the point of higher temperature, heat is absorbed while current flowing from the point of higher to lower temperature causes evolution of heat. The effect is, of course, reversed if the direction of the current is reversed.

In the case of iron, however, when it is heated at the point O and current is flowing from A to B, the point A shows higher temperature as compared to the point B [Fig. 4.14 (ii)]. It means that from A to O heat is evolved and from O to B, heat is absorbed. This is *negative Thomson effect*. Similar effect is found in platinum, cobalt, bismuth, nickel etc. So, by negative Thomson effect we mean that when current flows in a homogeneous conductor from the point of lower temperature to higher, heat is evolved while current flowing from the point of higher temperature to lower causes absorption of heat. Negative Thomson effect is also reversible.

It is important to note that *lead shows no Thomson effect*. If we take a rod of lead and heat it at the middle point, then two points equidistant from the mid-point on the two halves of the rod show the same temperature when current is flowing from one end to the other. For this reason lead is used as one of the metals to form a thermo-couple with other metals for the purpose of studying their thermo-electric properties.

Experimental demonstration of Thomson effect :

To show Thomson effect, Lord Kelvin used a thick strip of iron bent in a zigzag manner as shown in fig. 4.15. The free ends A and G of the strip were connected to a battery through a commutator. The end D was heated while the ends B and F were kept cold. Thermometers T_1 and T_2 were placed in holes drilled at C and E. Before passing current, the thermometers registered equal temperatures on account of uniform thermal conduction through the strip. When a current was passed from A to G, the thermometer T_1 indicated a higher temperature than the thermometer T_2. On reversing the direction of the current with the help of a commutator, the thermometer T_2 indicated a higher temperature than the thermometer T_1.

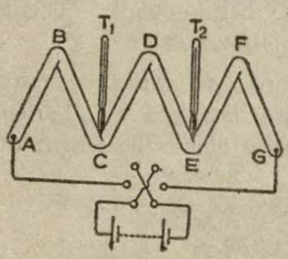


Fig. 4.15

4.17. Explanation of thermo-electric effects according to electronic theory :

Different metals have different electron densities *i.e.* the number of electrons per unit volume is different. When two different metals are brought in contact, diffusion of electrons takes place from the metal of higher electron density to the other. As a result, the metal of higher electron density becomes positive and the other negative due to more accumulation of electrons. This diffusion ceases when the e.m.f. developed is sufficient to prevent the passage of electrons from one metal to the other. Thus, we see that an e.m.f. is set up at the junction of two different metals.

(i) **Seebeck effect** : From the above consideration we see that a source of e.m.f. exists at each of the two junctions of two dissimilar metals forming a couple. The e.m.f. developed at the junction depends on the temperature of the junctions. When the two junctions of the thermo-couple are at same

temperature, the value of the e.m.f.'s at the junctions is the same. If one of the junctions is heated, it absorbs energy and the diffusion of electrons increases due to rise of temperature and hence the value of e.m.f. at this junction increases. As a result, the circuit, as a whole, will acquire a resusltant e.m.f. in a direction coinciding with the direction of e.m.f. at the hot junction.

Let us imagine the junctions are absent and in their place two imaginary cells are present. From fig. 4.16, it is clear that the cells are connected in opposition (according to the direction of diffusion of electrons). The cell A will have an e.m.f. e_1 in the direction from copper to iron and the other cell at B has e.m.f. e_2 also in the direction from copper to iron. But B being at a higher temperature than A, $e_2 > e_1$. Hence the resultant e.m.f. $e = e_2 - e_1$, acts in the direction as indicated in the figure which accounts for the current in the Seebeck effect. If the temperature at the junction A is higher than that at B, $e_1 > e_2$ and the direction of the current is reversed.

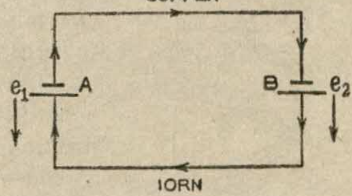


Fig. 4.16

(ii) **Peltier effect** : Again when the junctions are at the same temperature, the value of e.m.f.'s of the hypothetical cells at the junctions will be same and no resultant e.m.f. will act in the couple. Now, by inserting an additional cell at C, if current is sent in the same direction as in the Seebeck effect, then from the fig. 4.17 it is clear that the cell at A is charged by absorbing energy and the cell at B appears to be discharged by giving out energy which are externally manifested by a rise of temperature at the junction A and a fall of temperature at the junction B because the hypothetical cells at A and B are imagined to have a thermal origin. This accounts for Peltier effect.

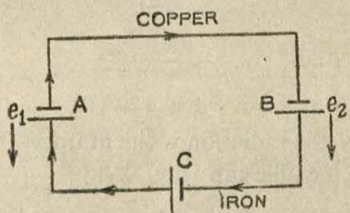


Fig 4.17

(ii) **Thomson effect** : For an explanation of Thomson effect, let us similarly imagine two hypothetical cells having thermal origin to exist at X and Y such that the e.m.f. of the cell X has direction from colder to hotter part and that of Y is also from colder to hotter part of the copper bar [Fig. 4.18]. If now a current passes from A to C, then the cell at X appears to be discharged by giving out energy and the cell at Y appears to be charged by absorbing energy which are externally manifested by a rise of temperature at Y and a fall of temperature at X. This explains Thomson effect.

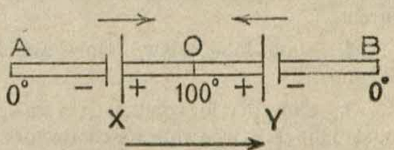


Fig. 4.18

If a homogeneous conductor is unequally heated, then an e.m.f. acts between two points having different temperatures because the free electrons of the conductor like a gas molecules tend to move from point of higher temperature to point of lower temperature. Conseqently, the high temperature point becomes positively charged

and the low temperature point negatively charged. In other words, an e.m.f. acts in the direction from colder to hotter part (conventional direction).

4.18. Application of thermo-electric effect ; Thermopile :

This is a very sensitive instrument for detection of thermal radiation. It consists of a number of thermo-couples joined in series. The thermo-couples are made of antimony (*A*) and bismuth (*B*) rods [Fig. 4.19]. The hot end of each couple is turned towards the radiation and the cold end is turned away from it. The two ends of the thermocouple are connected to a sensitive galvanometer *G*. If radiation falls on the hot junction of the thermopile, a temperature difference is created between the junctions, due to which a current flows from bismuth to antimony through the hot junction. The deflection shown by the galvanometer *G* is a measure of the intensity of the incident radiation.

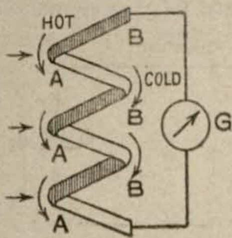


Fig. 4.19

In the improved form of a thermopile, there are a large number of thermo-couples consisting of antimony and bismuth bars, arranged in the form of a cube. The junctions are soldered but the bars are insulated from each other with mica strips. All the junctions on one side are coated with lamp-black so that the surface can absorb completely the radiation incident upon it. When not in use, the hot face is covered with a metallic cap [Fig. 4.20].

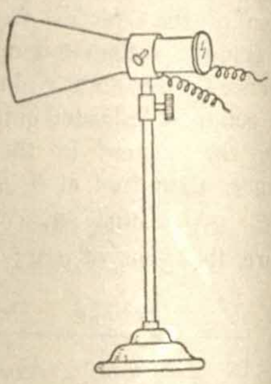


Fig. 4.20

Exercises

Eassy type :

1. State and establish Joule's law in connection with the production of heat by electric current. [*H. S. Exam. 1978*]

2. State Joule's law. How would you experimentally verify the law ? Draw neat diagrams. [*H. S. Exam. 1978, '82*]

3. Show that for conductors in series, the rate of heat dissipation in each is directly proportional to the resistance while for conductors in parallel, it is inversely proportional to the resistance.

4. Explain the principle of determining mechanical equivalent of heat by equating thermal and electrical energies. Give the necessary experimental procedure and circuit diagram.

5. What is the efficiency of an electrical circuit containing a load? Show that the efficiency of a circuit is 50% when the load resistance equals the internal resistance of the source of e.m.f.

6. Explain Seebeck effect, Peltier effect and Thomson effect. [*cf. H. S. Exam. 1979*]

7. Describe one experiment each for demonsrating Peltier effect and Thomson effect.

8. Draw a graph between the temperature and thermo-e.m.f. and point out the neutral temperature and temperature of inversion in the graph.

Short answer type :

9. A current is allowed to flow through a wire for some time. How does the heat produced in the wire depend on (i) the length of the wire (ii) the cross-section of the wire (iii) the resistivity of the wire (iv) the p.d. across the wire ? (In the first three cases, the p.d. across the wire may be assumed constant.) [*H. S. Exam. 1985*]

10. Explain what you mean by Joule, Watt and Kilowatt-hour.

11. On an electric bulb is written '230 volt-60 watt'. What is its significance ? What does B.O.T. mean ? [*H. S. Exam. 1978, '81*]

12. 'In paying electric bills we pay for electrical energy consumed'—Justify the statement.

13. What do you understand by the neutral temperature and the temperature of inversion of a thermo-couple ?

14. What is a thermopile ? For what purpose is it used ?

15. State the laws of thermo-electricity. What are the consequences of the law of intermediate metals ?

16. Two heating coils, made of same element, are connected parallel to the mains. The length and diameter of one of the coils are, each double than those of the other coil. Which one will produce more heat ?

[Hints : Resistance of the coil $R=\rho\frac{l}{\alpha}$; $\alpha=\frac{\pi d^2}{4}$; $R=\frac{4\rho l}{\pi d^2}$. The coil ($A$) whose length and diameter are double, has, according to this equation, a resistance less than the other coil (B). As the coils are connected in parallel, the (A) having less resistance will have greater current and hence will produce more heat.]

17. An electric heater continually produces heat but its temperature becomes steady after some time. What is the reason of it ?

[Hints : After some time, the temperature becomes steady because it loses heat at the same rate at which it generates it. At first, when electric energy is put in the heater, its temperature rises along with the generation of heat. As the temperature of the coil differs more and more from the surrounding air temperature, it loses more and more heat due to radition. Soon a time comes when the rate of loss of heat is equal to the rate of production of heat. Then the temperature of the heater becomes steady.]

18. A steady potential difference is acting between two points. Between the points, five wires of same length and diameter but of different materials are connected one after another. In which wire having the highest or having the lowest resistance is the rate of production of heat fastest ?

19. Two lamps, of 500 watt and 100 watt are connected to a p.d. of 110 volt. Filament of which lamp has the highest resistance ?

20. Two lamps, of 50 watt and 100 watt respectively are connected (i) series and (ii) in parallel with the mains. Which lamp glows brighter in the two cases? [*Jt. Entrance 1984*]

[Hints : In parallel connection, 100 watt lamp glows brighter and in series connection, the 50 watt lamp glows brighter. The resistance of 50 watt lamp is greater than that of the other. In series connection, the current in both of them is the same. Since brightness $\propto i^2R$, the 50 watt lamp glows brighter in series connection because of its greaterre sistance.

In parallel connection, both of them have the same terminal p.d. (V). Since brightness, in the case, is proportional to V^2/R, the 100 watt lamp glows brighter because of its less resistance.]

21. Two identical heating coils are enclosed in two identical glass bulbs. One of the bulbs contains some hydrogen gas while the other is vacuous. The bulbs are connected in series and current is sent through them. Which one gives brighter light ?

[Hints : As the bulbs are connected in series, same current flows through them. But the heat produced will be quickly conducted away in the bulb which contains gas. Hence the vacuum bulb gives brighter light.]

22. Two heating elements A and B are enclosed in two similar sealed glass tubes and are used as heaters. They are immersed in two identical calorimeters containing equal amount of water. When conected in series, the element A is found to generate more heat than the element B ; but when connected in parallel, the element B is found to generate more heat than A. Measurement of their resistances by a metre bridge gives same value. How can this happen?

[Hints : This can happen if the temperature coefficient of resistance of the element A is more than that of the element B. If α be the temperature co-eff^n of resistance of A, then $R_t = R_0(1+\alpha t)$. In series connection, $H_A \propto i^2.R_A.t$ and $H_B \propto i^2 R_B.t$. Since $R_A > R_B$. $H_A > H_B$.

In parallel connection, $H_A \propto R_A.t\left(\frac{R_B.i}{R_A+R_B}\right)^2$ and $H_A \propto R_B.t\left(\frac{R_A.i}{R_A+R_B}\right)^2$;

Since $R_A > R_B$, $H_B > H_A$.

In room temperature, $R_A = R_B$. So, the metre-bridge measurement gives same resistance.]

23. A metallic wire is heated by allowing a current to flow through it. In this condition, one half of the wire is immersed in water. It is found that the other half becomes more heated than the immersed half. Why ?

[Hints : Temperature of the half decreases due to immersion. So, its resistance falls. Heat produced being proportional to R, the immersed half now produces less heat].

24. What is a fuse ? Does the current-bearing capacity of a fuse depend on its length ?

25. The equation $P = i^2 R$ suggests that the rate of Joule heating in a wire is reduced if its resistance R is lowered. The equation $P = \frac{V^2}{R}$ suggests just the opposite. How do you reconcile this apparent paradox ?

26. What kind of wire would you prefer in demonetrating Peltier effect—thick or thin ?

27. What are the differences between Joule heating and Peltier heating ?

Objective type :

28. Mark the correct answer in the following cases :—

(a) A constant voltage is applied between the two ends of a uniform metallic wire. Some heat is developed in it. When is the heat developed double ?

Ans. Both the length and the radius of the wire are halved ; both the length and the radius of the wire are doubled ; the radius of the wire is doubled ; the length of the wire is doubled.

[*I.I.T. 1980*]

(b) A bulb is marked with 100 watt—200 volt ; what will be its resistance ?

Ans. 200 ohms, 400 ohms, 300 ohms.

c) How many joules are there in one Watt-hour ? Ans. 60 ; 3600 ; 36.

(d) For a constant potential difference, how will the Joule heating vary with the resistance ?

Ans. Heating $\propto R$; $1/R$; R^2

(e) When is the power delivered to a load by a source of e.m.f. maximum ?

Ans. When load resistance > internal resistance ; when load resistance < internal resistance ; when load resistance = internal resistance.

(f) What is the temperature of the hot end of a thermo-couple called when the thermo-e.m.f. developed has the mximum value ? Ans. Neutral temperature ; temperature of inversion, critical temperature.

(g) What kind of wire should you select for demonstrating Peltier effect ?

Ans. Thin wire, thick wire, short wire.

(h) When a current is sent round the circuit of a thermo-couple, heat is evolved at one junction and absorbed at the other. What is the phenomenon called ?

Ans. Thomson effect ; Seebeck effect ; Peltier effect.

Numerical Problems :

29. Calculate the amount of heat generated when a current of 2 amp flows for 5 minutes through a wire of 4.2 ohms resistance. [Ans. 1200 call]

30. An electric heater takes 5 amp current when connected to 110 volt supply line. How much calorie will it produce in 1 minute ? [Ans. 7920 cal.]

31. A cell of e.m.f. 2 volts and internal resistance 0.4 ohm is joined to two wires connected in parallel. If the resistance of the wire are 3 ohms and 7 ohms, compare the heat developed in them. [Ans. 7 : 3]

32. Calculate the resistance of '80 watt-120 volt' electric lamp in an incandescent state. [Ans. 180 ohms]

33. Two wires of resistance 80 ohms and 120 ohms are joined (i) in series (ii) in parallel with a 100-volt supply line. Determine in each case the power consumed in each resistance. [Ans. (i) 20 watts ; 30 watts (ii) 125 watts ; 83.3 watts]

34. Two resistors of 6 ohm and 9 ohm are connected in series and then the combination is put in parallel with a 5 ohm resistor. A resistor of 2 ohm is connected thereafter in series with the above combination. As a result of current flow, the rate of heat production in 5 ohm resistor is 21 cal/s. What is the rate of production of heat in 2 ohm resistor ? [Ans. 14.93 cal/s]

35. An electric iron when hot, has resistance 80 ohms. What will of the cost if it is connected across 200-volt line for two hours, charge being 36 paisa per unit ? [Ans. 36 paisa]

36. What will be the current flowing in a 400-watt electric lamp connected across 200-volt supply line ? What is its resistance ? What will be the cost for using the lamp for 100 hours, charge being 25 paisa per unit ? [Ans. 2 amp ; 100 ohms ; Rs. 10]

37. Two lamps—both to be operated at 200-volts, one consuming 200 watts and the other 100 watts are taken. The lamps connected in series are put across a 200-volts main supply. Calculate the power, in watt, consumed by each. [*Joint Entrance* 1976] [Ans. 22.2 ; 44.4]

38. Two similar wires of equal lengths have diameters in the ratio 1 : 2. They are joined in series. Compare the heat developed in the wires if a steady current passed through them for some time. [Ans. 4 : 1]

39. An electric stove which as a resistance of 55 ohms is connected to 220 volt mains. Calculate he time required to heat 1 kilogram of water from 34°C to 100°C. [Ans. 5 mnt 12 sec. (nearly)]

40. A heating coil of 10 ohm resistance is immersed in 40 gm of water at 0°C contained in a vesel whose water equivalent is 10 gm. The heating coil is connected to a battery and the *p.d* across the coil is 25 volts. How long after the connection is made, the water starts to boil ? [J=4.2 joules/cal[[Ans. 5 mnt 36 sec.]

41. An insulated wire of length 80 cm. and cross-section 0.16 sq mm is immersed in 250 gm of water and a current of 3 amp is passed through it. If the temperature rises by 17°C in 20 minutes, find the resistivity of the wire, assuming all the heat goes into the water. J=4.2 joules/cal. [Ans. 3.306×10^{-5} ohm-cm]

42. A 110 volt-500 watt heater is to be used on 220 volt mains. How much resistance is to be connected in series with the heater ? [Ans. 24.2 ohms]

43. A wire carrying a current of 0.5 amp. is immersed in an ice calorimeter and melts 1 gm. of ice per minute. Find its resistance in ohms. Laten heat of fusion of ice=80 cal per gm. [Ans. 22.2 ohms]

44. A current of 10 amp. passes through a resistor of 20 ohms for 5 minutes. Find (a) the charge flowing in coulombs (b) the energy expended in Joules and (c) the heat developed in calories. [Ans. (a) 3000 coulombs (b) 6×10^5 Joules (c) 1.44×10^5 calories]

45. If 20 coulombs of electric charge flows across a potential difference of 220 volts, how much work is done ? If this charge flows through a resistor, how much heat will be generated

in it ? If we assume that the current has flown for 1 minute in the above case, what is the resistance of the resistor ? J=4.2 joules/cal.

[*H. S. Exam. 1984*] [Ans. 4400 joule ; 1047.6 cal ; 665.1 ohm]

46. A 120-watt motor is used to operate an electric refrigerator. If the motor runs $\frac{1}{3}$ of the time, find the cost of operating it for a 30 day month if energy costs 8 paisa per KWh.

[Ans. R. 2.30]

47. The main-meter of a house is marked 10 amp-220 volts. How many 60 watt lamps may be used in the house with safety ? [*H. S. Exam. 1985*] [Ans. 36]

[**Hints :** $n \times 60 = 10 \times 220$]

48. In a hostel, there are 180 boarders. Each boarder uses an electric lamp of 60 watts for 5 hours a day. Find the amount of bill for the electrical energy consumed in a month of 30 days, if the cost of energy is 25 paisa per unit. [Ans. Rs. 405]

49. In a house 6 bulbs of 60 watt each and 2 fans of 40 watt each run for 6 hour a day. If one B.O.T. unit costs 50 paisa, what will be the monthly cost ? 1 month=30 days.

[*H. S. Exam. 1981*] [Ans. Rs. 39.60]

50. In a house, there are ten 40 watt lamps and three 100-watt fans, which run on an average, 5 hours a day. Calculate the cost of running the lamps and fans for 30 days, electric energy being charged at 20 paisa per B.O.T. unit. [Rs. 21]

51. In a house 6 bulbs, 60 W each burn for 5 hours a day. If one B.O.T. unit costs 50 paisa, what will be the monthly charge ? (1 month=30 days.) [*H. S. Exam. 1978*] [Ans. Rs. 27]

52. A thermocouple whose junctions are maintained at constant temperatures has a resistance of 5 ohms and its e.m.f. as measered by a potentiometer is 3.9 mv. What will be the reading on a millivoltmeter of resistance 60 ohms connected directly to the thermo-couple ?

[Ans. 3.6 mv]

Harder Problems :

53. In what time will 1 litre of water at 25.4°C be taken to its boiling point if the power supplied to water is equivalent to 1 H.P. ? [Ans. 7 minutes]

54. Two wires A and B have same length and cross-section but the wire A has specific resistance four times than that of B. What is the ratio of heat generated in respective wire if (i) the wires are connected in series to a constant voltage source (ii) the wires are connected in parallel to the same source (iii) compare the values of the total heat developed in the above two cases. [Ans. (i) 4:1 (ii) 1:4 (iii) 4 :25]

55. An electric kettle is rated at 220 volt 1 KW. It is desired to bring one litre of water from 86°F to the boiling point by it. If the water equivalent of the kettle is 100 gm, calculate the time required in the process of heating. Assume that 80% of heat produced goes into water. Calculate also the cost of electric energy consumed if 1 B.O.T. unit is charged at 25 paisa.

[*Jt. Entrance 1972*] [Ans. 6 m 44s ; 3 paisa (nearly)]

56. An electric kettle has two healing coils. When one of the coils is switched on, the liquid in the kettle begins to boil in 6 minutes and when the other is switched on the boiling begins in 8 minutes. In what time will the boiling begin if both coils are switched on simultaneously (i) in series and (ii) in parallel ? [*I.I.T. 1975*] Ans. (i) 14 mnt ; (ii) $3\frac{3}{7}$ mnt]

[**Hints :** $H=\frac{E^2}{R_1}.\,t_1$ for the 1st coil. $\therefore\ R_1=\frac{E^2}{H}.t_1$ similarly for the second coil $R_2=\frac{E^2}{H}.\,t_2$

(i) when in series $Rs=R_1+R_2=\frac{E^2}{H}(t_1+t_2)$. Now $H=\frac{E^2}{R_1+R_2}.\ t_2=\frac{E^2H.t_s}{E^2(R_1+R_2)}$

$$\therefore\ t_s=t_1+t_2=14 \text{ mnts}$$

(ii) When in parallel, $R_p=\dfrac{R_1R_2}{R_1+R_2}=\dfrac{\frac{E^2}{H}.\,t_1\times\frac{E^2}{H}.\,t_2}{\frac{E^2}{H}.\,t_1+\frac{E^2}{H}.t_2}=\frac{E^2}{H}\times\frac{t_1t_2}{(t_1+t_2)}$

Now $H=\frac{E^2}{R_p}\times t_p=\frac{E^2H(t_1+t_2)}{E^2.t_1t_2}.t_p=H\frac{(t_1+t_2)}{t_2t_1}.t_p \quad \therefore \quad t^1=\frac{t_1t_2}{t_1+t_2}=\frac{6\times 8}{6+8}=3\frac{3}{7}$ mnt.]

57. A p.d. of 130,000 volt is applied to power cables of total resistance 3 ohms. The current through the cables is 9 amp. Calculate (i) the input power to the cables (ii) power dissipated in the cables (iii) energy loss from the cables in 3 hours.

[Ans. (i) $1{\cdot}17\times 10^6$ watts (ii) 243 watts (iii) $2{\cdot}62\times 10^6$ joules]

58. A copper wire having cross-sectional area of 0.5 mm² and a length of 0.1 metre is initially at 25°C and is thermally insulated from the surrounding. If a current of 10 amp is set up in this wire find the time in which the wire starts melting. The change of resistance with the temperature of the wire may be neglected. What will this time be, if the length of the wire is doubled ? Melting point of copper=1075°C, Sp. resistance of copper=1.6×10^{-8} ohm-metre specific heat=9×10^{-2} kilo cal/kg/°C ; density=9×10^3 kg/metre³.

[*I.I.T. 1979*] Ans. (i) 9m 18 sec (ii) same time.]

59. A 2-metre long wire of resistance 4 ohms and diameter 0.64 mm is coated with plastic insulation of thickness 0·06 mm. When a current 5 amp flows through the wire find the temperature difference across the insulation in steady state. Thermal conductivity of plastic= 0.16×10^{-2} c.g.s. [*I.I.T. 1974*] [Ans. 2°C]

60. A wire of resistance p connects A and B, two points in a circuit, the resistance of the remainder is Q. If without any other change being made in the circuit, $(n-1)$ other wires, each having a resistance p are connected between A and B, show that the heat produced in the n wires will be greater or less than that produced orginally in the first wire according as p is greater or less than $\sqrt{n}.Q$. [*I.I.T. Kanpur, 1973*]

61. A fuse made of lead wire has an area of cross section 0.2 mm². On short-circuiting, the current in the fuse wire reaches 30 amp. How long after the short-curcuiting, will the the fuse begin to melt ? For lead, sp. heat=0.032 cal/gm/°C ; melting point=327°C ; density=11.34 gm/c.c. and resistivity=22×10^{-6} ohm cm. Assume that initial temperature of the wire at 20°C. Neglect heat losses. [*I.I.T. 1976*] [Ans. 0.1 sec]

62. Three equal resistors connected in series across a source of e.m.f. together dissipate 10 watts of power. What would be the power dissipated if the same resistors are connected in parallel across the same source of e.m.f. ? [*I. I. T. 1972*] [Ans. 90 watts]

63. Two liquids A and B of equal masses are kept in two vessels. Their sp. heats are 0.4 and 0.8 respectively. A heater of 1 ohm resistance is immersed in liquid A and another of resistance 2 ohm in liquid B. If current is sent after connecting the heaters in series, find the ratio of the rate of rise in the two liquids. What will be the value of the above ratio, if the heaters were joined in parallel ? [*I. I. T. 1965*] [Ans. 1 : 1 ; 4 : 1]

64. The walls of a closed cubical box of edge 50 cm are made of a material of thickness 1 mm and thermal conductivity 4×10^{-4} c.g.s. units. The interior of the box is maintained at 100°C above the outside temperature by a heater placed inside the box and connected across 400 volts d.c. Calculate the resistance of the heater. [*I.I.T.* 1971] [Ans. 6·4 ohms]

[Hints : $Q=0{\cdot}24\times\left(\frac{400}{r}\right)^2\times r$ per sec ; $Q=6\times\frac{4\times 10^{-4}\times(50)^2\times 100}{0.1}$ per sec]

65. It is desired to construct a 5-amp fuse from tin wire which has a melting point of 230°C and resistivity 22×10^{-8} ohm-metre at that temperature. Calculate the diameter of the wire required if the emissivity of its surface is 88×10^{-5} joule per cm² per second per °C excess temperature above the surrounding whose temperature is 20°C. [Ans. 1.1 mm]

[**Hints :** Area of the surface of the wire=$2\pi rl$. Heat energy radiated per sec=$2\pi rl.e\theta$ where e=emissivity. Again resistance R of the wire=$\frac{\rho l}{\pi r^2}$. Heat generated per sec $=i^2R=i^2\ \rho l/\pi r^2$. So, $2\pi rle\theta=i^2l\rho/\pi r^2$; $\theta=230-20=210$°C.]

66. A constant current of 1 amp flows through a platinum wire of resistance 5 ohm, stretched along the axis of a cylindrical tube through which a steady stream of water passes at a rate of 15 c.c. per minute. The steady difference between the temperature of water entering and leaving the tube is 4.8°C. Neglecting losses of heat, calculate the mechanical equivalent of heat.

[*Jt. Entrance 1966*] [Ans. 4.17 joule/cal]

67. A heating coil is connected in series with a resistance R across 240 volt mains. The coil is immersed in 1500 gm of water at 20°C. The water starts boiling in 15 minuites. When the experiment was repeated with R short-circuited the water started boiling in 9 minutes. Calculate the value of R. [Ans. 18 ohms (nearly)]

68. If the e.m.f. in a thermo-couple is 500 microvolt when the junction A is at 0°C and the junction B at 50°C and 600 micro-volts in the same direction as before when A is at 0°C and B at 100°C. What is the neutral temperature, given that the e.m.f. is proportional to the difference of temperature of the junctions and to the difference between the neutral temperature and the mean temperatures of the junctions ? [Ans. 87·5°C]

5

CHEMICAL EFFECT OF CURRENT AND ELECTROLYSIS

5.1. Introduction :

When eleetric current flows through a wire or any other solid conductor, the conductor becomes heated but no chemical action takes place in the conductor. Thus, when current flows through a copper wire, the wire gets heated but copper does not undergo any chemical change. But the same thing does not happen when current passes through a liquid conductor. If a current is allowed to pass through a solution of salts, bases or acids, a chemical action takes place in the liquid due to which the molecules of the solute substance are found to break up into two charged parts. This phenomenon is known as chemical effect of electric current. Chemical effect of current has wide applications in metallurgy, electroplating and in many other industrial spheres.

5.2. Some important terms :

(i) **Ion** : If an atom or a molecule has electrons more or less than the normal quota, they are called ions. Excess of electrons will make an atom or a molecule a negative ion while a deficit of electrons will make a positive ion. It is to be remembered that in normal condition, an atom or a molecule possesses as many electrons as it requires to neutralise the positive charge of its nucleus, and hence in normal condition it is neutral.

(ii) **Electrolyte** : A solution which conducts current through it and undergoes chemical decomposition is called an electrolyte. For example, copper sulphate solution, silver nitrate solution, acidulated water etc. are good electrolytes. But sugar solution although liquid, is not an electrolyte. Ordinarily, oils and pure water are not conductors of electricity. *Although mercury is a good conductor of electricity yet it is not considered as an electrolyte.* Solutions of salts, bases and acids are ordinarily good electrolytes.

(iii) **Electrodes** : The two plates at which the current enters and leaves the electrolyte are called electrodes. The electrode at which the current enters the electrolyte is called the *anode* and that by which it leaves, the *cathode*.

(iv) **Electrolytic cell** : The apparatus consisting of the vessel, electrolyte, electrodes etc in which the electrolysis is carried out is called an electrolytic cell or *voltameter* presumably because it can be used to measure the current delivered by a voltaic cell. The latter term should not be confused with voltmeter.

If the electrolyte is a solution of a silver or copper salt, the voltameter is called a silver or copper voltameter. If the electrolyte is acidulated water, then the voltameter is called a water voltameter, because when a current passes through it, the water and not the acid is electrolysed.

5.3. Some illustrations of electrolysis :

(i) **Electrolysis of copper sulphate solution** : Take some copper sulphate solution in a glass vessel and mix a few drops of sulphuric acid with it. Introduce two copper plates in the solution and connect a battery with the plates. Before passing current, make the cathode plate (C) clean and get its weight. Now pass current through the solution for some time. After that take out the cathode plate, make it dry and find its weight. You will see that the plate has gained some weight.

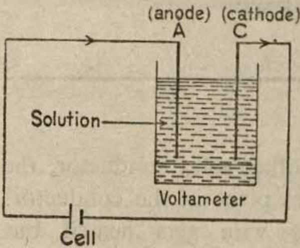


Fig. 5.1

When a solution of copper sulphate in water is prepared, every molecule of $CuSO_4$ is dissociated into Cu^{++} ions and SO_4^{--}ions. The positive Cu^{++}ions will move towards the cathode plate and get deposited on the plate as neutral copper molecules. The negative SO_4^{--}ions on the other hand, move towards the anode plate, react with Cu molecules of the plate and form neutral $CuSO_4$ molecules which dissolve in the solution and keep the strength of the solution unchanged. So, the net result is that Cu molecules from the anode get deposited on the cathode plate. Consequently, the anode plate will lose weight while the cathode plate will gain weight.

If instead of copper plates as electrodes, platinum plates or any inert metal plates are used, then at the cathode plate copper molecules will be deposited as before but at the anode plate, SO_4^{--}ions will react with H_2 molecules of water and will form H_2SO_4 and O_2. After some time, therefore, the cathode becomes covered with a reddish layer of pure copper and the copper sulphate solution loses its density.

Similarly, if silver nitrate solution is electrolysed with the help of silver electrodes, silver from anode will get deposited on the cathode.

(ii) **Electrolysis of water** : Hoffmann devised a very convenient apparatus for carrying out electrolysis of water. The apparatus consists of two graduated tubes made of glass, fitted with taps and provided with platinum electrodes [Fig. 5.2]. A short cross tube joins the graduated tubes at the bottom. The short horizontal tube is provided with an upright tube and a small reservoir.

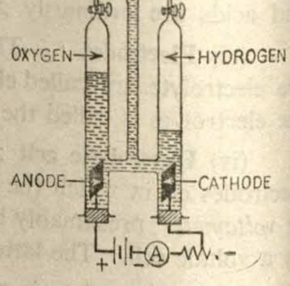


Fig. 5.2

The taps of the graduated tubes are first opened and then acidulated water is slowly poured in the reservoir until the graduated tubes are both full. Then closing the taps, current is passed from a battery. Streams of bubbles will come out of the electrodes which will collect in the graduated tubes by the displacement of water. When sufficient gas has collected, the current is stopped. It will be seen that the volume of gas collected in one tube is double than that in the other. Chemical tests will reveal that the gas of greater volume is hydrogen while the other is oxygen.

Here, every molecule of water breaks up into hydrogen and oxygen atoms due to electrolysis and they collect as neutral gases in the tubes. As hydrogen ions are positively charged, they collect at the cathode and oxygen ions being negatively charged collect at the anode. $H_2O \rightarrow (H^+ + H^+) + O^{--}$

5.4. Testing of polarity of an electric circuit by electrolysis :

Suppose we are to find out which of the leads coming from the mains is positive and which is negative. We can use electrolysis for the purpose.

Take some water in a vessel and dip the leads into water. Care should be taken so that the leads do not touch each other for, if they touch the whole line may be damaged due to short-circuiting. It will be seen that streams of bubbles are coming out of water from one lead. The bubbles are of hydrogen gas and the lead is negative because we know that due to electrolysis hydrogen gas is liberated at the cathode and oxygen at the anode and that in a certain time the volume of hydrogen liberated is double than that of oxygen.

5.5. Faraday's laws of electrolysis :

Michael Faraday carried out a series of experiments on electrolysis. From the results of his experiments, he succeded in establishing two laws, known as the laws of electrolysis.

Law 1. *The mass of ions liberated from an electrolyte is proportional to the quantity of electric charge that flows through the electrolyte.*

Law 2. *When same quantity of electric charge passes through different electrolytes, the masses of ions liberated at different electrodes are proportional to their chemical equivalents.*

Discussion of the first law : Suppose a current of I amp. flowing through an electrolyte for t seconds, liberates W gm. of an ion. Here, quantity of electricity passing through the electrolyte, $Q=I \times t$ coulomb. So, from the first law, it follows that $W \propto Q \propto I.t$ or, $W=ZIt$, where Z is a constant.

If $I=1$ amp. and $t=1$ sec., $W=Z$ *i.e.* Z is the mass of ion liberated when a current of 1 amp passes through an electrolyte for 1 second. This constant Z is known as *electro-chemical equivalent* (abbreviated as E.C.E.).

Definition : *The electrochemical equivalent of a substance is defined as the mass of it which is liberated during electrolysis by the passage of one coulomb* (*i.e.* 1 *amp current for* 1 *sec*) *of electricity.*

For example E.C.E. of silver is ·001118 gm/coulomb. It means that if 1 coulomb of electricity passes through a solution of silver salt, ·001118 gm of silver will be liberated.

Discussion of second law :

Definition : The chemical equivalent (abbreviated as C.E.) of a substance is the number of parts by weight of it which will combine with or replace 8 parts by weight of oxygen or the equivalent of any other element. It is numerically equal to the ratio of the atomic weight of the element and its valency *i.e.* chemical

$$\text{equivalent} = \frac{\text{atomic weight}}{\text{valency}}$$

Now, suppose, same quantity of electricity is passed through acidulated water, copper sulphate solution and a solution of silver salt. Due to electrolysis hydrogen, copper and silver respectively will be liberated in different cathodes. From Faraday's second law, we get that if 1gm of hydrogen is liberated, then the amounts of copper and silver liberated are $\frac{63\cdot5}{2}$ gm and 108 gm respectively ; for 1, $\frac{63\cdot5}{2}$ and 108 are the chemical equivalents of hydrogen, copper and silver respectively.

5.6. Relation between E. C. E. and C. E. of an element :

Consider a copper voltameter and a water voltameter through which 1 coulomb of electricity is passed. From Faraday's first law, we get, $W_1=Z_1$ and $W_2=Z_2$ where W_1 and W_2 are the masses of copper and hydrogen liberated and Z_1 and Z_2 are respectively their electro-chemical equivalents.

$$\therefore \quad \frac{W_1}{W_2}=\frac{Z_1}{Z_2} \quad \ldots \quad \ldots \text{(i)}$$

Again from Faraday's second law we, get, $\frac{W_1}{W_2}=\frac{C_1}{C_2}$ .. ..(ii)

where C_1 and C_2 are the chemical equivalents of copper and hydrogen respectively.

Hence, $\frac{Z_1}{Z_2}=\frac{C_1}{C_2}$ or $Z_1=\frac{C_1}{C_2}\times Z_2=C_1\times Z_2$ [$\because$ C.E. of $H_2=1$]

Therefore, E.C.E. of copper=C.E. of copper × E.C.E. of hydrogen.

In general we can say that E.C.E. of any element=C.E. of the element × E.C.E. of H_2.

5.7. Experimental verification of Faraday's laws :

First law : Take a copper voltameter and connect a battery, a rheostat, an ammeter and a plug key in series with it as shown in fig. 5.3. Before passing current through the voltameter, take out the cathode plate (K), thoroughly clean it with emery paper, wash it well under a running tap, make it dry and then find its weight. Adjust the rheostat so that the current does not exceed 0·2 amp/sq cm of cathode surface. Put the clean and dry cathode plate in its position and send the current through the voltameter. Suppose the current (I amp say) is allowed to flow for t_1 seconds. The ammeter will record the current. A stop watch is used to get the time of flow of current. After that the cathode is removed, washed

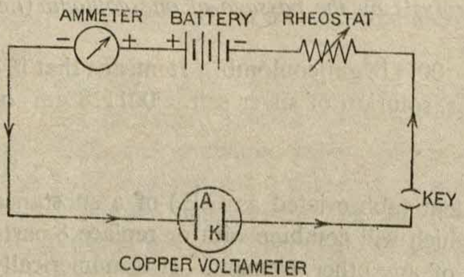


Fig. 5.3

and dried as before. Finally, it is weighed again. From these two weighings, the mass of copper deposited may be found out. Suppose it is W_1 gm. Place the cathode plate again in the voltameter and allow the same current to flow for a different time, say t_2 seconds. As before, find the mass of copper deposited. Let it be W_2 gm.

It will be seen from the results of the experiment that

$$\frac{W_1}{W_2}=\frac{I\times t_1}{I\times t_2}=\frac{Q_1}{Q_2} \text{ i.e. } W\propto Q.$$

Second law : Three different voltameters—a copper, a silver and a water voltameter—are connected in series with a battery, an ammeter, a rheostat and a plug key (Fig. 5.4). In water voltameter, a few drops of sulphuric acid are mixed with water. Since the voltameters are connected in series same current will

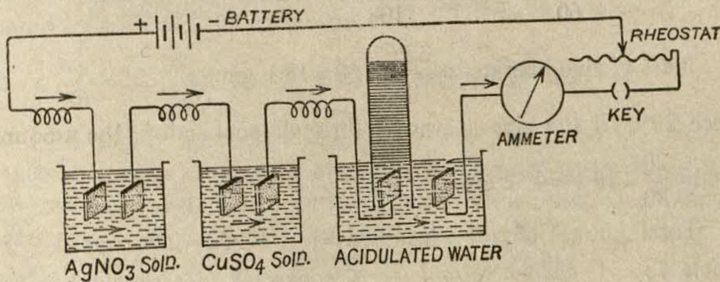


Fig. 5.4

flow through them. A suitable current is passed for a given time through the voltameters. Silver and copper will be liberated respectively at the cathodes of the silver and copper voltameters. In water voltameter oxygen will be collected at the anode. As before, find the masses of copper and silver deposited by weighing the cathode plates. For oxygen, find the volume of the gas collected at N.T.P. and from it the mass of oxygen. If the masses of elements liberated be W_1, W_2 and W_3 respectively, then it will be found that $W_1 : W_2 : W_3$ $=\frac{63\cdot5}{2} : 108 : 8$

Example 1 : *For what time must a current of* 2·5 *amp pass through a solution of zinc sulphate to deposit* 1 *gm of zinc ? E.C.E. of Zn*=·0003387 *gm/coulomb.*

Ans. We know $W=Z.I.t$; Here $W=1$ gm ; $Z=\cdot0003387$ gm/coulomb ; $I=2\cdot5$ amp.

$$\therefore \quad 1=\cdot0003387\times2\cdot5\times t \quad \text{or,} \quad t=\frac{1}{\cdot0003387\times2\cdot5}\text{ sec.}$$
$$=1181 \text{ sec.}=19 \text{ mnt } 41 \text{ sec.}$$

Example 2 : 1·2 *gm of copper are liberated when a current of* 1·5 *amp flows through a copper sulphate solution for* 40 *minutes. Find the electrochemical equivalent of copper.*

Ans. We have $W=Z.I.t.$; Here $W=1{\cdot}2$ gm ; $I=1{\cdot}5$ amp ; $t=40\times60$ sec.

$\therefore\ 1{\cdot}2=Z\times1{\cdot}5\times40\times60$

or, $Z=\dfrac{1{\cdot}2}{1{\cdot}5\times40\times60}={\cdot}00033$ gm/coulomb.

Example 3 : *How much zinc will be consumed in a battery the current from which deposits* 60 *gm of silver from a silver nitrate solution,* 20% *of the zinc being wasted through local action ? Chemical equivalent of silver*=108 *and that of zinc*=32·6.

Ans. From Faraday's second law, we have,

$$\frac{\text{mass of zinc liberated}}{\text{,, ,, silver ,,}}=\frac{\text{C.E. of zinc}}{\text{,, ,, silver}}$$

or, $$\frac{\text{mass of zinc liberated}}{60}=\frac{32{\cdot}6}{108}$$

$\therefore$ mass of zinc liberated$=\dfrac{32{\cdot}6}{108}\times60=18{\cdot}1$ gm.

Since 20% of the zinc is wasted through local action, the amount of zinc so wasted$=\dfrac{20}{80}\times18{\cdot}1=4{\cdot}52$ gm.

$\therefore$ Total amount of zinc consumed$=18{\cdot}1+4{\cdot}52=22{\cdot}62$ gm.

Example 4 : *A copper voltameter is connected in series with a water voltameter and* 1·5 *amp current is sent through them for* 20 *minutes. If* 0·636 *gm of copper is deposited in copper voltameter, what gram of hydrogen is liberated in the water voltameter ? At wt of copper*=63·6, *its valency*=2 ; *At.wt of hydrogen* =1·008 *and its valency*=1.

Ans. As the same current flows for the same time in both the voltameters, we get from Faraday's second law, $\dfrac{W_{CU}}{W_H}=\dfrac{C_{CU}}{C_H}$ where W_{CU} and C_{CU} are the mass of copper ion liberated and the chemical equivalent of copper resecptively ; similarly, W_H and C_H are the mass of hydrogen liberated and its chemical equivalent.

But $C_{CU}=\dfrac{\text{At. wt. of Cu}}{\text{its valency}}=\dfrac{63{\cdot}6}{2}$ and $C_H=\dfrac{\text{At. wt. of H}_2}{\text{its valency}}=\dfrac{1{\cdot}008}{1}$

$\therefore\ \dfrac{0{\cdot}636}{W_H}=\dfrac{63{\cdot}6}{2}\times\dfrac{1}{1{\cdot}008}\quad\therefore\ W_H=\dfrac{0{\cdot}636\times2\times1{\cdot}008}{63{\cdot}6}=0{\cdot}02$ gm. (approx.).

Example 5 : 1 *amp of current is required to be sent for* 3 *minutes to liberate* 20 *cc of hydrogen at N.T.P. If an atom of hydrogen weighs* $1{\cdot}6\times10^{-24}$ *gm, find the charge of an electron in coulomb. The density of hydrogen at N.T.P.*= 9×10^{-5} *gm/cc*

Ans. We have, $W=Z.I.t.$; Here $W=20\times9\times10^{-5}$ gm ; $t=3$ mnt$=3\times60$ sec. ; $I=1$ amp.

$\therefore\ Z=\dfrac{W}{I.t}=\dfrac{20\times9\times10^{-5}}{180}=10^{-5}$ gm/coulomb.

Suppose charge of an electron$=e$ coulomb. Since hydrogen is monovalent, charge of each hydrogen ion$=e$ coulomb. From the definition of electro-chemical equivalent, we may say, that 1 coulomb of charge liberates Z gm of hydrogen. So, e coulomb will liberate $Z.e.$ gm. Again e coulomb of charge liberates one hydrogen atom *i.e.* $1{\cdot}6\times10^{-24}$ gm. So,

$$Z.e.=1{\cdot}6\times10^{-24} \quad \text{or,} \quad e=\frac{1{\cdot}6\times10^{-24}}{10^{-5}}=1{\cdot}6\times10^{-19} \text{ coulomb}$$

Example 6 : *On sending a current through a silver voltameter for* 16 *mnt* 40 *sec* 2 *gm* 236 *mg of silver was deposited on the kathode plate. What amount of silver will be deposited if the current is doubled and sent for* 25 *minutes* ?

Ans. We know, $W=Z.I.t$; in the first case, $2{\cdot}236=Z.I\times1000$

[2 gm 236 mg=2·236 gm ; 16 mnt 40 sec=1000 sec.]

In the second case, $W=Z\times2I\times25\times60$

Dividing, $\dfrac{W}{2{\cdot}236}=\dfrac{2\times25\times60}{1000}$ $\quad\therefore\quad W=6{\cdot}708$ gm$=6$ gm 708 mg.

Example 7 : *A copper voltameter consists of two parallel copper plates* 6 *cm. apart and* 1 *metre square, immersed in copper sulphate solution of resistivity* $1{\cdot}2\times10^{-2}$ *ohm-metre. Calculate the potential difference which must be established between the plates to provide a constant current to deposit* 658 *gm of copper on the cathode in* 1 *hour. E.C.E. of copper*$=3{\cdot}29\times10^{-4}$ *gm/coulomb.*

Ans. We know $W=Z.I.t.$ or $I=\dfrac{W}{Z.t}=\dfrac{658}{3{\cdot}29\times10^{-4}\times3600}$ amp.

The resistance of the voltameter,

$$R=\frac{\rho l}{\alpha}=\frac{1{\cdot}2\times10^{-2}\times6\times10^{-2}}{(1)^2}=7{\cdot}2\times10^{-4} \text{ ohm}$$

Hence, required $\text{P.D.}=I.R.=\dfrac{658\times7{\cdot}2\times10^{-4}}{3{\cdot}29\times10^{-4}\times3600}=0{\cdot}4$ volt

5.8. Definition of ampere according to electrolysis :

The International Committee, in 1910, defined the practical unit of current on the basis of deposition of silver due to electrolysis in silver voltameter. This unit is called the *international ampere.*

It is defined as the steady current which when passed through a silver voltameter deposits 0·001118 gm of silver in 1 second on the cathode.

***5.9. Arrhenius' theory of electrolysis :**

To explain different phenomena in connection with electrolysis Arrhenius of Stockholm (1887) proposed a theory which became known as Arrhenius' theory of electrolysis.

Arrhenius suggested that the molecules of a solute break up into atoms when it is dissolved in a solvent and also there is a continuous interchange of atoms between the molecules. Thus, at an instant, in the process of molecular

interchange of atoms, there is a large number of free atoms. The behaviour of the free atoms is similar to those of undissociated molecules. Further it is also suggested that these atoms are *electrically charged* even before the application of a potential difference between the electrodes. All metallic and hydrogen ions have positive charge and are called *electro-positive* while all non-metallic ions have negative charge and are called *electro-negative*. In the case of a very dilute solution, the degree of dissociation is high and most of the molecules of the dissolved substance dissociate to form free ions. Thus a molecule of NaCl breaks up into a positively charged sodium ion and a negatively charged chlorine ion as follows :
$NaCl \rightarrow Na^+ + Cl^-$

The ionisation of electrolytes in the process of solution may be explained on the basis of electrostatic forces. The atoms of a molecule are held together by electrostatic forces of attraction. When the salt is immersed in a solvent, say water, due to high permittivity of water ($K=80$), the electrostatic forces of attraction $\left(F=\frac{q_1 q_2}{K r^2}\right)$ between the atoms are weakened and this results in the separation or ionisation of the atoms.

On the basis of dissociation theory, the application of the external *p.d.* merely directs the movement of the charged ions towards the respective electrodes. The positively charged ions move towards the cathode and hence are called *cations* and the negatively charged ions move towards the anode and hence are called *anions*. The charges carried by the ions are handed over to the electrodes. This explains electrolytic conduction through liquids and electrolysis.

5.10. Faraday and Avogadro number :

Faraday is a unit for electrical charge. From Faraday's first law we know that if Z be the E.C.E. of an element, 1 coulomb of electricity will liberate Z gm of the element from a solution of its salt by electrolysis. Hence, to liberate an amount of the element equal to its chemimal equivalent (or a mole), the quantity of electricity required$=\frac{\text{C.E of the element}}{Z}$

Now, E.C.E. of silver is ·001118 gm/coulomb and its C.E. is 108. Hence to liberate 108 gm of silver, the quantity of electricity equired$=\frac{108}{\cdot 001118}=$
$=96540$ coulombs (approx.).

If similar calculations are carried out for other elements like hydrogen, copper, zinc etc., it will be seen that same amount of electricity *viz.* 96540 coulombs is required to liberate a quantity of those elements equal to their chemical equivalents. This amount of electricity is known as one Faraday (F).

$\therefore$ **1 Faraday=96540 coulombs.**

Avogadro number : Taking the case of silver, say, we have seen that 96540 coulombs of charge are carried by the atoms in a mole of silver. As silver is monovalent, the charge carried by each silver atom (or ion) is e, where e is the electronic charge$=1{\cdot}6\times10^{-19}$ coulomb.

$\therefore$ Number of atoms in 1 mole$=\dfrac{96540}{1{\cdot}6\times10^{-19}}=6{\cdot}03\times10^{23}$ (approx.).

If similar calculations are carried out for other elements, it will be seen that the number of atoms in 1 mole of every element comes out as above. Hence it is a constant quantity and is known as *Avogadro number* (N_A).

It should be noted that 96540 coulomb is the charge on 1 mole of electrons *i.e.* $eN_A=F$.

5.11. Atomicity of electricity :

As a piece of matter is supposed to be composed of a large number smallest units of matter, known as atom, a certain quantity of electric charge, in the same way, is supposed to be made up of a large number of smallest unit, called an atom of electricity. This concept of atomicity of electricity comes from Faraday's laws of electrolysis. The charge contained by an atom of electricity can be obtained in the following way.

We know that the chemical equivalent of a substance$=\dfrac{\text{gram-atom}}{\text{valency}}$. Further a gram-atom of the substance contains N atoms, where N is the Avogadro number. If each of these N atoms carries a charge e in the ionised condition, then the total charge carried by the chemical equivalent of the substance

$$=\frac{\text{charge carried by } N \text{ atom}}{\text{Valency}}=\frac{N.e}{x} \text{ where } x=\text{valency of the substance.}$$

From Faraday's laws we know that one Faraday or 96540 coulombs of charge are necessary for liberation of chemical equivalent of the substance.

$$\therefore\ \frac{N.e}{x}=96540 \quad \text{or} \quad e=\frac{96540}{N}\times x=\frac{96540}{6{\cdot}03\times10^{23}}\times x=1{\cdot}6\times10^{-19}\times x \text{ coulomb.}$$

Since valency of a substance is essentially a positive integer (1, 2, 3, etc), the charge e carried by an ion must be an integral multiple of $1{\cdot}6\times10^{-19}$ coulomb. Monovalent ions, for which $x=1$, necessarily carry the smallest amount of charge *viz* $1{\cdot}6\times10^{-19}$ coulomb. Any charge smaller than this is inconceivable and hence this amount of charge is referred to as an atom of electricity.

Strangely enough, after about thirty years, when electron was discovered, it was found that the charge carried by an electron was $1{\cdot}6\times10^{-19}$ coulomb. Electronic charge was, therefore, considered as the smallest possible charge available and was taken as a unit of charge.

Example 1 : *The electro-chemical equivalent of silver is* $1{\cdot}118\times10^{-3}$ *gm/coulomb and the atomic weight of silver, which is monovalent, is* 108. *Calculate the ratio of the charge to the mass of a hydrogen ion (proton).*

Ans. By definition, the E.C.E of silver (Z) is the mass of silver liberated by one coulomb. Since silver is monovalent, the positive charge carried by each ion is the same as that on the proton *i.e.* e coulombs.

By Avogadro's hypothesis, the number of silver atoms in one gram-atom (*i.e.* 108 gm.) is equal to Avogadro number N_A. Hence, the number of silver ions deposited when charge of one coulomb passes$=\dfrac{Z.N_A}{108}$.

Hence, charge on each ion$=\dfrac{1}{Z.N_A/108}=\dfrac{108}{Z.N_A}$ coulombs.

1 gm. of hydrogen contains N_A atoms also, so that mass of one hydrogen atom is given by $M_H=\dfrac{1}{N_A}$ gm.

Hence, $\dfrac{\text{charge on proton } (e)}{\text{mass of proton } (M_H)}=\dfrac{108\times N_A}{Z\times N_A}=\dfrac{108}{Z}=\dfrac{108}{1{\cdot}118\times10^{-3}}$

$=9{\cdot}654\times10^4$ coulomb/gm.

Example 2 : *If* 96,540 *coulomb of charge liberate* 1 *gm equivalent of ion of an element, in what time will* 0·15 *amp liberate* 20 *mg of copper from copper sulphate solution ? Chemical equivalent of copper*=32.

Ans. We know, 1 Faraday or $96540=\dfrac{\text{chemical equivalent}}{Z}=\dfrac{32}{Z}$

$\therefore\quad Z_{cu}=\dfrac{32}{96540}$ gm/coulomb

Now, $W=Z.I.t$ or $20\times10^{-3}=\dfrac{32}{96540}\times0{\cdot}15\times t$

$\therefore\quad t=\dfrac{20\times10^{-3}\times96540}{0{\cdot}15\times32}$ sec=402 sec=6 mnt 42 sec (nearly)

5.12. Determination of current in a circuit by the application of electrolysis :

By the application of electrolysis, we can find the current in a circuit in the following way. It can also be used to find the error in a particular reading on an ammeter, if a standard ammeter is not available.

Take a copper voltameter. Make its cathode plate clean and dry. Find its weight in a balance. Now connect the circuit whose current is to be determined with the voltameter, taking care that the negative terminal of the circuit is joined to the cathode plate. Allow the current to flow through the voltameter for some time. Note the interval with the help of a stop watch. After stopping the current, remove the cathode plate. Wash the plate and make it dry. Weigh the plate again and find the mass of copper deposited. Let it be W gm.

We know, $W=Z.I.t.\qquad\therefore\quad I=\dfrac{W}{Z.t.}$

So, knowing W, t and Z, the current I can be calculated.

To correct the reading of an ammeter, the instrument is to be included in the circuit. The error in the ammeter is then the difference in the reading on the ammeter and the current calculated from the above equation. See example No. 2 below.

Example 1 : *A copper voltameter is connected in series with a battery and a current is passed for* 1 *hour. It is found that* 1·1 *gm of copper has been deposited on the cathode. If E.C.E. of copper is* ·00033 *gm/coulomb, what is the current flowing through the voltameter* ?

Ans. We know, $W=Z.I.t.$ or $I=\frac{W}{Z.t.}$

Here, $W=1{\cdot}1$ gm ; $Z={\cdot}00033$ gm/coulomb ; $t=60\times60$ sec.

$$\therefore \quad I=\frac{1{\cdot}1}{{\cdot}00033\times60\times60}=\frac{11}{{\cdot}33\times6\times6}=0{\cdot}926 \text{ amp.}$$

Example 2 : *The current which liberates* 0·85 *gm of copper in* 25 *minutes is allowed to pass through a copper sulphate solution taken in a copper voltameter. An ammeter connected in series with the voltameter reads* 1·7 *amp. What is the error in the reading of the ammeter* ? *E.C.E. of copper*$=3{\cdot}29\times10^{-4}$ *gm/coulomb.*

Ans. We know, $I=\frac{W}{Z.t.}$; here $W=0{\cdot}85$ gm ; $t=25\times60$ sec. and $Z=3{\cdot}29\times10^{-4}$ gm/coulomb.

$$\therefore \quad I=\frac{0{\cdot}85}{3{\cdot}29\times10^{-4}\times25\times60}=\frac{0{\cdot}85\times10^{3}}{3{\cdot}29\times25\times6}=1{\cdot}722 \text{ amp.}$$

$\therefore$ The error in the reading$=1{\cdot}722-1{\cdot}7=0{\cdot}022$ amp. (less).

5.13. Minimum potential difference required for electrolysis of water :

When one gm-molecule of water is electrolysed, 2 gm-equivalents of hydrogen is liberated at the cathode and one gm-equivalent of oxygen at the anode. The quantity of charge necessary to liberate 2 gm-equivalent of hydrogen= $2\times96{,}540$ coulombs.

If V be the p.d. between the electrodes, the amount of work done= $2\times96{,}540\times V$ joules. The work done must necessarily be greater than the amount of energy required to separate 1 gm-equivalent of oxygen from hydrogen in water. Form calorimetric studies, it is found that 68400 calories of heat is absorbed in the formation of 1 gm-molecule of water. Taking $J=4{\cdot}2$ joules/cal, 68,400 calories$=68{,}4000\times4{\cdot}2$ joules.

$$\therefore \quad 2\times96{,}540\times V=68{,}400\times4{\cdot}2 \quad \text{or} \quad V=1{\cdot}48 \text{ volts.}$$

This is the minimum potential difference necessary to maintain the current. It is, thus, clear why a single Daniel cell is not sufficient for electrolysis of water. It would, of course, start a current but when polarisation begins, the back e.m.f. due to it rises until it equals the e.m.f. of the cell, when, of course, the current will cease to flow.

5.14. Practical applications of electrolysis :

Electrolysis finds its application in many branches of industry, some of which are briefly discussed here.

(i) **Electroplating :** It is a process of giving base metals a coating of mo re expensive metals. It serves a double purpose. Not only does it protect the base metals from atmospheric corrosion but it also provides a more attractive appearance. Household cutlery and other articles made of nickel alloys are usually given a coating of silver by this process. The articles to be electroplated are generally suspended from a conduction rod in a vessel, which contains a solution of the salt of gold, silver etc. *i.e.* the substance with which the articles are to be electroplated. From another conduction rod is suspended a plate of pure silver, copper etc. whose coating is needed. If, now a battery be connected with the conducting rods, current will flow through the solution and the articles will get a layer of the desired metal on them [Fig. 5.5]. In this way, when iron is given a coating of zinc, it is called *galvanised* iron.

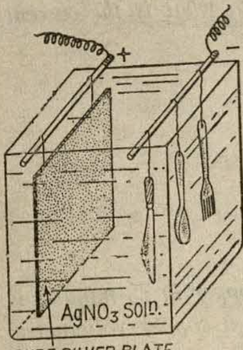


Fig. 5.5

(ii) **Electrotyping :** To obtain exact copies of engraved blocks on metals, the process of electrolysis is applied. For this purpose, the mould of the block is first prepared from a soft material like wax and the block is then pressed against the mould. The wax mould thus gets an impression of the block. The surface of the mould is coated with powdered graphite in order to make the surface a conductor of electricity. The mould forms the cathode, a copper plate the anode and copper sulphate solution the electrolyte in a copper voltameter. When current is passed, copper is deposited on the surface of the mould. When the thickness of the copper deposit is sufficient, it is removed from the mould. It can be used many times without any damage.

Gramophone records are also made in this way.

(iii) **Extraction and purification of metals :**

The process of electrolysis is widely applied in metallurgy in extracting metals like aluminium, sodium, potassium etc. from their ores and purifying extracted metals like, zinc, copper etc. It is also used in the manufacture of some useful chemicals like caustic potash etc. Details are available in any text book of Chemistry.

Example 1 : *A spoon having an area of* 20 *sq mm is to be coated with a silver coating to a thickness* 0·1 *mm. If a current of* 0·15 *amp is used, calculate the time for which it must flow. E.C.E. of silver*=·001118 *gm/coulomb ; density of silver*=10·5 *gm/cc*

Ans. Area of the spoon=20 sq mm.=0·2 sq cm.

Thickness of the coating=0·1 mm.=0·01 cm.

Volume of the silver liberated=$0{\cdot}2\times0{\cdot}01$ cc

$\therefore$ Mass of the silver liberated=$0{\cdot}2\times0{\cdot}01\times10{\cdot}5=0{\cdot}021$ gm.

We know, $W=Z.I.t.$ or $t=\frac{W}{Z.I.}$.

Here, $W=0{\cdot}021$ gm ; $Z=0{\cdot}001118$ gm/coulomb ; $I=0{\cdot}15$ amp.

$\therefore \quad t=\frac{0{\cdot}021}{0{\cdot}001118\times 0{\cdot}15}$ sec$=125$ sec.$=2$ mnt 5 sec.

Example 2 : *A plate of area* 10 *sq cm is to be electro-plated with copper (density* 9 *gm/cc) to a thickness of* 0·001 *cm on both sides, using a cell of* 12 *volts. Calculate the energy spent by the cell in the process of deposition. E.C.E. of copper* $=3\times 10^{-4}$ *coulomb/gm.* [*I.I.T. 1971*]

Ans. Since both the surfaces are to be plated, the total area$=10+10=$ 20 sq cm. Mass of copper to be deposited $W=20\times 0{\cdot}001\times 9=0{\cdot}18$ gm. From Faraday's law, the charge $Q=\frac{W}{Z}=\frac{0{\cdot}18}{3\times 10^{-4}}=600$ coulombs.

$\therefore$ the required energy$=$volt$\times$coulomb$=12\times 600=7200$ joule.

Example 3 : *A silver and a copper voltameter are connected in parallel to a* 12 *volt battery of negligible internal resistance. In* 30 *minutes,* 1 *gm of silver and* 1·8 *gm of copper are deposited. At what rate energy is being delivered by the battery ?* [*I.I.T. 1975*]

Ans. Let i_1 and i_2 be the current that passed through the silver and copper voltameters respectively. For silver voltameter, $1=11{\cdot}2\times 10^{-4}\times i_1\times 30\times 60$

or $i_1=\frac{10^4}{1800\times 11{\cdot}2}=0{\cdot}49$ amp.

For copper voltameter, $1{\cdot}8=6{\cdot}6\times 10^{-4}\times i_2\times 30\times 60$ or $i_2=\frac{1{\cdot}8\times 10^4}{1800\times 6{\cdot}6}=1{\cdot}52$ amp. Total current $i=i_1+i_2=0{\cdot}49+1{\cdot}52=2{\cdot}01$ amp.

$\therefore$ Rate of energy delivered$=$volt$\times$current$=12\times 2{\cdot}01=24{\cdot}12$ watt.

Exercises

Essay type :

1. How is a current conducted through an electrolyte ?
2. Describe briefly what happens when current is allowed to pass through a copper sulphate solution with (a) copper electrodes and (b) platinum electrodes.
3. State Faraday's laws in connection with electrolysis. How would you verify them experimentally ? Define 'chemical equivalent' and 'electro chemical equivalent.' [*H. S. Exam. 1979,' 81, '83*]
4. Describe how current flowing in a circuit may be determined by the application of electrolysis. Draw a neat diagram of the arrangement and clearly indicate the cathode and the anode.
5. Explain Arrhenius theory of electrolysis.
6. Mention some practical applications of electrolysis and give a brief description of them.

7. Write notes on the following : (a) relation between electro-chemical equivalent and chemical equivalent. (b) Faraday (c) Electroplating. [*H. S. Exam. 1981*]

Short answer type :

8. Explain the following terms : (i) Ion (ii) Electrolyte (iii) Electrolysis (iv) Electrolytic cell.

9. What is the difference between a voltaic cell and an electrolytic cell ?

10. What is an electrolyte ? Is sugar solution an electrolyte ? Do you consider mercury a good electrolyte ?

11. What do you understand by the statement that the electro-chemical equivalent of silver is 0·001118 gm./coulomb ?

12. What is the relation between the E.C.E. of an element and E.C.E. of hydrogen ?

13. What are the differences between the conduction of electricity through soild conductors and liquid conductors ?

Objective type :

14. Answer 'yes' or 'no' in the following cases :

(i) Is pure water regarded an electrolyte ?———

(ii) Is the weight of the anode plate increased when copper sulphate solution is electrolysed with copper electrodes ?—

(iii) Is Faraday a unit of electric charge ?——

(iv) Is hydrogen liberated at the anode when water is electrolysed ?———

(v) Is electrochemical equivalent of an element equal to the ratio of its atomic weight and valency ?———

Numerical problems :

15. How much silver will be deposited in 1 hour when a current of 0·1 amp is passed through silver nitrate solution ? *E.C.E.* of silver=0·001118 gm/coulomb. [Ans. 0·402 gm.]

16. A current of 2 amp passing for half an hour through a water voltameter liberates 423 cc of hydrogen measured at 13°C and 80 cm pressure. Calculate the E.C.E. of hydrogen given 1 litre of hydrogen at N.T.P. weighs 0·089 gm. [Ans. $1·05\times10^{-5}$ gm./coulomb]

17. In a copper voltameter the mass of copper deposited is 1·5 gm in 10 minutes. If E.C.E. of copper is ·000328 gm/columb, find the current flowing through the voltameter. [Ans. 7·62 amp]

18. A silver voltameter and a copper voltameter are connected in series with a battery and an ammeter. 1·8 gm of silver is liberated in 30 minutes by 0·89 amp of current. Find (a) the *E.C.E.* of silver and (b) the mass of copper deposited. [Chemical equivalents of copper and silver are 318 and 108 respectively.] [Ans. (a) 0·00112 gm/coulomb (b) 0·53 gm.]

19. How long would it take to deposit 0·1 mm thickness of copper on one side of a circular plate of metal of radius 2·5 cm, if the current passing is 1·25 amp ? *E.C.E.* of copper=0·00033 gm./coulomb ; density of copper=8·9 gm per cc [Ans. 1 hr 10 mnt 38 sec]

20. In a circuit, there are a battery of negligible resistance, a resistance box and a voltameter. When 5-ohm resistance is put in the resistance box, 0·36 gm of copper is liberated in the voltameter in 10 minutes. But when 10-ohm resistance is put, 0·48 gm of copper is liberated in 20 minutes. Calculate the resistance of the voltameter. [Ans. 5 ohms]

21. In a copper voltameter, copper is deposited on a cathode plate of 60 sq cm area by 3 amp of current. Find the thickness of the copper deposited in 30 minutes, given density of copper=9 gm/cc ; E.C.E. of copper=·000329 gm/coulomb. [Ans. 0·00329 cm.]

22. A current of 2 amp. is allowed to pass through a copper sulphate solution for 4 hours 27 minutes. Find the thickness of the copper deposited on one side of an electrode measuring 5 cm×6 cm E.C.E. of copper=0·00033 gm/coulomb ; density of copper=8·9 gm/cc [Ans. 0·0396 gm]

23. In the electrolysis of water 83·7 c.c. of hydrogen were collected at a pressure of 68 cm of Hg and at 25°C when a current of 0·5 amp had been passed for 20 minutes. What is the E.C.E. of copper in copper sulphate solution ? At wt. of Cu=63·57, At. wt of H_2=1·008, density of H_2 at N.T.P.=0·089 gm/litre. [*oint Entrance 1984*]
[Ans. $3{\cdot}241\times10^{-4}$ gm/coluomb]

24. If an electric current passes through a copper voltameter and a water voltameter in series, calculate the volume of H_2 which will be liberated in the latter at 25°C and 78 cm. of Hg pressure, while 5×10^{-2} gm of Cu is deposited in the former. E.C.E. of $H_2=1{\cdot}04\times10^{-5}$ gm/c; E.C.E. of Cu$=3{\cdot}3\times10^{-4}$ gm/c, density of $H_2=9\times10^{-6}$ gm/cc at N.T.P. [Ans. 18·6 cc]

25. In an experiment analysing an aqueous solution of $CuSO_4$, between copper electrodes, 0·477 gm of Cu are deposited on the cathode when a current of 0·8 amp flows for 30 minutes. Calculate the atomic weight of copper. Copper is diavalent. [Ans. 63·8]

[**Hints :** Apply the formula, $Z=\frac{A}{2}\times\frac{1}{F}$ where A=at.wt. and F=Faraday=96540 coulomb].

26. A steady current of 5 amp is passed through a silver voltameter in series with a coil of wire of 10 ohm resistance immersed in 200 gm of water. What will be the rise of temperature of the latter when 0·1 gm of silver has been deposited ? E.C.E. of Ag=·001118 gm/c ; J=4·2 joule/cal ; thermal capacity of the coil and vessel=10 cal. [Ans. 5·07°C]

[**Hints :** Apply the relations : $W=Z.I.t.$ and $m.s.\theta=I^2R.t/J.$]

27. A Daniell cell sends current 0·1 amp for 45 minutes. Calculate the change of mass of the copper and zinc electrodes, given that E.C.E. of copper=0·00033 gm/coulomb ; C.E. of Cu=31·8 ; C.E. of Zn=32·6. [*H. S. Exam. 1983*] [Ans. Zn plate loses=0·091 gm, copper plate gains=0·089 gm]

Harder Problems :

28. An unknown current sent through a copper voltameter liberates 0·000328 gm/coulomb on the cathode. What is the strength of the current ? [Ans. 1 amp]

29. In a voltameter there is silver nitrate solution. How much current is needed to liberate 0·805 gm of silver at the cathode in one hour ? E.C.E. of silver$=1{\cdot}118\times10^{-3}$gm/coulomb.
[Ans. 0·2 amp.]

30. If 1 amp of current liberates 0·65 gm of copper in 33 minutes from copper sulphate solution, what is the E.C.E. of hydrogen ? Atomic wt of Cu=63 and valency=2.
[Ans. $1{\cdot}042\times10^{-5}$ gm/coulomb]

31. A battery of negligible resistance, a resistance box and a copper voltameter are all connected in series. When a resistance of 2·5 ohm is put in the resistance box 0·72 gm of copper is liberated in the voltameter in 20 minutes. Again when 5 ohm is put in the box, 0·24 gm of copper is liberated in 10 minutes. Calculate the resistance of the copper voltameter.
[Ans. 2·5 ohms]

32. A 6 volt cell of negligible internal resistance is connected in series with a Joule's calorimeter and a copper voltameter. The heating coil of the calorimeter has a resistance of 3 ohm and the water equivalent of the calorimeter is 240 gm. The water equivalent of the voltameter and its contents is 600 gm. If in 12 minutes, the rise in temperature of the calorimeter is 4·2°C, find (*a*) the mass of copper deposited on the cathode of the voltameter (*b*) the resistance and the rise in temperature of the voltameter (*c*) If the level of the electrolyte in the voltameter is doubled and part of electrodes still remain above the liquid, explain how the resistance of the voltameter will be affected. [*I.I.T. 1968*] [Ans. (*a*) 0·332 gm (*b*) 1·3 ohm ; 0·72°C (*c*) halved]

33. What fraction of the electrical energy spent is converted into chemical energy when a 6-volt battery is used to electrolyse water ? E.C.E. of hydrogen$=1{\cdot}04\times10^{-5}$ gm/coulomb and combustion of 1 gm of hydrogen to form water liberates 35000 calories (1 calorie=4·2 joules).
[Ans. 0·255]

34. How many atoms of copper (II) will be liberated by 48250 coulombs of electricity in a copper voltameter? Avogadro's number $=6{\cdot}04\times10^{23}$ [*Jt. Entrance 1984*] [Ans. $1{\cdot}51\times10^{23}$]

35. A current is passed for 45 minutes through acidulated water and the hydrogen liberated is dried and collected over mercury. If the volume of hydrogen is 430 cc at 68 cm pressure and 15°C and an ammeter through which the current also passes reads 1·2 amp, what is the error of the ammeter? [Ans. $-0{\cdot}04$ amp.]

36. A charge of 10^4 coulomb is passed through a circuit containing a water voltameter and a copper voltameter in series. How much hydrogen will be liberated and how much copper will be deposited? E.C.E. of $H_2=1{\cdot}05\times10^{-5}$ gm/c, At. wt. of Cu$=63{\cdot}6$; Valency of Cu$=2$, C.E. of $H_2=1{\cdot}008$. [*Jt. Entrance 1982*] [Ans. 0·105 gm ; 3·34 gm]

6

ELECTROMAGNETISM

A. Action of electric current on magnet

6.1. Oersted's experiment :

Some of the most important technical applications of electricity depend on the fact that *a current produces a magnetic field* in its neighbourhood. This connection between magnetism and electricity was first discovered by the Danish Physicist Hans Christian Oersted in 1820. His experiment was as follows :

He took a wire *AB* through which a current can flow. The wire stretched north and south and below the wire was kept a magnetic needle. When no current flew through the wire, the needle remained in the north-south position (shown by dotted lines in fig. 6.1). As soon as the current passed along the wire, the needle got deflected and took up a position nearly perpendicular to the wire. When the direction of the current was reversed the needle again set itself at right angles to the wire but with its ends reversed.

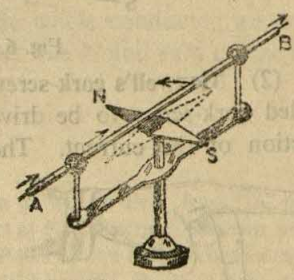


Fig. 6.1

Keeping the direction of current in the wire same, the deflections of the needle were found to be opposite when the wire is

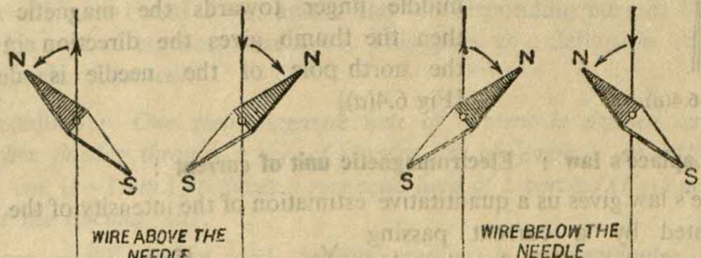


Fig. 6.2

above and below the magnetic needle. These different conditions are shown in fig. 6.2.

This experiment of Oersted conclusively proves that electric current produces a magnetic field surrounding it. It is worth-while to point out here that, *this magnetic field does not magnetise the conductor carrying the current* ; for if some iron filings be brought near the conductor, the filings will not be attracted by the conductor. It is also to be pointed out that if the wire carrying the current, is stretched at right angles to the axis of the magnetic needle the wire will not deflect the magnetic needle, although it will produce its own magnetic field.

6.2. Rules for the direction of deflection of the magnetic needle :

From the above experiment we have come to know that the direction of deflection of the needle depends on the direction of the current and also on whether the wire is above or below the needle. The following rules may be applied to find the direction of deflection of the needle due to a current :

(1) **Ampere's swimming rule** : Let the observer imagine himself to be swimming along the wire in the direction of the current with his arms outstretched and facing the needle. Then the *N*-pole of the needle will be deflected towards his left hand. [Fig. 6.3].

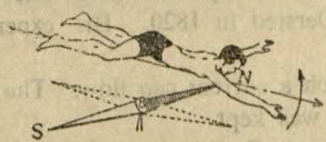


Fig. 6.3

Fig. 6.4

(2) **Maxwell's cork-screw rule** : Imagine a right-handed cork-screw to be driven along the wire in the direction of the current. The direction in which the thumb rotates gives the direction of deflection of the *N*-pole of the needle. [Fig. 6.4].

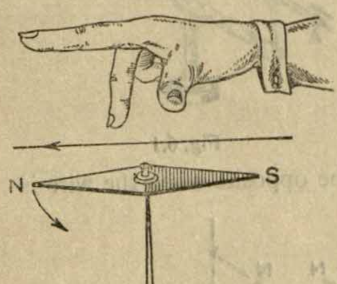


Fig. 6.4(*a*)

(3) **Thumb rule** : Stretch the first three fingers of your right hand so that they are at right angles to each other. If the forefinger points to the direction of current and the middle finger towards the magnetic needle, then the thumb gives the direction in which the north pole of the needle is deflected. [Fig 6.4(*a*)]

6.3. Laplace's law : Electromagnetic unit of current :

Laplace's law gives us a quantitative estimation of the intensity of the magnetic field created by a current passing through a conductor. The law is also known as *Biot-Savart's Law*.

Consider a current i flowing in a conductor XY of length l in the direction from X to Y (Fig. 6.5). Let us take a small element AB of length δl of the conductor. Now, the intensity of the magnetic field at a point P due to this current element according to Laplace's law is (i) proportional to the length δl of the element (ii) proportional inversely to the square

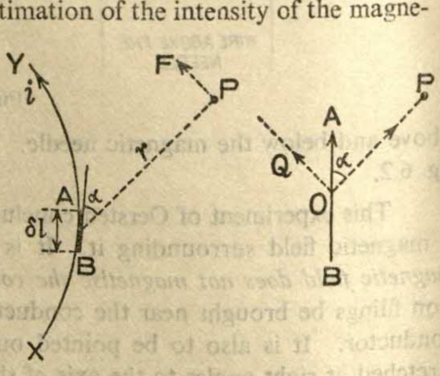


Fig. 6.5

of the distance r of the point P from the current element (this distance is called *radius vector*) (iii) proportional to the sine of the angle (α) between the direction of the current and the radius vector and (v) proportional to the strength of the current (i)

Hence if δF be the intensity of the magnetic field at P, due to the current element then, $\delta F \propto \frac{i.\delta l.\sin\alpha}{r^2}$ or, $\delta F = \frac{K.i.\delta l.\sin\alpha}{r^2}$ where K is the constant of proportionality. It depends upon the unit of current chosen.

This direction of this magnetic field is perpendicular to the plane containing the element δl and the radius vector. In the fig. 6.5 the plane in question is the plane of paper and hence the magnetic field at P is perpendicular to the plane of the paper. If the direction of the current be as shown in the diagram. the field will be directed from front to back. But if the point P is situated towards the left of the conductor the field is from back to front. It may be ascertained by the application of any of the rules mentioned earlier.

To find out the magnetic field at P due to the whole conductor we are to divide the conductor into a number of small elements like δl and sum up the total effects at P. Mathematically it can be written as

$$F = \sum \frac{K.i.\delta l.\sin\alpha}{r^2}$$

As to why the magnetic field at P is proportional to sin α, will be clear from fig. 6.5 (b). The current element AB may be resolved ,as far as its effect at P is concerned, into two components AB cos α along OP and AB sin α along OQ. Since P lies on the former component, the distance of P from the component is zero and therefore, the field is zero. Hence, it is the component AB sin α which is effective in producing the effect at P.

Electromagnetic unit of current : If in the Laplace's equation we put $\delta l=1$, $\alpha=90°$, $r=1$ and $F=1$, and if the corresponding current in the element be called a unit of current, then $K=1$. This gives us a definition of the *electromagnetic unit of current.*

Definition : *One eletromagnetic unit of current is defined as the current which when flowing through a wire of circular arc of length* 1 *cm.* ($l=1$ *cm.*) *and radius* 1 *cm.* ($r=1$ *cm.*) *produces a magnetic field of* 1 *oersted* ($F=1$ *oersted*) *at the centre of the circular wire.*

Here $\alpha=90°$ or, sin $\alpha=1$, because radius is perpendicular to a small element of the circular wire.

According to electromagnetic unit of current, $\delta F = \frac{i.\delta l.\sin\alpha}{r^2}$. and

$$F = \sum \frac{i.\delta l.\sin\alpha}{r^2}$$ where i is the current in e.m.u.

The relation between the practical unit of current 'ampere' and the electromagnetic unit of current is as follows :-

10 amperes=1 e.m.u. of current

6.4. Mapping of magnetic lines of force due to current :

(i) **Current in a straight conductor** : Since an electric current has a magnetic effect, we should expect it to be surrounded by lines of force. The following experiment demonstrates the lines of force due to a current flowing in a straight conductor.

Experiment : PQ is a straight wire passing through a hole on a piece of cardboard (Fig. 6.6). Some iron-filings are scattered over the cardboard which is held in a horizontal position. If a strong current is now sent through the wire and the cardboard is slightly tapped, the filings will be found to arrange themselves along concentric circles around the wire. This arrangement of iron filings represents the lines of force. If the current passes from P to Q, the direction of lines of force are such as shown in fig. 6.6. If the current is reversed, the lines of force are still circles but in the opposite sense. The direction may be obtained by applying any of the three rules mentioned earlier.

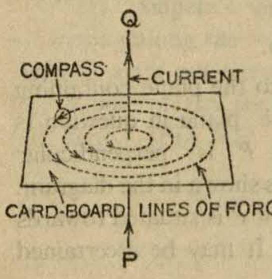


Fig. 6.6

If a current i amp. flows in a long straight conductor, the magnetic field F at a distance r from the conductor is given by $F=\dfrac{2i}{10r}$. .. .. (i)

If the current is expressed in e.m.u., $F=\dfrac{2i}{r}$.

(ii) **Current flowing in a circular conductor** : Fix a short circular wire of several turns to a thin piece of cardboard on which are sprinkled some iron filings [Fig. 6.7]. When a strong current is passed through the coil and the cardboard tapped gently, the iron-filings arrange themselves in a definite pattern as shown in the figure which represents the magnetic lines of force. If the lines are carefully observed it will be seen that near the wire, the lines are concentric circles around the wire but near about the centre of the coil, the lines are straight, parallel and perpendicualr to the plane of the coil. We can, therefore, conclude that the *magnetic field due to a circular current is uniform and perpendicular to the plane of the coil over a small area round the centre of the coil.*

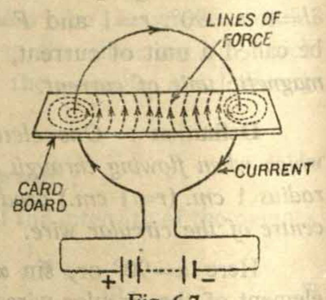


Fig. 6.7

Let i e.m.u. of current flow along a circular coil of radius r. Consider a small element of the coil of length dl [Fig. 6.8]. The magnetic field produced by the element at the centre O of the coil, according to Laplace's law, is

$$\delta F=\frac{i.\, dl.\, \sin \alpha}{r^2}$$

In the present case, dl and r being perpendicular to each other, $\alpha=90°$.

So $\sin\alpha=1$ Hence, $\delta F=\dfrac{i.\,dl}{r^2}$

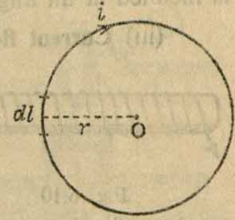


Fig. 6.8

Since i and r are constant over the entire length of the coil, the magnetic field at O due to the whole coil is

$F=\Sigma\dfrac{i.dl}{r^2}=\dfrac{i}{r^2}\,\Sigma\, dl=\dfrac{i}{r^2}\times 2\pi r=\dfrac{2\pi\, i}{r}$. If the coil has n turns, the field intensity at O is $F=\dfrac{2\pi ni}{r}$ .. (ii)

If i be expressed in amperes, $F=\dfrac{2\pi ni}{10r}$.

Such a uniform magnetic field is used in a tangent galvanometer which is described in art. 6.7.

Example 1 : *A circular coil of wire of radius* 0·1 *metre has* 200 *turns. If a current of* 5 *amp flows through the coil, find the intensity of magnetic field at the centre of the coil.*

Ans. The magnetic field at the centre of a circular coil is,

$F=\dfrac{2\pi ni}{10r}$ where i is in amperes.

Now, $n=200$; $i=5$ amp; $r=0{\cdot}1$ metre$=10$ cm.

$\therefore\quad F=\dfrac{2\times 3{\cdot}14\times 200\times 5}{10\times 10}=62{\cdot}8$ Oe.

Example 2 : *Two circular coils of wire, each of* 100 *turns are placed concentrically—one in the horizontal plane and the other in the vertical plane. The radius of the vertical coil is* 20 *cm while the radius of the horizontal coil is* 40 *cm. The current flowing in the vertical and horizontal coil are* $\frac{7}{22}$ *amp. and* $\frac{14}{22}$ *amp respectively. Find the magnitude and direction of the resultant magnetic field at the centre.*

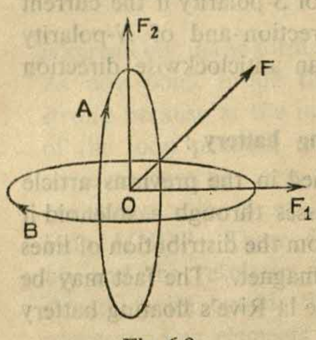


Fig. 6.9

Ans. Let the coil A be vertical and the coil B horizontal [Fig. 6.9]. O is their common centre. The field at O due to the current in the coil A is

$$F_1=\frac{2\pi ni}{10r}=\frac{2\times 22\times 100\times 7}{10\times 7\times 20\times 22}=1\text{ Oe}$$

Since, the plane of the coil is vertical, the intensity F_1 lies in the horizontal plane. Again the field at O due to the coil B is

$$F_2=\frac{2\pi ni}{r}=\frac{2\times 22\times 100\times 14}{10\times 7\times 40\times 22}=1\text{ Oe}$$

Since the plane of the coil B is horizontal, the intensity F_2 lies in the vertical

plane. If F be the resultant intensity at O, then $F=\sqrt{F_1{}^2+F_2{}^2}=\sqrt{2}$ Oe and it is inclined at an angle of 45° to the horizontal or the vertical,

(iii) **Current flowing in a solenoid** : If a long insulated wire is wound on an insulating cylinder in such a way that every turn of the coil is at right angle to the axis of the cylinder, then such a coil is called a **solenoid** (Fig. 6.10)

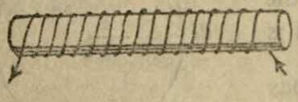


Fig. 6.10

When an electric current is passed through such a solenoid, the resultant magnetic field is very similar to that of a bar-magnet. This can be demonstrated in the following way.

A solenoid is fixed on horizontal cardboard in such a way that its axis lies on the cardboard. Some iron-filings are sprinkled on the cardboard and a current is passed through the solenoid. On tapping the cardboard gently, the iron filings will arrange themselves in a pattern suggesting the lines of force of a bar-magnet (Fig. 6.11). For this reason it is said that *a current-carrying solenoid behaves like a bar-magnet.*

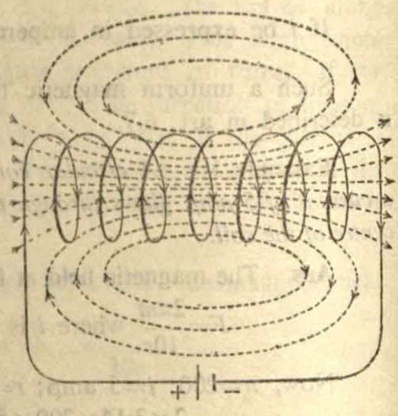


Fig. 6.11

Fig. 6.11 shows the nature of lines of force inside and outside the solenoid. Inside the solenoid, the lines of force are more crowded and parallel to the axis of the solenoid while outside the solenoid, the lines of force are similar to those of a bar-magnet. One end of the solenoid acts like a N-pole and the other like a S-pole,

If a current of i amp flows in a long solenoid, the magnetic field at a point on the axis of the solenoid and inside it is given by $F=\frac{4\pi ni}{10}$, where n=no. of turns per unit length of the solenoid.

If the current is expressed in e.m.u, $F=4\pi ni$.

Rule for the polarity of a solenoid carrying a current :

When viewing one end of the solenoid, it will be of S-polarity if the current flows in a clockwise direction and of N-polarity if the current flows in an anticlockwise direction [Fig. 6.12].

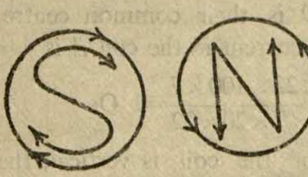


Fig. 6.12

De la Rive's floating battery :

It has been mentioned in the previous article that when a current passes through a solenoid it becomes a magnet and from the distribution of lines of force, it is seen that the solenoid behaves like a bar-magnet. The fact may be demonstrated by the following arrangement known as De la Rive's floating battery [Fig. 6.13].

Two plates of zinc and copper are dipped in dilute sulphuric acid con-

tained in a wide and fairly tall test tube. Two thick wires are then soldered to the plates and they pass through a piece of cork which closes the mouth of the test tube. A spiral wire forming a soleniod is connected to the ends of the stout wires. The whole arrangement is then allowed to float in water. To make it float upright some mercury is put in the test tube.

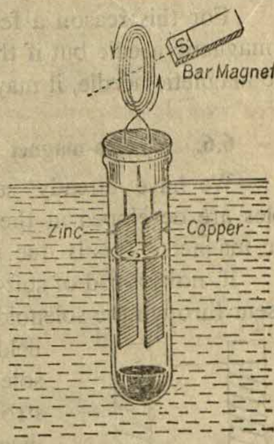


Fig. 6.13

When floated in water, the test tube will assume such a position that the axis of the solenoid is placed along north-south direction (shown by dotted lines in the diagram). If the test tube is disturbed, it will again take up the previous position after a few oscillatous just as a freely suspended magnet always takes up the north-south position. Why does it happen so ?

The plates of zinc and copper together with sulphuric acid form a simple cell. A current flows from copper plate to the zinc plate through the solenoid. The end of the solenoid where the current flows in anticlockwise direction, develops a north pole and the other end a south pole. As a result, whole arrangement turns until the solenoid sets itself along the magnetic meridian just as a freely suspended magnet does.

The development of *N*-polarity at the end of the solenoid when current is anticlockwise may be demonstrated by holding the *N*-pole of a magnet near that end and noticing the consequent repulsion.

6.5. Magnetic field due to a current in a loop of wire :

If the terminals of a battery are joined by a wire which is simply doubled back on itself as shown in fig. 6.14 then the current flowing through the wires will produce no magnetic field. Each element on the outward run of the wire, such as *MN*, produces a magnetic field equal in magnitude but opposite in direction to the field produced by the corresponding element *PQ* of the inward run and hence they cancel each other. But as soon as the wire is spread out in the form a rectangular loop say, its magnetic field appears [Fig 6.15].

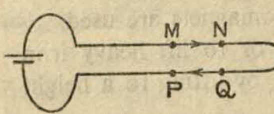


Fig. 6.14

At any point inside the loop, the field is very strong because at the inside point all the elements of the loop produce magnetic fields in the same sense as can be seen by applying Maxwell's corkscrew rule to each side of the rectangular loop *MNQP*. These fields are, therefore, added up and the resultant field becomes very strong. At any point outside the loop, but near *NQ* say, corresponding elements like *PM* and *NQ* produce opposing fields but the point being situated at

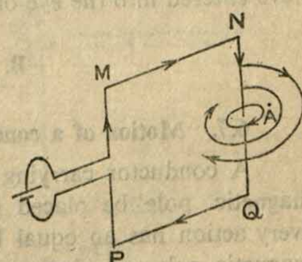


Fig. 6.15

different distances from the corresponding elements (*PM* is further away than *NQ*) the fields do not cancel ; a weak resultant field appears at the point.

For this reason a feeble current flowing in a straight wire may not deflect a magnetic needle but if the wire is bent in the form of a rectangular loop around the magnetic needle, it may produce deflection of the needle.

6.6. Electro-magnet :

We have already seen that when a current is sent through a solenoid, poles are developed at the two ends of the solenoid which then behaves like a bar magnet. This is due to the magnetic effect of electric current.

If, now, a rod of soft-iron be inserted into the solenoid, and a strong current is sent through the solenoid, the rod will become a strong magnet. The magnetic field produced by the current flowing through the solenoid has, in this case, magnetised the rod into a strong magnet. This type of magnet is called *electro-magnet.*

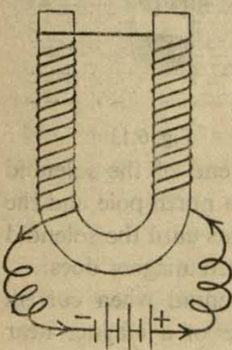


Fig. 6.16

The advantage of an electromagnet is that it remains a strong magnet as long as the current flows through the coil and becomes demagnetised as soon as the current is stopped. Further, the strength of the magnet can be increased conveniently by increasing the number of turns of the coil or by increasing the strength of the current.

The electro-magnets, in practical use, are generally bent in the form of the letter U. [Fig. 6.16]. *Note that the windings of the coil in the two limbs of the electro-magnet are in the opposite direction.* This helps to develop two opposite poles at the two faces of the electro-magnet.

The following are the important uses of electromagnet :

(i) In electric fan, relay, motor, dynamo etc. electromagnets are used.

(ii) Electromagnets are used in factories to move or to lift heavy iron materials and sometimes to break heavy chunks of iron by lifting to a height and then dropping them on to the ground.

(iii) Electromagnets are used to sort out particles of iron from a mixture of iron and other non-magnetic materials.

(iv) Physicians use electromagnets to pull out iron particles which might have entered into the eye of a man.

B. Action of magnet on current

6.7. Motion of a conductor carrying current and placed in a magnetic field :

A conductor carrying a current, produces a magnetic field around it. If a magnetic pole be placed in the field, it will experience a force. We know that every action has an equal but opposite reaction. According to this rule, the magnetic pole will also exert a force on the conductor, which if free to move, will be deflected from its position. This is the action of a magnet on current.

Consider a straight wire AB carrying a current in the downward direction. The concentric lines of force and their directions are shown in fig. 6.17. A magnetic north pole placed at P will have a tendency to move along PR perpendicular to the plane of the paper. If the magnetic pole at P is fixed and the wire AB is movable, the wire will experience a force in the direction OX perpendicular to the plane of the paper and will be deflected in that direction. This is due to the fact that action and reaction are equal and opposite. If the direction of current in the wire is reversed, the direction of deflection *i.e.* the direction of force is also reversed.

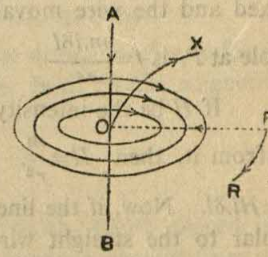


Fig. 6.17

6.8. Direction of deflection of wire : Fleming's left hand rule :

Depending upon the direction of current and the magnetic field, the direction of deflection of the wire can be found out by Fleming's left-hand rule.

Stretch the forefinger, second finger and the thumb of the left-hand mutually at right angles. If the forefinger points in the direction of the Field and the second finger in the direction of the Current, the thumb will point in the direction of the Motion (Fig. 6.18). It is also known as motor rule.

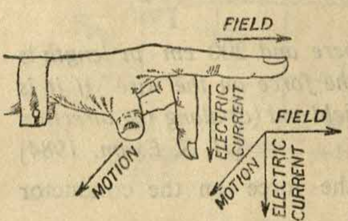


Fig. 6.18

6.9. Force on a straight current-carrying conductor placed in a uniform magnetic field :

Let a straight wire carrying a current i. e.m.u. be placed at a distance r from a magnetic pole of strength m placed at P. According to Laplace's formula, the magnetic field produced by an element δl at P is given by $\delta F = \dfrac{i\delta l . \sin\alpha}{r^2}$

If the element δl be at right angles to the point P, then $\alpha = 90°$. Hence the magnetic field due to the element δl at P is given by,

$$\delta F = \frac{i\delta l}{r^2}$$

Since the strength of the magnetic pole placed at P is 'm', the force experienced by the pole, f=pole strength$\times$intensity of the field$=m\times\delta F=\dfrac{m.i\delta l}{r^2}$

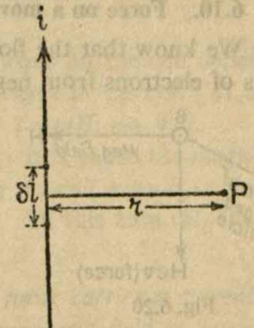


Fig. 6.19

Now, every action has an equal and opposite reaction. So, if the pole be fixed and the wire movable, the force experienced by the element δl due to the pole at P, is $f=\frac{m.i\delta l}{r^2}$

If H be the intensity of the magnetic field produced by the pole at a distance r from it, then $H=\frac{m}{r^2}$. So, the force on the element δl by the magnetic field $=Hi.\delta l$. Now, if the lines of force of the magnetic field be everywhere perpendicular to the straight wire, the force experienced by the conductor$=\Sigma H.i\delta l.=H.il$ dynes. [$\Sigma\delta l=l=$length of the wire]

If i be expressed in ampere, force$=\frac{1}{10}$ Hil dynes.

If there be 'n' turns in the conductor, each carrying a current i, then the force$=n.H.i.l.$ The direction of this force is given by Fleming's left hand rule. This force is utilised in a suspended coil galvanometer. [See art 6.17(iii)].

If the conductor be parallel to the direction of H, then $\alpha=0$ and in that case, the force $f=0$.

Example : *A wire carrying a current of* 10 *ampere and* 200 *cm in length is placed in a field of flux density* 0·15 *oe. What is the force on the wire if it is placed* (*a*) *at right angles to the field* (*b*) *to* 45° *at the field and* (*c*) *along the direction of the field.* [*H. S. Exam. 1984*]

Ans. (*a*) At right angles to the field, the force on the conductor $=\frac{Hil}{10}$ dynes. $\therefore$ force$=\frac{1}{10}\times 0{\cdot}15\times 10\times 200=30$ dynes.

(*b*) At an angle α to the direction of the field, the force on the conductor$=\frac{1}{10}$ $H.il.\sin\alpha$. If α is 45°, then force$=\frac{1}{10}\times 0{\cdot}15\times 10\times 200\times\sin 45°$

$=30\times 0{\cdot}707=21{\cdot}21$ dynes.

(*c*) Since $\alpha=0$, $\sin\alpha=0$; so the force on the conductor$=0$

6.10. Force on a moving charge :

We know that the flow of current in a conductor actually means the movements of electrons from negative to the positive end of the source of the current.

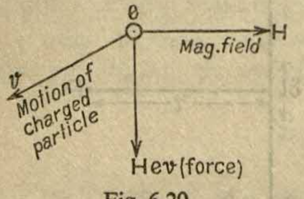


Fig. 6.20

Suppose that a charge e e.m.u. (say an electron) is moving with a velocity v cm/s at right angles to the direction of a uniform magnetic field of intensity H oersted. If the charge takes a time t to travel a distance l, then $t=\frac{l}{v}$ sec. The motion of the charged particle constitutes a current $i=\frac{e}{t}$ (rate of flow of charge).

$$\therefore\ i=\frac{e}{t}=\frac{e.v}{l}\ \text{or}\ i.l=e.v.$$

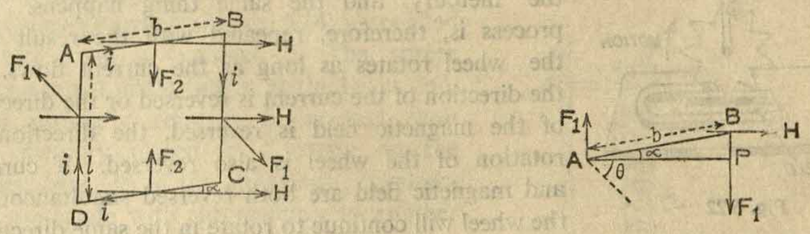
But the force on this current element$=H.i.l.$ dynes.$=H.e.v.$ dynes.

Hence a charged particle moving perpendicular to the direction of a uniform magnetic field experiences a force which is perpendicular both to the magnetic field and to its direction of motion (Fig. 6.20).

6.11. Torque on a coil in a uniform magnetic field :

Consider a rectangular coil of height $AD=l$ and width $AB=b$ placed in a magnetic field of intensity H [Fig. 6.21 (i)]. If the coil carries a current i e.m.u. each of its arms will experience a force.

Now, the forces F_1 on the vertical arms AD or BC are given by $F_1=H.i.l.$ And the forces F_2 on the horizontal arms AB or DC are given by $F_2=H.i.b \sin \alpha$. The forces on the horizontal arms tend to compress the coil and are resisted by



(i) Fig. 6.21 (ii)

the rigidity of the wire. The forces F_1, F_1 however, set up a couple, whose moment or torque is given by $T==F_1 \times AP=F_1.b \cos \alpha=H.i.l.b. \cos \alpha=H.i.A. \cos \alpha$. where A is the face area of the coil $[A=l \times b]$. If θ be the angle between the magnetic field and the perpendicular to the plane of the coil [Fig. 6.21(ii)], then $\theta=90^\circ-\alpha$

$$\therefore \quad T=H.i.A. \sin \theta.$$

If the coil has n turns, $T=nHiA \sin \theta$

Magnetic moment : From above we see that the torque on the coil depends on (i) the magnetic field (H) (ii) the current (i) (iii) area of the coil (A) and (iv) the number of turns (n). Putting $m=n.i.A$, we have $T=mH. \sin \theta$.
m is evidently a property of the coil and the current in it. It is called the *magnetic moment* of the coil. *It is defined as the torque exerted on the coil when it is placed with its plane parallel to the magnetic field of* 1 *oersted.* In this case, $H=1$, $\sin \theta=\sin 90^\circ=1$ and hence $T=m$.

Example : *A coil of face area* 5 *sq. cm. and* 30 *turns carries a current of* 5 *amp. It is placed with its axis at right angles to a magnetic field of intensity* 0·25 *oe. What torque is needed to keep it in equilibrium ?*

Ans. Magnetic moment of the coil$=n.i.A=30 \times \frac{5}{10} \times 5=75$

$$\therefore \quad \text{Torque } T=m.H. \sin \theta=75 \times 0{\cdot}25 \times \sin 90^\circ=18.75 \text{ dyne-cm.}$$

6.12. Experiment to demonstrate the action of magnet on current :

Barlow's wheel : It consists of a star-shaped wheel made of copper, capable of rotating in a vertical plane about a horizontal axis. Some mercury is kept in a slot cut on the wooden base. As the wheel rotates, its pointed teeth, one after another, come in contact with mercury. Across the slot are placed the poles of a strong horse-shoe magnet. By means of two terminals, current is led through the wheel and mercury and back to the battery.

When current flows down the spoke dipping into mercury, a force is exerted on it, the direction of which may be obtained from Fleming's left hand rule. If the current flows from upward to downward, the spoke will move forward in the direction of the arrow as shown in fig. 6.22. As the spoke moves forward, the next spoke, due to intertia of motion, dips into the mercury and the same thing happens. The process is, therefore, repeated with the result that the wheel rotates as long as the current flows. If the direction of the current is reversed or the direction of the magnetic field is reversed, the direction of rotation of the wheel is also reversed. If current and magnetic field are both reversed simultaneously, the wheel will continue to rotate in the same direction. Moreover the speed of rotation of the wheel will increase or decrease according as the strength of the current or the magnetic field is increased or decreased.

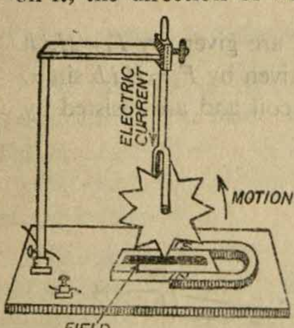


Fig. 6.22

This mechanical rotation by electric current is also known as the principle of motors.

C. Action of current on current

6.13. Introduction :

If we have two current carrying wires placed close to each other, the magnetic field due to one will exert a force on the other. In fig. 6.23 (*a*) and

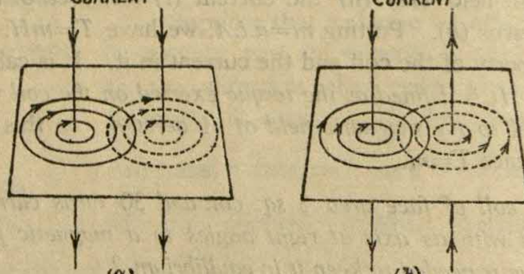


Fig. 6.23

(*b*) two straight and parallel wires having current flowing in them are shown. In fig. 6.23 (*a*), the currents are like and parallel while in fig. 6.23 (*b*), they

are unlike and parallel. If in both cases, Fleming's left hand rule be applied in order to find the direction of the force, it will be seen that there is a force of attraction in the first case while a force of repulsion takes place in the second case.

6.14. Law of parallel currents :

(i) Like parallel currents attract each other.

(ii) Unlike parallel currents repel each other.

6.15. Experimental verification of the law of parallel currents :

(i) **Roget's vibrating spiral** : By this simple experiment, the attraction between like parallel currents can be demonstrated. It consists of a closely wound helical spring C (Fig. 6.24) suspended vertically from a terminal T. The spring carries a small metal weight at the lower end, which normally remains in contact with some mercury placed in a cavity on the wooden base (B) of the apparatus. Connections are made with the mercury to another terminal T fixed on the wooden base. When a current is sent through the spring by joining a battery to the terminals, (T, T), two consecutive turns of spring will carry like parallel currents with the result that each two consecutive turns will attract each other. This will lift the small metal weight out of mercury and the circuit will break, stopping the flow of current. The attractive force between the turns is therefore removed and the loaded spring falls back into the mercury. The process is repeated and the spring vibrates up and down as long as the current flows.

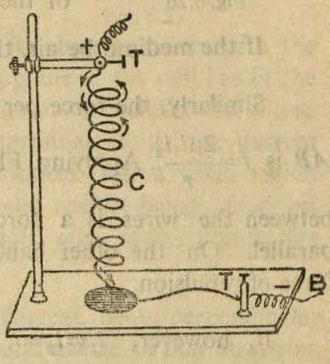
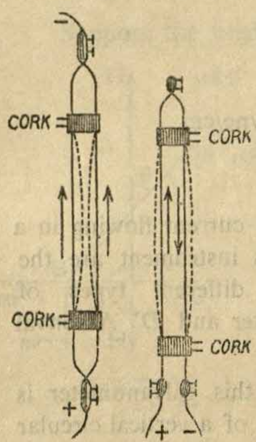


Fig. 6.24

(ii) Two straight copper wires are kept vertical and parallel by means of two pieces of cork as shown in the fig 6.25, so that they do not touch each other. When both the ends of the wires are joined above and below as in the fig. 6.25(i) and current passed, then these currents are parallel and like. Consequently, there will be attraction between them and the wires will be drawn to each other as shown by the dotted lines.

On the other hand, if the wires are joined only at the top and current is passed [Fig. 6.25(ii)], then the currents are parallel but unlike. Consequently there will be repulsion between them and the wires will bulge out as shown by the dotted lines.



(i) Fig. 6.25 (ii)

6.16. Forces of attraction and repulsion between two infinitely long parallel wires carrying current :

Let AB and CD be two long, straight and parallel wires carrying currents i_1 and i_2 e.m.u. respectively [Fig. 6.26]. The magnetic field intensity at P due to the current i_2 in CD is $H=\frac{2i_2}{r}$ [see eqn. (i), art 6.4].

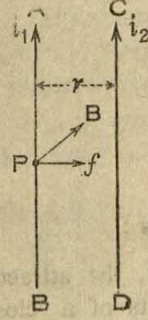


Fig. 6.26

The direction of this field is at right angles to the wire AB. If μ be the permeability of the medium in which the wires are placed, then, total field intensity at P is $B=\mu H=\frac{2\mu i_2}{r}$. Hence, force per unit length of the wire at P is $f=B.i_1=\frac{2\mu i_1 i_2}{r}$; this force lies in the plane of the paper and is perpendicular to the wire AB.

If the medium be air, then $\mu=1$ and hence $f=\frac{2i_1 i_2}{r}$

Similarly, the force per unit length on the wire CD due to the current i_1 in AB is $f=\frac{2\mu i_1 i_2}{r}$. Applying Fleming's left hand rule, it is found that the force between the wires is a force of attraction if the currents in the wires are like parallel. On the other hand, if the currents are unlike parallel, the force is a force of repulsion.

If, however, $i_1=i_2=i$, then $f=\frac{2\mu i^2}{r}$

Example : *Two infinitely long and parallel wires are carrying currents of* 3 *amp and* 4 *amp respectively. If the separation of the wires be* 10 *cm, what is the force exerted by one on the other* ?

Ans. We know, the force/unit length $f=\frac{2i_1 i_2}{r}$

Here $i_1=\frac{3}{10}$ e.m.u ; $i_2=\frac{5}{10}$ e.m.u. and $r=10$ cm.

$\therefore$ force per unit length $f=2\times\frac{}{10}\times\frac{5}{10}\times\frac{}{10}=0{\cdot}03$ dyne/cm.

6.17. Galvanometers :

Galvanometers are instruments by means of which current flowing in a circuit may be measured. The basic principles of this instrument are the interaction between electricity and magnetism. Of different types of galvanometers, the tangent galvanometer, the sine galvanometer and D' Arsonval galvanometer are very important.

(i) Tangent galvanometer : The basic principle of this galvanometer is the effect of current on magnet. It consists essentially of a vertical circular coil of several turns of insulated copper wire wound on a circular wooden

frame [Fig. 6.27]. The ends of the coil are connected to two screw terminals (T,T) fixed on the wooden base B. There are several screws (S, S) by means of which the base can be levelled. The circular wooden frame can rotate about a vertical axis passing through its centre. At the centre of the coil is mounted a small magnetic needle which can freely rotate in a horizontal plane. The magnetic needle has a long, light aluminium pointer attached at right angles to its axis. The pointer moves over a circular horizontal scale divided into four quadrants, each having graduations from 0° to 90°.

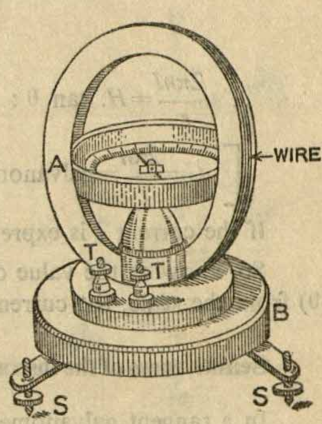


Fig. 6.27

Adjustment of the instrument : Before using the instrument, the base B is to be levelled with the help of the levelling screws (S, S). Then the frame carrying the coil is to be rotated until its plane coincides with the plane of the magnetic needle. In this condition, the plane of the coil lies in the magnetic meridian and the pointer points 0° – 0° on the scale. The circuit whose current is to be measured is then connected to the terminals T, T. The current will flow through the coil. The magnetic field it produces over a small region round the centre of the coil is uniform. The magnetic needle, being acted on by this uniform magnetic field, will be deflected. Both ends of the pointer are read to get the angle of deflection.

Theory : The magnetic field produced by the current, being perpendicular to the plane of the coil, will try to deflect the magnetic needle in that direction. Once deflected from the meridian position, the needle will be acted on by the earth's magnetic field which will try to restore the needle to its previous position. In other words, two opposite couples will act simultaneously on the needle which will finally come to rest when the moments of the couples balance each other.

Suppose the needle AB is at rest after suffering a deflection of an angle θ [Fig. 6.27 (a)]. If H be the horizontal component of earth's magnetic field, F the intensity of the magnetic field due to the current flowing in the coil and m the pole-strength of the magnetic needle, two couples (mH, mH) and (mF, mF) will act upon the needle (See figure). At the equililibrium position of the needle, the moments of the couples will be equal.

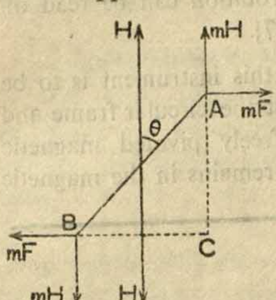


Fig 6.27(a)

Now, moment of mF-couple $= mF \times AC$

$= mF \times AB \cos\theta$.

and „ „ mH-couple $= mH \times BC = mH \times AB.\sin\theta$.

$\therefore\ mF \times AB \cos\theta = mH \times AB \sin\theta$.

or, $F = H.\tan\theta$.

If n=number of turns of the coil, I=current in e.m.u. flowing in the

coil and r=radius of the circular coil, then it can be proved that,

$$F=\frac{2\pi nI}{r}. \quad \text{[Art 6.4 (ii)]}$$

$$\therefore \quad \frac{2\pi nI}{r}=H.\tan\theta: \quad \text{or, } I=\frac{H}{2\pi n/r}\tan\theta=\frac{H}{G}.\tan\theta=K.\tan\theta$$

$$\left[\ G=\frac{2\pi n}{r}=\text{galvanometer constant and } K=\frac{H}{G}=\text{reduction factor}\ \right]$$

If the current I is expressed in ampere, then $I=10K.\tan\theta$.

So, knowing the value of the reduction factor (K) and reading the deflection (θ) from the scale, the current can be found out.

Sensitiveness of the instrument :

In a tangent galvanometer, $I=\frac{H}{2\pi n/r}.\tan\theta$ or $\tan\theta=\frac{2\pi n.I}{r.H}$

A galvanometer is sensitive if it can produce a large deflection for a small current *i.e.* the more the value of $\tan\theta$ for a given current I, the more sensitive the galvanometer is. Now, from the above expression of $\tan\theta$, it is clear that (i) a larger value of n and (ii) a smaller value of r will give, for a given value of I, a larger value of $\tan\theta$. For this reason, the number of turns of the galvanometer coil is taken as large and the radius as small as practicable, in order to make the instrument sensitive.

Accuracy of the instrument : Since the tangent of an angle varies from 0 to ∞, this galvanometer can be used to measure all currents varying from 0 to ∞. But it is found that the instrument gives accurate result when the deflection is near about 45°*. Ordinarily, the deflection is kept between 30° and 60°.

(ii) **Sine galvanometer :** So far as the construction is concerned, there is very little difference between a tangent galvanometer and a sine galvanometer. In this instrument, the circular wooden frame carrying the coil of wire can rotate about a vertical axis passing through its centre and the rotation can be read off from a circular scale attached to the platform B [Fig. 6.27].

Principle of action : Like tangent galvanometer, this instrument is to be levelled first and then the coil should be rotated so that the circular frame and the magnetic needle lie in the same plane. Since the freely pivoted magnetic needle remains in the magnetic meridian, the frame also remains in the magnetic

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*$I=10.k\tan\theta$; Differentiating, we get $dI=10k.\sec^2\theta d\theta$

$$\text{Dividing.} \quad \frac{dI}{I}=\frac{\sec^2\theta}{\tan\theta}.d\theta=\frac{d\theta}{\sin\theta\cos\theta}=\frac{2.d\theta}{\sin 2\theta}$$

In measuring the current I, if the error is dI, then $\frac{dI}{I}$ is the proportional error. To keep the proportional error minimum, it is necessary that 2θ should be maximum. It is possible if $\sin 2\theta=\sin 90°$ or $\theta=45°$. So, the proportional error in a tangent galvanometer is minimum when $\theta=45°$

meridian. But as soon as a current flows through the coil, the magnetic needle is deflected. Then the coil is also rotated unless and until it becomes co-planer with the needle. Suppose, the magnetic needle (N-S) and the plane of the coil (AB) make an angle θ with meridian direction [Fig. 6.28]. Here, the magnetic field produced by the current, being perpendicular to the plane of the coil, is also perpendicular to the magnetic needle. Each pole of the needle will be acted on by a force mF due to this field. On the other hand, earth's magnetic field will exert a force mH on each pole trying to restore it to the meridian direction. Consequently, as in a tangent galvanometer, here also two opposing couples will act on the magnetic needle which will remain in equilibrium at the deflected position under their influence.

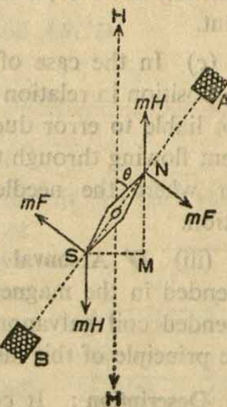


Fig. 6.28

In other words, in the equilibrium position of the needle, the moments of the two couples will be equal. Now the moment of mF couple$=mF\times NS=mF\times 2l$

[$2l$=length of the magnetic needle]

And, the moment of mH couple$=mH\times SM=mH\times 2l.\sin\theta$.

$$\therefore\quad mF\times 2l=mH\times 2l.\sin\theta.$$

$$\text{or}\quad F=H.\sin\theta.$$

If n=number of turns of the coil, I=current in e.m.u. ; r=radius of the coil, then the magnetic field at the centre of the coil, $F=\frac{2\pi nI}{r}$ [ref. art. 6.4 (ii)]

$$\therefore\quad \frac{2\pi nI}{r}=H.\sin\theta.\quad \text{or,}\quad I=\frac{H}{2\pi n/r}\sin\theta=\frac{H}{G}\sin\theta=K.\sin\theta.$$

If the current is expressed in amperes, then $I=10.\ K.\sin\theta$.

Since the current is proportional to $\sin\theta$, the galvanometer is called a sine galvanometer.

Comparison between a tangent and a sine galvanometer :

(*a*) For a feeble current, a sine galvanometer is more sensitive than a tangent galvanometer. To understand it, let us take an illustration. Suppose a current produces a deflection of 45° in a tangent galvanometer. The same current when passed through a sine galvanometer would give a deflection of 90°, because $\sin 90°=\tan 45°=1$. So, the same current produces a greater deflection in a sine galvanometer than in a tangent galvanometer. Hence, a sine galvanometer is *more sensitive* than a tangent galvanometer.

(*b*) Since $\sin\theta$ takes up values between 0 and 1, a sine galvanometer can measure currents varying between 0 and K e.m.u. where K is the reduction factor of the galvanometer. But a tangent galvanometer can measure current of any strength. Hence a tangent galvanometer has *wider range* than a sine galvanometer.

In practice, however, a tangent galvanometer is not used to measure a very strong current.

(c) In the case of a sine galvanometer, the needle always remains in the same position in relation with the coil. This has the advantage that the deflection is not liable to error due to non-uniformity of the magnetic field created by the current flowing through the coil. But this is not the case with a tangent galvanometer where the needle moves out of the plane of the coil in the deflected position.

(iii) **D' Arsonval galvanometer** : In this galvanometer, the coil is freely suspended in the magnetic field of a parmanent magnet and hence it is called a suspended coil galvanometer. The effect of magnet on current constitutes the basic principle of this instrument.

Description : It consists of a coil of copper wire of several turns wound on a rectangular frame *ABCD* made of brass or aluminium. The coil is suspended by a thin strip of phosphor-bronze in the annular space formed by fixing a soft-iron cylinder (*E*) between the cylindrical pole faces of a permanent magnet (*N-S*). The lower end of the coil is connected to a small spring *S*, made of phosphor-bronze which serves as a current lead. The phosphor-bronze suspension above serves the purpose of the other current lead. The upper end of the suspension is attached to a torsion head *T*. The current led through the phosphor-bronze suspension passes through the coil *ABCD* and goes back to the external circuit through the spring *S*. The phosphor-bronze suspension wire carries a small mirror *M* which reflects light from a lamp on to a scale placed a metre away. The reflected beam acts as a pointer in measuring the deflection of the coil. One advantage of this arrangement, known as *the lamp and scale arrangement*, is that the reflected beam turns through *twice* the angle through which the mirror turns.

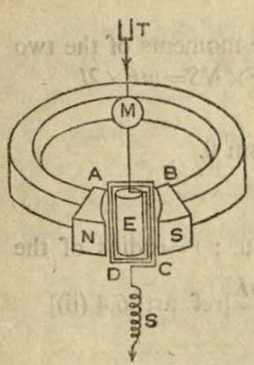


Fig. 6.29

The pole-pieces of the magnet are shaped cylindrical. Further placing a soft-iron cylinder *E* coaxially, the magnetic field is made intense and radial so that the lines of force are always parallel to the plane of the rectangular coil [Fig. 6.30].

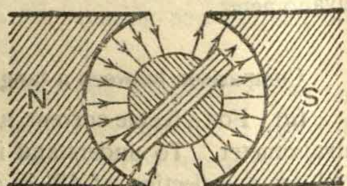


Fig. 6.30

Principle of action : When there is no current in the coil, the coil comes to rest in such a position that there is no twist in the suspension thread. But when current passes through the coil, two equal and opposite parallel forces act respectively on the two vertical sides of the coil in accordance with Fleming's left hand rule. These two forces together form a *deflecting couple* which causes the coil to rotate until the deflecting couple is just balanced by the opposing *control*

couple set up by torsion in the upper suspension thread. The position of the deflected spot of light on the scale gives the deflection of the coil.

Theory : Let, l=length of the vertical arm of the coil $ABCD$
b= „ „ „ horizontal „ „ „ „
H=magnetic field of the permanent magnet
n=number of turns of the coil
i=current flowing in the coil in e.m.u.

When current flows through the coil, according to the art. 6.7 each vertical arm of the coil experiences a force $F=nH.i.l.$ [Fig. 6.31] at right angles to the plane of the coil. Since the current flows in opposite directions in the two vertical arms, the forces acting on them are also opposite. The direction of force on any vertical arm may be obtained from Fleming's left hand rule. The couple produced by these parallel and opposite forces deflects the coil until the torsional couple set up by the twist in the suspension wire balances it. Suppose the deflection of the coil in the position of equilibrium is θ.

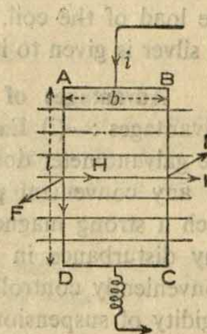


Fig. 6.31

Now, the moment of the deflecting couple$=F\times b$ $=n.H.i.l.b$

If the torsional couple for unit twist for the material of the suspension wire be C, then the torsional couple for a twist $\theta=C.\theta$.

Since the coil is at rest, $n.H.i.l.b.=C.\theta$. or $n.H.i.A=C.\theta$.
[A=face area of the coil$=l\times b$]

$$\therefore \quad i=\frac{C}{nHA}.\ \theta \text{ e.m.u.}=K.\ \theta, \text{ where } K=\frac{C}{nHA}=\text{a constant } i.e.,\ i\propto\theta.$$

So, we see that the current is proportional the the deflection of the coil.

The deflection (θ) of the galvanometer coil is usually measured by the displacement of a spot of light on a scale, light being reflected by the mirror M.

For this purpose, a fine beam of light from a source is focussed on the mirror M and the reflected light is received on a scale placed parallel to the plane of the mirror. If the mirror turns through an angle θ, the reflected light turns through 2θ. If this produces a displacement x cm of the spot of light on the scale, then $2\theta=\frac{x}{D}$, where D=distance in cm between the scale and the mirror [Fig. 6.32].

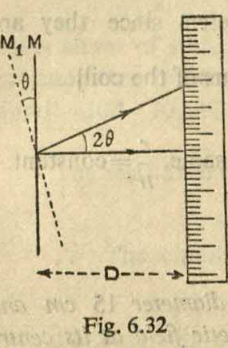


Fig. 6.32

$$\therefore \quad \theta=\frac{x}{2D}.$$

Measuring the displacement x and noting the distance D, the deflection θ can be determined. This arrangement is known as the lamp and scale arrangement.

Sensitiveness of the instrument :

We have seen that $i=\frac{C.\theta}{nHA}$ or $\theta=\frac{nHA}{C}.i$. The instrument will be very sensitive, if θ is large for a small value of i. From the above relation it is clear that to make θ large *i.e.* to make the instrument sensitive, (i) n, A and H should have large values and (ii) C should have a small value.

The value of C is sufficiently less if we select a phosphor-bronze or quartz fibre of rectangular cross section. The fibre is, at the same time, tough to carry the load of the coil. To make quartz fibre a conductor of electricity, a coating of silver is given to it.

Advantages of the galvanometer : This galvanometer has the following advantages :—(i) Earth's magnetic field has no effect on this galvanometer since the galvanometer does not contain any magnetic needle. Hence, it can be placed in any convenient position. (ii) The horse-shoe permanent magnet produces such a strong magnetic field that any stray external magnetic field cannot cause any disturbance in its action. (iii) The instrument can be made sensitive by conveniently controlling the number of turns, the face area of the coil, torisonal rigidity of suspension etc. and can be used to measure feeble current. (v) Since the current is proportional to deflection, a linear or evenly divided scale can be used for the measurement of current. (v) The instrument is 'dead beat' *i.e.*, the coil, once deflected, comes back to the position of rest quickly ; the coil does not oscillate for long.

Example 1 : *The coil of a tangent galvanometer has* 100 *turns and of mean radius* 10 *cm. What is the current flowing through the galvanometer when it shows a deflection of* 60° ? (H=0·37 *and tan* 60°=1·7321).

Ans. We know, $i=\frac{10\ H.r}{2.\pi.n}.\tan\theta=\frac{10\times0{\cdot}37\times10\times1{\cdot}7321}{2\times3{\cdot}14\times100}=0{\cdot}1$ amp.

Example 2 : *A current is sent through two tangent galvanometers in series. The deflection is seen to be the same in both the galvanometers. Compare the radii of the coils, if the number of turns is* 110 *in the first coil and* 25 *in the second.*

Ans. Same current will flow through the galvanometers since they are connected in series. Now, we know, $i=\frac{H.r}{2\pi n}.\tan\theta$ [r=radius of the coil]

From this we see that other factors remaining the same, $\frac{r}{n}$=constant.

$$\therefore\ \frac{r_1}{n_1}=\frac{r_2}{n_2}\quad\text{or,}\quad\frac{r_1}{r_2}=\frac{n_1}{n_2}=\frac{110}{25}=\frac{22}{5}$$

Example 3 : *The coil of a tangent galvanometer has diameter* 15 *cm and number of turns* 50. *What will be the intensity of the magnetic field at its centre when a current of* 0·1 *amp flows through it ? If* H=0·42 *Oe at the locality, what will be the deflection of the galvanometer needle ?*

Ans. At the centre of the coil, the field intensity $F=\frac{2\pi ni}{r}$

Here, $i=0{\cdot}1$ amp$=0{\cdot}01$ e.m.u. ; $r=\frac{15}{2}=7{\cdot}5$ cm. ; $n=50$

$$\therefore \quad F=\frac{2\times 3{\cdot}14\times 50\times 0{\cdot}01}{7{\cdot}5}=0{\cdot}42 \text{ Oe.}$$

Again, $F=H.\tan\theta$ or $\tan\theta=\frac{F}{H}=\frac{0{\cdot}42}{0{\cdot}42}=1 \quad \therefore \quad \theta=45^\circ$

So, the deflection of the galvanometer needle$=45^\circ$.

Example 4 : *A sine galvanometer with a short needle is used as a tangent galvanometer and when a given current is passed through it a deflection of* 30° *is produced. Find the deflection which the same current should produce if the instrument is used as a sine galvanometer.*

Ans. For a tangent galvanometer, $I=K.\tan\theta_1$; when used as a sine galvanometer we have $I=K.\sin\theta_2 \quad \therefore \quad \tan\theta_1=\sin\theta_2$ or, $\tan 30^\circ=\sin\theta_2$

$$\text{or, } \sin\theta_2=\frac{1}{\sqrt{3}}=0{\cdot}5774 \quad \therefore \quad \theta_2=35^\circ 18 \text{ ; (nearly)}$$

Example 5 : *A tangent galvanometer of resistance* 10 *ohms when included in a circuit of total resistance* 100 *ohms (including the galvanometer resistance), shows a deflection of* 60°. *What shunt should be used with the galvanometer in order that the deflection may be* 30° ?

Ans. Total resistance of the circuit being 100 ohms, the current through the galvanometer $I_1=\frac{E}{100}$ where $E=$battery e.m.f. From the principle of tangent galvanometer, we get,

$$I_1=10\ k\tan\theta=10.k.\tan 60^\circ \text{ ; } \quad \therefore \quad \frac{E}{100}=10.k.\tan 60^\circ \quad .. \quad \text{(i)}$$

Let a shunt of resistance S be connected with the galvanometer. Since the galvanometer resistance is 10 ohms, the resistance of the remaining part of the circuit$=100-10=90$ ohms.

The resistance of the shunted galvanometer$=\frac{10.\ S}{10+S}$ ohm

$\therefore$ The circuit resistance$=\left(90+\frac{10.\ S}{10+S}\right)$ ohm.

Total circuit current $I=\dfrac{E}{90+\dfrac{10.S}{10+S}}=\dfrac{E(10+S)}{900+100.S}$

Hence galvanometer current $I_2=\dfrac{S}{S+G}.\ I=\dfrac{S}{S+10}\times\dfrac{E(10+S)}{900+100S}$

$$=\frac{E.S}{900+100.S}$$

From the principle of tangent galvanometer, we get

$$I_2=10.k.\ \tan\theta=10.k.\ \tan 30°$$

$$\therefore\quad \frac{E.S.}{900+100.S}=10k\tan 30° \quad ..\ (ii)$$

Dividing (ii) by (i), $\dfrac{100.S}{900+100.S}=\dfrac{\tan 30°}{\tan 60°}=\dfrac{1}{\sqrt{3}}\times\dfrac{1}{\sqrt{3}}=\frac{1}{3}$

$$\therefore\quad S=4{\cdot}5 \text{ ohms.}$$

Example 6 : *In a circuit, there are, in series a battery, a silver voltameter and a tangent galvanometer of mean radius* 24 *cm and* 30 *turns. If* 1·8 *gm of silver is deposited in* 1 *hour in the silver voltameter, calculate* (*i*) *the reduction factor of the tangent galvanometer and* (*ii*) *the horizontal intensity of earth's magnetism. Deflection of galvanometer*=45° *and E.C.E. of silver*=$1{\cdot}1182\times10^{-3}$ *gm/coulomb.*

[*Joint Entrance 1975*]

Ans. (i) We know, $W=Z.I.t$ or $I=\dfrac{W}{z.t.}=\dfrac{1{\cdot}8}{1{\cdot}1182\times10^{-3}\times60\times60}$ amp.

Again from the principle of a tangent galvanometer, we know, $I=10.K.\tan\theta$

$$\therefore\quad \frac{1{\cdot}8}{1{\cdot}1182\times10^{-3}\times60\times60}=10\times K\times\tan 45°$$

$$\text{or } K=\frac{1{\cdot}8}{1{\cdot}1182\times10^{-3}\times60\times60\times60\times10}=0{\cdot}05 \text{ C.G.S. } [\tan 45°=1]$$

(ii) The magnetic field intensity at the centre of a circular coil is given by, $F=\dfrac{2\pi ni}{10.r}$ [i is in amp.] But $F=H.\tan\theta$. or $\dfrac{2\pi ni}{10.r}=H.\tan\theta$.

$$\therefore\quad \frac{2\times3{\cdot}14\times30\times1{\cdot}8}{10\times1{\cdot}1182\times10^{-3}\times60\times60\times24}=H.\tan 45°$$

$$\text{or,}\quad H=0{\cdot}351 \text{ C.G.S.}$$

Example 7 : *A small magnet is suspended at the centre of a vertical circular coil. When the coil carries a current of* 1·25 *amperes and makes an angle of* 30° *with the magnetic meridian the suspended magnet points east-west. If the number of turns of the coil is* 10 *and its radius is* 20 *cm., find the horizontal component of the earth's magnetic field.* [*I.I.T. 1971*]

Ans. See Fig. 6·33. The horizontal component H of earth's magnetic field acts along the magnetic meridian. AB is the plane of the coil, placed at an angle of 30° with the meridian. N-S is the magnetic needle pointing east-west [for clarity, the needle is shown magnified). Now, the magnetic field F at the centre of the

coil due to its current is perpendicular to the plane of the coil. Its magnitude

$$F=\frac{2\pi ni}{10\,r}=\frac{2\pi\times10\times1{\cdot}25}{10\times20}=3{\cdot}14\times0{\cdot}125 \text{ oe}$$

Now, a restoring couple (mH, mH) and a deflecting couple (mF, mF) act on the needle as shown in the figure. At the position of rest of the needle, their moments are equal. Now, the moment of the restoring couple$=mH\times NS$ and that of the deflecting couple$=mF\times CN$.

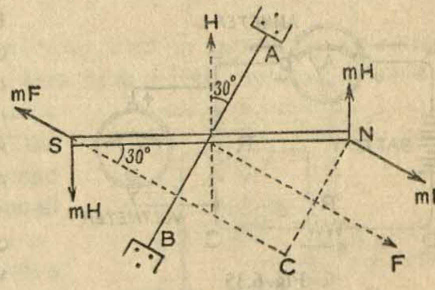


Fig. 6.33

$$\therefore\quad mH\times NS=mF\times CN$$

or, $$H=F\times\frac{CN}{NS}=3{\cdot}14\times0{\cdot}125\times\sin 30°=3{\cdot}14\times0{\cdot}125\times\tfrac{1}{2}=0{\cdot}196 \text{ Oe (nearly)}$$

Example 8 : *The coil of a D' Arsonval galvanometer has* 60 *turns, length* 2 *cm and breadth* 3 *cm. The coil is placed in a uniform radial field of* 500 *Oe. If the controlling couple due to torsion in the suspension wire be* 18 *dyne-cm, find the current flowing in the coil in milli-ampere.*

Ans. We know, $i=\frac{C.\theta}{nH.A}$ e.m.u.$=\frac{10\,C.\theta}{nHA}$ amp $=\frac{10\,C.\theta}{nHA}\times10^3$ milli-amp.

Here, $C.\theta=18$ dyne-cm ; $n=60$; $H=500$ Oe ; $A=3\times2=6$ sq cm.

$$\therefore\quad i=\frac{10\times18\times10^3}{60\times500\times6}=1 \text{ milli-amp.}$$

6.18. Ammeter and voltmeter :

Electrical instruments designed to measure an electric current in amperes are called *ammeters* and those designed to measure potential difference in volts, are called *voltmeters*. The principle upon which both of these devices operate is essentially the same as that of the suspended coil galvanometer.

A coil of fine copper wire is so mounted between two poles a permanent magnet *N-S* that its rotation is controlled by a hair spring. The farther the coil is turned from its equilibrium position or 0- mark of the scale, the greater is the controlling force. To this coil is fixed a long, light pointer at the end of which there is a scale, reading amperes if it is an ammeter, or volts if it is a voltmeter [fig. 6.34]. As the current through the moving coil of an ammeter or voltmeter increases, the resultant force acting on the coil increases which, therefore, turns the coil through a greater and greater angle until the deflecting couple is just balanced by the restoring couple brought about by the hair spring.

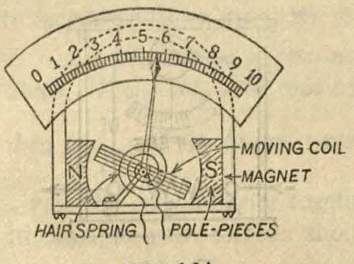


Fig. 6.34

Whenever an ammeter is connected to a circuit to measure its current, it must be connected in series so that the main current may pass through it. As

illustrated in fig. 6.35, the ammeter is so connected that all the current flows through it. To prevent a change in the current when the instrument is inserted in the circuit, *all ammeters must have a low resistance*. Most ammeters, therefore, have a low-resistance wire, called shunt, connected across the armature coil.

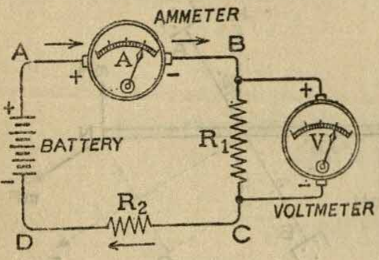


Fig. 6.35

A voltmeter, on the other hand, is connected across that part of the circuit whose potential difference is to be measured. If the p.d. between the ends of the resistor R_1 is wanted, the voltmeter is connected across the resistor as shown in fig. 6.35. If the p.d. across the resistor R_2 is to be measured, the voltmeter connections are made at C and D, whereas if the potential difference maintained by the battery is wanted, the connections are made at A and D. In order that the insertion of a voltmeter in the circuit does not change the circuit current, *the voltmeter must have a high resistance*. If the armature coil does not have a large resistance of its own, additional resistors are added in series with the coil.

Very delicate instruments are often used for measuring very small currents or potential difference. For example, a meter whose scale is calibrated to read thousandths of an ampere or a volt, is called *a milliammeter* or *a millivoltmeter*. One whose scale is calibrated in millionths of an ampere or a volt, is called a *microammeter* or *a microvoltmeter*.

Principle of ammeter and voltmeter :

A sensitive moving coil galvanometer used to measure current in amperes *i.e.* used as an ammeter, must have a parallel resistance as shunt whose value depends on two factors *viz* (i) the current it needs for full scale deflection and (ii) the maximum range of current it is required to measure.

Suppose the resistance of the galvanometer is G and the current it needs for full scale deflection is I_g. If the range of current to be measured is from 0 (zero) amp to I amp, then from the principle of shunt, $I_g=\frac{S}{S+G}\,I$, where S is the required shunt connected between the galvanometer terminals A and B [Fig. 6.36].

$$\therefore\quad \frac{S+G}{S}=\frac{I}{I_g} \quad \text{or} \quad 1+\frac{G}{S}=\frac{I}{I_g}$$

$$\text{or} \quad S=\frac{I_g \cdot G}{I-I_g}$$

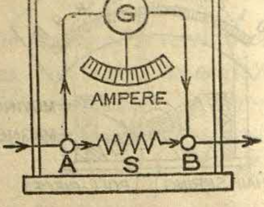


Fig. 6.36

If a shunt of above value be connected between the terminals A and B of the galvanometer, it will act as an ammeter of $0-I$ amp range. Usually the shunt has a very low resistance. Consequently, the effective resistance of the shunted galvanometer is also very low. For this reason, an ammeter when inserted in a

circuit, will not appreciably change either the circuit resistance or the circuit current, yet it will record the current.

Again, a sensitive moving coil galvanometer used to measure a potential difference in volts, *i.e.* used as a voltmeter, must have a high resistance in series with it. The value of the high resistance, in a similar way, depends on two factors *viz* (i) the current it needs for full scale deflection and (ii) the maximum range of voltage it is required to measure.

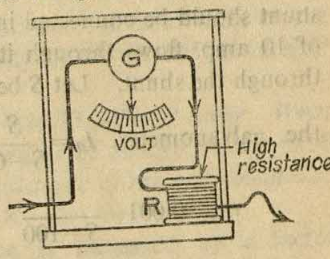


Fig. 6.37

Suppose I_g is the current which gives a full scale deflection in a galvanometer whose resistance is, say G. The high resistance R to be connected in series with it in order to convert it into a voltmeter capable of reading a maximum voltage V may be obtained in the following way [Fig. 6.37]

$$\text{We have } I_g=\frac{V}{R+G} \quad \text{or} \quad R+G=\frac{V}{I_g} \quad \therefore \quad R=\frac{V}{I_g}-G$$

If a high resistance of above value be connected in series with the galvanometer it will act as a voltmeter capable of measuring a voltage range of $0-V$ volts.

6.19. Calculation for increasing the range of ammeter and voltmeter :

To increase the range of an ammeter, a shunt, consisting of a low resistance, is to be joined across the ammeter terminals. The value of the shunt resistance required to increase a given meter range by a factor n may be found out as follows :

Let the ammeter resistance be R ohms and the required shunt resistance R_s ohms. If the full scale reading of the meter is I amp, the current which must pass through R_s when the range is nI amp is $(nI-I)$ amp. Referring to fig. 6.36, the p.d. across the metre is :

$$RI=R_s\,(nI-I) \quad \text{or} \quad R_s=\frac{R}{n-1} \quad \cdots \quad \cdots \quad \text{(i)}$$

Similarly to increase the range of a voltmeter, a high resistance is to be connected in series with the voltmeter. The value of the high resistance required to increase a given meter range by a factor n may be found out as follows :

Let the voltmeter resistance be R ohms and the required high resistance R_1 ohms. If the full scale reading of the meter is V volt and the current taken is I, then $I=\frac{V}{R}$. If the range is now made nV, then with a p.d. of nV volt applied across the galvanometer together with the high resistance R_1, the current passing through the galvanometer should remain unaltered. For that, we have,

$$I=\frac{nV}{R+R_1} \quad \therefore \quad \frac{V}{R}=\frac{nV}{R+R_1} \quad \text{or} \quad R+R_1=nR$$

$$\therefore \quad R_1=R(n-1) \quad \cdots \quad \text{(ii)}$$

Example 1 : *A galvanometer gives a full scale deflection when a current of* 0·001 *ampere passes through it. How can you convert it into an ammeter reading up to* 10 *amperes ? The resistance of the galvanometer is* 100 *ohms.*

Ans. In order to convert the galvanometer into an ammeter, a suitable shunt should be connected in parallel with the galvanometer so that 0·001 amp. out of 10 amp. flows through it, producing a full-scale deflection and the rest flows through the shunt. Let S be the required shunt. In this case, the current through the galvanometer, $I_g = \dfrac{S}{S+G} I$.

$$\text{or,}\quad 0{\cdot}001 = \frac{S}{S+100} \times 10 \quad \text{or,}\quad 1 + \frac{100}{S} = \frac{10}{{\cdot}001} = 10000$$

$$\therefore \quad S = \frac{100}{9999} = 0{\cdot}01 \text{ ohm (nearly)}$$

So, a shunt of 0·01 ohm connected in parallel with the galvanometer will convert it into an ammeter reading upto 10 amp.

Example 2 : *You are given a galvanometer having resistance* 100 *ohms. This galvanometer can measure current up to* 120 *micro-amp. How would you convert it to a voltmeter reading up to* 2·4 *volts* ?

Ans. In order to convert the galvanometer to a voltmeter, a suitable high resistance should be connected in series with it so that when 2·4 volt p.d. is applied at the ends of the combination, 120 micro-amp (*i.e.* 120×10^{-6} amp) current flows through the instrument and produces a full scale deflection. Let R be the required high resistance. Then, $120 \times 10^{-6} = \dfrac{2{\cdot}4}{100+R}$

$$\text{or}\quad 100 + R = \frac{2{\cdot}4}{120 \times 10^{-6}} = 20{,}000 \quad \therefore \quad R = 19900 \text{ ohms.}$$

So, a series resistance of 19900 ohms with the galvanometer will convert it into a voltmeter reading up to 2·4 volts.

Example 3 : *A milliammeter gives full scale deflection when* 5 *ma current passes through it. What will you do to convert it into an ammeter reading upto* 5 *amp ? Resistance of the coil*=20 *ohms.*

Ans. A suitable shunt is to be connected across the terminals of the instrument such that when 5 amp current passes through the circuit, the shunt takes up $(5-0{\cdot}005) = 4{\cdot}995$ amp and 0·005 amp *i.e.* 5 ma passes through the instrument, producing a full scale deflection. If S be the resistance of the shunt, then considering the p.d. across the coil of the instrument and that across the shunt, we can write, $S \times 4{\cdot}995 = 20 \times 0{\cdot}005$

$$\therefore \quad S = \frac{20 \times 0{\cdot}005}{4{\cdot}995} = 0{\cdot}02 \text{ ohm (nearly)}$$

So, to convert the milliammeter into an ammeter of 5 amp range, a shunt of 0·02 ohm has to be connected across it.

Now, the ratio of the current measured to the current through the coil is $\frac{i}{i_c}=\frac{5}{5\times10^{-3}}=1000$

The ratio is the same whatever the current i, because it depends only on the resistances S and R (the coil resistance) ; it may easily be shown that its value is $(S+R)/S$.

Example 4 : *You are given a millivoltmeter of* 1 *to* 30 *millivolt range. What will you do to use the instrument for measuring* (*i*) *a range of voltage from* 1 *to* 30 *volt* (*ii*) *a range of current from* 1 *to* 3 *amp ? The resistance of the instrument*=25 *ohms.*

Ans. (i) For measuring voltage, the range is increased by a factor

$$n=\frac{30}{30\times10^{-3}}=1000$$

From eqn (ii) of art. 6.19, we get $r=R(n-1)$; Here $R=25$ ohms and $n=1000$.

$$\therefore\ r=25\ (1000-1)=24975 \text{ ohms.}$$

(ii) When the instrument reads 30 millivolt, the maximum current that passes through the instrument$=\frac{30\times10^{-3}}{25}$ amp. For measuring current, the range is to be increased by a factor $n=\frac{3\times25}{30\times10^{-3}}=25{,}00$

From eqn (i) of art. 6.19, we get $R_s=\frac{R}{n-1}$; here $R=25$ ohms and $n=2500$

$$\therefore\ R_s=\frac{25}{2500-1}=\frac{25}{2499}=0{\cdot}01 \text{ ohm.}$$

Exercises

Essay type :

1. How would you demonstrate the effect of current on a magnet ? Explain the rule giving the direction of deflection of the needle.

2. How would you demonstrate that a linear conductor carrying a current produces a magnetic field ? Draw the lines of force of such a field and indicate their direction.

3. Describe an experiment illustrating the magnetic field produced by a circular coil of wire carrying a current. Draw the lines of force and indicate the direction of the field.

4. Explain with the help of neatly drawn diagrams the special features of the magnetic field produced by (i) a linear current, (ii) a circular current, (iii) a solenoidal current.

5. State Laplace's law. What is electromagnetic unit of current ? What is its relation with the practical unit of current ?

6. Describe suitable experiments to show that a current-carrying conductor when placed in a magnetic field is deflected. Explain the rule that gives the direction of deflection of the conductor.

7. Describe a few experiments to show that a magnetic field exerts a force on a current-carrying conductor. Explain the action and the principle of a Roget's vibrating coil.

8. Describe and explain the action of a Barlow's wheel. Draw a neat diagram.

9. (*a*) Describe and explain the action of a tangent galvanometer. What do you understand by 'reduction factor' and 'galvanometer constant' of a tangent galvanometer ? [*H. S. Exam. 1981*]

(*b*) Can a tangent galvanometer function at north or south magnetic pole of the earth ?

10. Describe the construction of a suspended coil galvanometer. Work out its theory. What are its advantages ? [*H. S. Exam. 1978, '83*]

11. Describe (*a*) an ammeter and (*b*) a voltmeter.

12. What are the differences between an ammeter and a voltmeter ? How are they connected in a circuit ? Draw a neat diagram.

13. Describe the principle of operation of a moving coil ammeter. How would you convert it into a voltmeter ? [*H. S. Exam. 1980*]

14. A straight current carrying conductor is perpendicular to the lines of force of a uniform magnetic field. What force is exerted on the conductor ? What is the direction of this force ? What force would act on the conductor if it is held parallel to the lines of force ?

Short answer type :

15. A small magnetic needle is pivoted on a vertical stand. If a wire, carrying a current, is placed along the axis of the needle, how will the needle place itself when the wire is stretched above the needle ?

What will be the positions of the needle in the following cases :—(i) the wire is above the needle (ii) the wire is below the needle (iii) the direction of current is reversed.

16. A copper wire is stretched over a magnetic needle in the east-west direction. How will the needle behave if the positive terminal of a battery is connected to the west end of the wire and the negative terminal to the east end ?

17. A conductor, even though it is carrying a current, has zero net charge. Why, then does a magnetic field exert a force on it ?

18. A coil of wire, carrying a current, is placed on a piece of cork and floated on water. A uniform magnetic field is applied on it. How will the coil set itself in the magnetic field ? How will it behave if a magnet is brought near it ?

19. The terminals of a battery are joined by a long wire which is folded into two halves lying very close to each other. A magnetic needle is placed near the wire. Will there be any deflection in the needle ? If the wire were bent into a loop around the magnetic needle, will the needle be deflected ?

20. A feeble current is flowing in a straight wire which is not capable of producing any deflection in a magnetic needle placed near the wire. What can you do to produce a deflection by the same wire carrying the same current ?

21. What effect will be produced on the rotation of a Barlow's wheel in the following cases :—(i) the magnetic field is reversed (ii) the current is reversed (iii) both the magnetic field and the current are reversed simultaneously ?

22. What effect will be produced on the rotation of a Barlow's wheel if (i) the current is increased (ii) the magnetic field is increased (iii) the magnetic field is removed.

23. What is the magnetic moment of a coil ? Upon what factors does it depend ?

24. What will happen in the case of a tangent galvanometer if (i) the plane of the coil does not lie in the magnetic meridian (ii) the plane of the coil is not vertical (iii) the magnetic needle is not small.

25. Equal currents are flowing in two very long parallel wires. Answer the following :—
(*a*) What will be the direction of magnetic field at a point on any wire due to the current in the other ? (*b*) What is the force on any wire due to the current in the other ? (*c*) How will the

force alter if the current in each wire is doubled ? (*d*) What is the direction of the magnetic field at a point equidistant from the two wires ? [*I.I.T. 1973*]

[**Hints** : (*a*) The direction of the magnetic field at a point on any wire is along the tangent to the radial distance from the point to the other wire, (b) As the currents are in the same direction, the force will be attractive, (*c*) The force of attraction is proportional to the product of the currents *i.e.* $F \propto i_1 i_2$. Hence when currents are doubled, the force is quadrupled, (*d*) At a point equidistant from the wires, the magnetic fields are equal and opposite. Hence, there will be no resultant field at that point].

26. A rectangular loop carrying a current i is situated near a long straight wire such that the wire is parallel to one of the sides of the loop and is in the plane of the loop. If a steady current I is established in the wire as shown in fig. 6.38, how will the loop move ?

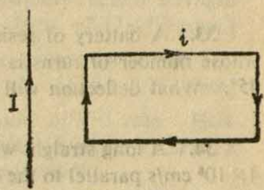


Fig. 6.38

27. A thin and flexible wire is kept on a table in the form of a rectangle and a strong current is sent through it. How will be the shape of the wire now ?

[**Hints** : Each two parallel sides of the rectangle carry parallel and opposite currents. They repel each other. Consequently, the wire takes up a circular shape.]

28. Equal currents i are flowing through two infinitely long parallel wires. Will there be a magnetic field at a point exactly half way between the wires when the currents in them are (i) in the same direction (ii) in the opposite direction ? [*I.I.T. 1978*]

29. Through mistake, an ammeter has been connected in parallel and a voltmeter in series in a circuit. What will happen to the instruments ?

[**Hints** : The instruments will be damaged. Voltmeter is a high resistance instrument. When connected in series, it will produce sufficient heat ($\propto i^2rt$) due to high resistance which will burn the coil. On the other hand, ammeter is a low resistance instrument. When connected in parallel, most of the circuit current will flow through the instrument and the heat that it will produce, will be sufficient to burn out the coil of the instrument.]

Objective type :

30. (i) When a current flows through a circular wire, the magnetic field it produces at the centre of the coil is (*a*) non-uniform (*b*) uniform (*c*) zero. Which is correct ?

(ii) A conducting circular loop of radius r carries a constant current i. It is placed in a uniform magnetic field $\vec{B}_0$ such that $\vec{B}_0$ is perpendicular to the plane of the loop. The magnetic force acting on the loop is (a) irB_0 (b) $2\pi irB_0$ (*c*) zero (*d*) πirB_0. Which is correct ? [*I.I.T. 1983*]

(iii) The wires in a horse-shoe type electromagnet are wound (*a*) in the same direction (*b*) in the opposite direction in the two arms of the magnet. Which is correct ?

(iv) Two parallel currents flowing in the same direction (*a*) repel each other (*b*) attract each other (*c*) exert no force on each other. Which is correct ?

(v) When a tangent galvanometer shows a deflection of (*a*) 40° (*b*) 45° (*c*) 50°, the reading is accurate. Which is correct ?

(vi) The principle of a suspended coil galvanometer is based on (*a*) the action of a magnet on current (*b*) the action of a current on magnet (*c*) the action of a current on current. Which is true ?

(vii) In a suspended coil galvanometer, it is advantageous to keep a soft-iron piece within the coil in the form of (*a*) a block (*b*) a rectangle (*c*) a cylinder. Which is correct ?

(viii) Two very close and parallel currents flowing in opposite directions produce (*a*) a very strong magnetic field (*b*) a very feeble magnetic field (*c*) no magnetic field at a point equidistant from the wires. Which is correct ?

Numerical Problems :

31. The coil of a tangent galvanometer has number of turns 7 and is of 11 cm radius. If the value of H be 0·2 Oe what current, in amperes, will produce a deflection of 45° in the galvanometer ? [Ans. 0·5 amp.]

32. A circular coil of wire of radius 20 cm and of 20 turns is kept vertical in the magnetic meridian. When current flows through the coil, a small magnet placed at the centre of the coil is deflected through 45°. Find the magnitude of the current. $H=0·38$ Oe.
[*I.I.T. 1974*] [Ans. 0·6 amp. (nearly)]

33. A battery of resistance 10 ohms, is connected in series with a tangent galvanometer whose number of turns is 100 and resistance 40 ohms. The deflection of the galvanometer is 45°. What deflection will be produced if 50 turns of the coil are connected to the battery ?
[Ans. 39°48]

34. A long straight wire carries a current of 2 amp. An electron travels with a velocity of 4×10^6 cm/s parallel to the wire 10 cm. from it and in a direction opposite to the current. What force does the magnetic field of current exert on the electron ? [Ans. $2·56 \times 10^{-15}$ dyne]

35. A current is passed for half an hour through a tangent galvanometer and a copper voltameter connected in series. 0·4 gm of copper is deposited in the copper voltameter. If the deflection in the tangent galvanometer be 30°, find the reduction factor of the galvanometer. $Z_{cu}=3 \times 10^{-5}$ gm/coulomb. [*Jt. Entrance 1974*] [Ans. 0·116]

36. A current is passed through two tangent galvanometers joined in series. The radius of the first coil is 3 times that of the second but the number of turns in both of them is equal. If the deflection recorded in the second galvanometer be 60°, what is it in the first one ?
[*Jt. Entrance 1970*] [Ans. 30°]

37. Two tangent galvanometers A and B, are identical in construction, except for the number of turns in the coil. They are connected in series and a current is sent through them. The deflection in A is 45° and that in B is 31°. Calculate the ratio of the number of turns in the two instruments. Tan 31°=0·60. [Ans. 5/3]

38. The radius of the circular coil of a tangent galvanometer is 10 cm. How many turns of the coil are required to produce a deflection of 45° when a current of 0·01 amp passes through the coil ? $H=0·18$ c.g.s. [Ans. 286]

39. An electric circuit contains a battery, a silver voltameter and a tangent galvanometer of 30 turns of wire of 2·4 cm. mean radius, all connected in series. If 1·8 gm of silver is deposited in an hour, find (i) the reduction factor of the tangent galvanometer and (ii) horizontal component of earth's magnetic field when the deflection of the galvanometer is 45° E.C.E. of silver=0·0011182 gm/coulomb. [*Joint Entrance 1975*] [Ans (i) 0·0447 c.g.s (ii) 4·352 c.g.s.]

40. (i) A galvanometer of resistance 750 ohms can measure current up to ·005 amp. What should you do to measure current up to 3 amp with it ? [Ans. 1·25 ohm shunt]

41. The range of a milli-ammeter is up to 0·15 milli-amp. It has a resistance of 5 ohms. What should be done to convert it to a voltmeter reading up to 0·75 volt ?
[Ans. 4995 ohms in series]

Harder Problems :

42. A 100 ampere current flows in opposite directions in two parallel wires, each of length 30 metres and separated by a distance of 20 cm. Calculate the force between them.
[Ans. 6×10^6 dynes]

43. A straight solenoid of length 75 cm. consists of 600 turns in which a current of 0·2 ampere flows. What is the magnetic flux through the solenoid, if its radius is 1·8 cm ?
[Ans. 20·5 c.g.s.]

44. A current i flows in a circular wire of radius a. A circular coil of radius $2a$ coplanar and concentric with the wire is added. It has 8 turns and the same current i flows in it in the

opposite sense. Show that the magnetic field at the centre is numerically three times as strong as it was before.

45. A current of 1 ampere flows round a wire bent into a circle of 20 cm in diameter. An equal current flows round a circle of 2 cm in diameter suspended at the centre of the larger coil. What couple is required to hold the small circuit with its plane at right angles to that of the large one ? [Ans. 0·0197 c.g.s.]

46. A circular coil of 18 turns of radius 12 cm carries a current of 3·5 amp. Calculate the value of the couple required to maintain it with its plane parallel to a magnetic field of strength 25 dynes/unit pole. [Ans. $7{\cdot}12\times10^4$ c.g.s.]

47. Two long straight parallel wires carry currents of 10 and 15 amperes respectively and are situated 12 cm apart. Find the force on 5 cm. length of each wire. [Ans. 1·25 dynes]

48. A circular coil of 100 turns and radius 5 cm. carries a current of 0·1 amp. How much work is required to turn it in an external magnetic field of strength 1·5 weber/m^2 from a position in which θ [refer to art 6.11] equals zero to one in which θ equals 180°?

[Ans. 0·24 joule]

[**Hints** : Work done$=\int_0^\pi nHiA \sin\theta.d\theta$]

49. Two coils each of 100 turns are held such that one lies in the vertical plane and other in the horizontal plane with their centres coinciding. The radius of the vertical coil is 20 cm and that of the horizontal coil is 30 cm. How would you neutralise the magnetic field of the earth at their common centre? What is the current to be passed through each coil ? H=0·349 Oe and dip angle=30°. [*I.I.T. 1968*] [Ans. 0·111 amp ; 0·097 amp]

50. A cell is connected in series with a tangent galvanometer of resistance 2 ohms and a resistance box. When a resistance of 8 ohms is inserted in the box, the galvanometer deflection is 60°. When the resistance in the box is 30 ohms, the deflection is reduced to 30°. Find the internal resistance of the cell [*Jt. Entrance 1979*] [Ans. 1 ohm]

51. A battery is connected with a tangent galvanometer and a resistance in series and the deflection of the galvanometer is 60°. On placing a resistance in parallel with the galvanometer, the deflection is reduced to 30°. If the resistance placed in parallel is $\frac{1}{8}$ th of the galvanometer resistance, calculate the ratio of the series resistance and the galvanometer resistance.

[*Jt. Entrance 1976*] [Ans. $R : G=2 : 7$]

52. A circular coil of 10 turns and radius 8 cm is placed with its plane at right angles to the magnetic meridian. If a suspended magnet at its centre makes 18 vibrations per minute with current in one direction and 30 vibrations per minute when the direction of current is reversed, what is the strength of the current, given that the field due to the coil is greater than H ? (Take $H=0{\cdot}18$) [Ans. 0·049 *e.m.u.*]

53. The coil of a tangent galvanometer has a radius of 15 cm and contains 50 turns. Assuming that it is only used to measure currents which give a deflection less than 60° and greater than 1°, determine the range of current in amperes for which it is available.

[Ans. 0·001504 to 0·1489 amp.]

54. A current flowing through a tangent galvanometer consisting of 10 turns of wire of radius 8 cm produces a deflection of 45° when the instrument is in a position where $H=0{\cdot}18$ Oe. What alterations would you make in the instrument so that it would give the same deflection for a current of $\frac{1}{1000}$ th of an ampere ? [Ans. Increase the number of turns to 2292]

55. The resistance of the coil of a moving coil galvanometer is 99 ohms and a current of 10 milli-amp produces full scale deflection of its pointer. How can the galvanometer be converted into instruments suitable for measuring (i) currents upto 1 amp and (ii) potential difference upto 100 volts ? [*H. S. Exam. 1984*] [Ans. 1 ohm shunt (ii) 9901 ohm in series]

56. A galvanometer of resistance 120 ohm shows full scale deflection at 5×10^{-4} ampere of current. What shunt should be used in parallel with it so that it may read a maximum current of 5 amp. What is the present resistance of the instrument ? [Ans. 0·012 ohm ; 0·012 ohm]

57. A shunted galvanometer is connected in series with a resistance box and a cell of negligible resistance. The resistance put in the box is R_1. After noting the deflection of the galvanometer the shunt is removed. It is seen that the resistance in the box has to be increased to R_2 in order to keep the deflection of the galvanometer unaltered. Prove that the resistance of the galvanometer is $\frac{S}{R_1}(R_2-R_1)$ where S is the shunt resistance connected with the galvanometer. [*I.I.T. 1968*]

58. A galvanometer having a coil of resistance of 100 ohms gives a full scale deflection when a current of 1 milliamp is passed through it. What is the value of the resistance which can convert this galvanometer into a meter giving a full scale deflection for a current of 10 amperes ?

When this modified galvanometer is connected across the terminals of a battery, it shows a current of 4 amp. The current drops to 1 amp when a resistance of 1·5 ohm is connected in series with the modified galvanometer. Find the e.m.f. and the internal resistance of the battery.

[*I.I.T. 1972*] [Ans. 0·001 ohm ; 0·49 ohm ; 2 volt]

59. A galvanometer having a resistance of 40 ohms gives a deflection of one scale division for a current of $\frac{1}{1001}$th ampere. Find the magnitude of resistance and show how it must be connected to change the galvanometer into (i) an ammeter reading 1 ampere per scale division and (ii) a voltmeter reading 1 volt per scale division.

[Ans. (ii) 0·04 ohm in parallel (ii) 961 ohms in series]

60. The coil of a moving coil galvanometer has a resistance of 50 ohms and gives a full scale deflection when a current of 10 milli-amp passes through it. How can you convert the instrument into (i) an ammeter reading upto 10 amp (ii) a voltmeter of maximum range 100 volts ? [*Jt. Entrance 1984*] [Ans. (i) 0·05005 ohm shunt (ii) 9950 ohms series]

7

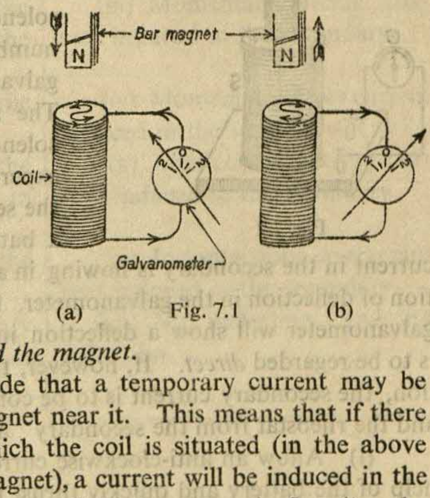
ELECTRO-MAGNETIC INDUCTION

7.1. Introduction :

Following Oersted's discovery in 1820 that magnetism could be produced by electricity, many scientists looked for the reverse effect—the possible production of electric current by means of magnetism. This was first accomplished by Michael Faraday in 1831. He found that a transient e.m.f. could be produced in a closed circuit by a magnet or by another current-bearing circuit. The transient e.m.f. is called induced e.m.f. and the phenomenon is known as *electro-magnetic induction.* Faraday's discovery is the basis of the electric generator, the transformer and other devices which for the first time made the commercial development of electricity possible.

7.2. Experiments to demonstrate electro-magnetic induction :

(A) **Current induced by a magnet :** Wind 150 or 200 turns of fine copper wire on a cardboard cylinder of radius about one inch and length about a few inches. Keeping the cylinder vertical, connect a sensitive galvanometer to the free ends of the coil. Now insert the N-pole of a bar magnet quickly into the cylinder. The galvanometer needle will show a sudden deflection [Fig. 7.1(a)]. If the magnet be now quickly pulled out, again a brief deflection will be obtained in the galvanometer but in the opposite direction [Fig. 7.1(b)]. As long as the magnet is held still in any position, no deflection in the galvanometer will be seen. Deflection is produced only when *there is a relative motion between the coil and the magnet.*



(a) Fig. 7.1 (b)

From this experiment we can conclude that a temporary current may be induced in a closed circuit by moving a magnet near it. This means that if there be any change in the magnetic field in which the coil is situated (in the above experiment, the magnetic field of the bar-magnet), a current will be induced in the coil. In other words, current is induced in a coil if the magnetic flux (*i.e.* the total number of lines of force) linked with coil changes.

We know that when a current flows through a solenoid, it behaves like a bar-magnet having two poles at its two ends. In the experiments described above, if the direction of current flowing in the solenoid during insertion and withdrawal of the magnet, be observed carefully, it will be seen that the solenoid has developed polarities at its ends. When N-pole is introduced into the solenoid, the upper end of the solenoid develops a N-polarity and when N-pole is pulled out, a south pole is developed at the upper end. If instead of N-pole, S-pole of the bar-magnet is introduced, south pole will be developed on the upper

end during insertion and a north pole during withdrawal. From these facts, Lenz had established a law, known as **Lenz's law** which states that *in all cases of electro-magnetic induction, the direction of the induced current is such as to oppose the very cause to which it is due.*

The magnitude of the induced current is found to increase with the strength of the magnet, with its speed of motion and with the number of turns of wire on the coil. All these cause an increase in the magnetic flux linked with the coil. Hence Faraday was able to establish the fact that the inductive effect depends upon the *rate of change* of magnetic flux linked with the coil. His law in this connection, known as **Faraday's law** states that *the induced e.m.f. in a circuit is proportional to the rate at which the magnetic flux linked with the circuit changes.*

(B) **Current induced by current** : Since a solenoid carrying a current behaves like a bar magnet, its lines of force will change if the current changes. If change of flux be applied on a nearby closed circuit, then according to Faraday's law of electro-magnetic induction, an e.m.f. will be induced in the circuit. It can be demonstrated by the arrangement shown in fig. 7.2.

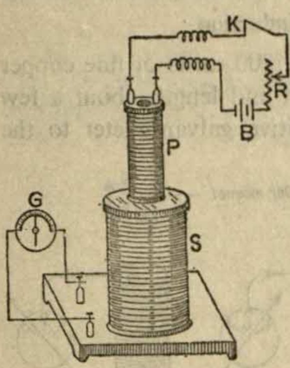


Fig. 7.2

P is a solenoid connected to a battery B, a rheostat R and a tap key K. S is another solenoid larger than P and it contains a large number of turns of insulated wire. A sensitive galvanometer G is connected to this solenoid. The solenoid P is called *the primary* and the solenoid S is called *the secondary*. Before the experiment proper, the direction of current in the secondary is to be ascertained by connecting a battery and a rheostat with it. Suppose, the current in the secondary is flowing in an anticlockwise direction. Note the direction of deflection in the galvanometer. In the subsequent experiments, whenever the galvanometer will show a deflection in this direction, the current in the secondary is to be regarded *direct*. If, however, the galvanometer shows an opposite deflection, the secondary current is to be considered *inverse*. Now, remove the battery and the rheostat from the secondary and perform the following experiments :

(i) Allow an anti-clockwise current to flow through the primary P with the help of the battery and quickly thrust the primary into the secondary. A sudden deflection in the galvanometer will be seen and the direction of deflection suggests that a brief *inverse current* has been induced in the secondary S. If the primary be now jerked away, there is again a brief current induced in the secondary, but the galvanometer deflection indicates that it is a *direct current*.

(ii) Insert the primary P into the secondary S, with the key K off *i.e.* with no current flowing in the primary. Now start primary current by pressing the key K. At once the galvanometer shows a sudden deflection indicating that a transient *inverse current* is induced in the secondary. If the primary current be increased suddenly by the rheostat, the same effect as before will be found in the secondary.

(iii) Now stop the primary current by suddenly releasing the tap key K. Again a brief current is induced in the secondary but it is a *direct current*. Same thing will be found if the primary current be suddenly decreased with the help of the rheostat.

To remember the above results, they may be tabulated as follows :

Experiment with	*Procedure*	*Observation and inference*
Magnet	(i) Suddenly inserting the N-pole. (ii) Suddenly taking out the N-pole, (iii) Keeping the magnet still,	(i) Momentary *inverse* current induced. (ii) Momentary *direct* current induced. (iii) No deflection and no induction.
Solenoid	(i) Suddenly inserting the primary in the secondary, (ii) Suddenly jerking out the primary from the secondary, (iii) Suddenly starting the primary current keeping the primary in the secondary, (iv) Suddenly breaking the primary current, (v) Suddenly increasing the primary current keeping it in the secondary, (vi) Suddenly decreasing the primary current, (vii) Keeping the primary current on while the primary is in the secondary,	(i) Momentary *inverse* current induced in the secondary. (ii) Momentary *direct* current induced in the secondary. (iii) Momentary *inverse* current induced in the secondary. (iv) Momentary *direct* current induced in the secondary. (v) Momentary *inverse* current induced in the secondary. (vi) Instantaneous *direct* current induced in the secondary. (vii) No deflection and no induction.

7.3. Magnetic flux and flux density :

Consider a closed surface of area A in a medium of permeability μ. Suppose that a uniform magnetic field of intensity H is acting perpendicular to the plane of the surface [Fig. 7.3]. Since the permeability of the medium is μ, total intensity of the field $=\mu H$. From the definition of the magnetic field we may say that the number of lines of force passing normally through unit area $=\mu H$. The number of lines of force referred to above is known as *magnetic flux density* and is usually denoted by the letter B. So, the magnetic flux density in the medium $B=\mu H$. It is needless to mention that B is a vector quantity and has a specified direction.

If ϕ be the total member of lines of force crossing the area A, then $\phi = A.B$. Here, ϕ is referred to as *magnetic flux* over the area A.

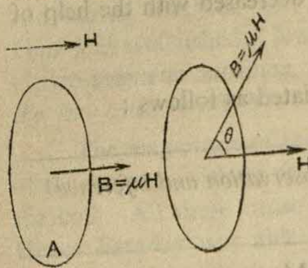


Fig. 7.3

If, however, the flux-density B is not perpendicular to the surface A, then the flux linked with the surface $\phi = B.A.\cos\theta$, where θ is the angle between the flux density B and the normal to the surface A. [Fig. 7.3(*b*)].

According to c.g.s. system, the unit for magnetic flux-density B is Oersted/cm^2 and that for magnetic flux is Oersted.

According to M.K.S. system (or S.I. system), the unit of B is Weber/metre^2 (or Tesla) and that of magnetic flux is Weber.

Positive and negative flux :

A normal can be drawn to a plane in two different ways. When the normal points in the direction of B (*i.e.* $\theta = 0$) the flux linked with the surface is taken as positive. When the normal points in the opposite direction (*i.e.* $\theta = 180°$) the flux$= BA.\cos 180° = -B.A$ and then the flux linked with the surface is taken as negative.

7.4. Explanation and the establishment of the laws of electro-magnetic induction :

The experiments on electromagnetic induction with a bar-magnet and a current-carrying coil as described in art. 7.2 may be explained with the help of magnetic flux and the laws of electro-magnetic induction, known as *Faraday's laws* may be established therefrom.

It is already known that a bar-magnet produces lines of force and a current-carrying coil behaves like a bar-magnet *i.e.* it also produces its own lines of force. When the north pole (say) of a bar magnet or a current-carrying closed coil of wire moves towards another closed coil, the lines of face of the former intersect the latter. As the N-pole or the current-bearing coil approaches the second coil, more and more lines of ferce get linked up with the second coil *i.e.* the magnetic flux linkage with the second coil increases gradually. Experiments show that under this circumstances, a current is induced in the second closed coil. On the other hand if the N-pole of the bar magnet or the current-carrying coil be removed further away from the second coil, the magnetic flux linkage with the second coil gradually decreases. It is found that under this circumstances also, a current is induced in the second coil. If, however, the pole or the current-carrying coil be kept at rest inside the second coil, there is no change (either increase or decrease) in the flux linkage with the second coil. Experiment shows that no current is induced in the second coil under this circumstances. From this Faraday came to the conclusion that the reason of induction of current in the second closed coil is the change of magnetic flux linkage with it and formulated the first law of electro-magnetic induction in the following way :

1st law : *An e.m.f. is induced in a closed circuit and a current flows in it whenever there is a change in the magnetic flux linked with it.*

If the bar-magnet or the current-carrying coil mentioned above be quickly moved towards or taken away from the second closed coil, the induced current in the coil becomes stronger. On the other hand, if the movement be slow, the induced current becomes feeble. Now, a quick rate of approach of the bar magnet or the current-carrying coil towards the second coil means an increase in the rate of change of flux linked with the second coil. Similarly, a slow rate of approach means a decrease in the rate of change of flux linked with the second coil. From this experimental result, Faraday formulated the second law of electromagnetic induction which is as follows :

2nd law : *The magnitude of induced e.m.f. in a closed circuit is proportional to the rate of change of magnetic flux linked with the circuit.*

If the flux linked with each turn of a coil be ϕ_1 and ϕ_2 respectively before and after an interval of time t, then the e.m.f. induced in eaeh turn is $e \propto \frac{\phi_2-\phi_1}{t} \propto \frac{\phi}{t}$ where ϕ is the flux change in time t. If there be n number of turns in the coil, the total e.m.f. induced in the coil $e \propto \frac{n\phi}{t}$. According to calculus notation, we can write $e \propto n.\frac{d\phi}{dt}$ .. (i)

It is to be noted that the two laws formulated by Faraday indicate (i) *the cause* (ii) *the duration* and (iii) *the magnitude* of the electromagnetic induction in a closed circuit.

Let us now see how the third law of electro-magnetic induction was established by Lenz. We know that when current flows through a solenoid, magnetic poles are developed at the two ends of the solenoid which, then, behaves like a bar-magnet. In the experiment described in art 7.2, when the bar magnet is introduced into or taken out from the closed coil, a current in induced in the coil, which develops polarity on the upper face of the coil. [ref. to the rule regarding solenoid in art 6.4(iii)]. The rule indicates that *N*-pole is created on the upper face due to the current flowing in it when *N*-pole of the bar magnet is inserted into the coil and *S*-pole is created when the bar-magnet is pulled out. If instead of *N*-pole, the south pole of the bar magnet is used, then *S*-pole will be created at the time of introduction and *N*-pole at the time of removal of the bar magnet. It is to be noted that the induced current in the closed coil develops such a polarity on the upper face of the coil that it always opposes the movement of the bar magnet *i.e.* it opposes the approach as well as the removal of the bar magnet.

Some explanation may be adduced in the case of a current-carrying coil. We know that like parallel forces attract each other and unlike parallel forces repel each other. Now, if a current-carrying closed coil be brought near another closed coil, a current is found to be induced in the second coil, the direction of which being opposite to the direction of inducing current. Again if the first coil is removed away from the second coil, a current is also induced in the second coil,

the direction of the induced current now being same as that of the inducing current. In both the case, the direction of the induced current is such that it opposes the approach or removal of the current-bearing coil. From this fact, Lenz proposed the third law of electro-magnetic induction, popularly known as *Lenz's Law*, which is as follows :

3rd Law : *In all cases of electro-magnetic induction, the direction of the induced current (or e.m.f.) is always such as to oppose the very cause which is responsible for inducing the current (or e.m.f.) in the circuit.*

Considering the third law, the equation (i) may be written as $e \propto -n.\frac{d\phi}{dt}$

[Negative sign is due to Lenz's law] or $e=-k.n.\frac{d\phi}{dt}$ where k is a constant.

Selecting the units of different quantities conveniently, k may be made equal to 1. In that case

$$e=-n.\frac{d\phi}{dt} \quad .. \quad \text{(ii)}$$

The above equation represents mathematically all the three laws of electro-magnetic induction.

7.5. Lenz's law and the law of conservation of energy :

In describing the experiments on electro-magnetic induction, it has been pointed out that there is a definite relation between the direction of the induced current and the direction of the action that causes it. This relation is formulated in the law, known as Lenz's law. This law is an application of the well-known law of conservation of energy. It can be proved in the following way :

Consider the experiment of fig. 7.4. If the magnet is pushed towards the coil, the flux through the coil increases and a current is induced in it. This current makes the coil magnetic. Suppose the direction of the induced current is such that the front face of the coil gets a N-pole. Then the approach of the bar-magnet will be helped by the attraction between the N-pole of the coil and S-pole of the bar. It would then be unnecessary to continue to push the bar-magnet into the coil ; the motion would reinforce itself and unlimited amount of mechanical as well as electrical energy would be produced without expending any further energy. According to the principle of conservation of energy, this cannot happen. So the front face of the coil must, instead, become an S-pole, whose repulsion would oppose the movement of the bar magnet. Similarly when the bar magnet is taken away from the coil, the induced current must be in the opposite direction, making the front face of the coil a N-pole whose attraction opposes the removal of the bar. This direction of induced current is given by Lenz's law. Hence we see that the law is simply an application of the law of conservation of energy.

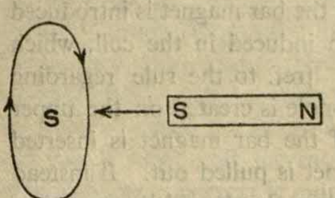


Fig. 7.4

Further, the phenomena of electro-magnetic induction show that a current may be set up in a closed circuit without the help of any battery. Apparently we find no source of energy to set up this current and the phenomena seem to be, at the first sight, contrary to the principle of conservation of energy. As a matter of fact, it is not so. Lenz's law shows that whether a magnet is made to approach to or recede from a closed conducting coil, an extra mechanical energy is to be spent against the force of repulsion at the time of approach of the magnet and against the force of attraction at the time of removal of the magnet. The extra mechanical energy spent is responsible for the corresponding electrical energy involved in setting up the induced current in the circuit.

Lenz's law from the law of conservation of energy :

It can be shown in the following way that Lenz's law follows from the principle of conservation of energy.

Suppose the coil (Fig. 7.4) in question is made to approach the *N*-pole of the magnet and its motion instead of being opposed, is facilitated. It would then follow that when the coil is just put in motion in a magnetic field it would continually move with increasing velocity which is contrary to experience and against principle of conservation of energy. What actually should happen is the prevention of approach of the coil and that accomplished by setting up *N*-polarity at the front face of the coil as demanded by Lenz's law.

Similarly when the coil is made to recede from the *N*-pole of the magnet, if its motion is somehow facilitated then it would go on moving with gradually increasing velocity away from the magnet which is also contrary to experience. In other words, its motion is prevented by setting up *S*-pole at the end of the coil facing *N*-pole of the magnet as demanded by Lenz's law.

Thus, we see that Lenz's law follows from the principle of conservation of energy.

7.6. Self induction :

When a current is started in a coil of wire, it produces a magnetic field, the lines of force of which become suddenly linked up with the coil *itself*, which gives rise to an induced e.m.f. opposing the growth of current in the coil. Similarly when the current in the coil is broken, the lines of force linked with the coil suddenly disappear which gives rise to an induced e.m.f. opposing the decay of current in the coil.

Further, a sudden change (either an increase or a decrease) in the current flowing in the coil also brings about a change in the number of magnetic lines of force linked with the coil and hence an induced e.m.f. in the coil which always opposes such change of current.

This effect in the coil itself is called **self-induction.** The coil, therefore, possesses a property called **self-inductance.**

Self-inductance, often called inductance, is a constant of the coil. It depends upon (i) the number of turns (ii) area of cross-section and (iii) permeability of the core material. The larger the number of turns, area of cross-section of a coil, the larger is its inductance. If the coil is wound over an iron core, the self-inductance

increases by a factor μ where μ is the permeability of iron. A coil having an appreciable self-inductance, is called an *inductor*.

Experiment to demonstrate self-induction of a circuit :

Prepare a solenoid L by winding a thick copper wire into several turns. It is connected in parallel with an electric lamp of 12 volts. A 12-volt battery B, a switch S and a resistor R are connected with the solenoid as shown in fig. 7.5.

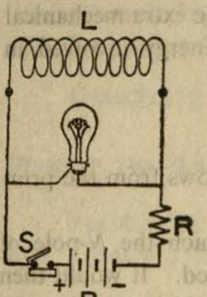


Fig. 7.5

When the switch S is closed, the lamp glows brightly for an instant and then becomes dim. When the switch is opened the lamp again glows brightly for a moment and then goes out. When the switch is closed, the back e.m.f. in the solenoid L, prevents the current from growing up rapidly through the solenoid. The solenoid therefore acts as though it had a very high resistance, so that practically all of the current passes through the lamp. When the current becomes steady and this happens in a very short time, there is no back e.m.f. in the solenoid L and a part of the current flows through the lamp and a part through the solenoid. When the switch is opened, the magnetic lines of force vanish, inducing a current in the solenoid. This current flowing through the lamp, causes it to light up momentarily to full brightness.

7.7. Co-efficient self-induction and its units :

Suppose, the current flowing in a coil at any instant is i and the total number of lines of force linked with the coil$=N$. In this case,

$$N \propto i \quad \text{or} \quad N=L.i \quad [L=\text{a constant}]$$

Here, L is a constant, known as the *co-efficient of self induction.* If $i=1$, $N=L$. So we can say that the flux linkage with a coil when unit current flows through it is numerically equal to its co-efficient of self-induction. From the above equation we get,

The rate of change of flux$=L\times$ rate of change of current.

or $\frac{dN}{dt}=L\frac{di}{dt}$ or $e=\frac{dN}{dt}=L\frac{di}{dt}$; Now when $\frac{di}{dt}=1$, $L=e$. This gives an alternative definition of the coefficient of self-induction.

The coefficient of self-induction of a circuit is defined to be numerically equal to the e.m.f. induced in the circuit due to unit rate of change of current in it.

Units :

(i) **Absolute unit** : It is the inductance of a coil (or a circuit) in which 1 e.m.u. of e.m.f. is set up by change of current at 1 e.m.u. per sec. It is called 1 e.m.u. of inductance.

(ii) **Practical unit** : It is the inductance of a coil (or a circuit) in which e.m.f. of 1 volt is set up by the change of current at 1 ampere per second. It is called henry.

1 henry$=10^9$ e.m.u. of inductance.

Example : *What is the e.m.f. induced in an inductor of* 10 *henry when the current through the inductor changes from* 10 *amperes to* 7 *amperes in* 0·9 *second ?*

Ans. : We know, the induced e.m.f. $e=-L\frac{di}{dt}=-L.\ \frac{(i_2-i_1)}{t}$

Hence, $L=10$ henry ; $i_2=7$ amp ; $i_1=10$ amp and $t=0.9$ sec.

$$\therefore \quad e=-\frac{10\times(7-10)}{0\cdot9}\ \text{volt}=33\cdot3\ \text{volts.}$$

7.8. Non-inductive coils :

The self-inductance of a coil depends upon its area of cross-section, shape, number of turns and manner of winding. In fact, every conductor, straight or bent, possesses self-inductance. A straight conductor possesses far less inductance than when it is wound in the form of a solenoid. Now, in the construction of resistance boxes, resistance coils, P.O. Boxes etc. coils of wire are used, which, obviously possess high self-inductance, which is undesirable in many experiments. To minimise the effect of self-inductance in such resistance coils, the coils are wound so as to set up extremely small magnetic fields. In this case the wire is doubly wound on itself before being coiled up. Such a coil is said to be non-inductive. Each turn is in close contact with a similar turn carrying current in the opposite direction. The magnetic effect of one is completely neutralised by the other (see art 6.5). Thus the resultant magnetic flux and the self-inductance are negligibly small. In the case of a post office box, or resistance box, all the resistance coils are non-inductively wound on bobbins (Fig. 7.6).

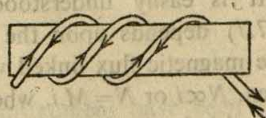


Fig. 7.6

7.9. Mutual induction :

Suppose we have two coils P and S, each of several turns of wire, placed near each other. A battery B and a key K are joined in series with the coil P and a sensitive galvanometer G with the coil S (Fig. 7.7). If current be started in the coil P by pressing the key K, there will be a momentary deflection in the galvanometer G connected with coil S. When the current in the coil P becomes steady, the needle of the galvanometer comes back to its zero position. The reason is as follows :

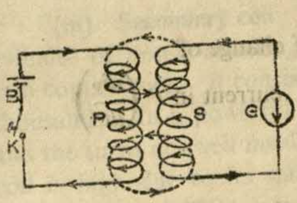


Fig. 7.7

When the current in the coil P is started, the lines of force of the magnetic field produced by it get linked up with the coil S (shown by dotted lines). This change of flux sets up an induced e.m.f. and hence a current in the coil S. It is to be noted that this momentary current in the coil S is opposite to the current in the coil P. If now the current in the coil P be stopped by releasing the key K there will again be a sudden deflection in the galvanometer G but in the opposite direction. The reason is that with the stoppage of current in the coil P, the

magnetic flux linked with the coil S disappears and this again sets up an e.m.f. in the coil S in the opposite direction.

Not only at the make and break of the current in the coil P, there will be a deflection in the galvanometer of the coil S whenever there is a change in the current (either an increase or a decrease in it). Same result will be obtained if the battery is connected to the coil S and the galvanometer to the coil P. In other words, the effect is really mutual.

This effect of setting up a momentary current in a coil by changing the current in another adjoining coil is called *mutual induction*.

The mutual inductance depends upon (i) the number of turns in the coils P and S (ii) their geometrical shape and (iii) their separation. It is maximum when the primary P and the secondary S are parallel to each other because the entire flux of the primary P then links with the secondary S. It is minimum when the coil P is perpendicular to the coil S. The mutual inductance is further increased if the primary coil is wound over an iron core, by a factor μ, where μ is the permeability of iron.

7.10. Co-efficient of mutual induction and its units :

It is easily understood that the magnetic flux linked with the coil S (Fig. 7.7) depends upon the current flowing in the coil P. If the lines of force *i.e.* the magnetic flux linked with the coil S be N when the current in the coil P is i then, $N \propto i$ or $N = M.i.$ where M is a constant and is known as the *co-efficient of mutual induction* between the coils.

If $i=1$, $N=M$. Thus, the coefficient of mutual induction between two coils is numerically equal to the flux linkage with one when unit current flows through the other.

From the aforesaid relation, we can write,

The rate of change of flux in the coil $S\left(\frac{dN_S}{dt}\right)$

$= M \times$ rate of change of current in $P\left(\frac{di}{dt}\right)$

i.e. the e.m.f. induced in the coil $S(e) = M \times$ rate of change of current in $P\left(\frac{di_P}{dt}\right)$

When $\frac{di_P}{dt}=1$, $e=M$

From this we get an alternative definition of the coefficient of mutual induction.

The coefficient of mutual induction between two coils is numerically equal to the e.m.f. set up in one coil due to unit rate of change of current in the other.

Units :

(1) **Absolute unit** : If one e.m.u. of e.m.f. is induced in a coil due to a rate of change of current at 1 e.m.u. per second in the other, the two coils are said to have 1 e.m.u. of coefficient of mutual induction between them and it is the absolute unit of the coefficient.

(2) **Practical unit** : If an e.m.f. of 1 volt is induced in a coil due to a rate of charge of current at 1 amp. per second in the other, the two coils are said to have 1 henry of coefficient of mutual induction between them and it is the practical unit of the coefficient.

7.11. Ruhmkorff's induction coil :

The most important example of the application of mutual inductance is Ruhmkorff's induction coil. It converts a large current at low voltage into low

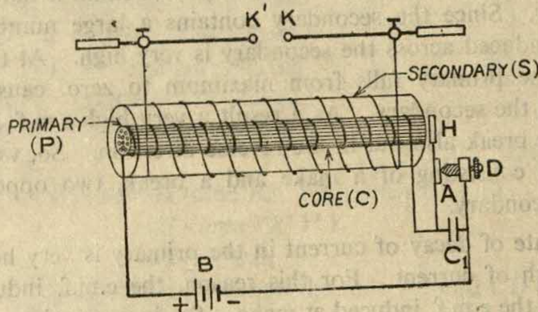


Fig. 7.8

current at high voltage. The essential parts of the instrument have been shown in fig. 7.8.

(i) **Core** : It consists of a bundle of soft iron strands insulated from each other and is placed inside a hollow cylinder made of some non-conducting material like vulcanite or ebonite (C).

(ii) **Primary coil** : It consists of a coil (P) of comparatively few turns of thick copper wire well insulated from each other and wound over the core. A battery is connected to the ends of the primary coil.

(iii) **Secondary coil** : The secondary coil (S) is wound over another hollow cylinder of non-conducting material, covering the primary coil and is made of thin copper wire. It consists of very large number of turns of the order of several thousands. In a powerful indution coil, the secondary runs into miles in length and the turns are well insulated from each other. The resistance of the secondary coil is high due to its small area of cross-section and large length. Since the number of turns of the secondary is large, it can produce high voltage due to change of flux linked with it. The ends of the secondary coil are connected to two discharging knobs K-K whose distance apart may be varied.

(iv) **Contact breaker** : It is an automatic make and break arrangement with which the primary current can be started and stopped. The soft-iron hammer H is supported by a spring which carries a soft-iron armature A. There is an adjustable screw D which has a fine platinum tip. When the tip of the screw D touches the armature A, current from the battery B flows through the primary coil. The magnetic field it produces magnetises the soft-iron strands of the core which attract the hammer H towards it. When the hammer H moves towards the core, the contact between the armature A and the screw D breaks and the

primary current stops with the result that the soft-iron strands are demagnetised. The hammer H relieved of the attractive force of the core, springs back to its original position due to spring action, and restores the contact between the armature A and the screw D. Again current flows through the primary and the whole process is repeated over and over again. It therefore automatically, makes and breaks the primary circuit.

Action : At the time of make, current passes through the primary and a large flux is set up in the secondary. As a result, an e.m.f. is induced in each turn of the secondary. Since the secondary contains a large number of turns, the resultant e.m.f. induced across the secondary is very high. At the time of break the current in the primary falls from maximum to zero, causing a maximum change of flux in the secondary. As a result a very high e.m.f. is induced in the secondary during break also but in the reverse direction. So, we see that during a complete cycle consisting of a make and a break, two opposite e.m.f.'s are induced in the secondary.

Now, the rate of decay of current in the primary is very high compared to the rate of growth of current. For this reason, the e.m.f. induced at break is much higher than the e.m.f. induced at make. So, in a complete cycle, a resultant e.m.f. is left in the secondary. Since the number of turns of the secondary are large and the frequency of make and break high, a large e.m.f. is developed across the sparking knobs in a short while.

Function of the capacitor : When the primary circuit is broken, an e.m.f. is induced due to self-inductance in the primary windings which causes a spark between the armature and the screw-tip when they are apart. This spark gradually damages the platinum tip of the screw D. In order to prevent it, a capacitor C_1 is used in parallel to the contact breaker. The capacitor absorbs the energy and avoids sparking. The capacitor actually *slows down* the fall of the primary current at the instant of break ; but in doing so, it prevents the induced e.m.f. in the primary from rising high enough to cause a spark. And the rate at which the primary current falls, in charging the capacitor, is greater than the rate at which it would fall if a spark were passing. Thus, with a capacitor, the secondary e.m.f. is less at the instant of break than without one, but it is greater throughout the rest of the fall of the primary current. Consequently, the average secondary voltage at break is higher with a capacitor than without it ; in practice, it is almost double. To get the greatest possible secondary voltage, the capacitance of the capacitor is so chosen that it just suppresses sparking at the contact between the armature and the screw-tip.

Use of induction coil : In the later part of nineteenth century, induction coils were widely used in the investigation of high voltages and in the study of electric discharge through rarefied gases. In the early years of the present century, induction coils were used to work a morse radio transmitter and also for operating X-ray tubes. Now-a-days, they are used in the laboratories for working small gas discharge tubes for spectroscopic works. They are also used in the coil-ignition system of motor-cars.

***7.12. E.M.F. induced in a conductor moving in a magnetic field :**

Consider a straight conductor XY lying on the parallel rails AB and CD of a conductor $ABCD$ [Fig. 7.9]. The frame $ABCD$ and the conductor XY lie in plane of the paper (say). A uniform magnetic field H is acting perpendicular to the plane of the paper. In this condition, the magnetic flux linked with the closed circuit $=H\times$ area $BXYC$

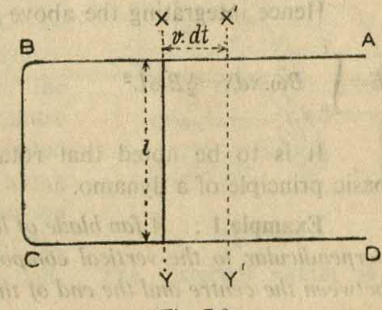


Fig. 7.9

If, now, the conductor XY moves to the right with a velocity v and goes to the position $X'Y'$ in a small interval of time dt, then the magnetic flux linked with the closed circuit in this position $=H\times$ area $BX'Y'C$

$$\therefore \text{ change of flux } d\phi=H(\text{area } BX'Y'C-\text{area } BXYC)$$
$$=H\times \text{area}XX'Y'Y$$
$$=H\times l\times v.dt$$

where l=length of the conductor between the rails AB and CD and $XX'=v.dt$.

This change of flux induces an e.m.f. in the circuit, given by $e=\dfrac{d\phi}{dt}=H.l.v.$

The direction of induced current in the circuit is given by Lenz's law.

Example : *Calculate the e.m.f. induced between the ends of an axle* 1·75 *metre long of a railway carriage travelling on level ground with a uniform speed of* 50 *km/hr. The vertical component of earth's field is* 0·5 *Oe.*

Ans. We know, the e.m.f. induced$=H.l.v$ e.m.u, when the quantities are expressed in c.g.s. units.

Here, $H=0{\cdot}5$ Oe ; $l=1{\cdot}75$ metre$=175$ cm and $v=50$ km/hr$=\dfrac{50\times10^5}{60\times60}$ $=\frac{5}{36}\times10^4$ cm/s.

$$\therefore \quad e=\frac{0{\cdot}5\times175\times5\times10^4}{36}\text{ e.m.u.}=\frac{0{\cdot}5\times175\times5\times10^4}{36\times10^8}\text{ volts}=1{\cdot}22\times10^{-8}\text{ volt.}$$

***7.13. E.M.F. induced between the two ends of a conductor rotating in a uniform magnetic field :**

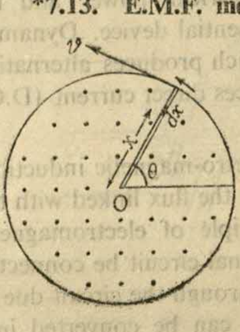


Fig. 7.10

Consider a conductor of length L rotating in the plane of the paper with an angular velocity ω about the point O. A uniform magnetic field B is acting perpendicular to the plane of the paper. As the conductor rotates, a change of flux will be associated with it which induces an e.m.f. in the conductor.

Now, consider a small length dx at a distance x from the centre O. If the element moves with a velocity v perpendicular to the magnetic field B, the e.m.f. induced on this element $dE=B.v.dx$ [See art 7.12]. The conductor may be considered to be made up of numerous such

small elements, the angular velocity of each element being ω. Since the element, under consideration, describes a circle of radius x, its linear velocity $v=\omega.x$.

$$\therefore \quad dE=B.\omega.x.dx.$$

Hence integrating the above over the whole length, the total e.m.f. induced

$$E=\int_0^L B\omega.xdx=\frac{1}{2}B\omega L^2$$

It is to be noted that rotation of an armature in a magnetic field is the basic principle of a dynamo.

Example 1 : *A fan blade of length 2l rotates with a frequency n cycles per sec. perpendicular to the vertical component B of earth's magnetic field. Find the p.d. between the centre and the end of the blade.*

Ans. The e.m.f. induced $E=\frac{1}{2}B.\omega L^2$.

Here $\omega=2\pi n$ and L=the length between the centre and the end of the blade$=l$.

$$\therefore \quad E=\tfrac{1}{2}.B.2\pi n.l^2=\pi Bn.l^2 \text{ e.m.u.}$$

Example 2 : *A conductor* 30 *cm long rotates about one end at* 1000 *revolutions per minute in a plane perpendicular to a magnetic field of intensity* 5000 *oersted. Find in volts, the e.m.f. induced between the ends of the conductor.*

[*Jt. Entrance 1984*]

Ans. We know $E=\frac{1}{2}B\omega L^2$

Here $B=5000\ Oe$; $\omega=2\pi n=2\pi\times\frac{1000}{60}$; $L=30$ cm.

$$\therefore \quad E=\tfrac{1}{2}\times 5000\times 2\pi\times\frac{1000}{60}\times 30\times 30 \text{ e.m.u.}$$

$$=235{\cdot}5\times 10^6 \text{ e.m.u.}=\frac{235{\cdot}5\times 10^6}{10^8} \text{ volt}=2{\cdot}35 \text{ volt.}$$

7.14. Dynamo :

Modern civilization is entirely dependent on electrical power and for large scale supply of electric power, dynamo is an essential device. Dynamos are of two types : (i) **Alternator** or A.C. dynamo which produces alternating current in a circuit and (ii) D.C. dynamo which produces direct current (D.C.) in a circuit.

The principle of action of a dynamo is based on electro-magnetic induction. If a closed coil be continuously rotated in a magnetic field, the flux linked with the coil will constantly change and according to the principle of electromagnetic induction, an e.m.f. will be set up in the coil. If an external circuit be connected to the rotating coil, then an alternating current will flow through the circuit due to the induced e.m.f. in the coil. The alternating current can be converted into direct current by the use of a commutator. Brief descriptions of the above two types of dynamo are given below.

(i) **A.C. dynamo** : Fig. 7.11 illustrates the construction of a simple form of A.C. dynamo. The machine consists of a rectangular or cylindrical coil of fine insulated copper wire wound on an iron frame. It is called the **armature** of the machine. It rotates in the magnetic field of a permanent magnet. Oil or steam engine is used to rotate the armature. In hydro-electric installation, water power is utilised for the rotation of the armature. The ends of the armature coil are connected to two metallic **slip-rings** mounted on the armature spindle. The slip-rings rotate along with the rotation of the coil. There are two carbon **brushes** which are made to press lightly against the slip-rings during the rotation of the armature coil. The external circuit is connected to the carbon brushes.

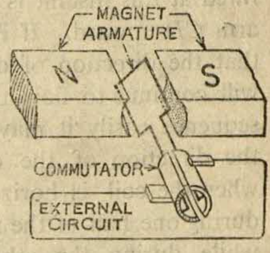


Fig. 7.11

When the armature is rotated, the coil intersects the lines of force of the magnetic field produced by the permanent magnet and according to the laws of electro-magnetic induction, an e.m.f. is induced in the armature coil. The direction of the induced e.m.f. may be obtained from *Fleming's right hand rule* or *Dynamo rule.* The rule is as follows :

Extend the thumb, the forefinger and the second finger of the right hand mutually at right angles (*Fig.* 7.12). *If the forefinger points in the direction of the field and the thumb in the direction of the motion, then the second finger will point in the direction of the induced e.m.f.*

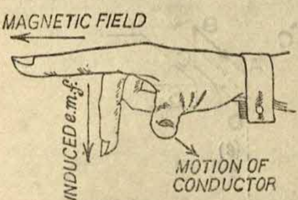
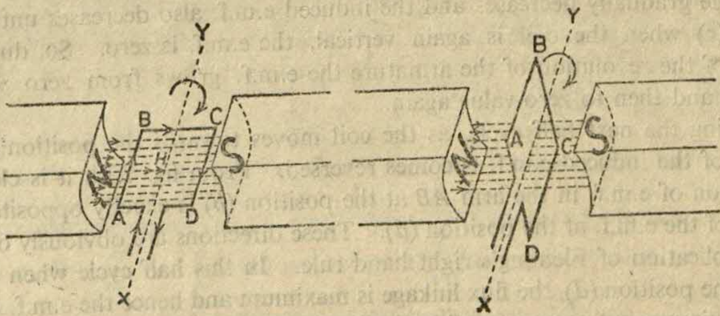


Fig. 7.12

Suppose the armature coil *ABCD* moves in the magnetic field of the magnet *N-S* about a horizontal line *XY* as axis. The direction of its rotation has been shown by the arrow head. As a starting point, consider the instant when the plane of the coil is horizontal [Fig. 7.13(i)]. Now, the arm *AB* will move upward and the arm *CD* downward. If Fleming's rule be applied on both the arms, it will be seen that the induced current



(i) Fig 7.13 (ii)

flows from *A* to *B* in the arm *AB* and from *C* to *D* in the arm *CD*. Considering the whole armature coil, the current flows along *ABCD*, until the plane of the coil

becomes vertical [Fig. 7.13(ii)]. In this position the arms of the coil move *along* the lines of force. No cutting of lines takes place and therefore the e.m.f. generated at this instant is zero. Next moment the arm *AB* goes downward and the arm *CD* upward. If Fleming's rule be again applied to the arms, it will be seen that the direction of current in the arms is now reversed. The reverse current will continue to flow till the armature coil is again vertical. To remember the sequence easily it may be said that (i) when the coil crosses the vertical position, the direction of the current is reversed and (ii) the current becomes maximum when the coil is horizontal and minimum when it is vertical. So, we see that during one half of the rotation of armature coil, the current flows in one direction while during the other half, the current flows in the reverse direction. If any external circuit be connected to the dynamo, the current through the circuit will be alternating.

7.15. Alternating Current :

Fig. 7.14 shows the different positions of the armature coil *ABCD* as it revolves in the magnetic field. No e.m.f. is induced when the coil is vertical [Fig. (*a*)]. As the coil rotates in the direction of the arrow towards the position (*b*), it cuts more and more lines of force and hence the induced e.m.f. also increases more and more. After rotating through full 90° when the coil comes to the horizontal position [Fig. (*b*)], the induced e.m.f.

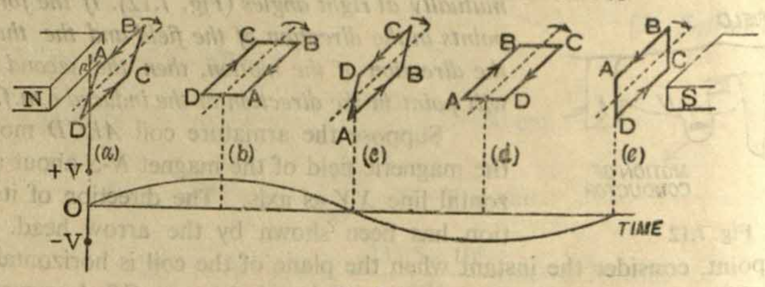


Fig. 7.14

becomes maximum. After this when the coil approaches the position (*c*), the flux linkage gradually decreases and the induced e.m.f. also decreases until at the position (*c*) when the coil is again vertical, the e.m.f. is zero. So, during the one half of the revolution of the armature the e.m.f. grows from zero value to maximum and then to zero value again.

During the next half cycle, as the coil moves towards the position (*d*), the direction of the induced e.m.f. becomes reversed. From fig. 7.14, it is clear that the direction of e.m.f. in the arm *AB* at the position (*b*) is exactly opposite to the direction of the e.m.f. at the position (*d*). These directions are obviously obtained by the application of Fleming's right hand rule. In this half cycle when the coil occupies the position (*d*), the flux linkage is maximum and hence the e.m.f. induced is also maximum but reverse. After that as the coil rotates towards the position (*e*) the induced e.m.f. gradually decreases. This cycle of changes will be repeated over and over again as long as the armature coil continues to rotate in the magnetic field.

This cycle of change of e.m.f. is graphically shown below the fig. 7.14. The graph clearly shows that the e.m.f. is *alternating*, that is, it goes first in one direction, then in the opposite. If, now, the coil is connected to an external circuit by means of slip-rings and brushes, the current furnished to this circuit will be *alternating current* (*A.C.*).

Such alternating voltage or e.m.f. may be represented by the following equation :

$$E=E_0 \sin 2\pi \frac{t}{T}.*$$

Here E denotes the e.m.f. existing in the circuit at any time t, E_0, the maximum value of the e.m.f. in the circuit (also known as *peak value*) and T the time-period of change of the e.m.f. (*i.e.* of the revolution of the coil).

At $t=0$, $T/2$ or T (*i.e.* the positions a, c and e in the fig. 7.14), $\sin \frac{2\pi}{T} t=0$ and hence $E=0$. At $t=T/4$ (*i.e.* the position b in fig. 7.14), $\sin \frac{2\pi}{T}.t=+1$ and hence $E=+E_0$. Again at $t=3T/4$ (*i.e.* the position d in fig. 7.14), $\sin \frac{2\pi}{T}.t=-1$ and hence $E=-E_0$.

The e.m.f. thus changes from 0 to 0 through $+E_0$ and again from 0 to 0 through $-E_0$ *i.e.* in the reverse direction during one complete rotation of the coil. *E.M.F.* of such alternating magnitude and direction is known as alternating e.m.f.

Such alternating e.m.f. acting in a circuit sends an alternating current of a similar sine equation which is as follows : $I=I_0 \sin 2\pi \frac{t}{T}$

Here also current assumes maximum value (or the peak value) $+I_0$ at $t=T/4$, $5T/4$, ..etc and $-I_0$ at $t=3T/4$, $7T/4$..etc

The current becomes zero at $t=0$, $T/2$, T..etc

Alternating e.m.f. or current is generally measured by its root mean square (abbreviated as *r.m.s.*) value. The r.m.s. value of an alternating quantity is the square root of the mean value of the squares of the quantity over a complete cycle.

---

*Suppose the coil has an area A and at any time t, its plane makes an angle θ with the direction of the magnetic field of intensity H. The flux through the coil$=A.H.\cos\theta$. If the coil has n turns, the total flux linkage $\phi=n.A.H.\cos\theta$.

If the coil rotates with a uniform angular velocity ω, then $\theta=\omega t$. So, flux linkage with the coil at the instant t is $\phi=nAH.\cos\omega t$.

According to Faraday's law, the e.m.f. induced E at the instant due to change of flux linkage, is given by, $E=-\frac{d\phi}{dt}=-\frac{d}{dt}(nAH.\cos\omega t)=-nAH\frac{d}{dt}(\cos\omega t)=nAH.\omega.\sin\omega t$

If T be the time period of revolution of the coil, then $\omega=\frac{2\pi}{T}$

$\therefore$ $E=nAH\omega.\sin\frac{2\pi}{T}t=E_0\sin\frac{2\pi}{T}t$, where $E_0=nAH\omega=$ maximum value or peak value of the induced e.m.f.

It can be proved that the r.m.s. value of e.m.f. $E=\frac{E_0}{\sqrt{2}}=0{\cdot}707\ E_0$ where E_0 is its peak value. Similarly, the r.m.s. value of current $I=\frac{I_0}{\sqrt{2}}=0{\cdot}707 I_0$ where I_0 is its peak current.

For example, an a.c. of 220 volts means that the r.m.s. value of the e.m.f. is 220 volts. So, its peak value $E_0=\sqrt{2}\times220=311$ volts. This shows why a shock from 220 volt a.c. is more fatal than a shock from 220 volt d.c. ; 220 volt d.c. remains 220 volt always but 220 volt a.c. momentarily becomes 311 volts when it assumes peak value and a shock from such high voltage is very dangerous for human bodies.

For long distance power transmission, A.C. has been found to be very economical. For this reason, more than 90% of the electrical energy used now-a-days for commercial purposes is generated as alternating current.

7.16. D.C. Dynamo :

If *A.C.* dynamo is provided with a commutator, it becomes a *D.C.* dynamo. Fig. 7.15 illustrates the construction of a *D.C.* dynamo.

In a *D.C.* dynamo, the ends of the armature coil are connected to a *single split ring* with two diametrically opposite carbon brushes. This is known as the commutator. The split ring rotates along with the armature. The external circuit is connected to the carbon brushes. As the armature rotates, an alternating e.m.f. is induced as in *A.C.* dynamo in the armature coil but the brushes are so arranged that when the coil is passing through the vertical position, the two halves of the split ring are just on the point of changing contact from one brush to another. Hence, although during the second half of rotation the current is reversed in the armature coil itself, the changeover between brushes and commutator halves ensures that one brush always remains positive and the other always negative, with the result that the current in the external circuit is unidirectional.

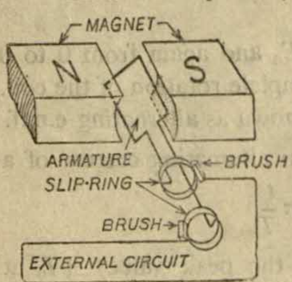


Fig. 7.15

Fig. 7.16 shows the actual appearance of a *D.C.* dynamo.

Advantage and disadvantage of a.c. and d.c. :

Direct currents (d.c.) are less easy to generate than alternating currents (a.c.) but alternating e.m.f's are more convenient to step up or to step down with help of a transformer. Moreover for distribution of electricity over a wide area, alternating current is found to be more economical. The national grid system, which supplies electricity to the whole country, is therefore fed with a.c. Alternating current is just as suitable for heating as is d.c., because heating effect of a current does not depend on the direction of the current ($H\propto i^2rt$). It is also equally suitable for lighting because filaments of electric bulbs depend on the heating effect and neon lamps function as well on a.c. as on d.c. Small motors of the size used in water pumps or refrigerators run satisfactorily on a.c. but large ones, as a general rule, do not. D.C. is, therefore, used on most tramway or electric railway systems.

For electrolysis purposes, however, a.c. is useless. The chemical effect of a current reverses its direction if a.c. is used and it would cause merely a small amount of the metal to be alternately

deposited and dissolved. For electroplating and for battery charging, direct current is always used. It may be pointed out that an a.c. may be converted into d.c. by a suitable rectifier (see art 2.3 in modern physics) and a d.c. into a.c. by a suitable convertor.

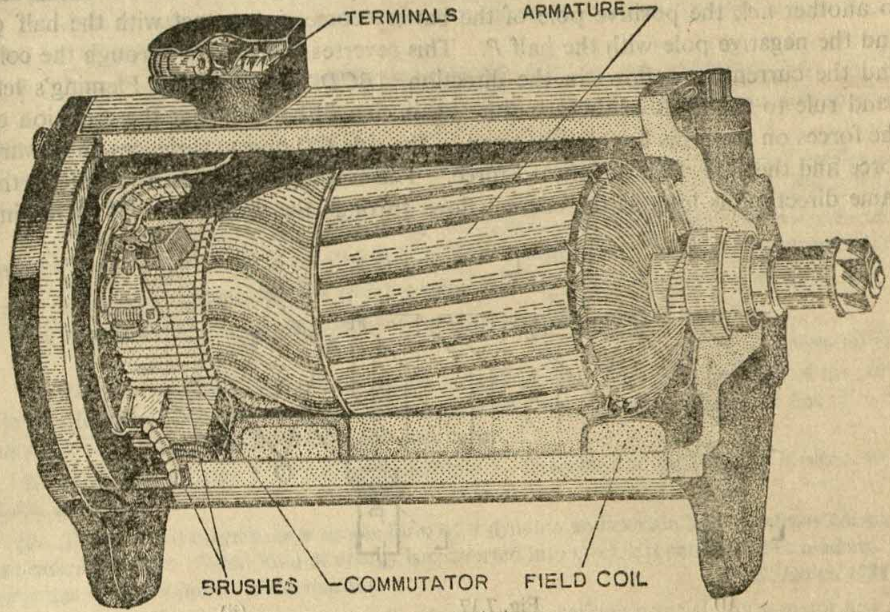


Fig. 7.16

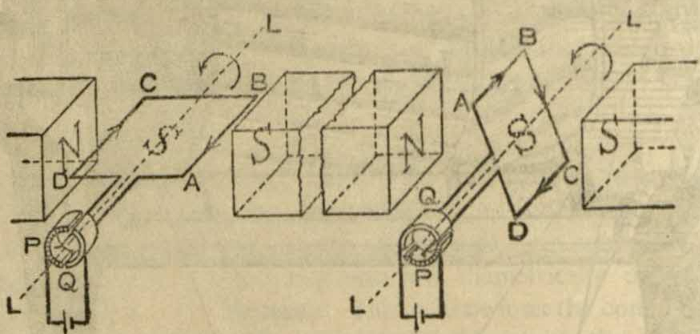
7.17. Motors :

The principle of action of electric motors is exactly opposite to that of dynamos. Dynamos convert mechanical energy into electrical energy but motors perform the reverse work *viz.* conversion of electrical energy into mechanical one.

Motors are of two types :—(i) *Direct current motors* or *D.C. motors* and (ii) *Alternating current motors* or *A.C. motors*. In the following paragraphs, we shall discuss, in brief, the construction and the action of a *D.C.* motor.

The construction of a *D.C.* motor is same as that of a dynamo, having armature, field magnet, carbon brush, commutator etc. Figs 7.17(i) and (ii) illustrate the action of a *D.C.* motor. *ABCD* is the armature coil. It is capable of rotating about a horizontal spindle *LL*, in the magnetic field produced by the field magnet *N-S*. At the starting point, suppose current from a battery flows in the direction *DCBA* through the armature coil which is in a horizontal position. In this condition, the positive pole of the battery is joined to the *P*-half of the commutator and the negative pole to the *Q*-half. By applying Fleming's *left hand rule* it will be seen that the side *AB* of the coil experiences an upward force and the side *CD* a downward force, for the direction of current in the two arms is opposite. These two forces constitute a couple which causes the coil to rotate about the axis *LL* in the direction of the arrow [Fig. 7.17(i)], until the coil reaches the vertical position [Fig. 7.17(ii)]. At this instant, the brushes touch the space

between the split-rings and the current is cut off. But due to inertia of motion, the rotation is continued and the coil goes past the vertical. When this occurs the two halves P, Q of the commutator automatically change contact from one brush to another *i.e.*, the positive pole of the battery comes in contact with the half Q and the negative pole with the half P. This reverses the current through the coil, and the current now flows in the direction $ABCD$. By applying Fleming's left hand rule to the arms of the armature again, it will be seen that the direction of the forces on the arms has also reversed. The side CD now experiences an upward force and the side AB a downward force. The coil thus continues to rotate in the same direction as long as the current flows through the armature. By increasing



(i) Fig. 7.17 (ii)

the current strength and using a powerful magnet, the armature may be made to rotate at a great speed. This is the working principle of a *D.C.* motor.

Electric motors are used in electric fans, tram cars, pumps, rolling mills etc.

Back e.m.f. in motors : When the armature coil of a motor rotates in the magnetic field of the field-magnets, the flux-linkage with it changes and an e.m.f. is induced in its windings. According to Lenz's law, this e.m.f. opposes the current which makes the coil turn. It is, for this reason, called a *back e.m.f.* If E be the e.m.f. of the battery connected to the armature coil, and V the back e.m.f. induced then the armature current is $i=\dfrac{E-V}{r}$ where r = the resistance of armature coil.

Generally the resistance of an armature is low—of the order of 1 ohm.

The back e.m.f. is proportional to (i) the strength of the magnetic field and (ii) the speed of rotation of the armature. When the motor is first switched on, the back e.m.f. is zero ; it rises as the motor gathers speed. In a large motor, the starting current would be dangerously high ; to limit it, a variable resistance is inserted in series with the armature which is gradually reduced to zero as the motor speeds up.

When a motor is running, the back e.m.f. (V) is not much less than the supply voltage (E). For example, a motor running off the mains (E=220 volts, say) might develop a back e.m.f. V=210 volts. If the armature has a resistance of 1 ohm, say, the armature current, during running condition, is 10 amp. When the motor was switched on, the armature current would be 220 amp., if no starting resistor is used. Such high current will burn out the coil of the armature.

Exercises

Essay type :

1. What is induced current ? Describe experiments to produce induced current by a magnet and a current carrying coil.

2. Describe, in brief, the experiments in connection with electromagnetic induction.

3. What are the laws of electromagnetic induction ? Explain them with suitable experiments.

4. State Faraday's laws of electromagnetic induction. How will you determine the direction of the induced e.m.f. ? [*H. S. Exam.* '83]

5. What is induced current ? Describe two experiments to produce induced current. Upon what do (i) the duration (ii) the direction and (iii) the magnitude of induced current depend ?

6. State Lenz's law and explain, with the help of the law the production of induced current.

7. What are self and mutual inductions ? What are their practical units ? [*cf. H. S. Exam. 1981*]

8. Describe and explain the action of an induction coil. Why is the primary of the coil made of a few turns of thick wire and the secondary of a large number of turns of fine wire ? What is the function of the condenser in this instrument ?

9. What is a dynamo ? What is its principle ? Describe and differentiate between a.c. and d.c. dynamos.

10. Draw a neat diagram of a simple form of a dynamo and explain how it delivers current to an external circuit. What kind of energy is converted into electrical energy in this machine ? How is this energy supplied to the machine ? [*H. S. Exam. 1983*]

11. What is the function of a motor ? Compare the working principles of a motor and a dynamo.

12. Explain what you understand by alternating current ? What are the peak value and r.m.s. value of an alternating current ? What is their relation ?

Short answer type :

13. You are given a coil of wire connected to a sensitive galvanometer. Explain, with reasons, what happens in the following cases :—(*a*) suddenly the N-pole of a bar magnet is inserted in the coil (*b*) the bar-magnet is kept steady in this position (*c*) the bar-magnet is suddenly withdrawn from the coil.

14. Explain, with the help of Lenz's law, what will be the direction of the induced current when (i) the S-pole of a bar-magnet approaches a coil (ii) the pole is removed far away from the coil.

15. Explain, with the help of Lenz's law, what will be the direction of induced current in a coil when (i) N-pole of a magnet is brought near the coil (ii) the pole is removed from the coil.

16. In the experiments mentioned below, state with reason in what direction current will flow in a given closed coil (i) the N-pole of a magnet is brought quickly near one end of the coil (ii) the S-pole of a magnet is rapidly moved away from that end of the coil (iii) what is meant by coefficient of self inductance of a coil ? What is the practical unit of it ? [*H. S. Exam. 1984*]

17. What is the source of energy of the induced current ? How can you establish Lenz's law from the principle of conservation of energy ? [*H. S. Exam. 1981*]

18. What are self and mutual inductions ? What are their practical units ?

19. A cylindrical bar-magnet is kept along the axis of a circular coil of wire. Will there be any current induced in the coil if the bar-magnet is rotated about the axis ?

[**Hints** : As the bar-magnet is cylindrical in shape, its rotation about its own axis makes no difference in the flux linkage with the coil. Consequently no current is induced in the coil.]

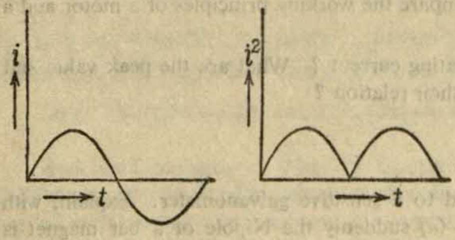
20. A copper ring is held horizontally and a bar magnet is dropped through the ring with its length along the axis of the ring. Will the acceleration of the falling magnet be equal to, greater than or less than that due to gravity ? [*I.I.T. 1974. Jt. Entrance '85*]

21. Two similar circular coils A and B are kept parallel to each other, their centres lying on the same axis. A current is flowing in the coil B and when looked from the coil-A, the current is clockwise. Looking from the coil B, what will be the direction of induced current in A when (*a*) the current in the coil is increased and (*b*) keeping the current in B steady, the coil B is moved towards the coil-A. [*I.I.T. 1971*]

[**Hints** : (*a*) When the current in the coil B is increased, the flux linked with the coil A increases. Consequently a current is induced in the coil, whose direction, according to Lenz's law will be such as to oppose the increase of flux *i.e.* its direction will be opposite to that of the current in the coil B. Since when looked at from the direction of B, its current appears anti-clockwise, the direction of the induced current in A. when looked from the same direction, will appear clockwise ; (*b*) when the coil B is moved towards the coil-A, the flux linked with the coil-A increases and similar to the case (*a*), the induced current in A will oppose the increase of flux. In other words, the direction of induced current, as in (*a*), will be clockwise.]

22. A bar-magnet is pulled quickly through a conducting loop along its axis with a uniform velocity such that its south pole enters the loop first. Draw qualitatively (*a*) the induced current and (*b*) the joule heating as a function of time. (Take induced current to be positive if it is clockwise when viewed along the path of the magnet.)

[**Hints** : (*a*) When the south pole is sufficiently away from the loop, a few lines of force intersect the loop. As the south pole approaches the loop, larger number of lines of force get linked with loop and hence the induced current increases. The direction of the induced current is such that the plane of the loop facing the south pole develops south polarity *i.e.* looking from the direction of approach of the south pole, the current is clockwise and hence positive. When the south pole reaches the centre of the loop, the positive induced current is maximum. Afterwards when the south-pole moves away from the loop, the current becomes anticlockwise and hence negative. This negative induced current for the same reason stated before,



(a) Fig. 7.18 (b)

gradually increases to a maximum value and then slowly reduces to zero. Represented qualitatively in a graph, it will look like fig. 7.18(*a*). (*b*) Since Joule heating is proportional to square of the current ($H \propto i^2$), the heat produced by the negative induced current is positive. So, if Joule heating be represented graphically as a function of time, it will be like fig. 7.18(*b*)].

23. A long wire, carrying a current, is folded back on itself, so that the currents in the two halves are very near to each other and oppositely directed. What will be the self-inductance of the wire ?

Objective type :

24. A few probable answers are given to each question below. Mark the answer you consider correct :

(*a*) A copper ring is held horizontally and a bar-magnet is dropped through the ring with its length along the axis of the ring. Will the acceleration of the falling magnet be equal to that due to gravity ? Ans. (i) equal to gravitational acceleration (ii) greater than gravitational acceleration (iii) less than gravitational acceleration.

(*b*) Two coils of wire are so connected in series that same current flows through them but

in opposite directions. Will the flux linked with any coil due to the current in the other be greater than when the currents are in the same direction ? Ans. (i) greater (ii) less (iii) same (iv) no lines of force will be linked.

(*c*) Does the self inductance of a coil depend on the medium in which it is placed ? Ans. (i) Depends (ii) Does not depend.

(*d*) Does the coefficient of mutual induction between two coils depend upon the number of turns of the coil ? Ans. (i) Increases with the increase of number of turns (ii) decreases with the increase of turns (iii) independent of the number of turns.

Numerical type :

25. A plane, having wing span of 30 metre, is flying horizontally with a speed of 100 metres/s. If the horizontal intensity of earth's magnetic field there be 5×10^{-5} Weber/m^2, find the p.d. developed at the ends of the wing ? [Ans. 0·15 volt]

26. How many volts are produced in a wire 12 cm long which cuts directly across a flux whose intensity is 14,000 gauss, if it moves at a speed of 2 metre/*s*. ? [Ans. 0·336*V*]

27. A square metal wire loop of side 10 cm and resistance 1 ohm is moved with a constant velocity v_0 in a uniform magnetic field of induction $B=2$ Weber/m^2 as shown in fig 7·19. The magnetic field lines are perpendicular to the plane of the loop (directed into the paper). The loop is connected to a net work of resistors each of value 3 ohms. The resistance of the lead wires *OS* and *PQ* are negligible. What should be the speed of the loop so as to have a steady current of 1 milliampere in the loop ? [*I.I.T. 1983*] Ans. 2 cm/*s*]

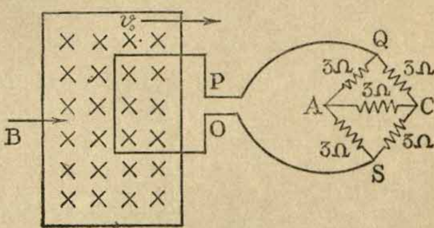


Fig. 7.19

[Hints : $e=B.l.v.=2\times\frac{10}{100}\times\frac{v_0}{100}=2\times10^{-3}\times v_0$ volt.

Circuit resistance$=3+1=4$ ohm $\therefore$ $e=1\times10^{-3}\times4$ volt]

28. The equation of an alternating current is $i=50\sin 400\pi t$. Find the frequency, peak value and r.m.s. value of the current. [Ans. 200 ; 50 amp. ; 35·36 amp.]

29. An alternating e.m.f. is represented by the equation $E=200\sin(100\pi t)$ volt. Calculate the frequency, peak value and r.m.s. value of the e.m.f. [Ans. 50 ; 200 volts ; 141·4 volts]

30. A simple electric motor has an armature of 0·1 ohm resistance. When the motor is running on a 50 volt supply the current is found to be 5 amp. Find the back e.m.f. developed. [Ans. 49.5 volt]

MODERN PHYSICS

1

CATHODE RAYS & X-RAYS

1.1. Starting of modern physics :

Modern physics means the advancement and development of Physics since the major discovery in 1890 till to-day. This advancement being principally with regard to atoms and the structure of matter, it is also called *Atomic Physics.* Now, the question is : what was that major discovery that took place in 1890, and marked the beginning of modern Physics ? During this time, some eminent Physicists, while carrying out experiments on the electrical conductivity of gases, discovered *electron* which finally brought about a total change in the conception of the structure of atom. Further more, during the last decade of nineteenth century some astounding discoveries like X-rays, radioactivity, etc. were made which called for a different outlook with regard to the structure of matter. For this reason, the last decade of nineteenth century is regarded as the starting point of modern Physics.

1.2. Electrical conductivity of gases :

A dry gas at normal pressure is a bad conductor of electricity. Normal air, for instance, is almost a perfect insulator. Had it not been so the study of electrical phenomenon would not have advanced very much. But gases can be made conductive by several processes. These are as follows :

(i) Passage of electromagnetic waves of short wave length like ultra-violet rays, X-rays, gamma rays etc. through the gas.

(ii) Passage of energetic particles like alpha, beta etc. given out by radio-active sources through the gas.

(iii) Heating the gas.

(iv) Passage of electric discharge through the gas in a rarified condition.

Experiments have revealed that the above processes render a gas conductive because they create *ions* in the gas. Normal atoms and molecules of a gas are neutral. But if one or more negatively charged electrons be detached from a neutral atom or a molecule, the residual atom or molecule will be positively charged and they constitute the positive ions. The detached electrons soon attach themselves to neutral atoms and molecules, which thereby become negative ions. The process in which positive and negative ions are produced is called *ionisation.*

1.3. Phenomena observed in a discharge tube :

Air or gas at normal temperature and pressure no doubt is an insulator, but not a perfect insulator. In 1900, C. T. R. Wilson demonstrated that a charged gold leaf electroscope got slowly discharged in spite of elaborate precautions taken to prevent leakage of electricity. The observed discharge of electroscope was ascribed to the partial conductivity of air medium. It has been experimentally found that the conducivity of air or gas increases with

the decrease of pressure of the gas. Such experiments were carried out by Willium Crookes, Lenard, J. J. Thomson and others. In course of their experiments, they observed interesting phenomena when electric discharge was carried out through a gas whose pressure was reduced step by step. These are known as phenomena in a discharge tube.

The tube used for studying the electric discharge in air at low pressure is shown in fig. 1.1. It is a cylindrical tube about 30 cm. long and about 4 cm. in diameter, made of hard glass. Both ends of the tube are closed and two electrodes

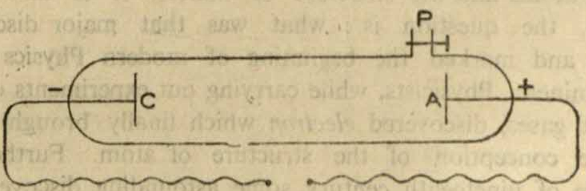


Fig. 1.1

(*C* and *A*) are sealed into the tube. There is an opening at the side of the tube with a stop cock *P* to which an exhaust pump and a manometer may be attached. To the electrodes are joined the secondary of an induction coil to apply a high potential difference. The electrode connected to the positive terminal of the induction coil is called the anode (*A*) and the other electrode is called the cathode (*C*). Although an induction coil does not deliver direct current, its characteristics are such that the potentials are higher on one half of the alternations that they are on the other and the two electrodes act nearly the same as if a high-voltage direct current were used. The phenomenon that happens in the discharge tube at various stages while the air is being pumped out is as follows :

When the pressure has dropped to about 8 mm. of mercury (about $\frac{1}{100}$ of an atmosphere), the first discharge appears at potential difference of about 10,000 to 15,000 volts between the electrodes. The discharge consists of long, thin and wavy streamers of violet-blue colour extending from one electrode to the other [Fig. 1.2(*a*)]. As the gas pressure drops to about 5 mm. of mercury, the discharge widens until it fills the whole tube with a pink diffuse glow as shown in Fig. 1.2 (*b*). This is known as *positive column*. At this time a low, continuous buzzing sound is also produced. The colour of the positive column depends primarly on the nature of the gas taken in the discharge tube. For example, with air, the colour is pink ; with hydrogen, the colour is blue or red ; with neon, it is deep red ; with helium, it is yellow, etc.

Ordinarily the neon signs that we see as advertisement display in shops and markets are nothing but these positive columns. Such commercial use of positive columns was first introduced by Geissler and hence, these tubes are known as '*Geissler tubes*'. At a still lower pressure of about 2 mm. the positive column leaves the cathode and advances towards the anode, leaving a dark region behind known as *Faraday dark space*. At this time a short bluish glow appears at the cathode. It is called *negative glow* [Fig. 1.2 (*c*)]. As the pressure drops still

further (*i.e.* about 0.1 mm. mercury), the Farady dark space grows in size and the positive column diminishes in length. With still further fall of pressure,

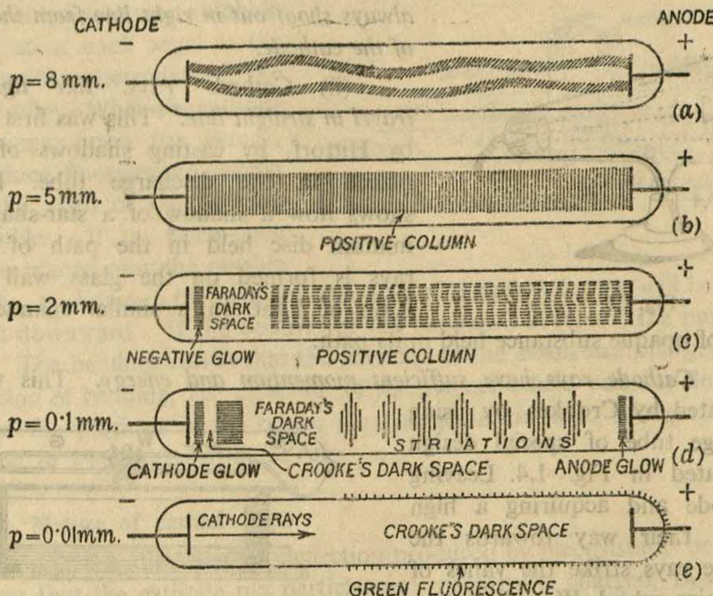


Fig. 1.2

the negative glow moves away from the cathode, producing another dark space between it and the cathode, known as *Crookes' dark space.* With the appearance of Crookes' dark space, the positive column divides into a number of equally spaced layers, called *striations* [Fig. 1.2(*d*)]. Also, a bluish glow, called *cathode glow* appears round the cathode and a pinkish glow, called *anode glow* round the anode.

As the pumping proceeds, the striations and the negative glow grow fainter and the Crooke's dark space widens until finally at a pressure of about 0.01 mm, Crookes' dark space fills the whole tube. At this point a new feature appears ; the whole glass tube itself glows with a faint greenish light. It is called *fluorescence* and is caused by the impact of extremely minute particles which emanate from the cathode and travel with tremendous speed towards the anode. These particles are called *cathode rays.*

1.4. Cathode rays :

The green glow in the final stage of the electric discharge just described was found to be fluorescence of glass caused by some invisible rays coming out of the cathode plate. Investigations carried out by Perin, Thomson, Crookes and others revealed that some very minute particles, carrying small amount of negative charge shoot out from the cathode surface normally and constitute the cathode rays. *Cathode rays are, therefore, nothing but streams of high speed electrons.*

1.5. Different properties of cathode rays :

(i) *Whatever may be the position of the anode the cathode ray particles always shoot out in right line from the surface of the cathode.*

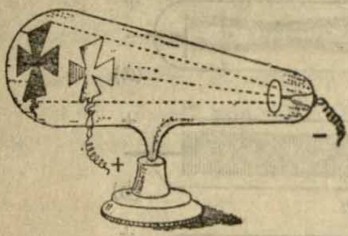


Fig. 1.3

(ii) *Cathode rays, like light rays, travel in straight line.* This was first revealed by Hittorf, by casting shadows of objects placed in the discharge tube. Fig. 1.3 shows how a shadow of a star-shaped aluminium disc held in the path of cathode rays is formed on the glass wall behind. Rays of light, in a similar manner, form shadows of opaque substance held in its path.

(iii) *Cathode rays have sufficient momentum and energy.* This was first demonstrated by Crookes by using a discharge tube of special design as illustrated in Fig. 1.4. Leaving the cathode and acquiring a high speed on their way towards the anode, the rays strike the vanes of a small mica wheel *W* and exert a force, causing it to turn and roll along a rail track towards the anode. When it reaches the anode, a reversal of the potential, making the plate *A* the cathode, will send it rolling back towards the plate *K* which is now the anode. From this experiment, Crookes concluded that cathode particles have momentum and energy.

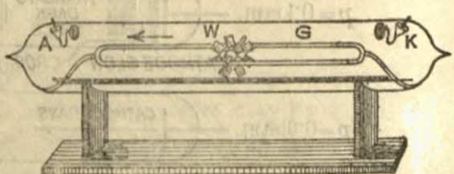


Fig. 1.4

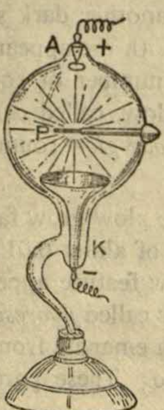


Fig. 1.5

(iv) *Cathode rays can excite fluorescence in some substances like zinc sulphide, barium platino-cyanide, etc.*

(v) *Like light, cathode rays can affect a photographic plate.*

(vi) *Cathode rays produce heat when they fall upon a body.* If a concave-shaped cathode (*K*) is used (Fig. 1.5)., it is found that a piece of platinum (*P*) held at the centre of curvature of the concave surface is heated to incandescence. It also illustrates that the particles of cathode rays possess sufficient kinetic energy.

(vii) *When cathode rays are made to pass through a gas, the gas gets ionised i.e., becomes a conductor of electricity.*

(viii) *If cathode rays are made to pass through an electric or a magnetic field, the rays are deflected.* This shows that the particles of cathode rays are electrically charged. From the direction of deflection, it may be said that the particles are charged negatively.

Fig. 1·6 shows a discharge tube of special design. The cathode rays coming out of the cathode plate pass through a narrow slit and form a narrow beam. A screen coated with zinc sulphide is placed at a small angle to the line joining the electrodes of the discharge tube. When the cathode rays impinge upon the screen, they produce fluorescence which renders the path of the cathode rays visible. If a horse-shoe magnet be placed over the outside of the tube, as shown in the diagram, the path of the cathode rays will be found to have bent downward. If the polarity of the magnet is reversed, the path is bent upward. The bending shows that the particles of the beam are charged. From the direction of bending, the direction of the magnetic field and the direction of motion of the particles in the beam, the charge is found to be *negative* by the application of Fleming's left hand rule.

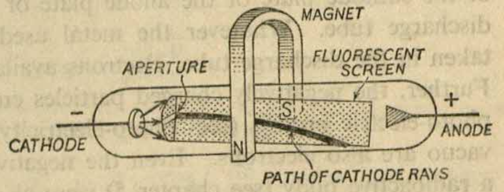


Fig. 1.6

1.6. Nature of cathode rays :

Although the direction of deflection produced by electrostatic or magnetic field shows that the cathode ray particles are negatively charged, this property was shown independently by Perin in the following way.

The apparatus is essentially a discharge tube, having two bulbs (Fig. 1·7). In the smaller bulb are sealed two electordes —the cathode C and the anode A. The larger bulb has a side tube in which there is an earth-connected cylinder. Inside this cylinder but without having any contact with it, there is another smaller cylinder, called Faraday cylinder or detector ($F.D.$) connected to a gold leaf electroscope. The two bulbs are connected by a narrow tube.

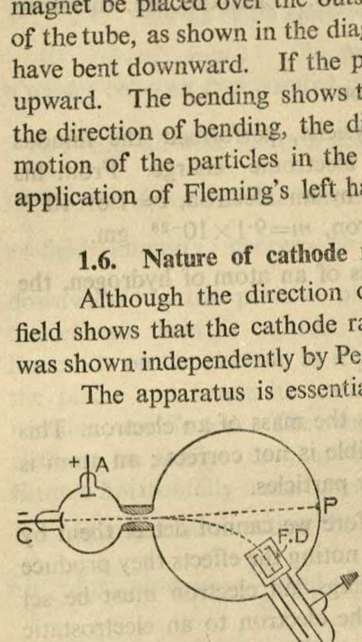


Fig. 1.7

When cathode rays pass through the narrow passage into the larger bulb, they form a narrow beam and travelling straight they produce a fluorescent spot at P. When this happens, the gold leaf electroscope shows no existence of charge in it and the leaves remain collapsed. If, with the help of a suitable magnetic field, the path of the cathode rays be deflected towards the Faraday cylinder, the leaves of the electroscope will, at once, diverge. On testing the electroscope in the usual way with a charged ebonite rod, it will be found to be negatively charged.

This experiment conclusively proves that cathode rays convey negative charge. This negatively charged particles of cathode ray have been named *electrons*.

Electron is a fundamental particle : To find out the nature of electron, Sir J. J. Thomson tried to measure the specific charge *i.e.* charge to mass ratio

of the electron. He found the value of the ratio as $17 \cdot 6 \times 10^7$ e.m.u./gm. Many subsequent experiments proved that the specific charge of an electron is independent of the cathode plate or the anode plate or even the nature of the gas inside the discharge tube. Whatever the metal used for the plates or whatever the gas taken in the discharge tube, electrons available from the tubes are all identical. Further, the negatively charged particles emitted from the surface of a metal by photo-electric process (see 'Photo-electricity in chapter 3) or by hot filaments in vacuo are also electrons. Even the negatively charged beta-particles emitted by a radioactive body (see chapter 5) were also identified as electrons. As a consequence it was assumed that the particles like cathode rays were constituents of all atoms and they were generally called *electrons*. Electrons are, therefore, regarded as fundamental particles.

1.7. Charge, mass and energy of an electron :

In 1913, R.A. Millikan, an American Physicist, performed the famous oil-drop experiment on the measurement of electronic charge. From his experiment, it was found that the charge carried by an electron, $e = 1 \cdot 6 \times 10^{-20}$ *e.m.u.* or $4 \cdot 8 \times 10^{-10}$ *e.s.u.* The mass of an electron, $m = 9 \cdot 1 \times 10^{-28}$ gm.

It is significant to mention here that the mass of an atom of hydrogen, the lightest of all elements, is $M = 1 \cdot 67 \times 10^{-24}$ gm.

$$\therefore \quad \frac{M}{m} = \frac{1 \cdot 67 \times 10^{-24}}{9 \cdot 1 \times 10^{-28}} = 1835 \text{ (nearly)}$$

i.e., the mass of hydrogen atom is about 1835 times the mass of an electron. This shows that the ancient idea that an atom is indivisible is not correct ; an atom is divisible and by breaking atoms, we may get smaller particles.

Electrons are too tiny to be visible and therefore we cannot detect them by ordinary means. Their presence can be detected by noting the effects they produce *viz.*, fluorescence, ionisation, etc. To do these things, an electron must be set in motion. This is done generally by subjecting the electron to an electrostatic field. The energy of such a moving electron is usually expressed in *electron-volt*. It is defined in the following way : *An electron-volt is the kinetic energy acquired by an electron when it is made to move in an electric field of* 1 *volt potential gradient*. It is written as *ev*.

Erg is the absolute unit of energy while electron-volt is the practical unit. Their relation may be established in the following way :

1 ev = charge of an electron in *e.s.u.* $\times$ 1 volt.

$= 4 \cdot 8 \times 10^{-10} \times \frac{1}{300}$ [1 volt $= \frac{1}{300}$ *e.s.u.* of *p.d.*]

$= 1 \cdot 6 \times 10^{-12}$ ergs.

So far as practical cases are concerned, the energy corresponding to one electron-volt is very small. Bigger units, like kilo-electron volts (*Kev*) and million-electron volts (*Mev*) are very often used.

1 $Kev = 10^3$ ev and 1 $Mev = 10^6$ $e.v.$

Example : *In a vacuum tube, the accelerating voltage is* 200 *volts. What velocity do the electrons attain in this tube ? Mass of an electron*$=9{\cdot}1\times10^{-28}$ *gm. and charge*$=4{\cdot}8\times10^{-10}$ *e.s.u.*

Ans. For an electron, $\frac{1}{2}mv^2=e.V$ or, $v^2=\sqrt{\dfrac{2eV}{m}}$

Here, $e=4{\cdot}8\times10^{-10}$ *e.s.u.* ; $m=9{\cdot}1\times10^{-28}$ gm. ; $V=\frac{200}{300}$ *e.s.u.*

$$\therefore\quad v^2=\sqrt{\frac{2\times4{\cdot}8\times10^{-10}}{9{\cdot}1\times10^{-28}}\times\frac{200}{300}}\quad\text{or,}\quad v=2\times10^9\sqrt{\frac{48}{3\times91}}=8{\cdot}4\times10^8\text{ cm./sec.}$$

1.8. Motion of an electron in an electric and magnetic field :

(i) *Electric field* : Suppose an electron moving horizontally with velocity v passes between two parallel plates P_1 and P_2. If the *p.d.* between the plates be V and their distance apart is d, then the field intensity between the plates $E=\dfrac{V}{d}$.

Hence the force on the electron of charge e=field intensity$\times$charge$=Ee=\dfrac{eV}{d}$ and is downward to the positive plate P_1. Since the electric field is vertical, no horizontal force acts on the electron as it passes between the plates and hence its horizontal velocity remains unaffected. The motion of the electron is similar to that of a projectile thrown horizontally under gravity.

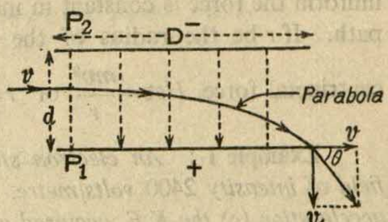


Fig 1.8(a)

In a vertical direction the downward displacement $y=\frac{1}{2}ft^2$ where f=acceleration of the electron$=\dfrac{Ee}{m}$ and t is the time. [m=electronic mass] $\therefore\ y=\frac{1}{2}.\ \dfrac{Ee}{m}.\ t^2$

In a horizontal direction, the displacement $x=v.t.$

Eliminating t, we obtain, $y=\frac{1}{2}.\left(\dfrac{Ee}{m}\right).\ \dfrac{x^2}{v^2}=\dfrac{Ee}{2mv^2}\ x^2\ \ldots.(i)$

The equation (i) shows that the path of the electron through the electric field is parabolic.

When the electron just passes the plates, $x=D$ and the vertical displacement at that moment is $y=\dfrac{Ee}{2mv^2}.\ D^2$ .. (ii). The electron then moves in a straight line as shown in fig. 1.8(a). The time for which the electron remains in the electric field, $t=\dfrac{D}{v}$. Thus the component of velocity v_1 gained in the direction of the field during this time, is $\quad v_1=f\times t=\dfrac{Ee}{m}\times\dfrac{D}{v}.$

Hence the angle θ at which the electron emerges from the field is given by

$$\tan\theta=\frac{v_1}{v}=\frac{Ee}{m}\times\frac{D}{v}\times\frac{1}{v}=\frac{Ee.D}{mv^2}=\frac{2y}{D}.\quad \text{[from equ. (ii)]}$$

(ii) *Magnetic field* : Now suppose that the electron moving with velocity v enters a uniform magnetic field of intensity H, acting perpendicular to the direction of motion. The force on the electron due to the magnetic field, we know, $F=H.e.v$ (see chapter six). The direction of the force is perpendicular to both H and v. Consequently unlike the electric field, the magnetic field simply deflects the electron without changing its speed.

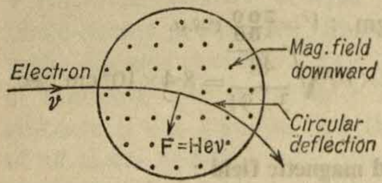


Fig. 1.8(b)

The force $H.e.v.$ is always normal to the path of the beam. If the field be uniform the force is constant in magnitude and the beam then travels in a circular path. If r be the radius of the circle, then since the force $F(=Hev)$ is the centripetal force, $Hev=\frac{mv^2}{r}$ or $r=\frac{mv}{H.e}$.

Example 1 : *An electron starts from rest and moves freely in an electric field of intensity* 2400 *volts/metre. Determine* (*a*) *the force on the electron* (*b*) *its acceleration* (*c*) *the K.E. acquired and* (*d*) *the velocity attained if the electron moves through a potential difference of* 90 *volts. Charge of an electron*$=1{\cdot}6\times10^{-20}$ *e.m.u.* ; *mass*$=9{\cdot}13\times10^{-28}$ *gm.* 1 *volt/cm*$=10^8$ *e.m.u./cm.*

Ans. (*a*) The force on the electron = field intensity × charge

$$=\left(\frac{2400}{100}\times10^8\right)\times(1{\cdot}6\times10^{-20})\text{ dynes}=3{\cdot}84\times10^{-11}\text{ dynes.}$$

(*b*) acceleration of the electron $f=\frac{\text{force}}{\text{mass}}=\frac{3{\cdot}84\times10^{11}}{9{\cdot}13\times10^{-28}}$

$$=4{\cdot}21\times10^{16}\text{ cm./sec}^2$$

(*c*) P.E. lost by the electron = K.E. gained.

Potential energy $=e.V=1{\cdot}6\times10^{-20}\times90\times10^8$ ergs $=1{\cdot}44\times10^{-10}$ ergs.

$\therefore$ $K.E.=\frac{1}{2}mv^2=eV=1{\cdot}44\times10^{-10}$ ergs.

(*d*) $v=\sqrt{\frac{2eV}{m}}=\left(\frac{2{\cdot}88\times10^{-10}}{9{\cdot}13\times10^{-28}}\right)^{\frac{1}{2}}=5{\cdot}62\times10^8$ cm/s (nearly)

Example 2 : *An electron moves at right angle to a magnetic field of* 150 *oersted and enters it with a velocity of* $0{\cdot}6\times10^{10}$ *cm./s. Find the radius of its circular path.* $e/m=1{\cdot}7\times10^7$ *e.m.u./gm.*

Ans. We know, $r=\frac{mv}{H.e}=\frac{1\times0{\cdot}6\times10^{10}}{1{\cdot}7\times10^7\times150}=2{\cdot}35$ cm.

Example 3 : *A narrow horizontal beam of electrons passes symmetrically between two metal plates mounted one on each side of the beam. The velocity of the electrons is* 3×10^9 *cm/s, the plates are* 3 *cm long and* 1 *cm apart. It is found that when a battery of* 568 *volts is connected to the plates, the electron beam just strikes the end of one of them. Calculate the value of e/m of electrons.*

Ans. When the beam of electrons enter the plates P_1 and P_2 they will be deflected so that travelling along the path AB, they just strike the end B of the plate P_2 [Fig. 1.8(c)].

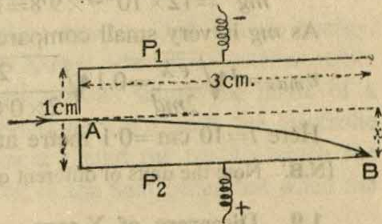


Fig. 1.8(c)

In this case, the deflection $x=\frac{1}{2}ft^2$.

But acceleration $f=\dfrac{\text{force}}{\text{mass}}=\dfrac{Xe}{m}$ and $t=\dfrac{D}{v}$

$$\therefore \quad x=\frac{1}{2}\cdot\frac{Xe}{m}\left(\frac{D}{v}\right)^2=\frac{1}{2}\times\frac{568\times10^8\times e}{m}\times\left(\frac{3}{3\times10^9}\right)^2$$

But $x=\frac{1}{2}$ of the distance apart of the plates P_1 and P_2=0·5 cm.

$$\therefore \quad 0{\cdot}5=\frac{1}{2}\times568\times10^8\times\frac{e}{m}\times\left(\frac{3}{3\times10^9}\right)^2$$

or, $$\frac{e}{m}=\frac{0{\cdot}5\times2\times9\times10^{18}}{568\times10^8\times9}=1{\cdot}7\times10^7 \text{ e.m.u./gm.}$$

Example 4 : *A particle having a charge of* $1{\cdot}6\times10^{-19}$ *coulomb enters midway between the plates of a parallel plate condenser. The initial velocity of the particle is parallel to the plane of the plates. A p.d. of* 300 *V is applied to the capacitor plates. If the length of the plates is* 10 *cm and they are separated by a distance of* 2 *cm, calculate the greatest initial velocity for which the particle will not be able to come out of the condenser plates. The mass of the charged particle is* 12×10^{-24} *kg.* [*I.I.T.* 1976]

Ans. See Fig. 1.8(c). P_1 and P_2 are the condenser plates. The electric field intensity between the plates $X=\dfrac{\text{volt}}{\text{distance}}=\dfrac{300}{2}=150$ volt/cm$=150\times100$ volt/metre.$=15\times10^3$ volt/metre.

The electric field will exert a vertical downward force on the particle. The horizontal velocity of the particle will remain unchanged as there is no horizontal force on it. Consequently, the particle will be deflected along a parabolic path. Now, the vertical downward acceleration on the particle $f=\dfrac{eX+mg}{m}$ and its deflection at t when it will arrive at the end of the plates $x=\frac{1}{2}ft^2=\frac{1}{2}f\left(\dfrac{l}{v}\right)^2$ where l=length of the plate and v=horizontal velocity of the particle.

Now, the condition for the particle not to come out of the plate is $x \geq d$ [d=half the distance apart of the plates]

i.e. $\frac{1}{2}f\left(\frac{l}{v}\right)^2 \geq d$ or $\frac{l}{v} \geq \sqrt{\frac{2d}{f}}$ or $\frac{v}{l} \leq \sqrt{\frac{f}{2d}}$

So, the maximum velocity of the particle is given by $v_{max}=l\sqrt{\frac{f}{2d}}=l\sqrt{\frac{eX+mg}{2d.m}}$

Now $eX=1 \cdot 6 \times 10^{-19} \times 15 \times 10^3$ Newton$=24 \times 10^{-16} N$.

$mg \ =12 \times 10^{-24} \times 9 \cdot 8 = 11 \cdot 76 \times 10^{-23} N$.

As mg is very small compared to eX, we can write

$$v_{max}=l\sqrt{\frac{eX}{2md}}=0.1\sqrt{\frac{24 \times 10^{-16}}{2 \times 0 \cdot 01 \times 12 \times 10^{-24}}}=10^4 \text{ metre/s.}$$

Here $l=10$ cm$=0 \cdot 1$ metre and $d=\frac{1}{2} \times 2$ cm$=0 \cdot 01$ metre.

[N.B. Note the units of different quantities in the problem].

1.9. Discovery of X-rays :

One of the most interesting episodes in the history of modern science is the accidental discovery of X-rays by Wilhelm Rontgen. In 1895, while experimenting with a cathode-ray tube, Rontgen found with amazement that a flourescent screen, placed near the tube became luminous and that a thick metal plate, placed between the tube and the screen cast a dark shadow on the latter. When a light substance, like a plate of aluminium or a sheet of paper or wood was interposed, the shadow cast was faint. Rontgen reasoned that some kind of invisible, yet penetrating rays of an unknown kind were being given out by the discharge tube. He subsequently found that this unknown radiation was coming from the glass wall of the tube at the place where it was struck by the cathode rays. In general, whenever high speed electrons are suddenly stopped by an obstacle, their energy is transformed into such radiation of high penetrative power. As the nature of the radiation was unknown to Rontgen, he called it *X-rays*, the letter *X* meaning, as it so often does in algebra, an unknown.

1.10. Production of X-rays :

(1) X-ray tube :

The discharge tube used for the production of X-rays is usually called an X-ray tube. Fig. 1.9 illustrates a common type of X-ray bulb or tube.

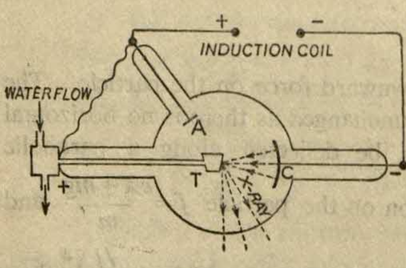


Fig. 1.9

The bulb is simply a cathode ray discharge tube with certain modifications. The cathode *C*, made of aluminium, is concave in shape while the anode, *A* is usually put in a side tube. In front of the cathode, there is another electrode, called *anticathode* or *target* (*T*), whose front surface facing the cathode is inclined at an angle of 45° to the axis of the concave cathode. The

anticathode is connected to the anode. The anticathode is so placed that the cathode ray stream, leaving the cathode surface normally, is focussed on to the anti-cathode. For general use of the X-ray bulb, the target is made of a metal of high melting point, like platinum, molybdenum or tungsten. With the gas pressure in the tube near about 0.001 mm. of mercury and with a potential difference of 30,000 to 50,000 volts applied between the electrodes by means of an induction coil, high speed electrons, emanating from the cathode, are suddenly stopped by the anticathode and their energy is partially converted into X-rays.

The electrons emitted by the cathode surface possess high kinetic energy, part of which is converted into heat and part into X-rays when the electrons strike against the anti-cathode. Consequently, the anti-cathode should be made of a metal of high melting point so that heat generated may not melt the electrode easily. There is also water circulation arrangement round the rod to which the anti-cathode plate is fixed. This takes away much of the heat generated when the bulb is worked.

Now-a-days, X-rays are produced by more improved type of devices like coolidge tube, betatron, etc.

(2) **Coolidge tube** : X-rays are produced, now-a-days, with a coolidge tube which is vacuum-field. Fig. 1.9(a) shows a sketch of the tube.

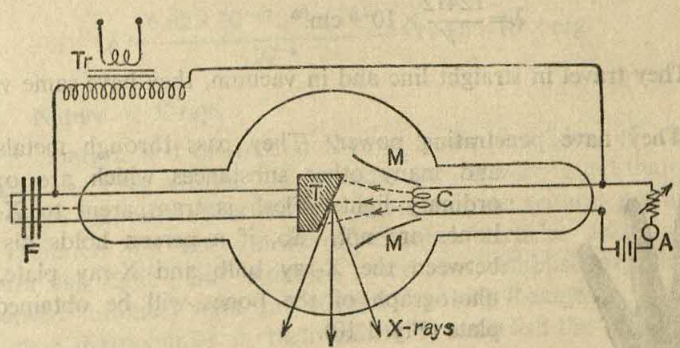


Fig. 1.9(a)

The cathode *C* is a thin filament of tungsten which is heated by current supplied by a battery or a step-down transformer. The heated filament emits copious thermions which are directed towards the target *T* by a high potential difference set up between the cathode *C* and the target *T* by means of a step-up transformer (*Tr*). The electrons are focussed properly by a molybdenum cylinder (M-M) which is kept at a negative potential with respect to the cathode. The target is a lump of metal such as tungsten which has a high melting point. A long, hollow handle made of copper is attached with it. When high-speed electrons from the cathode *C* strike against the target *T*, a lot of heat is produced which is dissipated by a flow of cold water through the hollow copper handle. Some fins *F* are also attached at the back of the handle for the same purpose.

The electrons in the coolidge tube which strike against the target acquire their energy from the applied high p.d. between the cathode and the target. Part

of this energy is converted into the energy of X-ray photon. The greatest advantage of this tube lies in the fact that the electron beam intensity and hence the X-ray intensity can be controlled by varying the filament heater current. The quality of X-ray output is also controlled by altering the p.d. across the tube.

The main disadvantage of the coolidge tube is that it does not function continuously ; its functioning is intermittent ; for the transformer connected between the cathode and the target develops alternating p.d. Usually a p.d. of 200 volt is applied to the primary and the secondary develops an alternating p.d. of about 50,000 volts. During the half-cycle when the filament gets a negative potential and the target a positive one, the tube generates X-ray but during the next half cycle, the potential reverses and the tube does not function. This disadvantage was, however, removed in later days by rectifying the a.c. with the help of rectifier valves.

1.11. Properties of X-rays :

(i) X-rays are not deflected by a magnetic or an electric field and hence they are not charged particles. They are electromagnetic waves of very small wave-length of the order of 10^{-8} cm. The shortest wave length of X-rays given out by a bulb depends on the applied voltage. If the bulb is operated on a voltage V volts, the shortest wave length of X-rays given out by the bulb is

$$\lambda=\frac{12412}{V}\times 10^{-8}\text{ cm.}^{*}$$

(ii) They travel in straight line and in vacuum, they have same velocity as light.

(iii) They have penetrating power. They pass through metals, ebonite and many other substances which are opaque to ordinary light. Flesh is transparent to X-rays but bones are not. So, if a person holds his hand in between the X-ray bulb and X-ray plate, a clear photograph of the bones will be obtained on the plate (Fig. 1.10).

Fig. 1.10

If the applied potential difference between the anode and the cathode be low, the X-ray produced is called *soft X-ray* which has got low penetrative power. On the other hand, if the potential difference be high, *hard X-rays*, having high penetrative power are produced.

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*When an electron passes across a potential difference V, its energy$=eV$. The maximum energy of a photon liberated by a collision of the above electron against an atom, according to quantum theory is given by $E_{max}=h\nu_{max}$ where h is the Planck's constant and ν_{max} the maximum frequency of the photon.: $\therefore$ $h\nu_{max}=eV$; Now $\lambda_{min}=\frac{c}{\nu_{max}}=\frac{ch}{eV}$ [c = velocity of light]

$h=6{\cdot}624\times10^{-27}$ erg sec ; $e=4{\cdot}8\times10^{-10}$ $e.s.u$; $c=3\times10^{10}$ cm/s.

So $\lambda_{min}=\frac{6{\cdot}624\times10^{-27}\times3\times10^{10}}{4{\cdot}8\times10^{-10}\times V/300}=\frac{12412}{V}\times10^{-8}$ cm (nearly)

(iv) They excite flourescence on barium platinocyanide and zinc sulphide.

(v) X-rays can ionise a gas.

(vi) They affect photographic plates.

(vii) They produce photo-electric effect *i.e.*, they liberate electrons when they fall on metallic substances.

(viii) X-rays exhibit like light, interference, diffraction and polarisation.

Example 1 : *An X-ray bulb is operated on 50 KV. Find the shortest wave-length of the X-rays given out by the bulb.*

Ans. The shortest wave length $\lambda=\frac{12412}{V}\times 10^{-8}$ cm.$=\frac{12412}{50\times 10^{3}}\times 10^{-8}$ cm.

[1 $KV=10^{3}$ V]

$=0{\cdot}25\times 10^{-8}$ cm (nearly)

$=0{\cdot}25$A° [1A°$=10^{-8}$ cm.]

[N.B. The wave length is usually expressed in *Angstrom unit* (symbol A°)].

Example 2 : *What is the energy of an X-ray photon of wavelength 10^{-8} cm. ? Given Plank's constant $h=6{\cdot}62\times 10^{-27}$ erg. sec ; Velocity of light $c=3\times 10^{10}$ cm/sec.* [*H. S. Exam. 1984*]

Ans. We know, energy of the photon $E=h\nu=h.\frac{c}{\lambda}$

$$\therefore \quad E=\frac{6.62\times 10^{-27}\times 3\times 10^{10}}{10^{-8}}=19{\cdot}86\times 10^{-9}\text{ erg.}$$

1.12. Nature of X-rays :

When Rontgen first discovered X-rays, he had no idea of their nature. So, he called the rays X-rays. Afterwards when it was found that electric and magnetic fields had no effect on the radiation, the natural presumption was that the radiation did not consist of charged particles and that it was a wave-motion like light. But Rontgen could not establish experimentally the identity between ordinary light and X-rays. Not only Rontgen, several scientists, later on, were disappointed in their efforts to establish the identity.

Success in this respect was first achieved by a German scientist Von Laue. He was successful in exhibiting diffraction phenomenon in X-rays with the help of a crystal. Afterwards, scientists like Bragg, Compton, Barkla, and others, demonstrated that X-rays can be reflected, refracted, polarised like ordinary light. In this way it was finally shown that the nature of X-rays is similar to the nature of light. For this reason, X-rays are regared as a kind of 'invisible light.'

1.13. Applications of X-rays :

To-day X-rays are used in hundreds of different ways. Some of the important applications of X-rays are mentioned below :

(i) **In medical science :** X-rays are widely used both in medical diagnosis and cure. X-ray photographs, called *radiographs*, are used to study fracture of bones and location of foreign bodies like bullets, pins etc. in the human body, for the diagnosis of tumors, tuberculosis, stones in kidneys and in gall bladder,

Very hard X-rays are used by physicians in the treatment of malignant growth, as it is found that cancer cells may be destroyed by an exposure of X-rays. Nevertheless precaution is always taken to avoid unwanted doses of X-rays, as these have harmful effects on normal cells.

(ii) **In industry** : In industry radiographs are used to reveal hidden flaws in metal castings and welded joints, beams and joists, cracks in wood, nails, procelain and other insulators, to detect defects in automobile tyres, tennis and golf balls, to detect hidden gems in shells, etc.

(iii) **In scientific research** : X-rays are widely used in higher scientific research, in studying the structure of crystals, structure and properties of atoms and various other research works.

(iv) **In police department** : Customs departments use X-rays in curbing smuggling activities and police departments in curbing criminal activities. They use X-rays to detect contraband goods hidden in wooden or metal boxes. Sometimes, criminals swallow contraband ornaments or coins to escape detection. In these cases, X-rays are very helpful in detecting the crime. Besides, the forensic department is now-a-days using X-rays to investigate criminal cases like murder, assassination etc.

*1.14. Effect of X-rays on human body :

Because of their ability to ionise atoms, X-rays are a source of danger to human body. The biological effects of ionising radiations is not fully understood, but it is known they are capable of causing destruction of cells. This property not only causes serious lesions to form on healthy parts of the body but is paradoxically used to heal cancerous growth on diseased part of the body. The effects are dependent on the intensity of the radiation and time of exposure *i.e.* **dose**=intensity$\times$time.

The absolute unit of dose *i.e.* the quantity of radiation received is *Rontgen.*

Definition : *The Rontgent (R) is that quantity of radiation, which on passing through* 1 cm^3 *of air at N.T.P. causes ionisation such that total charge on the ions, irrespective of sign, is* 1 *e.s.u. of charge* (*i.e.* 3×10^{-19} *coulomb*).

Exercises

Essay type :

1. Describe, in steps, the phenomenon observed in a discharge tube when the pressure of the air in the tube is reduced gradually.

2. What are cathode rays ? What are their properties ? How are these particles produced ? [*cf. H. S. Exam. 1979*]

3. Show that the radius of curvature of a charged particle moving at right angles to a magnetic field is proportional to its momentum.

4. What are X-rays ? Describe an X-ray bulb and state the principal properties of X-rays. [*H. S. Exam. 1979 '80*]

5. Why are X-rays called a kind of 'invisible light' ? State two of their properties which are similar to those of visible light. [*H. S. Exam. 1984*]

6. What are the main characteristics of X-rays ? Describe an arrangement for the production of X-rays in the laboratory. [*H. S. Exam. 1982*]

Short answer type :

7. Under what circumstances does air behave like a conductor of electricity ?

8. Why is reduction of pressure necessary for an electric discharge to pass through air ?

9. What is electron-volt ? What is its relation with erg ?

10. Who discovered—*X*-rays ? "*X*-rays have more energy than light"—Explain.

11. What is the basic difference between *X*-rays and cathode rays ?

12. What are the applications of *X*-rays ?

13. What is the charge of an electron ? How would you prove that this charge is negative ?

14. Why is electron called a fundamental particle ?

15. At what pressures are Faraday dark space and Crooke's dark space formed in a discharge tube ?

16. How much is the mass of a hydrogen atom heavier than that of an electron ?

17. What are hard and soft *X*-rays ?

18. Along what path an electron will travel in (i) a prependicular electric field and (ii) a perpendicular magnetic field ?

19. (*a*) If an electron is not deflected in passing through a region of space, can we be sure that there is no magnetic field in that region ?

(*b*) If a moving electron is deflected sideways in passing through a region of space, can we be sure that a magnetic field exists in the region ?

(*c*) An electron moving with a constant velocity passes through a region of space without any change in its direction. If E and B represent the magnetic and electric fields respectively, this region of space may have (i) $E=0$, $B=0$ (ii) $E=0$; $B\neq 0$ (iii) $E\neq 0$; $B=0$ (iv) $E\neq 0$; $B\neq 0$.

20. A proton and an electron are moving with same K.E. along the same direction. When they pass through a uniform magnetic field perpendicular to the direction of their motion, will they describe circular paths of same radius ? [*I.I.T. 1985*]

Objective type :

21. (i) A beam of X-rays and a beam of cathode rays are successively passed through a magnetic field and an electric field. There will be deflection (i) in both case (ii) in the first case only (iii) in the second case only. Which is correct ?

(ii) A beam- of *X*-rays consist of (i) negatively charged particles (ii) positively charged particles (iii) electromagnetic waves of short wave length. Which is correct ?

(iii) The X-ray beam coming from an X-ray tube will be (a) monochromatic (b) having all wavelengths smaller than a certain maximum wavelength (c) having all wavelengths larger than a certain minimum wavelength (d) having all wavelengths lying between a minimum and a maximum wavelength. Which is correct ? [*I.I.T. 1985*]

[**Hints** : (d).]

(iv) Is electronic mass heavier than the mass of hydrogen nucleus ? Ans, (*a*) heavier (*b*) lighter (*c*) equal. Which is correct ?

Numerical Problems :

22. An electron is moving with a velocity 10^7 metres/s. Find its kinetic energy in electron-volts. $m=9{\cdot}1\times 10^{-28}$ gm ; $e=4{\cdot}8\times 10^{-10}$ e.s.u. and 1 volt$=\frac{1}{300}$ e.s.u. [Ans. $2{\cdot}84\times 10^{2}$ *ev*]

23. After passing through a potential difference of 60 KV, starting from rest, an electron acquires a velocity of $1{\cdot}46\times10^{10}$ cm/s. Find the ratio of charge to mass of an electron in coulomb per gram. [*Jt. Entrance 1981*]

[Hints : $\frac{1}{2}mv^2=eV$ or $\frac{e}{m}=\frac{v^2}{2V}=\frac{(1{\cdot}46\times10^{10})^2}{2\times60\times10^3\times10^7}=1{\cdot}78\times10^9$ coulomb/gm.]

24. Two parallel plates 5 cm. apart are given a p.d. of 500 volts. An electron starts from rest and travels through the plates. Calculate the velocity attained and the distance traversed by it in $6{\cdot}6\times10^{-9}$ sec. $m=9{\cdot}11\times10^{-28}$ gm ; $e=1{\cdot}603\times10^{-20}$ e.m.u.

[Ans. $1{\cdot}16\times10^9$ cm/s ; 3·8 cm.]

25. Electrons move at right angles to a magnetic field of 100 oersted with a velocity of $0{\cdot}5\times10^{10}$ cm./s. Calculate the radius of circular path. $e/m=1{\cdot}76\times10^7$ e.m.u./gm.

[Ans. 2·84 cm. (nearly)]

26. An X-ray bulb is working at 30 kv. Find the shortest wavelength of X-rays produced by the bulb. [Ans. 0·41A°]

27. The shortest wavelength given out by an X-ray bulb is 1A°. What is the p.d. between the cathode and the anode of the bulb ? [Ans. $12{\cdot}4\times10^3$ volt]

28. What should be the minimum *p.d.* at the end of a coolidge tube so as to produce X-rays of wavelength 0·8A° ? $h=6{\cdot}62\times10^{-34}$ joule-sec ; $e=1.60\times10^{-19}$ coulomb.

[*Jt. Entrance 1983*] [Ans. 15.5KV]

29. An electron of charge $1{\cdot}6\times10^{-19}$ coulomb and mass $9{\cdot}1\times10^{-31}$ kg. travels along X-axis with velocity 10^6 metre/s and enters into a uniform electric field which acts perpendicular to the X-axis. The intensity of electric field is 10^2 volt/metre. If the field extends over a distance of 2 cm along the X-axis, find the deflection of the electron along the direction of the field when it comes out of the field. [*Jt. Entrance 1983*] [Ans. 3·51 cm.]

30. An electron is accelerated through 15,000 volts and is then allowed to circulate at right angles to a uniform magnetic field with $H=250$ gauss. What is its path radius ? Mass of electron$=9{\cdot}13\times10^{-28}$ gm and charge$=1{\cdot}6\times10^{-20}$ e.m.u. [Ans. 1·6 cm (nearly)]

31. Electrons bombarding a coolidge tube produce X-rays of wave length 1A°. Find the energy of the X-ray photon. $h=6{\cdot}62\times10^{-27}$ erg-sec ; $c=3\times10^{10}$ cm/s. [Ans. $19{\cdot}86\times10^{-9}$ erg]

32. A particle of mass 1×10^{-26} kg. and charge $+1{\cdot}6\times10^{-19}$ coulomb travelling with a velocity 1.28×10^6 m/s in the $+x$ direction enters a region in which a uniform electric field E and a uniform magnetic field B are present such that $Ex=Ey=0$ and $Ez=-102{\cdot}4$ kv/m and $Bx=Bz=0$, $By=8\times10^{-2}$ weber/m^2. The particle enters this region at the origin at $t=0$. Determine the location ((x, y, z co-ordinates) of the particle at $t=5\times10^{-6}$s.

[*I. I. T. 1982*] [Ans. $x=6{\cdot}4$ m, $y=0$, $z=18{\cdot}48$ m]

33. Starting from rest, the velocity of an electron became $1{\cdot}46\times10^{10}$ cm/s while passing through a *p.d.* of 60 K. V. Find the ratio of e/m of electron in coulomb per gm unit.

[*Jt. Entrance 1981*] [Ans. $1{\cdot}78\times10^6$]

34. An oil drop of radius $1{\cdot}2\times10^{-4}$ cm is given a charge of $4{\cdot}77\times10^{-10}$ e.s.u. The drop is to be kept steady by creating an electric field between two plates A (upper) and B (lower). What plate should be positively charged and what should be the intensity of electric field ?

[*Jt. Entrance 1976*] [Ans. upper plate ; 12·6 e.s.u./cm]

2

THERMIONIC EMISSION AND ITS APPLICATIONS

2.1. Introduction :

Metals, in general, contain copious free electrons which are not bound to any particular molecule of the metal. They move freely in the material. With the increase of temperature, their motion increases. For this reason, when a piece of metal is placed in a vacuum and heated, some of the free electrons reach speeds high enough to enable them to break away and form a cloud near the surface. This 'boiling-out' of electrons from a metal is called **thermionic emission.** Edison, in 1885, noticed this effect when he was experimenting with filament lamps. The electrons emitted from the hot surface are called **thermions** and the current they produce is called **thermionic current.**

As gas can exert pressure, in the same way, the free electrons in a metal are also capable of exerting pressure. This is known as electron-gas pressure. Inspite of the electron-gas pressure the electrons are not ordinarily emitted from the metal surface because the force with which they are held bound to the surface predominates over this pressure. Electrons will have to do some work, if they want to leave the metal surface. This work is referred to as the *work-function* of the metal surface. Work-function is different for different metal surfaces. On this theory, an electron escapes from a metal surface when it has sufficient kinetic energy to overcome the surface forces. At low temperatures, a negligible number of free electrons have sufficient energy to escape. When the temperature is increased, the free electrons gain kinetic energy by collision with more energetic metal atoms and some are able to escape. The process is somewhat similar to the evaporation of molecules from a liquid surface. As a matter of fact, with the help of this analogy and applying the laws of kinetic theory and thermodynamics, Richardson gave a formula for the saturation thermionic current at absolute temperature T as follows :

$$I=AT^2 e^{-b/T}$$

where A is a constant which has a value of about 60 for pure metals such as tungsten or tantalum but varies widely for other practical emitters and b is a measure of the work an electron must perform when leaving the emitter surface. The above equation is known as *Richardson's formula*. The formula states that in order to get copious supply of electrons, the temperature of the metal surface must be increased or the emitter should be selected in such a way so as to have low work function. Experiments show that any metal coated with barium or strontium, gives copious supply of electrons at about 800°C.

Work function of a metal : The constant b in the above formula is a characteristic of a metal. Theory shows that b is related to the minimum kinetic energy required by an electron to escape from the surface ($\frac{1}{2}mv_0^2$), by the formula,

$$b=\frac{\frac{1}{2}mv_0^2}{k}$$

where k is known as Boltzmann constant.

The minimum escape energy of the electron is usually expressed in the form : $\frac{1}{2}mv_0^2=e\phi$ where e is the charge on the electron in coulomb and ϕ is the p.d. in volts through which an electron must pass starting from rest, in order to acquire the same amount of kinetic energy. Hence, $b=\frac{e\phi}{k}$. The constant ϕ is known as the *work function* of the metal ; it is a measure of the work that the electron must do in overcoming the surface forces on escaping.

2.2. Two-electrode thermionic valve or Diode :

Following Edison's experiment, Fleming, in England, showed that electrons sent out from the hot filament could be attracted to a positively charged plate near by. Fleming thus devised the first thermionic valve, known as Fleming's valve at the beginning but latter on as '*Diode*'.

It consists of a filament F and a plate P enclosed in a glass bulb which is highly evacuated (Fig. 2.1). When the filament is heated to incandescence by means of a battery B_1, it emits electrons by thermionic process. Now, if the plate P is given *a positive potential* with respect to the filament F by another battery B_2, the electrons are attracted towards the plate and a current passes from the filament to the plate as indicated by a milliammeter A included in the plate circuit. But if the terminals of the battery B_2 be reversed, giving the plate P *a negative potential* with respect to filament, no current flows. So, the current flowing between the filament and the plate is always uni-directional. For this reason, it is called *a valve.*

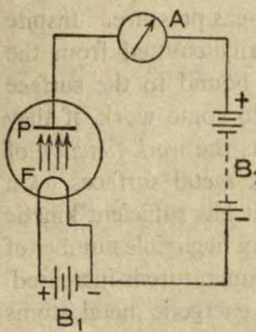


Fig. 2.1

Ordinarily, the voltage of the battery B_1 is not very high. On the other hand, the voltage of the battery B_2 is high. For this reason, the former is called *a low tension* battery and the latter *a high tension* one.

Fig. 2.1(a) shows the construction of a diode. Generally, the plate P of the diode is called **anode** or simply **plate** and the electron-emitting filament is called **cathode.** The cathode of a diode valve may be directly heated (Fig. 2.1) or indirectly heated. In a directly heated type of diode, the filament is directly connected to the battery. It is made of a ribbon of very fine wire and has a resistance such that when proper voltage is applied to it, the filament soon acquires the requisite temperature for emitting copious electrons.

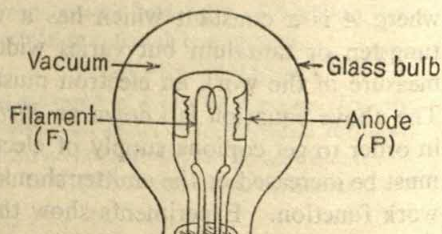


Fig. 2.1(a)

An indirectly heated type of diode has the advantage that it can be used with a D.C. or A.C. supply.

The schematic symbol used to represent this cathode diagrammatically and its real appearance are shown in figures 2.2(a) and 2.2(b). Here C is a nickel cylinder having a layer of oxide on its outer surface. There is a heating wire H inside this cylinder but without any contact with it. This wire is generally made of tungsten. The wire is heated by current supplied by either a d.c. or an a.c. source of supply and the heat radiated or conducted by the wire heats up the cylinder C. Heated in this way, when the cylinder attains a suitable temperature, electrons are emitted from the oxide coating. It is clear that both a.c. and d.c. may be used in this cathode. Further, the potential in this cathode is everywhere same and it has a long life.

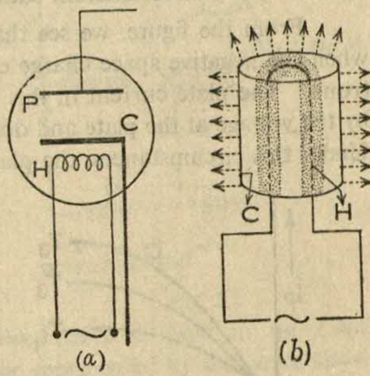


Fig. 2.2

2.3. Space charge :

The total number of electrons emitted by a cathode of a diode depends on the effective temperature of the cathode. The plate voltage has no effect on the number of electrons emitted by the cathode. The function of positive plate voltage is simply to attract the electrons towards the plate. The plate voltage, therefore, determines the number of electrons reaching the plate. When the p.d. between the plate and the cathode of a diode is low, all the electrons emitted from the cathode, cannot reach the plate. Some of the electrons are found to gather near the cathode and form a cloud of electrons. This cloud that is formed in the inter electrode space between the cathode and the plate is called the **space charge.**

The repulsive force exerted by the space charge retards the emission of electrons from the cathode ; some of the emitted electrons are even sent back to the cathode. With the gradual increase of p.d. between the plate and the filament, increasing number of electrons reach the plate. Consequently, the plate current increases. If the plate voltage is made sufficiently high, a point is reached eventually when all the electrons emitted from the cathode are attracted to the plate and the effect of space charge is completely overcome. Further increase of plate voltage cannot increase the plate current and the emission from the cathode limits the maximum current flow. This maximum current is called the **saturation current.** The effect of space charge on the function of a diode is considerable.

2.4. Characteristic curves of a diode :

The relation between the plate current (I_p) in a diode and the plate voltage (V_p) for a constant cathode temperature, can be represented by a curve, known as the **characteristic curve** of the diode. Keeping the cathode at different

temperatures, a family of such curves may be drawn. From these characteristic curves we may get valuable information regarding the behaviour of the valve. Fig. 2.2(*c*) shows the nature of characteristic curves obtained by plotting V_p against I_p for three different cathode temperatures T_1, T_2 and T_3 ($T_3 > T_2 > T_1$).

From the figure, we see that all the curves are similar at low plate voltage when the negative space charge effect is predominant in limiting the flow of electrons. The plate current in this low plate voltage region is completely controlled by the voltage at the plate and does not depend on the temperature of the cathode. Under this circumstances, the plate current is said to be **space charge limited.** As the plate voltage is gradually increased, an increasingly greater portion of the total supply of emitted electrons are attracted to the plate and the effect of the space-charge is eventually overcome. This is evidenced by the gradual flattening of the characteristic curves as the plate voltage is increased. When the plate voltage is sufficiently high the entire supply of emitted electrons reaches the plate and the current attains a constant value. This is called *saturation current* which is independent of plate voltage. The horizontal portion of the characteristic curves indicates that the saturation current does not change when the plate voltage is increased.

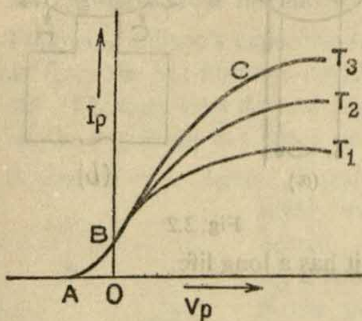


Fig. 2.2(c)

The portion *AB* of the characteristic shows that some plate current (*OB*) is available even if the plate voltage is zero. This proves that a few electrons can reach the plate even when there is no attractive force of the positively charged plate. This happens because when electrons are ejected from the cathode surface, they are ejected with some initial kinetic energy. This initial kinetic energy helps a few electrons to reach the plate and a small plate current is produced.

Langmuir-Child's Law :

An interesting relation holds for diodes that are operated in the plate-voltage region where space-charge limits the value of the plate current. From the characteristic curves, it is found that for plate potential, the current is less than the saturated current and is approximately proportion to $\frac{3}{2}$ power of the voltage between the plate and the cathode. This may be expressed mathematically by the formula $I = kV^{\frac{3}{2}}$ where k is a constant depending on the shape and distance apart of the electrodes. This relation is known as '*Three halves law*' or the Langmuir-Child's law.

The saturation current is not quite constant in practice and I increases slowly with increasing p.d.'s. One important reason for this is the reduction in work function of the cathode as the electric field strength increases at its surface.

2.5. Use of diode ; rectification :

A diode is now widely used for the purpose of rectification *i.e*,, for converting A.C. into D.C. The rectification process can be understood from the following description.

Fig. 2.3 shows the plate P and the filament F of a diode. A low tension battery ($L.T.$) is used to heat the filament. If an A.C. voltage is now applied between the plate and the filament, the plate will acquire alternatively positive and negative potential with respect to the filament. During that half cycle when the plate is positive, it attracts the electrons emitted by the filament and a current flows through the plate-filament circuit *i.e.*, through the resistor R. During the next half cycle, when the plate is negative, it repels the electrons and no current flows through the resistor R. So, we see that through the resistor, an intermittent but always *unidirectional* current flows. If the input A.C. voltage be represented by the wavy curve as shown in fig. 2.4(a), the output current through the resistor R may be represented by the curve shown in fig. 2.4(b). From the figure, it is clear that during the upper half of the A.C. supply only, current flows through the resistor R but no current flows during the lower half. This is known as *half-wave rectification.*

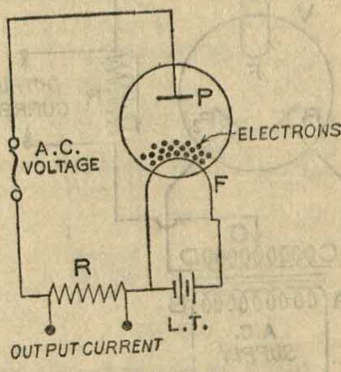


Fig. 2.3

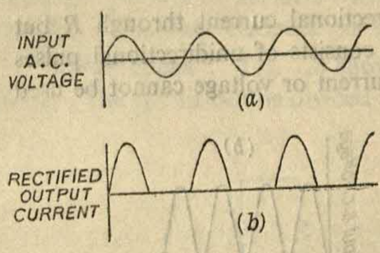


Fig. 2.4

The current in R consists of a series of separate pulses which flow during alternate half-cycles of the mains supply. If the diode characteristic was perfectly linear, the pulses would consist of half-sine waves. Suppose a.c. input voltage is given by $E=E_0 \sin \omega t$ and the equation of the diode characteristic is $I=K.V$ where K is a constant. The p.d. across R at a time t during a half-cycle when the plate P is positive relative to the filament F, is given by $V=I.R.$. Hence, the p.d. across the diode at this instant is $V'=E-I.R=E_0 \sin \omega t-I.R.$

So, $I=KV'=KE_0 \sin \omega t-K.I.R$

$$\text{or } I=\frac{K.E_0 \sin \omega t}{1+K.R.}$$

i.e. the current is sinusoidal during half-cycle [Fig. 2.4(b)].

By using two diodes, both loops of the A.C. cycle can be used and by passing the output through suitable circuits consisting of resistors and capacitors, the pulsations can be smoothed out almost completely into continuous direct current. This process is known as *full wave rectification.*

The valve used for full-wave rectification is called **duo-diode** (fig. 2.5). It employs two plates with a common filament in one enclosure. P_1 and P_2 are the plates. They are connected to the two ends C and D of the secondary of a

transformer. The mid-point O of the secondary is connected to the filament F through a resistor R. When an alternating current is supplied to the primary of the transformer, the plates P_1 and P_2 become alternately positive and negative with respect to the filament F. During one half-cycle when P_1 is positive and P_2 negative with respect to the filament, the electron current flows from the filament to P_1 but during the other half-cycle when P_2 is positive and P_1 negative, the electron current flows from the filament to P_2. It is to be noted that in both the cases, *the conventional direction of current* (shown by arrow head) *through the resistor R is unidirectional.* In this way, full-wave retification can be done with a duo-diode valve.

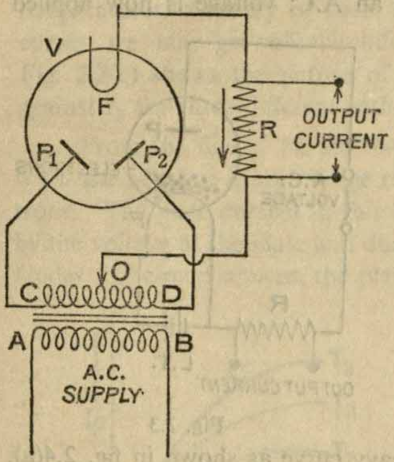


Fig. 2.5

The duo-diode, no doubt, produces unidirectional current through R but the current is not smooth. The output current consists of unidirectional pulses as shown in Fig. 2.6(b). Such a pulsating d.c. current or voltage cannot be used

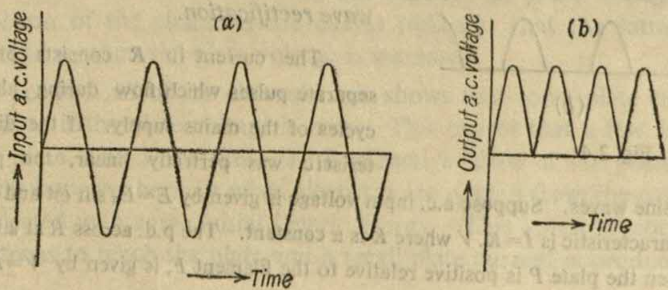


Fig. 2.6

directly for operating any electronic circuit. The pulsations can however be easily smoothed out by filters. Some rectifying valves contain a little mercury vapour. When electrons flow through them they ionise the mercury atoms. The ions and the electrons thus produced make the valve a good conductor and reduce the voltage drop across it ; they therefore, allow more of the voltage from the input to appear across the load R.

2.6. Three-electrode valve or Triode :

In 1907, the American experimenter Lee De Forest made an important addition to the diode by inserting a third electrode between the plate and the filament. This additional electrode is known as *grid* (G). Introduction of grid has vastly extended the usefulness and the applicability of the valve. It is called

a *triode* because of the fact that it contains three electrodes [Fig. 2.7(a)]. The triode valve consists of a highly evacuated glass bulb containing (i) a filament F usually made of platinum, tungsten or tantalum and is coated with a substance like barium oxide to give a good emission of electrons at low temperature, (ii) a grid G which may be a flat metel gauze placed above the filament or a spiral of wire mounted with the filament as axis. The grid is generally placed closer to the filament than to the plate and (iii) a plate P which is in the form of a flat plate if the grid is flat or in the form of a cylinder surrounding the grid if the latter is spiral. A triode is diagrammatically represented as shown in fig. 2.7(b).

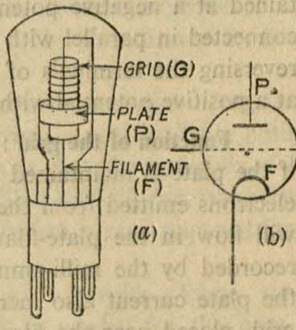


Fig. 2.7

The filament is provided with two external leads for connecting it to a low tension battery which supplies the heating current. The grid and the plate are each provided with a single external lead so that they may be given any desired potential—positive or negative—with respect to the filament by the use of suitable batteries. The three electrodes of the triode are well insulated from one another.

Triode circuit : Fig. 2.8 shows a simple circuit arrangement for the use of triode. The circuit can be divided into three parts :

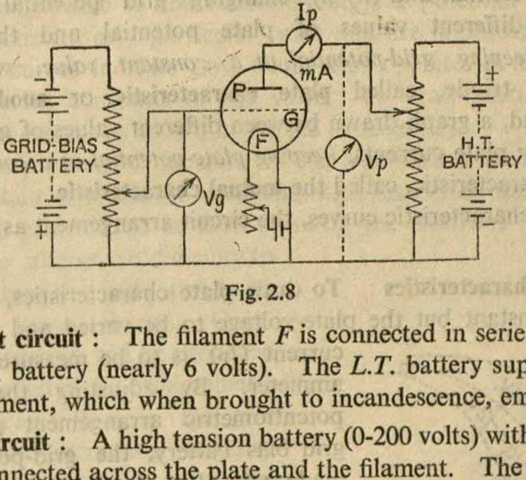


Fig. 2.8

(i) **Filament circuit :** The filament F is connected in series with a rheostat and a low tension battery (nearly 6 volts). The *L.T.* battery supplies the heating current to the filament, which when brought to incandescence, emits electrons.

(ii) **Plate circuit :** A high tension battery (0-200 volts) with a potentiometer arrangement is connected across the plate and the filament. The positive terminal of the *H.T.* battery is connected to the plate P and the negative terminal to the negative of the filament. By this arrangement the plate can be given positive potential from 0 to 200 volts with respect to the filament. A miliammeter (*mA*) in series with the plate measures the plate current (I_p) and a voltmeter (V_p) across the plate and the filament measures the potential applied to the plate.

(iii) **Grid circuit :** Another battery, known grid-bias battery, of lower voltage (0—20 volts) with a potentiometer arrangement, is connected to the grid (G) and the filament. The negative terminal of grid bias bettery is connected to

the grid and the positive terminal to the filament. As a result, the grid is maintained at a negative potential with respect to the filament. A voltmeter (V_g) is connected in parallel with the grid to measure the voltage given to the grid. By reversing the terminals of the grid bias battery, the grid can also be maintained at a positive potential with respect to the filament, whenever necessary.

Function of the grid : From the above circuit arrangement it is obvious that if the plate is maintained at a positive potential with respect to the filament, the electrons emitted from the filament will be able to reach the plate and a current will flow in the plate-filament circuit. This is known as *plate current* which is recorded by the milli-ammeter (*mA*). With the increase of the plate potential, the plate current also increases until it reaches a maximum value. Now, if the grid, placed near the filament, be given a negative potential with respect to the filament, the grid will repel the electrons back to the filament, thereby decreasing the plate current. On the other hand, if the grid is maintained at a positive potential with respect to the filament, the grid will attract the electrons which will quickly move towards the plate, thereby increasing the plate current. In this way, the plate current can be increased or decreased at will, by suitably varying the grid-potential.

2.7. Static characteristics of a triode :

In a triode, the plate current may be changed in two ways : (i) by changing plate potential and (ii) by changing grid potential. If we draw a graph between different values of plate potential and the corresponding plate currents, *keeping grid-potential at a constant value*, we get one static characteristic of triode, called **plate characteristic** or **anode characteristic.** On the other hand, a graph drawn between different values of grid-potentials and the corresponding plate currents, *keeping plate-potential at a constant value*, gives another static characteristic, called the **mutual characteristic.**

To get the characteristic curves, the circuit arrangement as shown in fig. 2.8, is necessary.

(i) **Plate characteristics** : To draw plate characteristics, grid-voltage (V_G) is to be kept constant but the plate-voltage to be varied and at each step plate current (I_P) is to be measured from the milli-ammeter. By adjusting the slider of the potentiometric arrangement provided with the grid bias battery, the grid-potential at first, is made zero ($V_G=0$) and in this condition, the plate is given various positive potentials, starting from zero value to a high value (say, 200 volts) and at every stage current is noted from the milliammeter. If a graph is now drawn between ($V_P - I_P$) a curve like the curve no. 1 [Fig. 2.9] will be obtained. When the plate-potential is low, the plate-current is also low. The reason, as has been mentioned earlier, is space charge. With the increase of plate

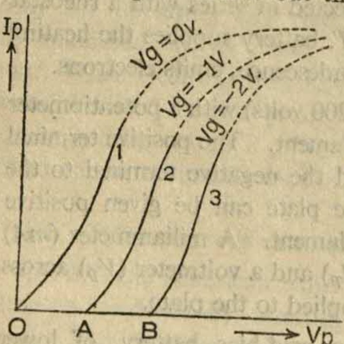


Fig. 2.9

potential, the space charge effect decreases and hence the plate current increases. Finally when the plate-potential is sufficiently high, the space charge effect is totally absent and the plate current becomes saturated. This is indicated by the top horizontal portion of the characteristic curves.

The curve no. 2 shows the $V_P - I_P$ characteristic curve when grid-voltage is kept constant at -1 volt. Figure shows that plate current is not available until the plate potential increases from 0 to A. The reason is that the negative grid potential repels some of the electrons emitted by the filament and increases the space-charge effect. So, a greater plate potential than before is now required for commencement of plate current.

Curve no. 3 indicates the $V_P - I_P$ curve when $V_G = -2$ volts. Since the grid is more negative, a still greater plate voltage (OB) is needed for the commencement of plate-current.

Fig. 2.9 shows that the characteristic curves, in general, are bent at the lower portion but are straight and parallel at the upper region. The plate characteristics help us to find the effective resistance, called *plate resistance* (R_P) of the triode. This is the internal resistance of the device for the given grid and plate voltage and can be obtained from the slope of the characteristic curves. The plate resistance R_P is given by $R_P = \left(\frac{\delta V_P}{\delta I_P}\right) V_G = \text{const.}$

(ii) **Mutual characteristics :** Mutual characteristics give an idea as to how plate current changes due to the alteration of grid potential when the plate-potential is kept constant at a particular value and are important for more than one reason. The curves indicate how triode can operate as an amplifier and give several important constants of the valve. To draw mutual characteristic curves, circuit arrangement as shown in fig. 2.8 is necessary.

Adjusting the position of the slider of the plate-circuit rheostat, the plate is given, say, a constant potential of $+80$ volt with respect to the filament. If in this condition, the grid-voltage be zero ($V_G = 0$), the electrons emitted from the filament, will have almost no difficulty to reach the plate, except the little resistance offered by the space charge. Consequently, a plate current will be produced which will be recorded by the milliammeter in the plate circuit. If now, the grid be given gradually increasing negative potential by adjusting the rheostat in the grid circuit, the electrons will face increasing difficulty to reach the plate. As a result, the plate current will diminish slowly. Fig. 2.10 shows this change of

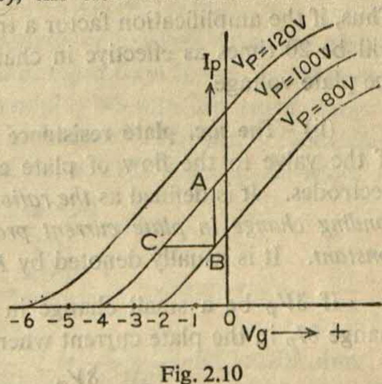


Fig. 2.10

plate current with the change of grid voltage (when plate voltage is kept constant at $+80$ volts). Figure shows that the plate current stops when grid-voltage is -2 volts. This value of grid voltage is called the *cut off point*. If the grid be given increasing positive potential, instead of negative, the plate current also

increases till it becomes saturated. The portion of the graph, then, becomes parallel to the V_g axis.

A family of such curves may be drawn by keeping the plate at different constant potentials like 100 volt, 120 volt etc. The nature of the curves has been shown in fig. 2.10. These curves are known as *mutual characteristic curves.* From a study of these curves, we can say (i) the plate current increases proportionally with the increase of plate potential ; the reason is simple ; with the increase of plate potential, the space charge diminishes and more and more electrons can reach the plate.

(ii) With the increase of plate potential, the cut-off point of grid bias also (negative) increases.

(iii) The top and bottom portions of the curves are bent but the middle portions are straight and parallel.

2.8. Valve constants :

The most important design factors of a triode or the valve constants are (i) the amplification factor (μ) (ii) the a.c. plate resistance (R_p) and (iii) the mutual conductance (g_m).

(i) **The amplification factor** : It is a measure of the relative effectiveness of the control grid in overcoming the electrostatic field of the plate. It is defined as the *ratio of a small change in plate voltage to the small change in grid voltage required to produce the same change in the plate current.* It is usually denoted by μ.

If δV_p be the small change in plate voltage and δV_g the small change in grid voltage for the same change in plate current (essentially holding it constant), then

$$\mu=\frac{\delta V_p}{\delta V_g} \text{ (when } I_p \text{ is constant)} \ldots \text{(i)}$$

Thus, if the amplification factor a triode be 20, a change in grid (or signal) voltage will be 20 times as effective in changing the plate current as the same change in the plate voltage.

(ii) **The a.c. plate resistance** : It is a measure of the internal opposition of the valve to the flow of plate current, when a.c. voltages are applied to the electrodes. It is defined as *the ratio of a small change in plate voltage to the corresponding change in plate current produced thereby, when the grid voltage is kept constant.* It is usually denoted by R_p.

If δV_p be a small change in plate voltage causing a corresponding small change δI_p in the plate current when the grid voltage (V_g) is kept constant, then

$$R_p=\frac{\delta V_p}{\delta I_p} \text{ (when } V_g \text{ is constant)} \quad \ldots \quad \ldots \quad \text{(ii)}$$

Different types of valves have different a.c. plate resistance, 6J5 triode, which is commonly used as an amplifier, has a.c. resistance of about 7700 ohms.

(iii) **The mutual conductance** : It is a measure of the effectiveness of the control grid in securing changes in the plate current and hence signal output of the valve. It is defined as *the ratio of a small change in plate current to the small change in grid voltage producing it, when the plate voltage is kept constant.* It is also known as *transconductance* and is denoted by g_m.

If δI_p be a small change in plate current caused due to a small change δV_g in the grid voltage when the plate voltage (V_p) is kept constant, then,

$$g_m=\frac{\delta I_p}{\delta V_g} \text{ (when } V_p \text{ is constant)} \qquad \ldots \qquad \ldots \qquad \text{(iii)}$$

From equ.s (ii) and (iii) on multiplication, we get

$$R_p \times g_m=\frac{\delta V_p}{\delta I_p}\times\frac{\delta I_p}{\delta V_g}=\frac{\delta V_p}{\delta V_g}=\mu.$$

So, the valve constants are inter-related and knowing any two, we can find the third.

Example 1 : *A triode passes a plate current of* 5 *m.a. at plate voltage* 150 *volts and grid voltage* $-2V$. *If the grid voltage is changed to* $-3{\cdot}5V$, *the plate current falls to* 3·2 *m.a. but can be restored to* 5 *m.a. by increasing the plate potential to* 195 *V. Find the mutual conductance and a.c. resistance. Hence find the amplification factor of the valve.*

Ans. We know, $g_m=\frac{\delta I_p}{\delta V_g}$; Here $\delta I_p=5-3{\cdot}2=1{\cdot}8$ m.a.$=1{\cdot}8\times10^{-3}$ amp. and $\delta V_g=-2-(-3{\cdot}5)=1{\cdot}5$ volt $\therefore\ g_m=\frac{1{\cdot}8\times10^{-3}}{1{\cdot}5}=1{\cdot}2\times10^{-3}$ amp/volt.

Again, we know, $R_p=\frac{\delta V_p}{\delta I_p}$. Here $\delta V_p=(195-150)=45$ volts and $\delta I_p=5-3{\cdot}2=1{\cdot}8$ m.a.$=1{\cdot}8\times10^{-3}$ amp. $\therefore\ R_p=\frac{45}{1{\cdot}8\times10^{-3}}=25000$ ohms.

Further, since $\mu=R_p\times g_m$, we have $\mu=25000\times1{\cdot}2\times10^{-3}=30$.

Example 2 : *In the study of static characteristic of a triode valve, the plate current increases by* 4 *m.a. when the plate voltage is changed from* 90 *volts to* 115 *volts at constant grid potential. Keeping the plate potential of* 90 *volts, the same change in plate current is achieved when the grid potential is changed by* 1·5 *volts. Calculate* (*i*) *amplification factor* (*ii*) *mutual conductance and* (*iii*) *a.c. resistance of the valve.*

Ans. (i) Amplification factor $\mu=\frac{\delta V_p}{\delta V_g}=\frac{115-90}{1{\cdot}5}=16{\cdot}6$

(ii) Mutual conductance $g_m=\frac{\delta I_p}{\delta V_g}=\frac{4\times10^{-3}}{1{\cdot}5}=2{\cdot}6\times10^{-3}$ mho

(iii) A.C. resistance $R_p=\frac{\delta V_p}{\delta I_p}=\frac{115-90}{4\times10^{-3}}=\frac{25}{4\times10^{-3}}=6{\cdot}25\times10^{3}$ ohm.

2.9. Triode as an amplifier :

We have seen earlier that a triode acts as an amplifier because the plate current is affected to a much greater degree by a change in the grid

voltage than by a change in the plate voltage when the grid is given a suitable bias. Often we find microphones, record player heads, and many other electrical devices produce such feeble electrical voltages that it is necessary to magnify or amplify them by a large factor before they are used to operate loud-speakers. Amplifiers are electronic circuits which take in small voltages and amplify them, say A times. A is called the *amplification factor* or *gain* of the amplifier. Triode, by virtue of the property mentioned above, can be suitably used as devices for amplification.

Consider the linear portion ACB of the mutual characteristic of a triode [Fig 2.11]. Let the mid-point C of the straight portion be selected as the working point of the valve *i.e.* the grid is given a negative potential $(-V_g)$ represented by the point E corresponding to the mid-point C. If the grid potential is maintained at that value, a constant plate current of the amount OL will be obtained. If an alternating potential be now applied to the grid, its potential will vary correspondingly about the point E. During the positive half-cycle of a.c. supply, the grid potential changes from OE to OD (*i.e.* the grid becomes less negative than at start) and during the negative half-cycle, the grid-potential changes from OE to OF (*i.e*, the grid becomes more negative than at start). Due to this change in grid potential, the plate current also changes and the nature of change in plate current is shown in fig. 2.11. The plate current changes in the same manner as the change in grid potential but the amplitude of current variations is much larger. Under these conditions, the triode amplifies without any distortion, the input signal given at the grid. From the characteristic curve, it is clear that the variations of the current in the plate circuit are greater, the steeper the curve ACB. To obtain large amplification, therefore, the steepest part of the curve should be used and the working point of the grid should be fixed at the middle of the steepest part.

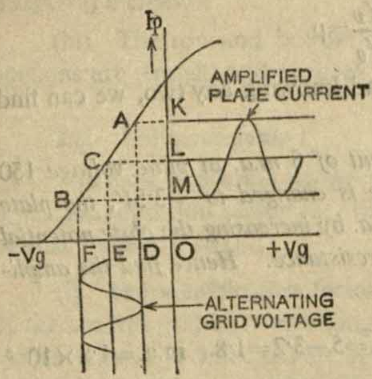


Fig. 2.11

If we are to see whether any amplification has resulted by the use of the valve, we must convert the change of plate current into a change of potential which can then be compared with the original potential change applied to the grid. This can be done by inserting a resistor R, known as 'load' in the plate-filament circuit as shown in fig. 2.12. A change of grid potential δV_g produces, say, a change of δI_p in the plate current. Then the change of p.d. across the load R is $\delta E = R.\delta I_p$.

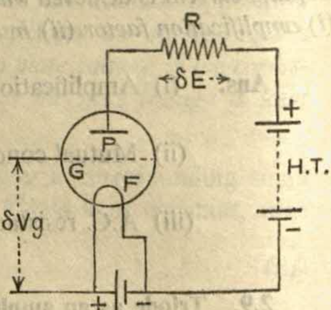


Fig. 2.12

$\therefore$ Amplification produced $= \dfrac{\delta E}{\delta V} = \dfrac{R.\delta I_p}{\delta V_g}$ (i)

Now, if μ be the amplification factor of the valve, then a change of δV_g in the grid potential

produces a change of $\mu\delta V_g$ in the plate potential which acts across the a.c. resistance R_p as well as the external load R.

$$\therefore \quad \mu\delta V_g = (R+R\ )\delta I^p \quad \text{or} \quad I_p = \frac{\mu.\delta V}{R+R_p}$$

So, the voltage amplification produced $A=\dfrac{\delta E}{\delta V}=\dfrac{R\mu\delta V_g}{(R+R_p)\delta V_g}=\dfrac{\mu R}{R+R_p}=\dfrac{\mu}{1+R_p/\mathrm{R}}$

$$\therefore \quad A=\frac{\mu}{1+R_p/R}$$

From the above relation, it is clear that the voltage amplification—sometimes it is called *stage gain* —can never be more than the amplification factor (μ) of the valve. When the external load is infinity (*i.e.* $R=\alpha$), the voltage amplification A becomes maximum and is equal to μ, the amplification factor, because $R_p/R=0$. But the plate current diminishes when R becomes very high. For this reason, generally the load R is made equal to a.c. resistance R_p and in that case, the voltage amplification $A=\mu/2$.

When the valve is used as a voltage amplifier, it is important to safeguard that the output is not distorted. To ensure that the amplifier operates without distortion *i.e.* the wave-form of alternating potential across the load R is exactly of the same shape, though of greater amplitude, as the wave-form of the grid input, (i) *the characterestic must be straight and linear and* (ii) *the working point must be fixed at the middle point of the steepest part of the characteristic.*

Example : *A triode with an amplification factor of* 10 *and an a.c. resistance of* 10,000 *ohms is connected in series with a load resistance of* 50,000 *ohms and a small voltage v is applied between grid and filament. What change in voltage takes place across the load resistance* ?

Ans. We know, $A=\dfrac{\mu}{1+R_p/R}$; here $\mu=10$; $R_p=10{,}000$ ohms and $R_,=50{,}000$ ohms

$$\therefore \quad A=\frac{10}{1+\dfrac{10{,}000}{50{,}000}}=8{\cdot}33.$$ Hence the change in voltage across the load $=8{\cdot}33.v$.

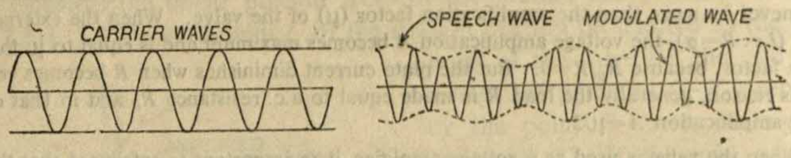
2.10. Elementary principles of radio :

Transmission and reception of radio programmes from one end of the world to the other within a moment is, indeed, an amazing achievement of modern science. Radiophysics, as a matter of fact, is now-a-days so vast and extended that a detailed study of the subject is not possible within a limited scope. We shall, therefore, discuss in the following paragraphs, the elementary principles of transmission and reception of wireless waves.

(i) **Transmission system** : Programmes of songs, music, speech etc. that are broadcast over the radio have extensive range of frequencies, varying from 50 to 20,000 cycles/sec. These signals have different amplitudes too. When a person sings or talks before a microphone, the microphone transforms the sound energy into alternating current whose frequency and amplitude vary according to the frequency and amplitude of the original sound. In other words the microphone converts the sound energy into electrical energy. The different frequencies

of the transformed electrical energy are called *audio-frequencies.* For various reasons, this audio-frequency electric waves representing the original speech or music, cannot be transmitted from one place to another through aether and for that reason, a distant station can not receive them or reproduce them.

In transmission system, a *carrier* is needed to carry through aether the audio-frequency waves to distant receiving stations. These *carrier* waves are nothing but high frequency electromagnetic waves of constant amplitude [Fig. 2.13(i)]. A triode can produce such carrier waves and radiate them into space from a high



(i) Fig. 2.13 (ii)

antenna installed in the transmitting station. When no message is sent, the amplitude of these carrier waves remains constant. The wave length of the carrier waves used in transmission of radio programmes varies from 10 to 500 metres and their frequencies from 600 kilo-cycles to 30 mega-cycles per second.

It has been mentioned before that the carrier waves do not convey any signal. The audio-frequency waves representing the programme—these are sometimes known as *speech waves*—are impressed on the carrier waves. The process is known as *modulation.* In *amplitude modulation*, the amplitude of the carrier is changed in the tempo of the sound waves. In the *frequency modulation* system, the carrier wave has a constant amplitude but its frequency is changed according to the pattern of the sound waves. The antenna radiates the modulated carrier waves in space as electromagnetic waves [Fig. 2.13(ii)], which travel in all directions with the speed of light. When they reach the aerial of a receiving station, the later detects the signals, speech or music.

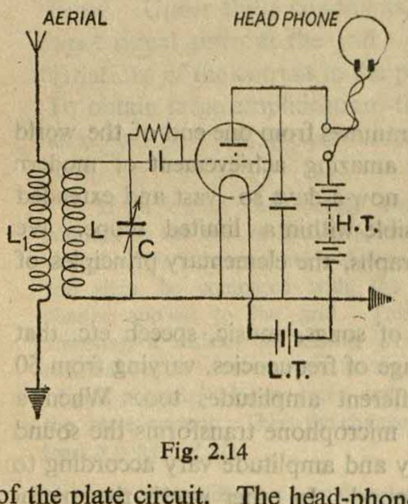


Fig. 2.14

(ii) **Receiving system** : The basic purpose of the receiving system is to detect and reproduce the signals broadcast from the transmitting station. A diagram of a very simple receiving circuit is shown in fig. 2.14. When a signal from a transmitting station arrives at the aerial of the receiver, high-frequency oscillations are set up in the inductance L_1 of the aerial. With the help of tuned *L-C* circuit these oscillations are fed into the grid-circuit of a triode, which separates the speech waves from the modulated carrier waves and sends them to the head-phone of the plate circuit. The head-phone reproduces the original signal or music.

Exercises

Essay type :

1. Describe a vacuum tube diode and show the characteristic (current *Vs.* voltage). Explain them. [*H. S. Exam. 1979*]

2. Explain with the help of diagrams how alternating current may be rectified using diode valve. [*H. S. Exam. 1978*]

3. What is the advantage of full wave rectification as compared with half wave rectification ?

4. Give the constructional detail of a triode. Draw the plate characteristics at different grid voltages. Explain the use of triode as an amplifier. [*H. S. Exam. 1980*]

5. What are valve constants ? Write the relation between them.

6. What are (i) Amplification factor and (ii) Voltage amplification ? Show that

$A = \frac{\mu}{1+R_p/R}$ where symbols have usual meaning.

7. Write a short note on wireless transmission and reception.

8. Write notes on : (*a*) trode (*b*) carrier wave (*c*) modulation (*d*) amplification (*e*) space charge.

Short answer type :

9. What is thermonic emission ? What is the nature of these ions ?

10. What is space charge ? What is its effect on the plate current ?

11. What is the significance of using the third electrode—the gird in a vacuum tube ? Where does the added energy in an amplifier come from ?

[**Hints :** Triode gets the added energy from the high tension battery connected to the plate.]

12. Define : (*a*) amplification factor (*b*) mutual conductance (*c*) a.c. plate resistance.

13. Why is the grid made perforated ?

[**Hints :** It will collect very little electrons and a small grid current will flow. This will cause a small wastage cf energy.]

14. Why voltage amplification is always less than the amplification factor of a triode ?

Objective type :

15. A. Which of the statements below are true and which are untrue ?

(*a*) Triode valve may be used to rectify an alternating current.

(*b*) For a given plate voltage, the plate current in a triode valve is maximum when the potential of (i) the grid is positive and plate is negative (ii) the grid is zero and the plate is positive (iii) the grid is negative and the plate is positive (iv) the grid is positive and the plate is positive. Which is correct ? [*I.I.T. 1985*]

(*c*) To use the triode as an amplifier, the working point should be fixed at the non-linear portion of the characteristic curve.

(*d*) The process of superposing the speech waves on the carrier wave is called the modulation.

(*e*) The grid in a triode is a very efficient controller of space charge.

(*f*) The plate resistance of a triode valve is 3×10^{3} ohms and its mutual conductance is $1{\cdot}5\times10^{-3}$ amp/volt. The amplification factor of the valve is (i) 5×10^{-5} (ii) 4·5 (iii) 2×10^{6}. Which is correct ? [*I.I.T. 1983*]

B. Select the correct statements from the following :

(i) A diode can be used as a rectifier (ii) A triode can not be used as a rectifier (iii) The current in a diode is always proportional to the applied voltage (iv) The linear portion of the I-V characteristic of a triode is used for amplification without distortion.

Numerical Problems :

16. A diode is connected to a H.T of 200 volts and a cell of 6 volts is included in the filament circuit. An ammeter A included in the plate circuit reads 400 mA. Two ammeters A_1 and A_2 are included in the filament circuit. A_1 in the negative part of the filament supply and A_2 in the positive part. The ammeter A_2 reads 2 amp. What in the reading in A_1 ? [Ans. 2·4 amp]

17. The following data were obtained in a triode valve experiment :

Plate Voltage (V_p)	200V	200V	250V	250V	250V
Grid Voltage (V_g)	−1V	−1·5V	−1V	−1·8V	2·3V
Plate current (I_p)	5*ma*	2 *ma*	9·8 *ma*	5 *ma*	2 *ma*

Calculate (i) the amplification factor (ii) a.c. plate resistance (iii) the mutual conductance [Ans. (i) 62·25 (ii) 10418 ohm (iii) 6×10^{-3}]

18. A triode valve has amplification factor 30 and mutual conductance $1{\cdot}2\times10^{-3}$. Determine the a.c. plate resistance of the valve. [Ans. 25×10^{3} ohms]

19. In a triode, if plate voltage be changed from 100 volt to 150 volt and at the same time, grid voltage is charged from −0·35 to −0·9 volt, the plate current remains constant at 5*ma*. Find from these data, the amplification factor of the valve. [Ans. 91]

20. A triode has mutual conductance of 3×10^{-3} amp/volt and a.c resistance 15,000 ohms. Find the load which must be inserted in the plate circuit in order to obtain a voltage amplification of 30. [Ans. 30,000 ohms]

21. A triode has the following characteristics : $\mu=40$; a.c. resistence=11,00 ohms and load in the plate circuit R=40,000 ohm. Find the voltage amplification. [Ans. 31.3]

3

PHOTO ELECTRICITY

3.1. Discovery of photo-electricity :

It has been found that some substances, chiefly metals when irradiated by radiations of suitable wave-length, emit electrons as long as the radiation falls on them. This phenomenon is known as **photo-electricity.**

It was first observed by the celebrated German physicist Hertz in 1887. He was producing electromagnetic waves by passing sparks between two electrodes. He found that when ultra-violet rays fell on the spark gap, the sparks passed more easily. Hertz, however, did not proceed much with the phenomenon. A year later, *i.e.* in 1888, Hallwachs, Elster and Gietel carried out some experimental investigations about the phenomenon. They introduced two zinc plates in an evacuated quartz bulb and connected the plates to the positive and negative terminals of a battery. They found that when ultraviolet light was made to fall on the $-ve$ plate, a current started flowing in the circuit and stopped as soon as the ultra-violet beam was stopped. They also found that no current flew if the beam was made to fall on the $+ve$ plate. Lenard, in 1900. proved that the phenomenon was due to the emission of *electrons* from the metal surface exposed to ultra-violet light. As the effect is produced under the influence of light, it is called *photo-electric effect* (*photo* means light), and for the same reason, the electrons and the resulting current are known as *photoelectrons* and *photoelectric current.*

The alkali metals like lithium. sodium, potassium etc. are found to be very photo-sensitive. They emit electrons even when ordinary visible light falls on them. Experiments show that with light of suitable wavelength (gamma rays, X-rays or ultra-violet rays), almost all metals exhibit photo-electric phenomenon.

3.2. Experimental study of photo-electricity :

For experimental study of photoelectric phenomenon, an arrangement as illustrated in fig. 3.1 may be used. It consists of two metal plates K and A sealed into an evacuated quartz bulb. A potential difference may be set up between the plates with the help of a battery and a variable resistor, the plate K having been raised to a negative potential and the plate A to a positive potential. With the help of the potentiometer arrangement, the potential difference between the plates A and K may be increased or decreased or may even be reversed. A sensitive galvanometer G is included in the circuit.

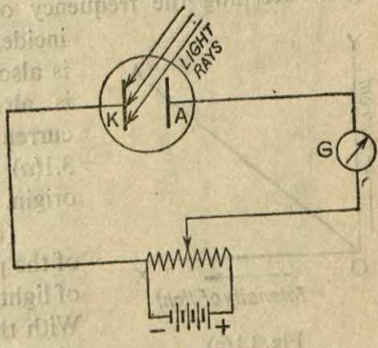


Fig. 3.1

If the plate K is coated with an alkali metal and if light be allowed to fall on

it, a current flows through the circuit and a deflection is produced in the galvanometer G. When the potential difference between the plates is increased, the current increases with the *p.d.* until it reaches a maximum value, called the saturation current.

If, however, the plate K is given a small $+ve$ and the plate A a $-ve$ potential and if light of a given wavelength is incident on K, the slower of the emitted electrons are drawn back and fail to reach the plate A. Consequently, the current will decrease appreciably. As the $+ve$ potential of the plate K is increased, more and more electrons are drawn back and the current decreases sharply until it becomes zero at some positive potential of the plate K, called the *stopping potential* for the particular metal. It is superfluous to mention that at the stopping potential, even the fastest electrons are turned back to the plate K. By measuring the stopping potential, the maximum velocity or the energy with which photoelectrons are emitted may be estimated.

If, however, on making the plate K negative and the plate A positive, we increase the intensity of the incident light, the wave length remaining the same, the photoelectric current increases but the stopping potential remains unchanged. Hence, we can conclude that the photo-electric current depends on the intensity of the light falling on the photo-metal but the velocity of the photo-electrons does not depend upon the intensity.

If, again, keeping the intensity of light constant, we vary the wave-length or the frequency, the stopping potential changes. The higher the frequency of the incident light (or shorter the wave length), the greater is the stopping potential and therefore, greater is the velocity of the emitted electrons.

Further, if the frequency of the light falling on the plate K be decreased gradually, it will be found that the photo-electric effect disappears at a certain minimum frequency, called the *threshold frequency*, whatever may be intensity of the light. The threshold frequency is, of course, different for different metals.

Characteristics of photo electric phenomenon : From the results of the above experiment, we may summarise, as follows, the salient features of photo-electric effect :

(i) The photo-electric current is proportional to the intensity of the incident light. Keeping the frequency of the light unchanged, if the intensity of the incident light be doubled, the photoelectric current is also doubled. If the intensity be zero, the current is also zero. The variation of the photoelectric current with the intensity is graphically shown in Fig. 3.1(a). It is a straight line OA passing through the origin.

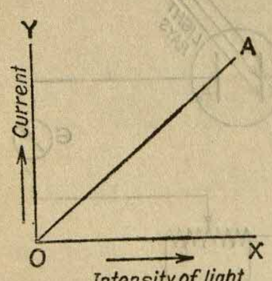


Fig. 3.1(n)

(ii) The initial velocity or the kinetic energy of the photoelectrons does not depend on the intensity of light but depends on the frequency of the light used. With the increase of frequency (*i.e.* with the decrease of wavelength) the initial velocity or the kinetic energy of the electrons increases and vice versa.

Fig. 3.1(*b*) shows the dependence of the maximum energy of the ejected photo-electrons on the frequency of the incident light and fig. 3.1(c) shows that

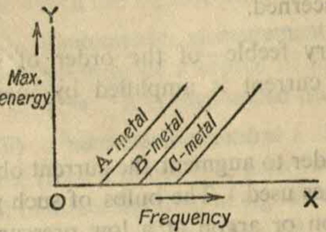


Fig. 3.1(b)

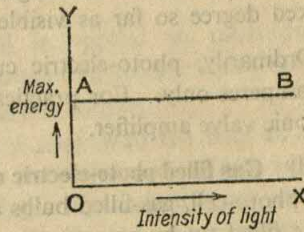


Fig. 3.1(c)

the maximum kinetic energy is independent of the intensity of the right. It is a straight line *AB* parallel to the intensity axis, indicating that whatever be the intensity, the maximum kinetic energy remains unchanged.

(iii) For every metal, there is a minimum frequency, below which no photo-electric effect is produced by the metal. This minimum frequency is called the threshold frequency which is different for different metals. In fig. 3.1(*b*), the intercepts made by the straight lines on the frequency axis give the threshold frequencies of the metals *A*, *B* and *C*.

(iv) Photo-electric effect is an instantaneous effect *i.e.* emission of electrons starts as soon as the light is incident on the metal and stops as soon as the light is cut off.

(v) For a given frequency of light, the initial kinetic energy with which the electrons are ejected varies from zero to a maximum value.

3.3. Photo-electric cells :

A photo-electric cell is a device for converting light energy into electric energy, utilising the photo-electric effect. There may be different types of photo-electric cell, *viz.* (i) high vacuum photo-emission cell (ii) gas-filled photo-emission cell (iii) photo-voltaic cell etc.

(i) **High vacuum photo-emission cell :** It consists of an evacuated glass or quartz bulb (Fig. 3.2). For visible light, the bulb is made of glass and for ultraviolet light, the bulb is made of quartz. It contains a semi-cylindrical plate *K*, having a large surface-area, known as the emitter or the cathode of the cell. A wire *A* or a frame of wire is fixed along the axis of the cylinder. This is known as the collector or the anode of the bulb. When visible light is used, the emitter is given a coating of sodium, potassium or cesium because these alkali metals are photo-sensitive so far as ordinary visible light is concerned. To get a profuse supply of electrons, various compounds are now-a-days used as cathode. A

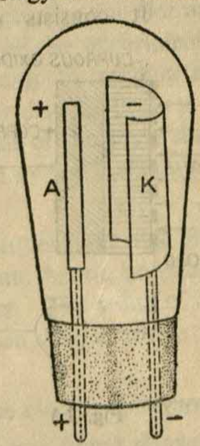


Fig. 3.2

coating of cesium and silver oxide, antimony-cesium alloy etc. are some of the compounds now used for making the cathode of a photo-cell. Very recently, a mixture of bismuth, silver, oxygen and cesium is found to be photo-sensitive in a marked degree so far as visible light is concerned.

Ordinarily, photo-electric current is very feeble—of the order of several micro-amperes only. For practical use, the current is amplified by means of thermionic valve amplifier.

(ii) **Gas filled photo-electric cell** : In order to augment the current obtained from a photo-cell, gas-filled bulbs are sometimes used. The bulbs of such photo-cells are filled up by some inert gas, like neon or argon at a low pressure (few millimeters of mercury). When the p.d. between the anode and the cathode is increased the photo-electrons emitted from the surface of the cathode travel towards the anode with considerable speed and colliding against the atoms and molecules of the gas, bring about ionisation of the gas. This process gives rise to a large number of electrons, which flowing towards the anode along with the mainstream, increase the photo-electric current to a considerable extent.

In a vacuum photo-cell, the photo-electric current is strictly proportional to the intensity of the incident light but in a gas-filled type of photo-cell, it is not so. For this reason, gas-filled photo-cell is never used for standardisation or for any quantitative work. They are used for other purposes.

(iii) **Photo-voltaic cell** : In photo-emission cells, a p.d. needs be applied between the anode and the cathode for driving electrons to the anode, which sets up the photo-electric current and for establishing the p.d., a battery is required to be joined with the cell. But a photo-voltaic cell does not require the assistance of an auxiliary battery like the above, because in this case, the ejected electrons themselves set up the necessary p.d. between the plates for driving current in the external circuit.

It consists of a thin layer of cuprous oxide coated on a disc of copper (Fig. 3.3). The oxide film has sputtered silver or gold film on its upper surface. When light passing through the thin sputtered film, falls on the oxide layer, electrons are emitted from it not into the surrounding air but into the gold or silver film. The oxide layer thus becomes positively charged and the film negatively. An e.m.f. is thus set up. If an external circuit R together with a sensitive galvanometer G is introduced between them a feeble current will flow through the circuit and the galvanometer will show a deflection. The current available from a photo-voltaic cell is proportional to the intensity of light.

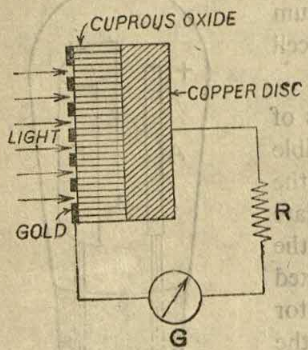


Fig. 3.3

Photo-voltaic cells are used as light-intensity meters as well as rectifier.

3.4. Practical applications of photo-electric cells :

Photo-electric cells have a number of important practical applications, some of which are mentioned below :

(i) **Photometric measurement** : This cell is applied in measuring the luminous intensity of a source of light or for comparing the luminous intensities of two sources. It is then called a direct-reading photometer.

(ii) **Photo-electric control** : By working a relay with the help of current available from a photo cell, many control works are performed. Thus, automatic control of street lighting, of signals at level crossings, of gates and doors, automatic counter, burglar's alarm, fire alarm etc. are rendered easy and effective by the use of photo-cells.

(iii) **Talking films** : Recording and reproduction of sound in cinema are, at present, made with the help of photo-cells.

(iv) **Television** : The transmitter of a television—the iconoscope—is a combination of photo-cell and cathode-ray oscillograph.

(v) **Belinogram system** : Belinogram is the process of transmission of pictures to distant places within a very short time by telegraphy. The photo-electric cells find an important application in this art.

3.5. Particle nature of radiation : Quantum theory :

Besides having useful practical applications, such as those described above, the photo-electric effect played an important part in the development of one of the outstanding ideas of modern physics—the Quantum theory.

Before the establishment of Quantum theory, all cases of energy transfer, according to the classical wave theory, were considered to be continuous because wave-motion was continuous. It was supposed possible to measure out and transport any desired amount of energy, with available limits, just as one might measure out a quantity of liquid. But at the beginning of this century, the German physicist Max Planck found that a theoretical explanation of the experimental observation in black-body radiation could be worked out only by making the unusual assumption that the energy is given off in separate or discrete packets of energy, known as *quanta* of energy. These quanta are not all of the same size ; instead the amount of energy carried by each is proportional to the frequency of the radiation in question.

If the energy (heat) radiated by a black body be of frequency ν, then each quanta of the radiation will carry, according to the Quantum theory, an energy equal to $h\nu$, when h is a constant known as Planck's constant. The value of h is $6{\cdot}62 \times 10^{-27}$ erg-sec ; if E be the energy of the radiation, then $E = h\nu$, where ν is the frequency of radiation.

Failure of classical wave theory of light in explaining photo electric phenomena :

The classical electromagnetic wave theory of radiation was found altogether incapable of explaining the experimentally well established facts about photo-electric effect. According to wave theory, the energy of light wave is distributed

uniformly over the entire wavefront. It is very difficult to understand how such uniformly distributed energy could suddenly concentrate on extremely small cross-section of an electron. Calculations on the basis of wave theory show that the time required for an atom to gather sufficient energy from such wave front so that an electron may escape the surface, may in some cases, run into several days. But experiment shows that the effect is instantaneous. Further application of wave theory to photoelectric effect compels us to conclude that the initial velocity of electric emission should depend on the intensity of the incident light which is contrary to experimental findings. The existence of threshold frequency, in each case, is also contrary to the conditions of wave theory, for it seems quite without reason why a sufficiently intense beam of low frequency fails to liberate an electron while a feeble beam of high frequency succeeds in doing so.

For a satisfactory explanation of these facts, Einstein applied the Quantum theory to the photo-electric process. According to this theory, radiation is regarded as a shower of quanta or photon—the name 'photon' was inducted by Einstein—each photon of energy $h\nu$ moving in space with the velocity of light. As a piece of matter is made up of numerous discrete and discontinuous atoms, radiation, likewise is made of numerous discrete and discontinuous photons. According to Einstein, the collision between a photon and an atom of a metal is an elastic collision in which the atom absorbs all the energy of the photon or none. With this assumption, Einstein established an equation known as *photo-electric equation* which successfully explained all the experimental facts of photo-electric effect and thus established Quantum theory on sound footing.

Subsequently, a few sub-atomic processes were found which necessitated the application of Quantum theory for a complete explanation. In this way, the Quantum theory as proposed by Max Planck was found to be applicable in all cases of radiation and became an accepted part of modern science.

Wave length of photon :

Suppose, the energy of a photon$=E$ and its frequency$=\nu$. From Quantum theory, $E=h\nu$. If λ be the wavelength of the photon then $\nu.\lambda=c$, where c is the velocity of the photon (*i.e.* light)

$$\therefore \quad E=h\nu=\frac{h.c}{\lambda} \text{ or } \lambda=\frac{h.c}{E}.$$

This shows, the more the energy of a photon, the less is its wave length.

Example 1 : *The energy of a photon is* 5 *electron-volt. What is its wave-length ?* $1\ ev=1{\cdot}6\times10^{-12}$ *erg* ; $h=6{\cdot}62\times10^{-27}$ *erg-sec.*

Ans. The energy $E=5\ ev=5\times1{\cdot}6\times10^{-12}$ erg$=8\times10^{-12}$ erg.

$$\text{Now} \quad \lambda=\frac{h.c.}{E}=\frac{6{\cdot}62\times10^{-27}\times3\times10^{10}}{8\times10^{-12}}=2{\cdot}48\times10^{-5}\text{ cm.}=2480\text{Å}$$

Example 2 : *Find the energy, in electron-volt, of a photon whose wavelength is* 4950A°. $1A^\circ=10^{-8}$ *cm and* $h=6{\cdot}62\times10^{-27}$ *erg-sec.* [*Jt. Entrance 1981*]

Ans. We know, energy of a photon $E=\dfrac{h.c}{\lambda}=\dfrac{6{\cdot}62\times10^{-27}\times3\times10^{10}}{4950\times10^{-8}}$ erg

Now, $1\ e.v. = 1 \cdot 6 \times 10^{-12}$ erg.

$$\therefore \quad \text{Energy } E = \frac{6 \cdot 62 \times 10^{-27} \times 3 \times 10^{10}}{4950 \times 10^{-8} \times 1 \cdot 6 \times 10^{-12}}\ ev = 2 \cdot 5\ ev.$$

3.6. Einstein's photo-electric equation :

According to the quantum theory, radiation is regarded as a shower of *photons*, each of energy $h\nu$, moving with the velocity of light. Assuming that an elastic collision between a photon and an electron results in the complete absorption of the energy of the photon by the electron of the atom. Einstein established an equation, known as *photo-electric equation*, which satisfactorily explained all the experimental observations in connection with photo-electricity.

The minimum amount of work or energy to take a free electron out of the surface of a metal against the attractive forces of the positive ions is known as the *work function* ω_0 of the metal. The work function depends upon the nature of the metal used. When light of sufficiently high frequency is incident on the metal, an amount ω_0 of the incident energy $h\nu$ is used to liberate the electron, leaving an excess energy $(h\nu - \omega_0)$ which is given to the ejected electron in the form of kinetic energy ; if the electron does not lose energy by internal collisions as it escapes from the metal, it will exhibit it all as kinetic energy after it emerges. Thus $\frac{1}{2}mv^2_{max}$ represents the maximum K.E. that the photoelectrons can have outside the surface ; in nearly all cases it will have less energy than this because of internal losses. The maximum kinetic energy $\frac{1}{2}mv^2_{max}$ of the photo-electrons is thus, according to Einstien's theory,

$$\tfrac{1}{2}mv^2_{max} = h\nu - \omega_0$$

The above equation is called Einstein's photo-electric equation.

It is noteworthy that when classical wave theory of light failed to explain satisfactorily all the characteristics of photo-electric phenomenon mentioned in art 3.2, Einstein applied his equation for the purpose. His explanation was as follows :

(i) Since ω_0 is a constant for a particular metal, it is clear from the above equation that velocity (v) or the K. E. ($\frac{1}{2}mv^2$) of the photo-electrons on emergence is proportional to the frequency ν of the incident light.

(ii) If the frequency ν of the incident photon be gradually reduced, the K. E. of the photo-electron becomes less and less, reaching zero at a value ν_0 (say) so that $h\nu_0 = \omega_0$. Photoelectric emission is, therefore, not possible at any frequency lower than ν_0 which is called the *threshold frequency* of the metal.

(iii) The initial K. E. or the velocity of the photoelectrons should not depend upon the intensity of the incident light because according to the quantum theory increase of intensity of light means increase of the number of photons. If the frequency ν remains unchanged, the energy ($h\nu$) of each photon remains unchanged and hence the initial velocity or K. E. of the photo-electrons also remains unchanged. But increase in the number of photons will cause an increase in the number of ejected electrons which means the photo-electric current will increase. The experimental results corroborate it.

(iv) In the above equation, the collision between a photon and an electron is regarded as an elastic collision and as such the transference of energy is instantaneous. So, there should not be any time-lag between the incidence of photon and the emergence of photo-electrons.

Example 1 : *Ultra-violet light of wavelength* $3000A^{\circ}$ *is incident an a metal surface whose work function is* 2·28 *electron-volt. What will be the maximum velocity with which electrons are ejected from the surface ?* $m=9{\cdot}1\times10^{-28}$ *gm.* ; $h=6{\cdot}56\times10^{-27}$ *erg-sec.* ; $1\ eV=1{\cdot}61\times10^{-12}$ *erg.*

Ans. The maximum energy of the photo-electrons is given by,

$$\tfrac{1}{2}mv^2_{max}=h\nu-\omega_0.$$

Here work function $\omega_0=2{\cdot}28\ eV=2{\cdot}28\times1{\cdot}61\times10^{-12}\ erg=3{\cdot}67\times10^{-12}$ erg.

$$\nu=\frac{c}{\lambda}=\frac{3\times10^{10}}{3000\times10^{-8}}=10^{15}\quad [1A^{\circ}=10^{-8}\text{ cm}]$$

Hence, $\tfrac{1}{2}mv^2_{max}=6{\cdot}56\times10^{-27}\times10^{15}-3{\cdot}67\times10^{-12}=(6{\cdot}56-3{\cdot}67)\times10^{-12}$ ergs.

$$=2{\cdot}89\times10^{-12}\text{ ergs.}$$

$$\therefore\quad v^2_{max}=\frac{2\times2{\cdot}89\times10^{-12}}{9{\cdot}1\times10^{-28}}=64\times10^{14}\text{ (nearly)}$$

or $v_{max}=8\times10^{7}$ cm./sec (approx.)

Example 2 : *The photoelectric threshold wavelength for a metal is* $3000A^{\circ}$. *What is the maximum energy, in eV, of the ejected photoelectrons when light of wavelength* $1000A^{\circ}$ *falls on the metal ?* $h=6{\cdot}55\times10^{-27}$ *erg-sec* ; $1\ eV=1{\cdot}61\times10^{-12}$ *erg.*

Ans. If ν_0 be the threshold frequency, then $\omega_0=h\nu_0$. In that case,

$$E_{max}=h\nu-h\nu_0=h(\nu-\nu_0)=h.c.\left(\frac{1}{\lambda}-\frac{1}{\lambda_0}\right)$$

Now, $c=3\times10^{10}$ cm/sec ; $\lambda=1000\times10^{-8}=10^{-5}$ cm; $\lambda_0=3000\times10^{-8}=3\times10^{-5}$ cm.

$$\text{Hence, } E_{max}=6{\cdot}55\times10^{-27}\times3\times10^{10}\left(\frac{1}{10^{-5}}-\frac{1}{3\times10^{-5}}\right)$$

$$=6{\cdot}55\times10^{-27}\times3\times10^{10}\times10^{5}\times(\tfrac{1}{1}-\tfrac{1}{3})$$

$$=6{\cdot}55\times10^{-27}\times3\times10^{10}\times10^{5}\times\tfrac{2}{3}$$

$$=13{\cdot}1\times10^{-12}\text{ erg.}$$

$$=\frac{13{\cdot}1\times10^{-12}}{1{\cdot}61\times10^{-12}}=8{\cdot}1\ eV.$$

Example 3 : *Cesium has a work function of* 1·9 *electron volts. Find the maximum energy of the liberated electrons when the metal is illuminated by light of wave length* $4{\cdot}5\times10^{-5}$ *cm.* $1\ eV=1{\cdot}60\times10^{-12}$ *erg* ; $h=6{\cdot}56\times10^{-27}$ *erg. sec.*

Ans. Maximum energy, $E_{max}=h\nu-\omega_0$

$$\text{Here,}\quad \nu=\frac{c}{\lambda}=\frac{3\times10^{10}}{4{\cdot}5\times10^{-5}}=66\times10^{13},$$

$$\text{Hence } E_{max} = 6{\cdot}56\times10^{-27}\times66\times10^{13} - 1{\cdot}9\times1{\cdot}6\times10^{-12}$$
$$= 4{\cdot}32\times10^{-12} - 3{\cdot}04\times10^{-12}$$
$$= 1{\cdot}28\times10^{-12}\ erg.$$
$$= \frac{1{\cdot}28\times10^{-12}}{1{\cdot}6\times10^{-12}}\ eV = 0{\cdot}8\ eV.$$

Example 4 : *Photo electrons are emitted by sodium when ultra violet light of wavelength* 3×10^{-8} *metre falls on the surface. Calculate the velocity of photo-electrons assuming the work function of sodium to be negligibly small. Mass of electron*$=9{\cdot}1\times10^{-31}$ *kg ; Planck's constant*$=6{\cdot}6\times10^{-34}$ *joule-sec.* [*I.S.C. Exam. 1978*]

Ans. Since work function of sodium is negligible, we have from Einstein's equation. $\frac{1}{2}mv^2 = hv = h\frac{c}{\lambda}$ or $v = \sqrt{\frac{2hc}{\lambda . m}}$

Here, $h = 6{\cdot}6\times10^{-34}$ joule-sec ; $c = 3\times10^{8}$ m/s ; $\lambda = 3\times10^{-8}$ m ; $m = 9{\cdot}1\times10^{-31}$ kg.

$$\therefore\quad v = \sqrt{\frac{2\times6{\cdot}6\times10^{-34}\times3\times10^{8}}{3\times10^{-8}\times9{\cdot}1\times10^{-31}}} = 3{\cdot}8\times10^{6}\ m/s$$

[Note that the above sum has been given in M.K.S. system.]

Exercises

Essay type :

1. What is photo-electric effect ? How was it discovered ?
2. Mention the salient features in connection with photo-electric effect. Describe suitable experiments to demonstrate them.
3. How many photo-electric cells are there ? Write, in brief, their description and uses.
4. Write what you know about the quantum theory. What do you understand by Planck's constant ?
5. State how the basic facts of the photo-electric effect may be explained with the help of Quantum theory. [*H. S. Exam. 1978*]
6. Explain Einstein's equation in connection with photo-electricity. What do you mean by photo-electric work function ?
7. What is photo-electricity ? How did Einstein explain the photo-electric emission ? [*H. S. Exam. 1984*]

Short answer type :

8. What are stopping potential and threshold frequency ?
9. In connection with photo-electricity, draw graphs between the following quantities : (*a*) Photo-electric current and intensity of light (*b*) the maximum kinetic energy of the ejected electrons and the frequency of the incident light (*c*) the maximum energy of the ejected electrons and the intensity of light used.
10. On what factor does the photo-electric threshold frequency of a metal depend ?
11. Two metal plates are sealed into an evacuated glass bulb. Utra violet light is made to fall on a plate which is given a positive potential with respect to the other plate. Will there be any current available ?

12. What do you mean by threshold frequency and work function ? What is the relation between them ? [*H. S. Exam. 1987*]

13. In the photo-electric effect, why does the existence of a threshold frequency speak in favour of the quantum theory and against the wave theory ?

14. A radiation of feeble intensity but of high frequency can liberate an electron from a metal surface but a radiation of high intensity but low frequency can not do so. Why ?

15. When light is incident on a meatal plate electrons are emitted only when the frequency of the light exceeds a certain value. How has this been explained ?

16. The energy required to remove an electron from sodium is 2·3 ev. Does sodium show a photo-electric effect for orange light whose $\lambda=6800A^\circ$? [Hints : No.]

17. Einstein's equation for the emission of photo-electrons from a metal due to radiation of frequency ν is $h\nu=\frac{1}{2}mv^2+w_0$ where m is the mass of electron, v is the maximum speed with which an electron can leave the metal and w_0 is a constant. (*a*) What does this equation imply as to the nature of light ? (*b*) What is the constant w_0 and how it is interpreted ? (*c*) What does this equation suggest regarding the minimum value of ν which is capable of ejecting electrons ?

Objective type :

18. Mark the correct answer in the following questions :—

(*a*) What is the nature of the particles that are ejected from a metal surface when ultraviolet light falls on it ? *Ans.* Positively charged protons ; neutral particles ; negatively charged electrons.

(*b*) Upon what does the energy of the liberated particles depend when ultravolet light falls on a metal plate ? *Ans.* intensity of light ; frequency of light ; mass of the particles.

(*c*) Which theory of light does the photoelectric phenomena support ? *Ans.* Wave theory ; Corpuscular theory ; Quantum theory.

(*d*) Every metal has a minimum frequency less than which it is not capable of ejecting any photo-electrons. What is that frequency called ? *Ans.* Threshold frequency ; Stopping frequency.

(*e*) The maximum *K.E.* of electrons emitted in photo-electric effect is linearly dependent on .. of the incident radition. Fill up the blank. [*I. I. T. 1984*]

Numerical problems :

19. The work function of molybdenum is 4·2 volt. What will be the maximum velocity of the liberated electrons when light of wave length 10^{-5} cm. falls on it ? $h=6{\cdot}62\times10^{-27}$ erg. sec ; $e=4{\cdot}8\times10^{-10}$ e.s.u. ; $m=9{\cdot}12\times10^{-28}$ gm. [Ans. $1{\cdot}7\times10^{8}$ cm./s (approx)]

20. Light of wave length 2000 A° falls on an aluminium surface. In aluminium 4.2 ev are required to remove an electron. What is the *K.E.* of (i) the fastest and (ii) the slowest emitted photo-electrons ? (iii) what is the stopping potential ? (iv) what is the threshold wavelength for alluminium ? [Ans. (i) 2 ev (ii) zero (iii) 2 volts (iv) 3000 A°]

21. The threshold wave length of a metal is 4000A°. What is the maximum energy of the liberated electrons when light of wave length 2000A° is incident on it ? 1 $eV=1.6\times10^{-12}$ erg. [Ans. 3.1 eV]

22. In an experiment using light of wave length 4×10^{-5} cm., the maximum electron energy was observed to be $1{\cdot}4\times10^{-19}$ joule. With light of wavelength 3×10^{-5} cm., the maximum energy was $3{\cdot}06\times10^{-10}$ joule. Derive a value for the Planck's constant.

[Ans. $6{\cdot}64\times10^{-27}$ erg. sec]

23. A metal has the value of photo-electric work function=1·32 electron-volt. What is the longest wave length that can cause photo-electric emission from the metal surface ? Electronic charge $=4{\cdot}8\times10^{-10}$ e.s.u. ; Plank's constant $=6{\cdot}6\times10^{-27}$ erg. sec.

[*Jt. Entrance 1984*] [Ans. 9.4×10^{-5} cm]

24. Cesium has photo-electric work function of 1.9 electron volt. What is the minimum frequency of light that can cause photo-electric emission from cesium ? $e=4.8\times10^{-10}$ e.s.u. ; $h=6.6\times10^{-27}$ erg. sec. [Ans. 45×10^{13} (nearly)]

25. Photo-electric threshold wave length of tungsten is 2300A°. What will be the maximum kinetic energy of photo-electrons if light of wave length 1800A° is incident on tungsten ? $1ev=1.6\times10^{-12}$ erg and $h=6.6\times10^{-27}$ erg. sec. [Ans. 1·49 *ev*]

[*Hints* : $\frac{1}{2}mv^2_{max}=hv-hv_0=\frac{hc}{\lambda}-\frac{hc}{\lambda_0}=\frac{hc(\lambda_0-\lambda)}{\lambda\lambda_0}$]

26. A radiation of frequency 5×10^{14} *Hz* liberates electrons from a metal surface with energy 2.31×10^{-19} joule. What will be the wave length of ultra-violet light required to emit photo electrons from the metal surface with energy 8.93×10^{-19} joule? $h=6.62\times10^{-34}$ joule-sec ; $c=3\times10^{8}$ m/s. [Ans. 2×10^{-7} m]

27. Ultra violet light of wavelength 800A° and 700A° when allowed to fall on hydrogen atoms in their ground state is found to liberate electrons with *K.E.* 1.8 *ev* and 4.0 *ev* respectively. Find the value of Planck's constant. [*I.I.T. 1986*] [Ans. 6.27×10^{-27} erg. sec]

[*Hints* : $E_1-E_2=hc\left(\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\right)$]

28. (*a*) A metalic surface when illuminated with light of $\lambda=3333$A° emits electrons with energies upto 0.6 *ev* ; when illuminated with light of $\lambda=2400$A°, it emits electrons upto 2.04 *ev*. Calculate the Planck's constant and the work function of the metal.

[Ans. 6.58×10^{-27} erg-sec ; 3.1 *ev*]

(*b*) A metal sheet (threshold frequency$=1.5\times10^{15}$) is given a negative charge of 96 e.s.u. How many photons of ultra violet light are requared to completely discharged the sheet ? What is the minimum amount of energy that must be absorbed in oder to effect this discharge ? $e=4{\cdot}8\times10^{-10}$ e.s.u,, $h=6{\cdot}625\times10^{-27}$ ergs-sec. [Ans. 2×10^{11} ; 1·99 ergs]

[*Hints :* To take out a charge of $4{\cdot}8\times10^{-10}$ e.s.u., no of photon required$=1$]

4

ELECTRONIC STRUCTURE OF ATOM

4.1. Atomic structure of matter :

Philosophers of ancient time were known to have expressed curiosity about the structure of matter. They gave various opinions about it. *Kanad*, the ancient Indian sage, was of opinion that matter consists of very small particles. As far back as the 6th century B.C. the greek philosophers opined that matter is composed of discrete, discontinuous and tiny particles. The word '*atomos*' according to Greek vocabulary, means indivisible. Hence they called the tiny particles of matter 'atoms'. But these are all conjectures of ancient people—not substantiated by actual experimentation. They did nothing more and left the idea vague. In the early part of the nineteenth century, a noted English chemist, John Dalton, first developed a scientific theory regarding the structure of matter, partly from the ancient guess-work and partly from some modern experimental results. It is known as *Dalton's atomic theory*. According to this theory, every substance is composed of very minute, indivisible particles, known as atoms.

Till the end of nineteenth century, scientists believed that atom is the smallest, indivisible state of matter, that atoms cannot be subdivided and that two or more atoms combining together produce a molecule and two or more molecules produce a piece of matter. But during the last decade of the nineteenth century, Sir J. J. Thomson, Crookes, Lenard and others carried out some experiments with electric discharge through gases at low pressure and discovered a particle which is about 1835 times lighter than the atom of the lighest element hydrogen. It, at once indicated that the ideas of scientists about the atom being the smallest and indivisible particle are not correct. Atoms are divisible and still smaller particles may be available by breaking up an atom. The most significant characteristic of these tiny particles are : (i) they are charged with negative electricity and (ii) they are alike—*i.e.* from whatever atoms they are collected, they have identical mass, diameter and electrical charge etc. These particles came to be called **electrons.** If electrons are collected from hydrogen atom, oxygen atom, chlorine atom or from any compound, they will be identical in all respects. Since electron enters into the constitution of all atoms, it is considered by the scientists as *a fundamental particle in the structure of matter.*

If a complete atom obtained either from an element or from any other substance be examined in respect of its electrical charge, it will be found to have no charge, either positive or negative ; a complete atom is always neutral but the electrons obtained from an atom show negative charge. It means that in an atom equal amount of positive charge must be present somewhere—otherwise how can an atom be electrically neutral ? How and where does this positive charge exist in an atom ? The answer to this question came first from the celebrated physicist Sir J. J. Thomson who proposed a model of atom, known as Thomson's model of

atom. According to this model, the atom was supposed to be a sphere filled with positively charged matter of uniform density in which just sufficient number of electrons were embedded to balance the positive charge [Fig. 4.1]. Since the electrons embedded within the positive charge resembles the plums in a, pudding, the atom model is sometimes referred to *plum-pudding model.* The electrons were further supposed to possess vibratory motion about their equilibrium position so as to account for the emission of light. When no light was being emitted, the electrons were supposed to be at rest. In 1906, Lord Rutherford, the eminent British physicist performed some experiments from which it became apparent that Thomson's atomic model was not tenable.

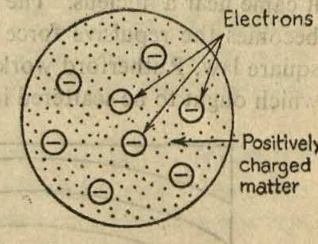


Fig. 4.1

In this experiment Rutherford was observing the course of fast alpha particles which were directed towards a thin sheet of gold. Alpha particles are a kind of positively charged particles emitted by a radioactive substance, having mass about seven thousand and a half times greater than that of an electron. Gold foils can be obtained very thin and the foil with which Rutherford was experimenting, was only 10^{-4} cm. thick. Rutherford noticed that the particles passed easily through the gold foil without making holes in it as a bullet might. This led him to suspect that, quite possibily, there was sufficient space in atoms through which the alpha particles went right through rather than pushing atoms out of the way.

Rutherford also noticed something else which was even more significant. He found that some of the alpha particles were deflected more than 90° from their straight line paths as they went through the foil [fig. 4.2]. Rutherford's brilliant mind thought it highly probable that this was caused by the electric

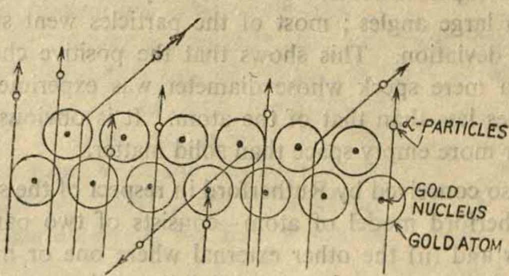


Fig. 4.2

repulsion between the positive charge of gold atom and positive charge of alpha particles. Later on, he, along with Geiger and Marsden, carried out a series of experiments on the scattering of alpha particles by thin foils.

These experimental findings could not be accounted for on the basis of Thomson atom model which supposed that the positive charge of the atom was evenly distributed through it. Rutherford next made a new set of calculations on the assumption that all positive charge of an atom is concentrated in a tiny nucleus

at the centre. The idea behind this approach to the problem was that the force of repulsion and consequent deflection of an alpha particle would depend on how close it came near a nucleus. The closer the alpha particle comes to the nucleus, greater becomes the repulsive force and hence the deflection. Using Coulomb's inverse square law, Rutherford worked out a formula giving the number of alpha particles which ought to be scattered in a particular direction. Gieger and Marsden verified

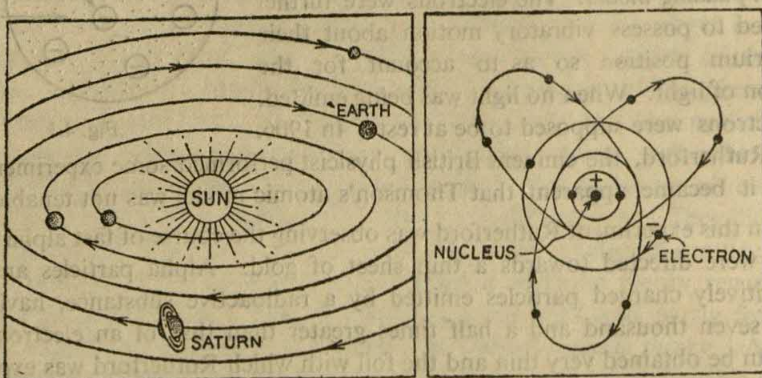


Fig. 4.3

the formula experimentally in 1911. In this way, the truth of Rutherford's assumption that the positive charge of an atom is concentrated in a small nucleus at its centre was verified. He said that the structure of an atom is like the structure of solar system. The solar system consists of a central sun with planets revolving round it in their respective orbits. Atom is very similar to this ; it has a central nucleus with electrons revolving round it in their respective orbits [fig. 4.3].

Rutherford's experiment showed that a very few of the alpha particles were deflected through large angles ; most of the particles went straight through the foil without any deviation. This shows that the positive charge of an atom is concentrated in a mere speck whose diameter was experimentally found to be about 10,000 times less than that of the atom. It is obvious, therefore, that an atom contains far more empty space than solid matter.

The model so conceived by Rutherford in respect of the structure of an atom —known as Rutherford model of atom—consists of two parts (i) one internal, known as *nucleus* and (ii) the other external where one or more electrons exist. The volume occupied by the nucleus is negligibly small compared to the volume of the atom but almost whole of the mass of the atom is deposited in the nucleus.

4.2. Introduction of Bohr's theory :

Very soon some of the drawbacks of Rutherford model of atom came to limelight. According to Maxwell's electromagnetic theory it was known that a charged particle, in accelerated motion, always radiates electromagnetic radiation. Now, an electron, revolving round a nucleus, has always a radial acceleration directed towards the nucleus, due to which the electron will radiate

energy. This energy can only come from the atomic system which will, therefore, steadily lose energy. As a result the radius of the orbit of electron will gradually shrink and finally the electron will fall on the nucleus. Thus the orbital motion of the electron destroys the stability of the atom. Rutherford model of atom when judged according to the laws of classical physics, poses this formidable problem. The dilemma was solved in 1913 by Niels Bohr, the famous physicist of Denmark.

To solve the problem of stability of the structure of atom and to explain the emission of discrete and definite line spectra, Bohr applied Plank's Quantum theory of radiation to Rutherford's atomic model and made the following assumptions :

(i) *That within the atom, an electron can revolve in closed orbits without radiating any energy. This orbital rotation without radiation of energy follows the classical mechanics as well as the laws of electrostatics.*

(ii) *That certain orbits are "permissible" i.e. the atom as a whole can exist only in certain definite states called stationary states. For the possible or the privileged orbits in which an electron can revolve without radiation of energy, the total angular momentum of the electron is an integral multiple of $h/2\pi$, where h is Planck's constant.*

(iii) *That when the electron in an excited state jumps from an outer orbit to an inner orbit of less energy, then alone it radiates a definite amount of energy $h\nu$.*

According to Bohr's first assumption, the electrons cannot revolve in all possible orbits round the nucleus as suggested by the classical theory, but only in certain definite orbits, known as *privileged orbits*. These orbits are such that as long as an electron revolves in any one of these privileged orbits, it will not radiate any energy, although its motion is governed by the ordinary laws of mechanics and electrostatics. In the realm of atoms, hydrogen atom is the simplest, having one proton in the nucleus and one orbiting electron.

According to the second assumption, the privileged orbits along which electrons can revolve without radiation of energy, are those in which the angular momenta of the electrons are integral multiples of a constant quantity. In other words, if r be the radius of a privileged orbit, v be the orbital velocity of the electron in that orbit and m the mass of the electron (Fig. 4.4), then according the Bohr's second postulate $mvr=n.\frac{h}{2\pi}$, where h is Planck's constant. Putting $n=1, 2, 3$, etc., we may get the radii of different privileged orbits. Applying this condition, if we calculate the energy of electrons in different privileged orbits, we will see that these energies are also of discrete values.

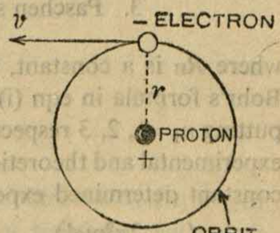


Fig. 4.4

In his theory Bohr assumed that the electrons in an atom may be *excited* by supplying energy from some external source. At ordinary temperatures, the electrons remain in their normal orbits and the atom is said to be in its lowest

energy state or *ground state.* If an atom absorbs energy from some external source, then one or more of the electrons may jump from one orbit into a higher one. In this condition, the atom is said to be *excited* but its stay in an excited condition is momentary. The disturbed electrons, soon jump back into lower orbits and in so doing they emit energy in the form of a electromagnetic wave pulse called a *photon.* According to the wave length of the emitted photon, the radiation may be visible light, ultraviolet light or even *X*-rays. According to Bohr's third postulate, the frequency ν of the absorbed or emitted radiation may be obtained from the relation, $E_2-E_1=h.\nu$, where E_1 and E_2 are respectively the total energy of the electron in the two orbits and h, the Planck's constant.

Bohr showed that in the case of hydrogen atoms, if the electron jumps from an orbit of quantum number n_2 to another of quantum number n_1 of less energy, the radiation will have an energy given by,*

$$E_2-E_1=h\nu=\frac{2\pi^2me^4}{h^2}\left(\frac{1}{n_1{}^2}-\frac{1}{n_2{}^2}\right)$$

If the wavelength of the radiation emitted be λ, then $\nu=\frac{c}{\lambda}$

$$\therefore \quad \frac{1}{\lambda}=\frac{2\pi^2me^4}{ch^3}\left(\frac{1}{n_1{}^2}-\frac{1}{n_2{}^2}\right) \quad .. \qquad .. \quad \text{(i)}$$

where m and e are the mass and charge of an electron.

From eqn (i), we can explain the origin of spectral lines of hydrogen.

Before Bhor proposed his theory of hydrogen spectrum, it had been found that wavelengths of hydrogen spectrum could be arranged in the form of different series, named after their discoverers. Among them were :

1. Lyman series $\rightarrow \frac{1}{\lambda}=R_H\left(\frac{1}{1^2}-\frac{1}{n^2}\right)$
2. Balmer series $\rightarrow \frac{1}{\lambda}=R_H\left(\frac{1}{2^2}-\frac{1}{n^2}\right)$
3. Paschen series $\rightarrow \frac{1}{\lambda}=R_H\left(\frac{1}{3^2}-\frac{1}{n^2}\right)$

where R_H is a constant, known as *Rydberg constant* and n is an integer. From Bohr's formula in eqn (i) it follows that all the spectral series can be obtained by putting $n_1=1, 2, 3$ respectively and $n_2=n$. Further (*a*) the agreement between the experimental and theoretical values of the wavelengths is excellent and (*b*) Rydberg's constant determined experimentally agrees with the value obtained from Bohr's theory $\left(R=\frac{2\pi^2me^4}{ch^3}\right)$.

Thus Bohr-Rutherford model of atom explains very satisfactorily the origin of line spectra of hydrogen & hydrogen like atoms and many other allied phenomena. The theory, as a consequence, came to be accepted by all very soon.

* See Appendix 1

It is to be noted that although hydrogen atom has only one electron, its spectrum when suitably excited, shows many lines instead of one. What is the reason ? It should be remembered that when we excite some quantity of hydrogen gas, millions of atoms are excited. But all these atoms do not absorb equal quantity of energy from the source of excitement. Electrons of these atoms move to different energised orbits according to the energy absorbed and radiate different quantities of energy when they return to their normal orbits. For this reason, the spectrum of hydrogen exhibits many lines.

Example 1 : *Find the radius of first Bohr orbit of hydrogen atom. Given* $h=6{\cdot}63\times10^{-27}$ *erg-sec; mass of electron*$=9{\cdot}1\times10^{-28}$ *gm; charge*$=4{\cdot}8\times10^{-10}$ *e.s.u.*

Ans. The radius of nth orbit $r_n=\dfrac{n^2h^2}{4\pi^2me^2.z}$ [See Appendix 1]

For H_2, $z=1$ and for 1st orbit $n=1$,

$$\therefore \quad r_1=\frac{(1)^2\times(6{\cdot}63\times10^{-27})^2}{4\times(3{\cdot}14)^2\times9{\cdot}1\times10^{-28}\times(4{\cdot}8\times10^{-10})^2}=0{\cdot}53\times10^{-8}\text{ cm. (nearly)}$$

Example 2 : *The energy of an electron revolving in the first Bohr orbit of hydrogen atom is* $-13{\cdot}6$ *eV. What will be the energy of the photon emitted when there is the transition of an electron from the second to the first Bohr orbit of the atom ?* [*H. S. Exam 1983*]

Ans. From Bohr's theory we know, the difference of energy between n_1 orbit and n_2 orbit is given by $E_2-E_1=\dfrac{2\pi^2me^4}{h^2}\left(\dfrac{1}{n_1^2}-\dfrac{1}{n_2^2}\right)$ [For H_2, $z=1$]

If $n_2=\infty$ and $n_1=1$ *i.e.* if transition takes place from an orbit at infinite distance to the 1*st* orbit, $E_\infty-E_1=\dfrac{2\pi^2me^4}{h^2}\left(\dfrac{1}{1^2}-\dfrac{1}{\infty}\right)$ or $E_1=-\dfrac{2\pi me^4}{h^2}$

$(\because \quad E_\infty=0)$

According to the problem $E_1=-13{\cdot}6$ *ev*. So, $\dfrac{2\pi^2me^4}{h^2}=13{\cdot}6$ *ev.*

Now if transition takes place from 2nd to 1st orbit

$$E_2-E_1=\frac{2\pi^2me^4}{h^2}\left(\frac{1}{1^2}-\frac{1}{2^2}\right)=13{\cdot}6\times\tfrac{3}{4}=10{\cdot}2\ ev$$

So, the energy of the photon emitted$=10{\cdot}2$ *ev.*

Example 3 : *An electron of energy 20 ev comes into collision with a hydrogen atom in its ground state. The atom is excited into a state of higher internal energy and the electron is scattered with a reduced velocity. The atom subsequently returns to its ground state with the emission of a photon of wavelength* $1{\cdot}216\times10^{-7}$ *metre. Determine the velocity of the scattered electron. Given* $h=6{\cdot}625\times10^{-34}$ *joule-sec ;* $m=9{\cdot}1\times10^{-28}$ *gm ;* $c=3\times10^8$ *m/s and* 1*e.v.*$=1{\cdot}6\times10^{-12}$ *erg.*

Ans. The energy of the radiated photon $E=\dfrac{hc}{\lambda}=\dfrac{6{\cdot}625\times10^{-34}\times3\times10^8}{1{\cdot}216\times10^{-7}}$ joule

$$=\frac{6.625\times10^{-34}\times3\times10^{8}}{1.216\times10^{-7}}\times10^{7}\text{ erg.}$$

$$=\frac{6.625\times10^{-34}\times3\times10^{8}\times10^{7}}{1.216\times10^{-7}\times1.6\times10^{-12}}\;ev.$$

$$=10.22\;e.v.$$

Hence the K.E. of the scattered electron

$$=20-10.22=9.88\;ev.$$

$$=9.88\times1.6\times10^{-12}\text{ erg.}$$

If v be the velocity of the scattered electron, then

$$\tfrac{1}{2}.mv^2=9.88\times1.6\times10^{-12}$$

$$\text{or }\tfrac{1}{2}\times9.1\times10^{-28}.v^2=9.88\times1.6\times10^{-12}$$

$$\therefore\; v^2=\frac{2\times9.88\times1.6\times10^{-12}}{9.1\times10^{-28}}$$

$$\text{or }v=1.86\times10^{8}\text{ cm/sec.}$$

Example 4 : *A single electron orbits round a stationary nucleus of charge $+ze$. where z is a constant and e is the magnitude of electronic charge. It requires 47·2 ev to excite the electron from the 2nd Bohr orbit to the 3rd. Find (i) the value of z (ii) the energy required to excite the electron from the 3rd to the 4th orbit (iii) the radius of 1st Bohr orbit (iv) K. E. of the electron in the 1st Bohr orbit. Given ionising energy of hydrogen atom=13·6 ev. ; Planck's constant 6.6×10^{-27} erg-sec. : $m=9.1\times10^{-28}$ gm. ; $e=4.8\times10^{-10}$ e.s.u.* [*I.I.T. 1981*]

Ans. (i) From Bohr's theory, we know, $E_3-E_2=\dfrac{2\pi^2me^4z^2}{h^2}\left(\dfrac{1}{n_2^2}-\dfrac{1}{n_3^2}\right)$

or, $47.2=z^2\times13.6\times\dfrac{5}{36}$ $\left[\text{Ionising energy}=\dfrac{2\pi^2me^4}{h^2}\text{ and }n_2=2\;;\;n_3=3\right]$

$$\therefore\; z^2=\frac{47.2\times36}{5\times13.6}=25\text{ (nearly)}\quad\therefore\quad z=5$$

(ii) Energy reqd. to excite electron from 3rd to 4th orbit

$$=\frac{2\pi^2me^4z^2}{h^2}\left(\frac{1}{3^2}-\frac{1}{4^2}\right)=25\times13.6\times\frac{7}{144}\,ev=16.52\;e.v.$$

(iii) Radius of 1st Bohr orbit $r_1=\dfrac{h^2}{4\pi^2me^2z}$

$$=\frac{(6.6\times10^{-27})^2}{4\times(3.14)^2\times9.1\times10^{-28}\times(4.8\times10^{-10})^2\times5}=0.1\times10^{-8}\text{ cm.}$$

(iv) K. E. of the electron in the 1st orbit

$$\tfrac{1}{2}mv^2=\tfrac{1}{2}\frac{ze^2}{r}=\tfrac{1}{2}\times\frac{5\times(4.8\times10^{-10})^2}{0.1\times10^{-8}}=0.576\times10^{-9}\text{ erg.}$$

4.3. Electron configuration in an atom :

The difference in the number of electrons in atoms attributes difference in physical and chemical properties of those atoms and this is why we see varieties of substances around us. But how are these electrons in an atom situated ? It

goes without saying that these electrons revolve in orbits defined by Bohr's condition. One or more such orbits constitute an electron shell and Bohr named them as *K, L, M, N* etc. shells. The maximum number of electrons that a particular shell can contain is fixed. For example, the shell nearest to the nucleus—the *K* shell—can contain, at best, two electrons and not more than that. The next shell—the *L* shell—can contain 8 electrons, the *M* shell 18 electrons, etc. If a shell contains all the electrons that it can possibly contain, the shell is said to be *complete* or *saturated*. These numbers may appear haphazard at first sight but they are not so. If the shells are numbered as 1, 2, 3 etc. then the square of the number of any shell multiplied by 2 will give the greatest number of electrons that the shell can contain.

The hydrogen atom is the simplest of all atoms ; it has only one electron. It goes without saying that this single electron of the hydrogen atom resides in the *K*-shell. Next comes the helium atom which has two electrons. The *K*-shell cannot hold more than 2 electrons. Hence, 2 electrons of helium atom revolve in the *K*-shell which, therefore, becomes complete. Next comes lithium atom having three electrons. Two of these electrons complete the *K*-shell and the third

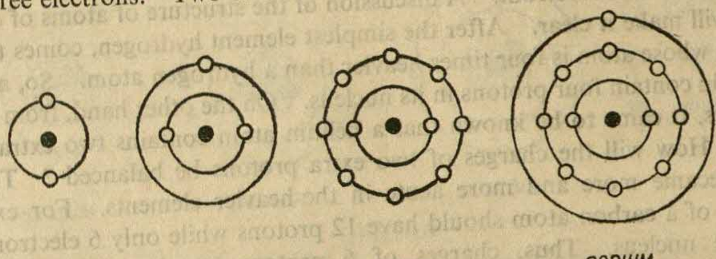


Fig 4.5

goes to *L*-shell which can hold, as we have seen earlier, 8 electrons in the maximum. In this way, if we come to neon atom, we find that it has 10 electrons. Out of these 10 electrons, two go to complete the *K*-shell and the remaining 8 electrons go to complete the *L*-shell. So, the first two shells of the neon atom are complete just as the first shell of the helium atom is. Next comes sodium atom (11 electrons) which requires *M* shell to provide position for its electrons (fig. 4.5).

It may be mentioned in this connection that chemical properties of an atom are mainly determined by the electrons in the outermost orbit of it. These electrons are known as '*valence electrons*'. We have seen that the atoms of gaseous elements like helium, neon, argon etc. have respectively their *K*-shell, *L*-shell, *M*-shell complete. There is no vacancy in those shells which means that these atoms have no valence electrons. Curiously enough, their chemical properties are also similar—each one of them is an inert gas. Again, atoms of lithium, sodium, potassium etc. have one valence electron each and the valence electrons of these elements reside respectively in *K, L, M* shells. There is also striking similarity in their chemical properties.

4.4. Structure of the nucleus :

One of the important parts of the atomic model conceived by Rutherford is the positively charged nucleus. It contains almost the whole mass of the atom and is a tightly packed concentrated mass, occupying a very small volume of the atom. One or more electrons of the atom move round the nucleus in different shells in their own orbits. But the question is : what is the structure of this significant part of the atom ?

The simplest of all atoms—the hydrogen atom—is found to have only one electron. Its nucleus, therefore, should have positive charge equal in amount to the negative charge of an electron because a complete atom is always electrically neutral. This nucleus of hydrogen atom came to be known as **proton.** Like electron, proton is also a fundamental particle in the structure of matter. It was, therefore, told at first that the nucleus is composed of protons only and because a complete atom is always electrically neutral, the number of electrons in the atom is equal to the number of protons in the nucleus.

But this structure of nucleus is not without defects. Very soon its shortcomings came in the forefront. A discussion of the structure of atoms of different elements will make it clear. After the simplest element hydrogen, comes the inert gas helium whose atom is four times heavier than a hydrogen atom. So, a helium atom should contain four protons in its nucleus. On the other hand, from various experiments, it came to be known that a helium atom contains two extranuclear electrons. How will the charges of two extra protons be balanced ? This discrepency became more and more acute in the heavier elements. For example, the nucleus of a carbon atom should have 12 protons while only 6 electrons exist outside the nucleus. Thus, charges of 6 protons remain unbalanced. The heaviest element uranium should have 238 protons in the nucleus, while only 92 electrons revolve round the nucleus. So, charges of $(238-92)=146$ protons are yet to be balanced. In all these cases, in order to balance the charges of extra protons and to give a complete atom a neutral character, the scientists agreed to accept the existence of requisite number of electrons *in the nucleus*. Thus, the nucleus of helium atom, according to the new conception, contains 2 electrons, carbon atom 6 electrons, uranium atom 146 electrons and so on. Electrons, being exceedingly small in mass, will not contribute, by their presence, anything to the mass of the nucleus and hence to the mass of an atom but will give an atom its much-needed neutral character. So, the structure of the nucleus now stands as follows : it is composed of electrons and protons but in the case of hydrogen only the nucleus does not contain any electron ; it is composed of a proton only.

But this model, too did not last long. A number of serious difficulties cropped up. Finally, after the discovery of stable isotopes, it appeared almost certain that electrons cannot exist inside the nucleus for the following reasons. Substances with identical chemical properties but different atomic masses are ealled *isotopes*. This meant that they had to be placed in the same position in the periodic table and for this reason, the substances are called 'isotopes', a word derived from the greek and means occupying the same place. Identity of chemical properties between an element and its isotopes necessarily means that

their atoms have equal number of extra nuclear electrons because these electrons are responsible for the chemical behaviours of an atom. Consequently, these atoms should have an equal number of protons, too. How then, can we account for the difference in their atomic weights ? Besides, some other important factors, like nuclear spin, magnetic moment etc. go against the hypothesis that electrons exist inside the nucleus.

The dilemma was solved in 1932, when another fundamental particle was discovered by Prof. James Chadwick who gave the name *neutron* to this new particle. Neutron is a neutral particle having mass almost same as that of a proton. Immediately after the discovery of neutron, scientists generally accepted the proton-neutron theory of the structure of the nucleus and rejected the proton-electron theory. According to this theory, the number of extra-nuclear electrons is equal to the number of protons in the nucleus and the extra electrons in the nucleus according to the proton-electron theory are to be replaced by an equal number of neutrons. Thus, in an atom of helium, there are 2 extra-nuclear electrons revolving round the nucleus which is composed of 2 protons

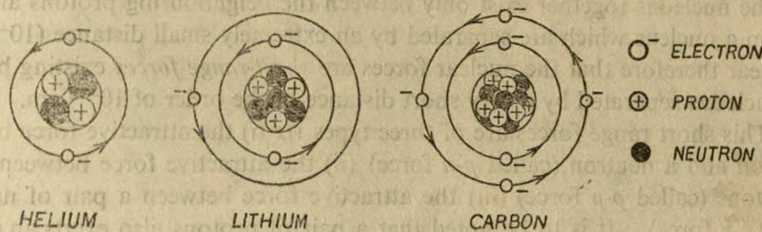


Fig. 4.6

and 2 neutrons. In lithium atom, there are 3 protons and 4 neutrons in the nucleus and 3 electrons outside (fig. 4.6), in a carbon atom, there are 6 extra-nuclear electrons and 6 protons and 6 neutrons in the nucleus, in uranium atom, 92 electrons outside the nucleus and 92 protons together with $(238-92)=146$ neutrons in the nucleus. Only hydrogen atom does not contain neutron. Regarding the structure of isotopic nuclei, it is told that the atoms of isotopes contain the same number of protons but different number of neutrons in their nuclei. Different number of neutrons in the nucleus would give different atomic weight but because of identical number of protons and hence of electrons, their chemical properties will precisely be the same. For example, in the case of neon isotopes of atomic masses 20 and 22, each atom of the isotopes will have 10 electrons and 10 protons but in the nucleus of the first, the number of neutron is 10 while in the second, the number is 12.

Removing all other difficulties that cropped up in connection with electron-proton theory, the newly proposed proton-neutron theory thus came to be accepted unanimously as the structure of atomic nuclei.

4.5. Nuclear Forces :

Having understood the structure of nucleus, question may now be raised as to why the protons and neutrons are held bound tightly in a nucleus ?

Protons, we know, carry positive charge and according to the laws of the electrostatics, there ought to be forces of repulsion (Coulomb's law) among the protons ; Neutrons on the other hand, are not charged particles and hence there is no such electrical forces acting among the neutrons. Nor there can be any Coulombian force between a proton and a neutron. Only other force that is possible is the force of gravitation between neutrons and protons. But nuclear forces can not be due to gravitational reason only because the gravitational forces acting inside the nucleus is too feeble to account for the powerful attractive forces between nucleons. It is, therefore, suggested that the attractive forces which operate among the nucleons are of a new type having no similarity with the ordinary forces that we see around us. These attractive forces are called nuclear forces.

We shall now try to investigate into some of the characteristics of this entirely new type of force. Electrostatic forces of attraction or repulsion operate between any pair of charges and varies inversely as the square of the distance between the charges, whatever may the distance be—small or large. But the nuclear forces which hold the nucleons together exist only between the neighbouring protons and neutrons in a nucleus which are separated by an extremely small distance (10^{-12} cm). It is clear therefore that the nuclear forces are *short-range forces* existing between two nucleons separated by a very short distance of the order of 10^{-12} cm.

This short range forces are of three types *viz* (i) the attractive force between a proton and a neutron (called *p-n* force) (ii) the attractive force between a pair of protons (called *p-p* force) (iii) the attractive force between a pair of neutrons (called *n-n* force). It is to be noted that a pair of protons also exerts an electrostatic force of repulsion between them but this force of repulsion is very weak compared to the strong nuclear attractive force. Experiments have shown that $(p-p)$, $(p-n)$ and $(n-n)$ forces are approximately equal. This is referred to as *charge independent character* of nuclear force.

According to *H.* Yukawa, the Japanese Nobel Laureate, the short range attractive forces among the nucleons arise due to the exchange of meson between protons and neutrons in a nucleus.

4.6. Mass number and atomic number :

Atoms being exceedingly small and light, their weights are determined in comparison with the atom of a particular element as a standard. Originally the mass of hydrogen atom was taken as the standard but since 1961 it has been agreed to use 1/12 *th* of the mass of the most commonly occurring isotope of carbon as the standard. According to this standard, the atomic weight (as a matter of fact, it should be atomic mass) of hydrogen is 1.00813, that of helium 4.00387, that of aluminium 26.99014 etc. This means that an atom of hydrogen is about $\frac{1}{12}$ times heavier than a carbon atom. Similarly, a helium atom is $\frac{4}{12}$ or $\frac{1}{3}$ times and an aluminium atom about $\frac{27}{12}$ times heavier than a carbon atom.

Now, the **mass number** of an atom denotes the integer nearest to the value of the atomic mass of that atom. Thus, the mass number of hydrogen atom is 1,

that of helium 4, of aluminium 27 etc. The mass number also represents the total number of protons and neutrons in the nucleus of an atom. For example, the mass number of helium is 4 but the helium nucleus is composed of two protons and two neutrons *i.e.* of 4 particles. In the same way, the nucleus of aluminium atom contains 27 particles, carbon atom 12, uranium atom 238 etc. Ordinarily, the mass number is denoted by the letter A. So, we can write,

A=*number of protons+neutrons in the nucleus.*

On the other hand, the **atomic number** of an atom denotes the number of electrons outside the nucleus of a neutral atom. It has got another important significance. Long ago, the celebrated Russian chemist Mendeleev showed that if chemical elements are arranged in order of their atomic masses, they show a regularly occurring sequence in their chemical properties. Such an arrangement is known as *periodic table.* Interestingly enough, the atomic number of an element coincides with its position in the periodic table. For example, the first position in the periodic table is occupied by hydrogen whose atomic number is 1. In the second position, comes helium and its atomic number is 2. In this way, the heaviest element uranium occupies 92 *th* position in the periodic table while its atomic number is 92.

Ordinarily the letter Z represents the atomic number. So, we can say, Z=*the number of electrons outside the nucleus in an atom.*

Since the number of protons in the nucleus equals the number of electrons outside the nucleus, it may also be said that Z=*the number of protons in the nucleus of an atom.* From this, we can also say that,

A—Z=*the number of neutrons in the nucleus of an atom.*

So, it is seen that if the mass number and the atomic number of an element are known, we can form an idea about the atomic structure of that element.

The notation usually adopted to designate a given atom is $_ZX^A$, where X stands for the chemical symbol of the element to which the atom belongs, A the mass number and Z the atomic number of the element. For example, $_{17}Cl^{35}$, $_{10}Ne^{20}$, $_{13}Al^{27}$ etc. In these, Cl represents chlorine atom which has 17 electrons, 17 protons and (35−17)=18 neutrons. Likewise, Al stands for aluminium atom and it has 13 electrons, 13 protons and (27−13)=14 neutrons.

Semi-Conductors

4.7. Conductor, insulator and semi-conductors :

Substances through which electricity can pass easily are called conductors. Metals like copper, silver, gold, aluminium etc. are good conductors of electricity. Substances through which electricity cannot pass easily are called insulators. Quartz, mica, sulphur, ebonite etc. are the examples of insulators. The conducting or the non-conducting property of substances can be explained on the basis of electronic structure of matter.

The crystalline structure of the common highly conducting metals as mentioned above is such that the outermost valence electrons of their atoms can easily

move to the neighbouring atoms and can fill up the vacancies in their outer most shells. The valence electrons are therefore free to wander through the substance. The complete inner shells of electrons are, however, bound to their individual mucleus. The valence electrons in incomplete shells by their free movement through the substance, conduct electricity and make the substance a good conductor. In contrast with good conductors, substances which are good insulators or good non-conductors, have practically no free electrons. All of the electrons in those substances remain bound to their respective atoms.

There are, however, a large number of solids that are neither good conductors of electricity nor good insulators. Their conductivity lies between those of good conductors and good insulators. These substances are called *semiconductors.* In them, electrons are capable of being moved by the application of heat, strong light or strong electric field. Researches are now being carried out with various semi-conductors—specially with germanium and silicon because with these semiconductors transistors have been devised. Appearance of transistors has made a great stir in the field of radio-physics. The transistor was first discovered in 1948 by three American physicists of the Bell Telephone Laboratories—Bardeen, W. Shockley and W. Brattain and they were jointly awarded the Nobel prize in physics in 1956.

Motion of charge carriers in a semi-conductor :

Silicon and germanium atoms each have four valence electrons. The atomic pattern of atoms in silicon and germanium crystals is a tetrahedral structure such that each atom shares one of its electrons with each neighbour and the neighbour in turn shares one of its four with it. It is called co-valent bond. Since there is no free electrons in absolutely pure germanium, there can be no conduction of electricity at low temperatures. Such pure crystals are called *intrinsic crystals*. When the crystal is raised to room temperature, however, the thermal energy of a valence electron may become greater than the energy binding to its nucleus. The bond is then broken. The electron leaves the atom and becomes a free electron. The free electron wanders through the crystal. Where an electron has broken free, a '*hole*' has been created (upper right and lower left of the fig. 4.7). Since that part of the crystal was neutral beforehand, it now lacks an electron and the vacant 'hole' is equivalent to a net positive charge.

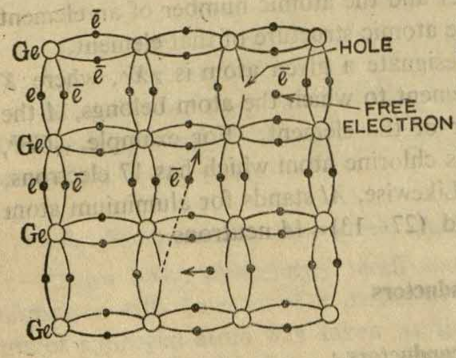


Fig. 4.7

Due also to thermal agitation, *a bound* electron next to a hole can move across to fill the gap, the net motion of the negative charge from one bounded position to another being in effect equivalent to the motion of a hole in the opposite direction. Thus the holes move as if they were carriers with a positive charge $+e$,

where e is the numerical value of the charge on an electron. This action has been shown at the lower part of the fig. 4.7.

Increase of temperature of a semi-conductor crystal, therefore, increases the thermal energy of valence electron which may break the co-valent bond and may become a free electron. This produces greater number of *electron-hole* pair and hence greater number of charge carriers. Consequently conduction of electricity through the crystal is facilitated. So, we can say that the resistance of a semiconductor crystal decreases with the rise of temperature. In this respect, a semi-conductor behaves exactly opposite to a pure metallic substance for, the resistance of a metallic substance, we know, increases with the rise of temperature. By this behaviour, we can easily distinguish between a pure metal and a pure semi-conductor. It is to be remembered that whatever may be the temperature, a pure semi-conductor always contains equal number of electrons and holes.

4.8. N-type and P-type crystals :

A pure or *intrinsic* semi-conductor has charge carriers which are thermally generated. These are relatively few in number. By 'doping' a semiconductor with a tiny amount of impurity, thus forming a so-called *extrinsic* semiconductor, considerable increase can be made to the number of charge carriers. Thus, both *N*-type and *P*-type crystals may be prepared by mixing suitable quantities of certain impurities with pure germanium. In *N*-type crystals, conduction of electricity is done by negative electrons only, while in *P*-type crystals, it is done by positive holes. The conductivity of these crystals is much more than that of pure semiconductors.

N-type crystals : To prepare *N*-type crystals, extremely small quantities of *arsenic* (one part in a million) are added to extremely pure germanium. Arsenic atoms have five valence electrons each and when an atom of arsenic is added to a germanium crystal, the atom settles in a *lattice site* with four of its electrons shared with neighbouring germanium atoms [Fig. 4.8]. The fifth electron may thus become free to wander. From the figure, it is seen that each arsenic atom (*As*) donates one free electron to the system. For this reason arsenic here is known as *a donor*. With arsenic present in quantities of one to a million, there are about 10^{16} donor atoms and 10^{16} free electrons present per cubic centimetre. In a good conductor like copper, there are approximately 10^{23} free electrons per c.c.

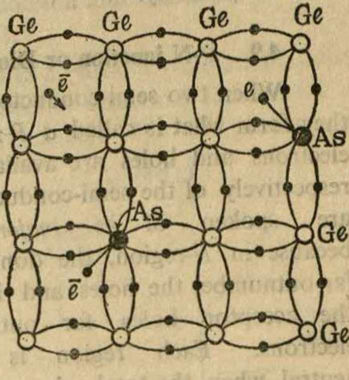


Fig. 4.8

Now due to thermal agitation, when a few bound electrons are broken free, an equal number of holes are thereby created and the free electrons donated by the impurity rush to fill up the gaps. Since, in this case, the number of donor electrons far exceeds the number of unbound electrons or holes, the conduction

of electricity is done mainly by the electrons. In this condition, the crystal is called an *N-type crystal, N* representing the negative charge on an electron.

P-type crystals :

If, on the other hand, germanium crystals are formed with *aluminium* as an impurity, we get a *P*-type crystal. Aluminium atoms have three valence electrons each and getting at a lattice site each aluminium atom attracts an electron from a neighbouring atom, thereby completing the four valence bonds and creating a hole in the neighbouring atom. Such a crystal is therefore one in which each aluminium atom provides one hole to the system that is free to accept an electron (fig. 4.9). For this reason, aluminium here is known as an *acceptor*. By thermal agitation, some of the bound electrons are shaken loose and an equal number of holes are produced. Since by far the majority of the charge carriers are holes and these act like positive charges, the conduction of electricity, in this case, is done mainly by the holes. In this condition, the crystal is called a *P-type crystal, P* representing the positive charge on a hole.

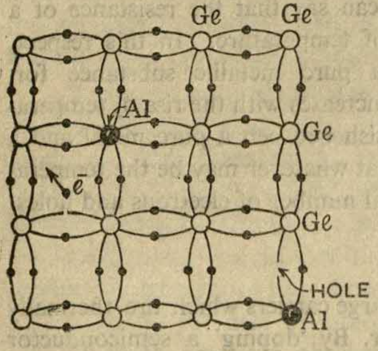


Fig. 4.9

It is to be noted that none of these crystals have a net charge. The surplus of free negative charges of the electrons in an *N*-type crystal and the positive charges of the holes in a *P*-type crystal are compensated for by the positive charges on the arsenic nuclei and the deficiency in positive nuclear charge of aluminium nuclei respectively.

4.9. P-N junction or Diode :

When two semi-conductors of the *P* and *N*-type are brought into contact, they form what is called *a P-N junction* or *diode*. In a junction thus formed, electrons and holes are available as carrier of charges in the *N* and *P*-regions respectively of the semi-conductors. They are spoken of as *majority carriers* because in *N*-region, the donor electrons far outnumber the holes and in *P*-region, the acceptor holes far outnumber the electrons. Each region is electrically neutral when the total charges of all the atoms are considered. If, now, a potential is applied with the help of a battery to make the *P*-region positive (Fig. 4.10) there will be a flow current in the circuit. The reason is as follows. The holes in the *P*-region are repelled by the positive pole of the battery while the electrons in the *N*-region are also repelled by the negative pole of the battery.

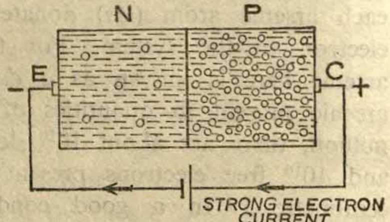


Fig. 4.10

Hence both the holes and the electrons drift towards the junction. The electrons jump across the junction to fill up the adjacent holes and at the same time, holes that migrate to the junction replenish the hole supply at the point. Hence a current flows in the circuit as long as the e.m.f. is applied. Under these conditions the junction is said to be biased in *the forward direction*, offering low resistance to the passage of current. It is to be noted that when the junction is biased in the forward direction, holes are pulled from where there are lots of holes and electrons from where there are lots of electrons, with the result that a large current flows in the circuit.

If, on the other hand, the *P*-region is made negative and the *N*-region positive, by reversing the direction of the applied e.m.f., both holes and electrons are attracted towards the respective terminals and away from the junction. Here, we are trying to pull holes from where there are only a few holes and electrons from where there are only a few electrons. As a result, the circuit current is very low and the junction appears to offer *high resistance* to the passage of current. In this condition, the junction is said to have a *backward bias*.

Since the *P-N* junction is able to pass more current in one direction than in the other, like a *diode* rectifier, the junction may be said to have rectifying properties. The junction diode has advantages over a diode valve ; for example, (*i*) it needs only a low voltage battery to function ; (*ii*) it does not need time to warm up ; (*iii*) it is less bulky and (*iv*) it is cheaper to manufacture in large numbers. For these reasons, the *P-N* junction is now-a-days widely used for rectifying purposes.

4.10. Characteristic curve of a semi-conductor diode :

It appears from the previous section that a forward bias applied to a *P-N* junction allows a large current to flow across the junction. If we apply various voltages—forward as well as backward—to the junction and measure current at each step, we shall get a curve as shown in fig. 4.11.

The graph shows that a slight voltage applied in a particular direction allows a large current to flow but when applied in the reverse direction, the current is very feeble even when the voltage is increased appreciably. For this property, a *P-N* junction is used to rectify an alternating current. Experiments show that as little as one to two volts forward bias permits current of about 20 to 100 milliamps which increases rapidly with increasing battery voltage. But in the case of reverse bias, the current grows to only a few micro-amperes.

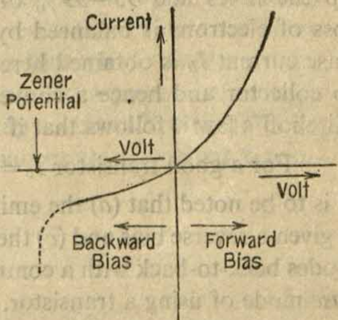


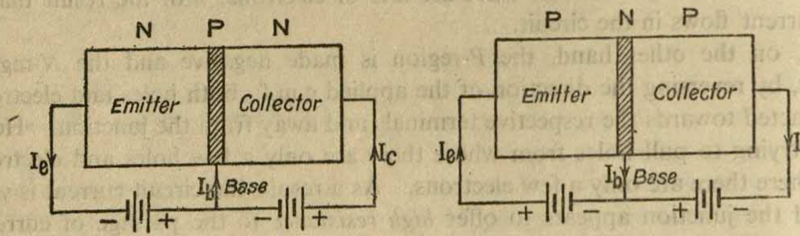
Fig. 4.11

If, however, the reverse bias is made very high, the covalent bonds near the junction break down and a large number of electron-hole pairs will be liberated ; the reverse current then increases abruptly to a relatively large value as shown in fig. 4.11. This phenomenon is known as **Zener effect** and the

particular backward bias at which the reverse current abruptly increases is known as **Zener potential or Voltage.** Zener effect is applied in voltage stabilisation.

4.11. Semiconductor triode or Transistor :

A transistor or a semiconductor triode is composed of three semiconductor elements, two of the *N*-type crystals and one of the *P*-type [Fig. 4.12(a)]. This combination is referred to as an *N-P-N* type transistor. We can also



(a) Fig. 4·12 (b)

have a *P-N-P.* type transistor [Fig. 4.12(b)] composed of two *P*-type and one *N*-type crystals. The circuit connections are made as shown in the figure 4.12(*a*) and (*b*).

Current in a transistor :

With the *N-P-N* type [Fig. 4.12(*a*)], when the diode formed by the left-hand *N-P* junction is biased in the forward direction by applying a small e.m.f. as shown in the figure, electrons from the *N*-layer readily travel towards the *P*-layer which is rich in holes. In this case, the *N*-layer emits electrons like a heated filament and hence it is called the *emitter.* The *P*-layer is called the *base* of the transistor. On entering the *P*-layer, some of the electrons go to fill up the holes, but most of them come under the influence of the positively biased *N*-layer of the right hand *N-P* junction which is connected to a battery of high voltage and are collected by it. For this reason, the right hand *N*-layer is called the *collector* of the transistor. Thus a current I_c, flows in the collector circuit. Since the hole-rich central *P*-layer is very thin (nearly ·001″) only a few of the electrons go to fill up the holes and 95—99% of the emitted electrons reach the collector. The loss of electrons is balanced by electron flow in the base circuit so that a small base current I_b is obtained here. So a strong electron current flows from emitter to collector and hence a conventional current from collector to emitter. From Kirchoff's law it follows that if I_e is the emitter current, $Ie=I_b+Ic$

For a good transistor $I_c=0{\cdot}99\ I_e$ and $I_b=0{\cdot}01\ I_e$

It is to be noted that (*a*) the emitter is always given a forward bias (*b*) the collector is given a reverse bias and (*c*) the transistor is formed by joining two semiconductor diodes back-to-back with a common base. For this reason it is called the *common-base* mode of using a transistor.

The circuit connections are reversed in the case of *P-N-P* type transistor [Fig. 4.12(*b*)]. In this case, the left hand *P-N* diode is biased in the forward direction so that its *P*-layer becomes the *emitter* according to the general rule

stated above, while the right hand *P-N* diode is biased in the backward direction and its *P*-layer becomes the *collector*. The base being common, it is also *common-base* mode. The hole-rich emitter injects hole into the *N*-type *base* region (*i.e.* attracts electron from the base). These holes attract electrons from the negatively biased collector and cause a current to flow from emitter to collector. Apart from the different polarities and directions of current flow, the *N-P-N* transistor is identical with the *P-N-P* transistor.

Here also $Ie = I_b + Ic$

Similarity with triode valve :

From the above discussion, it is clear that a transistor has the following three basic components :—(1) *emitter* (ii) *base* and (iii) *collector*. They represent respectively the filament, grid and plate of a triode. The emitter, like the filament of a triode, sends the carrier into the base (electrons in the *N-P-N* type and holes in the *P-N-P* type) and the collector like the plate of a triode attracts carriers through the base and causes a current to flow through the transistor. The basic principles of a transistor triode are, therefore, analogous to those of a vacuum-tube triode, the base functioning very much as a control grid. *N-P-N* and *P-N-P* type transistors are diagrammatically represented by symbols as shown in fig. 4.13. Arrow head shows the direction of current flow in the two cases.

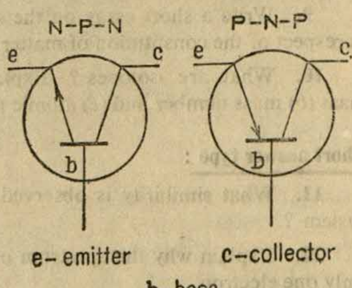


Fig. 4.13

Advantages over vacuum valve :

In recent years the transistors—an entirely new type of electron device—has come into its own and bids to replace the bulky electron tubes in most, if not all applications. Transistors are far smaller than vacuum tubes, have no filament and hence need no heating power and may be operated in any position. They are mechanically rugged, have practically unlimited life and can do some jobs better than electron tubes while catching up fast in other respects. In contrast to an electron tube which utilises the flow of free electrons through a vacuum or gas, the transistor relies for its operation on the movement of charge carriers through a solid substance. Furthermore, it is extremely cheap to manufacture. For all these advantages, all electronic instruments and appliances are now being transistorised.

Exercises

Essay type :

1. Write what you know about the electronic structure of an atom. How did Rutherford introduce the idea of a nucleus ? What was the defect in Rutherford's idea ? [*H. S. Exam. 1982*]

2. What is Bohr's contribution in the structure of an atom ? What are 'valence electrons' ?

3. Describe Bohr-Rutherford structure of atom. Write the assumptions of Bhor. How can you explain the origin of spectral lines by Bohr's theory ? [*H. S. Exam. 1979, '81, '83*]

4. What is the difference between a conductor, a non-conductor and a semi-conductor, so far as electronic theory is concerned ? What are *N*-type and *P*-type semi-conductor crystals ? [*H. S. Exam. 1981*]

5. What is a semi-conductor ? How does a semi-conductor diode act as a rectifier ? Explain. [*H. S. Exam. 1982*]

6. Mention the similarities between (i) *P-N* junction and a valve diode (ii) a transistor and a valve triode.

7. What is the difference between *n*-type and *p*-type semi-conductors ? Explain how *p—n* junction acts as a rectifier. What do you mean by forward bias and reverse bias ? [*H. S. Exam. 1983, '84*] [*Jt Entrance 1983*]

8. Explain briefly the action of a *p—n—p* transistor.

9. Write a short essay on the structure of nucleus. What are the fundamental particles in respect of the constitution of matter ?

10. What are isotopes ? Explain with illustrations the following terms : (*a*) atomic mass (*b*) mass number and (*c*) atomic number.

Short answer type :

11. What similarity is observed between the structure of an atom and that of the solar system ?

12. Explain why the spectrum of hydrogen has many lines although a hydrogen atom has only one electron.

13. Can a hydrogen atom absorb a photon whose energy exceeds its binding energy viz. 13·6 *ev* ?

14. Below are given some atomic notations. Find the number of electrons, protons and neutrons in them : (*a*) ${}_6C^{12}$ (*b*) ${}_{88}Ra^{226}$ (*c*) ${}_7N^{14}$ (*d*) ${}_8O^{16}$.

15. Two kinds of atoms have identical atomic number but different atomic masses. They contain (i) equal number of neutrons and equal number of protons (ii) equal number of protons but different number of neutrons (iii) equal number of neutrons but different number of protons. Which is correct ?

16. From the following pick out the correct statements :

In Bohr model of hydrogen atom (i) the radius of *n*th orbit is proportional to n^2 (ii) total energy of the electron in the *n*.th. orbit is inversely proportional to *n* (iii) the angular momentum of the electron in any orbit is an integral multiple of $h/2\pi$ (iv) the magnitude of potential energy of the electron in any orbit is greater than its *K.E.* [*I. I. T. 1984*]

17. Explain the terms : isotope ; atomic mass, mass number and atomic number. Give examples.

18. Who takes the major role in the conduction of electricity in *N*-type semi-conductor crystal ?

19. Starting with a pure semi-conductor, explain briefly how a *P*-type and a *N*-type semi-conductor are made.

20. Draw a sketch of the characteristic of a *P-N* junction diode. Explain, in terms of the movement of carriers, why the resistance of the diode is low in one direction and high in the reverse direction.

21. Resistance of a conductor increases with the increase of temperature but the resistance of a semi-conductor decreases with the increase of temperature. Explain.

22. What are the advantages of a transistor over a vacum tube ?

23. Is there any difference between *P-N-P* and *N-P-N* transistors ?

Objective type :

24. Fill in the blanks with suitable words :

(*a*) The nucleus of an atom is composed of —— and ——.

(*b*) By doping germanium with a little quantity of pentavalent atoms, we get —— crystals.

(*c*) The position of an element in the periodic table indicates the —— of its atom.

(*d*) The mass number of an atom is 48 and atomic number 23. The atom contains —— electrons, —— protons and —— neutrons.

(*e*) An atom has 6 electrons and 6 neutrons. Its mass number is —— and atomic number is ——.

(*f*) A junction diode offers greater resistance in —— bias than in —— bias.

(*g*) Consider the spectral line resulting from the transition $n=2 \rightarrow n=1$ in the atoms and ions given below. The shortest wave length is produced by ——.

(i) hydrogen atom (ii) deuterium atom (iii) singly ionised helium atom (iv) doubly ionised lithium.

[*Hints* : $\frac{1}{\lambda}=\frac{2\pi e^4 mz^4}{ch^3}\left(\frac{1}{1^2}-\frac{1}{2^2}\right)$; As z for lithium is highest, λ will be minimum] [*I.I.T. 1983*]

Numerical Problem :

25. A single electron orbits round a stationary nucleus of ze where z is a constant and e is the magnitude of electronic charge. It requires 47.2 *ev* to excite the electron from the second Bohr orbit to the third Bohr orbit. Find the value of z. Given $e=4{\cdot}8\times10^{-10}$ *e.s.u.* ; $h=6{\cdot}55\times10^{-27}$ erg-sec ; $m=9{\cdot}06\times10^{-28}$ gm and 1 $ev=1{\cdot}6\times10^{-12}$ erg. [*Jt. Entrance 1985*] [Ans. 5]

26. A doubly ionised lithium atom is hydrogen-like with atomic number 3. Find the wave length of the radiation required to excite the electron in Li^{++} from first to the third Bohr orbit. Ionisation energy of hydrogen atom = 13·6 e.v. [*I.I.T. 1985*] [Ans. 113·7 $A°$]

5

RADIOACTIVITY

5.1. Discovery of radioactivity :

Radioactivity may be defined as a spontaneous disintegration of the nucleus of one or more atoms. The phenomenon was discovered originally by Henry Becquerel, a French physicist, in 1896. The story of the discovery of radioactivity has been told as follows. During the research period of 1890's, many experimenters were studying the phenomenon of fluorescence and phosphorescence. It was well known at that time that various chemical compounds of uranium formed some of the best fluorescent materials with which to experiment.

One day Becquerel opened a drawer in his work-table and removed a photographic plate from a cardbord box and used the plate to take a picture. Upon development of the plate, however, he found an unaccounted for image of a key in the centre of the picture. Thinking back, he recalled that a key was lying on top of the box of plates when he removed it from the drawer. Curious as to how the image could have transferred to the plate, he noticed uranium ores on the table top only a few inches above the box.

Roentgen had just discovered *X*-rays and their penetrating power and Becquerel wondered if the uranium ore was giving off penetrating radiation. He set up the experiment as it was and placed a key on top of a box containing a fresh photographic plate. Above that he placed some uranium ore and after some time, developed the plate and found surely enough, an image of the key ! Becquerel's surmise was correct. Uranium spontaneously emitted very penetrating radiations, which were, at first named as Becquerel rays after the name of the discoverer. The name was later changed to *radioactivity*.

Becquerel's discovery caused an unprecedented commotion among the scientists at that time. Continuing researches with different fluorescent substances Madame Curie and her husband Pierre Curie in Paris, found that all thorium compounds emit, like uranium, powerful radiations. But the most significant discovery made by them was a mineral of uranium—known as *Pitch blende* which was many times more active than uranium. Learning this, the then Austrian Government made them a gift of a ton of pitch blende residues from the uranium refineries in Bohemia. After many weeks of painstaking toil, the Curies managed to extract from it a small quantity of hitherto unknown radioactive element. This they called *polonium* in honour of Madame Curie's native land, Poland. Continuing their investigations, they subsequently isolated in 1902 another new element which was more active still, and to this they gave the name *radium*. The activity of radium is more than a million times that of uranium for equal weights. Madame Marie Curie and Pierre Curie were awarded the Nobel Prize in 1903 for their remarkable discovery.

Subsequently, many other radioactive substances were discovered by other workers.

5.2. Nature of the rays emitted by radioactive substances :

Various workers tried to analyse the nature of radiations emitted by radioactive substances by utilising the different effects they produced such as penetrative power, scintillations on fluorescent screen, action on photographic plate etc. In 1899, Rutherford and his associates studied the ionising power of the rays given off by uranium and found that the rays were of two types, one possessed positive charge and were called α-particles while the other possessed negative charge and were called β-particles. Moreover, β-particles were found to be 100 times more penetrating than α-particles. In 1900, Villard showed that there was a third type of radiation in addition to α and β particles. As these were not affected by a magnetic field, they possessed no charge and were called γ-rays.

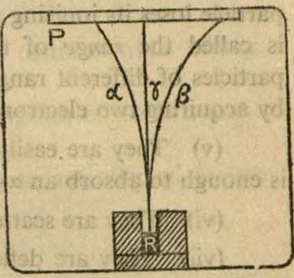


Fig. 5.1

The actual existance of these distinct types of radiation was definitely demonstrated by a simple experiment, first devised by Mme. Curie., which is diagrammatically illustrated in fig. 5.1. A small sample of radium R is dropped to the bottom of a small drill-hole made in a block of lead. This produces a narrow beam of rays emerging from the top of the block, since rays entering the walls of lead are absorbed before reaching the surface. A photographic plate P is held at some distance above the hole and the whole arrangement is placed in an air tight chamber, kept evacuated. A strong magnetic field is applied at right angles to the plane of the paper and directed away from the reader.

After a fairly long exposure, the plate P was developed and three distinct spots were found on it. This shows that the paths of some rays are bent to the left, some to the right and some are not bent at all. Considering the direction of magnetic field, the direction of motion of the rays and the direction of bending, it may be said that the paths bending to the left indicate positively charged particles, called *α-rays* or *α-particles*, those bending to the right indicate negatively charged particles, called β-rays or *β-particles*, and those going straight ahead indicate no charged particles and are called *γ-rays*. *γ-rays* are nothing but electro-magnetic waves like visible light or *X*-rays and are, therefore, not affected by electric or magnetic field.

Further, the curvature of the path bending to the right is more than the curvature of the other path. It shows that β-particles are lighter than α-particles.

5.3. Properties of radioactive rays :

Properties of alpha rays : (i) Alpha particles come out of radioactive bodies with high velocity. Different radioactive bodies emit alpha particles with different velocities which range between $1{\cdot}4\times10^9$ cm./sec to $1{\cdot}7\times10^9$ cm./sec.

(ii) α-particles can produce fluorescene on some substances like zinc sulphide etc. If the fluorescence is examined with a low power microscope, it is found to consist of individual scientillations which show that α rays are made up of discrete particles.

(iii) They affect a photographic plate.

(iv) They cause intense ionisation in gases. In air, it is found that an α particle loses its ionising power after proceeding a certain distance. This distance is called the *range* of the α-particle. Different radioactive substances emit α particles of different range. At the end of the range, an α-particle is neutralised by acquiring two electrons *i.e.* it becomes a neutral helium atom.

(v) They are easily absorbed by matter. A 0.01 cm. thick aluminium foil is enough to absorb an α-particle.

(vi) They are scattered while passing through thin metal or mica sheet.

(vii) They are deflected by electric and magnetic fields. This proves that particles of the rays are charged. From the direction of deflection, it is known that they are positively charged.

Properties of beta rays : (i) Beta rays are deflected by electric and magnetic fields and the direction of deflection shows that the particles are negatively charged. As a matter of fact, beta particles are nothing but fast-moving electrons.

(ii) β-particles are emitted by radioactive substances with very high velocity which ranges from 0·3 to 0·98 times the velocity of light.

(iii) They have smaller energy than α-particles of the same velocity and hence they possess smaller ionising power.

(iv) These particles do not show any definite range in air, and they follow an irregular path unlike α-particles which move in straight lines.

(v) On photographic plates, β-rays produce greater effect than α-particles.

(vi) They produce fluorescence on fluorescent substance.

(vii) Their penetrating power is greater (about 100 times) than that of α-particles. They penetrate through aluminium foil of about 1 cm. thickness.

Properties of γ rays : (i) These rays are not deflected by even very strong electric or magnetic fields. Hence they are believed to be of same nature as X-rays and possess no charge.

(ii) Their velocity is same as the velocity of light *i.e.* 3×10^{10} cm/sec.

(iii) Their penetrating power is very great. They can penetrate several centimetres of lead without being absorbed.

(iv) They have got ionising power—but not so intense as β or α-rays.

(v) They produce fluorescence and affect a photographic plate.

(vi) They possess all optical properties like reflection, refraction etc. As a matter of fact they are electromagnetic waves like ordinary light but of very small wave length.

5.4. Characteristics of radioactivity :

After the discovery of radioactivity several scientists in course of systematic preliminary survery of the phenomenon established the following characteristics of radioactivity :—

(i) Radioactivity is essentially a nuclear phenomenon and it has nothing to do with the electrons outside the nucleus.

(ii) Only those elements whose atomic weights are more than 206, exhibit the phenomenon of radioactivity.

(iii) The activity consists in the emission of three distinct kinds of rays known as alpha, beta and gamma rays.

(iv) The activity results in the spontaneous disintegration of nucleus and a natural transformation of one element into another.

(v) The activity is spontaneous in the sense that it is not affected by any external agent whether physical or chemical.

5.5. Alpha particles are the nuclei of helium atoms :

It has been mentioned earlier that alpha particles are deflected by electric and magnetic fields and that the direction of deflection suggests that the particles are charged positively. By measuring the deflection of alpha particles in electric and magnetic fields, the specific charge e/m of an alpha particle can be found out in a similar way by which e/m of an electron was determined. Experiments show that e/m of an alpha particle is just half of e/m of hydrogen atom *i.e.* $\left(\frac{e}{m}\right)_\alpha=\frac{1}{2}\left(\frac{e}{m}\right)_H$

Later on, measurement of charge carried by each α-particle showed that each particle carries positive charge equal in magnitude to the combined charge of two electrons *i.e.* twice the charge of hydrogen atom. Symbolically it may be represented as $e_\alpha=2e_H$; therefore, $m_\alpha=4m_H$.

Thus, mass of an α-particle is four times the mass of one hydrogen atom and charge is twice the charge of one hydrogen atom or an electron (or a proton). What do we know of the nature of α-particle from this ? According to modern atomic structure, a helium atom consists of a positively charged nucleus at its centre surrounded by two revolving electrons. The nucleus is composed of two positively charged protons and two neutrons. So the mass of a helium nucleus is four times the mass of a hydrogen atom and charge is twice because a hydrogen atom consists of one proton in its nucleus and one revolving electron. From this similarity, scientists came to this conclusion that an alpha particle is the nucleus of a helium atom.

5.6. Comparson between α- and β-particles :

Mass : The mass of an alpha particle is four times the mass of a hydrogen atom while the mass of a beta particle is equal to the mass of an electron. We have seen earlier that the mass of a hydrogen atom is 1835 times the mass of an electron. So, mass of an α-particle$=4\times$mass of a hydrogen atom

$=4\times1835\times$mass of a β-particle

$=7340\times$mass of a β-particle.

Charge : An alpha particle contains charge same as the charges of two protons. Again, a beta particle contains charge same as an electron or a proton. So, comparing the two we get,

$$\frac{\text{charge of an } \alpha\text{-particle}}{\text{,, ,, a } \beta\text{-particle}} = \frac{\text{Total charge of 2 protons}}{\text{,, ,, ,, 1 proton}} = 2$$

Velocity : Velocity of α-particles ranges from $1{\cdot}4 \times 10^9$ cm/sec to $1{\cdot}7 \times 10^9$ cm/s whereas the velocity of β-particle ranges from $0{\cdot}3 \times 10^{10}$ cm/s to $0{\cdot}68 \times 10^{10}$ cm/s.

Penetrating power : β-particles possess greater penetrating power than α-particles.

5.7. Radioactive decay :

The atoms of radioactive substances are not stable ; they disintegrate and during disintegration, the atoms give off either alpha or beta particles. Gamma rays arise from the energy changes produced when a nucleus throws out a particle. When a radioactive element decays, the product may itself be radioactive and the process can be traced through a series of elements. Each time a nucleus disintegrates and gives off a particle, it becomes a different nucleus and the end product of all such breakdowns is usually some isotope of lead.

5.8. Laws of disintegration :

Analysing different radioactive decay products, Rutherford and Soddy established two laws in connection with radioactivity. They are as follows :

(i) *When an atom disintegrates, emitting an α-particle a new atom is formed whose mass number is smaller by* 4 *and atomic number smaller by* 2 *than the parent atom.*

Since an alpha particle is composed of two protons and two neutrons, ejection of an alpha particle from a radioactive nucleus will cause the formation of a new nucleus whose mass number and atomic number will be less by 4 units and 2 units respectively than the corresponding values of the original nucleus. Thus,

$${}_{88}Ra^{226} \xrightarrow[\alpha]{} {}_{86}Rn^{222} \xrightarrow[\alpha]{} {}_{84}RaA^{218}$$

Radium disintegrates into radon whose mass number (222) and atomic number (86) are less by 4 units and 2 units respectively than the corresponding values of radium because an alpha particle is ejected in the disintegration. Similarly radon breaks down into radium-*A*, ejecting an α-particle and the mass number and atomic number are correspondingly reduced.

In general, it may be written as :

$${}_{Z}X^{A} \xrightarrow[\alpha]{} {}_{Z-2}Y^{A-4} + {}_{2}He^{4} \quad (\textit{i.e. } \alpha\text{-particle})$$

(ii) *When an atom disintegrates, emitting a β-particle, a new atom is formed whose mass number remains the same but the atomic number is increased by* 1.

The explanation is as follows :

A neutron, is regarded as a combination of a proton and an electron ($n^\circ \rightarrow p^+ + e^-$). When such an electron is ejected from a nucleus in the form of a beta particle, the product nucleus gets an extra proton instead of a neutron and so its atomic number is raised by one unit, although the mass number remains the same. Thus,

$$_{82}RaB^{214} \xrightarrow[\beta]{} {}_{83}RaC^{214}$$

In general, it may be written as :

$$_{Z}X^{A} \xrightarrow[\beta]{} {}_{Z+1}Y^{A} + {}_{-1}e^{0} \quad (i.e.\ \beta\text{-particle})$$

Hence the complete chain of disintegration from radium to radium C may be written as :

$$_{88}Ra^{226} \xrightarrow[\alpha]{} {}_{86}Rn^{222} \xrightarrow[\alpha]{} {}_{84}RaA^{218} \xrightarrow[\alpha]{} {}_{82}RaB^{214} \xrightarrow[\beta]{} {}_{83}RaC^{214}$$

The process of disintegration goes on until the stable product is reached, which is usually an isotope of lead having atomic number 82.

The above explanation also accounts for the expulsion of beta-particles (*i.e.* electrons) from a radioactive nucleus although a nucleus does not contain electron.

Emission of gamma-rays :

When a radioactive nucleus emits an alpha particle or a beta particle, the product nucleus ordinarily is raised to an excited state. The excited nucleus within a moment returns to a lower energy state or straight to its normal state. The energy difference between these two states is radiated in the form of a gamma-ray photon. A particular radioactive nucleus may, thus, emit γ-rays of different energies. Thus Thc″ emits five γ-rays of five different energies. It is to be noted that γ-ray emission does not cause any change in the mass number or atomic number of the nucleus.

5.9. Half-life and average life :

Now, among the disintegrating atoms of a radioactive element, some may have only a short existence, while others may remain unchanged for a long time—why we do not know. The disintegration of a particular atom is entirely a chance incidence. Experiments show, however, that every radioactive element has a definite rate of disintegration which may be conveniently represented by its *half-life period.*

The half-life of a radioactive element is the time required for half of a given quantity of that element to disintegrate into a new element. For example, half-life of radium is 1622 years. This means that if we start with 1 gm. of radium to-day, then 0.5 gm. of it will have disintegrated in 1622 years. After another 1622 years,

half of what remains will have disintegrated, leaving 0.25 gm. and so on. If this rate of disintegration is represented graphically, it will be like a graph as shown in fig. 5.2. From the figure, it is seen that infinite time will be required for all the atoms to disintegrate. As a matter of fact, decay curves of all the radio-active substances follow the same rule and are of same type with the only difference that the half-lives of different substances are different. For example, half-life of radium is about 1622 years but the radioactive element obtained by the disintegration of radium—it is called radon—has a half-life of about 4 days only.

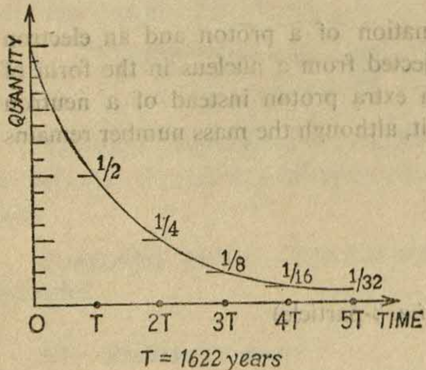


Fig. 5.2

[Suppose N_0=no. of atoms of a radioactive substance present at $t=0$.

N=no. of atoms of the radioactive substance present at $t=t$

and dN=a small number of atoms which disintegrate during a small interval dt after t.

Then, $\frac{dN}{dt}$=rate of disintegration at that instant. Experiment shows that the rate of disintegration is proportional to the number of atoms present at that time. Hence, $\frac{dN}{dt} \propto N$ or, $\frac{dN}{dt} = -\lambda.N$, where λ is a constant, called *decay constant* of the radio-element under consideration. Negative sign shows that with the increase of time, the number of atoms decreases as a result of disintegration. Integrating the above equation we get, $\int \frac{dN}{N} = \int -\lambda.dt$

or, $\log N = -\lambda t + k$ ..(i) [k is the constant of integration]

Now we know that at $t=0$, $N=N_0$. Hence putting this value in the above equation, $\log N_0 = 0 + k = k$.

Substituting this value of k in eqn. (i), we get,

$$\log N = -\lambda t + \log N_0 \quad \text{or,} \quad \log \frac{N}{N_0} = -\lambda t$$

$$\text{or,} \quad \frac{N}{N_0} = e^{-\lambda t} \quad \text{or,} \quad N = N_0 e^{-\lambda t} \quad .. \text{(ii)}$$

The equation (ii) shows that the disintegration of radioactive atoms follows an exponential law as graphically represented in fig. 5.2.

If, after a time $t=T$, the number N becomes equal to the half of the original

number N, then T is called the *half-life* of the radio-element under consideration. Putting these values in eqn. (ii), we get,

$$\frac{N_0}{2}=N_0e^{-\lambda T} \quad \text{or,} \quad \frac{1}{2}=\frac{1}{e^{\lambda T}}$$

$$\therefore \quad e^{\lambda T}=2 \quad \text{or,} \ \lambda T=\log_e 2 \quad \text{or,} \quad T=\frac{\log_e 2}{\lambda}=\frac{0{\cdot}693}{\lambda} \quad \ldots \quad \text{(iii)}$$

So, half-life of a radio-element is inversely proportional to its decay constant.

Average life : The disintegrations of radioactive nuclei are subject to statistical fluctuations. Nothing can be said about the decay of an individual atom. Among the numerous radioactive atoms of a substance, some have only a short existence, while others remain intact for a long time for reasons unknown to us. Why a particular atom decays at a given instant, its neighbouring atoms remaining unchanged, is a mystery. The past history of an atom—specially the fact that it has remained stable for a long time, seems to have no effect at the moment of its disintegration. In other words, depending upon chance, the actual life of an individual atom may be anything between *zero* and *infinity*. In such cases, the average life of the atoms can be calculated by adding the possible lives of all the atoms and then by dividing it by the total number of atoms present at the beginning. It can be shown that the average life $\tau=\frac{1}{\lambda}$ .. (iv).

Thus, the average life of a radio-active atom is the reciprocal of the disintegration constant of the atom.

From eqn. (iii) and (iv), we have, $T=\frac{0{\cdot}693}{\lambda}=0{\cdot}693$.

Example 1 : *The mass number and atomic number of uranium are respectively* 238 *and* 92. *It disintegrates successively into lead emitting eight* α*-particles and six* β*-particles. What will be the mass number and the atomic number of the lead isotope formed* ?

Ans. We know that when an α-particle is ejected, the product nucleus has its mass number reduced by 4 units and atomic number by 2 units. So ejection of 8 α-particles will give the product nucleus a reduction of $4\times8=32$ units in mass number and $8\times2=16$ units in atomic number.

But when a β-particle is ejected, the mass number of the product nucleus remains same as that of the parent nucleus while its atomic number is raised by one unit. Hence, ejection of 6 β-particles will give the product nucleus a rise of $6\times1=6$ units in its atomic number only.

So, the lead isotope will have a mass number$=238-32=206$ and an atomic number$=92-16+6=82$.

Example 2 : *The half-life of radon is* 4 *days. What is the value of its decay constant* ?

Ans. We know, $T=\frac{0{\cdot}693}{\lambda}$ or $\lambda=\frac{0{\cdot}693}{T}$ $\therefore$ $\lambda=\frac{0{\cdot}693}{4}=0{\cdot}173$ per day.

Example 3 : *At a certain instant, a piece of radioactive material contains* 10^{12} *atoms. How long will elapse before* 10^4 *atoms remain ? The half-life of the material is* 30 *days.*

Ans. We know, $N=N_0 . e^{-\lambda t}$

Here $N_0=10^{12}$ atoms ; $N=10^4$ atoms ; $\lambda=\frac{0{\cdot}693}{30}$

$$\therefore \quad 10^4=10^{12}\times e^{-\frac{0\cdot693}{30}\cdot t} \quad \text{or} \quad 10^{-8}=e^{-\frac{0\cdot693}{30}\cdot t}$$

Taking logs to the base 10, we have, $-8=-\frac{0{\cdot}693}{30}\times t\times \log e$

$$\therefore \quad t=\frac{8\times30}{0{\cdot}693\times\log e}=64 \text{ days (approx.)}$$

Example 4 : *The half-life of a radioactive substance is* 60 *years. Calculate its decay constant and average life.*

Ans. We know, $\lambda=\frac{0{\cdot}693}{T}=\frac{0{\cdot}693}{60}=1{\cdot}155\times10^{-2}$ per year.

Further $\tau=\frac{T}{0{\cdot}693}=\frac{60}{0{\cdot}693}=86{\cdot}5$ yrs (nearly)

5.10. Activity of a radioactive element :

The activity of a radioactive element is defined as the number of disintegrations per sec. If A_0 be the activity of a radioactive substance at $t=0$ then its activity A after a time t is given by

$$A=A_0 e^{-\lambda t}$$

where λ is the disintegration constant of the element. This shows that activity varies exponentially with time.

By international agreement, the unit of activity is **Curie** (symbol C_i) which is equal to $3{\cdot}7\times10^{10}$ disintegrations per sec. This is, however, the activity of 1 gm. of pure radium.

'*Rutherford*' is also taken as a unit of activity. It represents a rate of disintegration of 10^6 nuclei per second. Its symbol is R.

5.11. Artificial transmutation :

Transmutation, in general, means the conversion of one element into another *i.e.* one type of atom into another. Natural radioactivity is an illustration of transmutation. If the transmutation is brought about by any artificial means, it will be called an artificial transmutation. In the middle ages, a group of people, known as *alchemists* had a strong belief in the existence of the *philosopher's stone* which would on touch convert base metals into noble ones. The alchemists' life long search for this wonderful agent of transmutation resulted in disappointment chiefly due to their vague and meagre idea

regarding the structure of matter. Several centuries of arduous labour brought the scientists close to the fundamental constituents of matter. Finally with the discovery of radioactivity, it became clear that the individuality of an atom rests in its nucleus and that transmutation will be possible if the nucleus of an atom

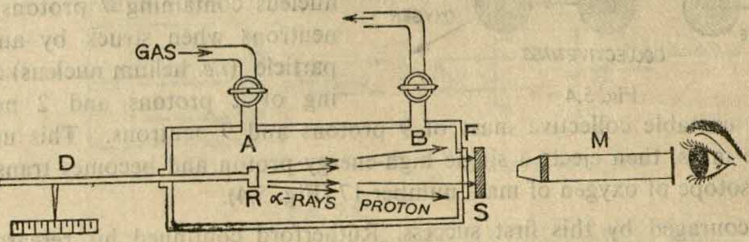


Fig. 5.3

can be disrupted by any artificial process. Guided by this idea and noticing at the same time, the great speed with which α-particles are ejected from natural radioactive substances, Rutherford made up his mind to use α-particles as tools for smashing nucleus. He performed a series of experiments in this direction and finally, in 1919, succeeded in transmuting elements thus realising the cherished dream of the alchemists. Rutherford's arrangement of apparatus is shown in fig. 5.3.

One side of a long glass tube had an opening which was covered with a thin sheet of silver foil *F*. A zinc sulphide screen *S* was placed close to the silver foil and the scintillations on it, when they occurred, were observed by a microscope *M*. Through one side tube at *A*, gases may be introduced in the glass tube while through another side tube at *B*, the gas may be pumped out. The source of α-particles *RaC*, was deposited on a small disc *R* fixed on one end of a movable rod *D*.

Rutherford, at first, placed the disc *R* close to the foil *F*. Alpha particles given off by the source *RaC*, struck the zinc sulphide screen *S* after penetrating through the silver foil *F* and produced scintillations. He then increased the distance of the source from the screen by pulling out the rod *D* until it was too far away for the alpha particles to reach the screen. Rutherford then introduced oxygen in the tube but found no scintillations on the screen. In any case, he did not expect any scintillation because he realised that alpha particles were not in a position to project much heavier oxygen nuclei so much. Next, he introduced nitrogen in the tube and immediately scintillations appeared on the screen. Rutherford made some tests and found that the particles which produced scintillations were the nuclei of hydrogen atom *i.e. protons*. Now the small amount of hydrogen known to be present in the tube as impurity was quite insufficient to account for the large number of scintillations observed. Where did, then these protons come from ?

Rutherford came to the conclusion that these protons had been knocked out of the nitrogen nuclei by the fast α-particles and in the process, the nitrogen nuclei were transmuted into oxygen nuclei. It was, indeed, an exciting discovery. The nuclear reaction which occurred in Rutherford's nitrogen experiment may be represented as follows : ${}_7N^{14}+{}_2He^4 \rightarrow {}_8O^{17}+{}_1H^1$

Here $_2He^4$ represents an alpha particle because α-particle is essentially a nucleus of helium atom and $_1H^1$ represents a proton which forms the nucleus of hydrogen atom. Further, the above equation states that a nitrogen nucleus containing 7 protons and 7 neutrons when struck by an alpha particle (*i.e.* helium nucleus) consisting of 2 protons and 2 neutrons form an unstable collective mass of 9 protons and 9 neutrons. This unstable collective mass, then ejects a single high-energy proton and becomes transmuted into an isotope of oxygen of mass number 17 (Fig. 5.4).

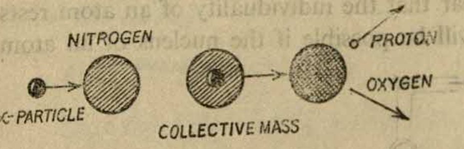


Fig. 5.4

Encouraged by this first success, Rutherford continued his researches in collaboration with Chadwick and showed that many other elements could be transmuted by bombardment of α-particles.

5.12. Discovery of neutron :

Rutherford's experiment on artificial transmutation evoked unprecedented enthusiasm in different laboratories and scientists all over the world became intensely active in studying the results of α-ray impact on light neulei. The extensive study of disintegration of light elements by α-particles led to the discovery of neutron, a fundamental particle with no charge.

In 1930 Bothe and Becker, in Germany, found that a very penetrating radiation, capable of penetrating a few inches thick of lead, was produced when α-particles were incident on light elements like beryllium. Since the radiation showed no deflection in magnetic and electric fields, it was thought to be γ-radiation of high energy. In 1932, Curie-Joliot placed a paraffin block in front of the penetrating radiation and found that protons of considerable range were ejected from the paraffin wax. The energy of the radiation was calculated from the range of ejected protons and it was then found to be exceptionally high for all known γ-rays. Further it was not known that γ-rays are capable of ejecting high-energy protons from hydrogenous substances like paraffin etc.

From these experimental findings, it appeared in the mind of Chadwick that the radiation was not γ-rays. He showed that the discrepancies with regard to the energy of the radiation could be removed if it could be supposed that it was not a γ-ray photon but an uncharged particle. To substantiate his assumption, Chadwick made the following experiment.

He used polonium as a source of α-particles and the unknown radiation obtained by the impact of α-particles with beryllium was then allowed to be incident on a slab of paraffin wax [Fig. 5.5]. The energy of the protons emitted from the paraffin could be found from their range in air, which was determined by placing mica sheets of various thickness in front of ionisation chamber until no effect was produced in the chamber. By previous calibration of the thickness of mica in terms of air thickness, the range in air and hence the energy of the protons was calculated.

He then applied the laws of conservation of linear momentum and energy to the collision with protons, assuming that the unknown radiation was a *particle* carrying no charge and that the collision was elastic. From the equation that

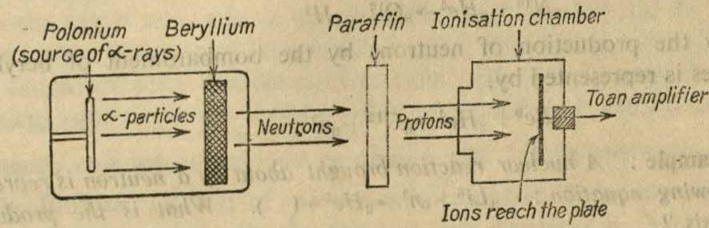


Fig. 5.5

followed, he calculated the mass of the particle and found it to be almost equal to the mass of a proton. Chadwick called the new particle, a *neutron*. It is now a well-established fact that all most all nuclei contain protons and neutrons.

The emission of neutrons as a result of the bombardment of beryllium with α-particles may be represented by the following equation :

$$_4Be^9 + {}_2He^4 \rightarrow {}_6C^{12} \rightarrow {}_0n^1 \text{ (neutron)}$$

The mass number of neutron is 1 and being neutral, its atomic number is zero ; its symbol is $_0n^1$.

Nuclear reactions : During radioactive disintegration or artificial transmutation involving emission of a particle like α, β or proton etc, the original atom, called the *parent*, changes into a new atom, called the *daughter*. These nuclear changes are represented by equations similar to those of chemical reactions. Rutherford and Soddy proposed in 1903 that the nature of the daughter atom can always be known from the nature of the parent atom and the particle emitted. They gave the following rules, known as Rutherford-Soddy rules for balancing the nuclear reaction equation :

(1) *The sum of mass numbers on the L.H.S. must be equal to the sum of mass numbers on the R.H.S.*

(2) *The sum of atomic numbers on the L.H.S. must be equal to the sum of atomic number on the R.H.S.*

The first law signifies that the number of nucleons taking part in the reaction is unchanged. The second law expresses the conservation of electric charge.

As an illustration of natural radioactive disintegration, let us consider a uranium atom with atomic number 92 and mass number 238 which disintegrates emitting an alpha particle. The daughter atom must have mass number (238 − 4)=234 and atomic number (92−2)=90 and it is an atom of a new element *thorium*. The reaction equation can be written as :

$$_{92}U^{238} \xrightarrow[\alpha]{} {}_{90}Th^{234} + {}_2He^4 \text{ (α-particle)}$$

In the first artificial transmutation brought about by Rutherford, the reacting particles were a nitrogen nuclei ($_7N^{14}$) and an α-particle ($_2He^4$). A proton ($_1H^1$)

was emitted during the reaction so that the daughter atom has a total mass number $=14+4-1=17$ and total atomic number $=7+2-1=8$. These values identify the daughter nucleus as an oxygen isotope ($_8O^{17}$). Hence,

$$_7N^{14}+{}_2He^4 \rightarrow {}_8O^{17}+{}_1H^1$$

Similarly the production of neutrons by the bombardment of beryllium with α-particles is represented by,

$$_4Be^9+{}_2He^4 \rightarrow {}_6C^{12}+{}_0n^1.$$

Example : *A nuclear reaction brought about by a neutron is represented by the following equation* : $_3Li^6+{}_0n^1 \rightarrow {}_2He^4+(\ \ )$. *What is the product in the parenthesis* ?

Ans. The mass number of the product $=6+1-4=3$ and the atomic number $=3+0-2=1$. So, the product is an isotope of hydrogen $_1H^3$ (tritium). The equation, therefore, is $_3Li^6+{}_0n^1 \rightarrow {}_2He^4+{}_1H^3$.

5.13. Equivalence of mass and energy :

In describing Rutherford's experiment on artificial transmutation stated earlier, it was pointed out that the total mass is the same before and after the change. However, if the exact mass values calculated from mass spectrograph measurement are used, this is found to be no longer quite true. The total mass of proton and oxygen atom that are formed turns out to be 0.0013 mass unit *more* than the total mass of the original alpha particle and nitrogen atom. This difference is small, but still very much bigger than the expected error of measurement. Also it is found that the total kinetic energy of the particles formed is very slightly but unmistakably *less* (by about 0.000002 erg) than the energy of the particles to begin with.

Einstein's theory of relativity offers an explanation of this discrepency. According to this theory, mass and energy which were hitherto considered to be quite independent things, are equivalent to each other and are mutually inter-convertible. Matter can, under certain circumstances, be converted into energy and the—other way around—energy can be converted into matter. The relation between the two is given by the famous *mass-energy equation* : $E=mc^2$ where E is the quantity of energy, in ergs, m is the equivalent amount of mass in grams, and c is the velocity of light, in centimeters per second.

One result of this relation is that whenever energy is given to a body (by heating it, by setting it into motion etc.) its mass must increase. But for any ordinary physical process, this increase will be far too small to detect. This is because the factor c^2 by which the energy must be divided to obtain the equivalent mass, has the enormous value of 9×10^{20} ! But conversely, the destruction of even a tiny amount of matter produces tremendous amount of energy. If the atoms of a piece of coal could be completely destroyed, the energy produced would be about 3 thousand million times that obtained by merely burning the piece. However, complete destruction of matter has not yet been attained ; even the atomic bomb could not do it.

Now, let us come back to Rutherford's nitrogen experiment. Einstein's mass-energy relation explains the observed energy loss. If the loss of energy in the above reaction be converted into equivalent mass with the help of the mass-energy equation, it will come to be equal to 0.0013 mass unit *i.e.* the lost energy has appeared as the mass of the product nuclei. This has also been found to be true in dozens of other nuclear changes examined. In this way, the theory of mass-energy equivalence has become firmly established as a physical law.

Atomic mass unit (a.m.u.) and its relation with electron volt :

The fundamental unit of mass usually employed in atomic physics is the *atomic mass unit* (a.m.u.). On the physical scale, this mass is $\frac{1}{16}$ of the mass of the atom of the oxygen nucleus $_8O^{16}$. More recently, the atomic mass unit is taken as $\frac{1}{12}$ of the mass of the atom of carbon nucleus $_6C^{12}$.

$$1 \text{ a.m.u} = 1{\cdot}6598 \times 10^{-24} \text{ gm.}$$

The corresponding energy in ergs is given by $E=mc^2$ *.i.e* $E=1{\cdot}6598\times10^{-24}\times(3\times10^{10})^2=1{\cdot}4918\times10^{-3}$ erg [velocity of light $c=3\times10^{10}$ cm/sec]

Now, 1 electron-volt $(e.v.)=4{\cdot}8\times10^{-10}\times\frac{1}{300}$ erg$=1{\cdot}602\times10^{-12}$ erg.

$$\left[e=4{\cdot}8\times10^{-10} \text{ e.s.u. and } 1 \text{ volt}=\frac{1}{300} \text{ e.s.u}\right]$$

$$\therefore \quad 1 \text{ a.m.u}=\frac{1{\cdot}4918\times10^{-3}}{1{\cdot}602\times10^{-12}} \text{ ev}=931.2 \text{ Mev.}$$

Binding energy of a nucleus :

Einstein's mass-energy equation also provides us a way to understand the **binding energy** of a nucleus. In all nuclei, except the hydrogen nucleus, there are a number of protons and neutrons. They are commonly called the *nucleons.* The nucleons are held bound to the nucleus with a tremendous energy known as the binding energy of the nucleus. Where does the binding energy of a nucleus come from ?

If we carefully observe the constitution of a nucleus we shall see that in general its mass is less than the sum of the masses of the constituent particles in free state. The decrease of mass is called the *mass-difference* of the nucleus. According to Einstein when a nucleus is formed by the combination of several particles, some energy is liberated and this energy comes from the mass difference. If 'm' be the mass difference *i.e.* the difference between the sum of the masses of nuclear particles and the mass of the nucleus, then the energy liberated is given by $E=m.c.^2$. This energy appears as the binding energy of the nucleus. Greater the binding energy of a nucleus, greater is its stability.

Let us take the case of $_3Li^7$ nucleus, which has 3 protons and 4 neutrons. Taking the mass of each free proton as 1·007277 a.m.u., the total mass of 3 protons $=3\times1{\cdot}007277=3{\cdot}021831$ a.m.u. Taking the mass of each free neutron as 1·008665 a.m.u., the total mass of 4 neutrons$=4\times1{\cdot}008665=4{\cdot}034660$ a.m.u.

Their combined mass in free state $=3{\cdot}021831+4{\cdot}034660$

$=7{\cdot}056491$ a.m.u.

But lithium-7 nucleus has a mass of 7·016005 a.m.u. Here the mass-difference $=7{\cdot}056491-7{\cdot}016005=0{\cdot}040486$ a.m.u. This shows that when a lithium-7 nucleus is formed by the union of 3 protons and 4 neutrons, 0·040486 a.m.u. of mass is lost and the energy it liberates appears as the binding energy of the lithium nucleus.

Now, from equation $E=mc^2$, we know that a loss of 1 a.m.u. of mass liberates 931·2 Mev of energy. Hence, the binding energy of lithium-7 nucleus $=0{\cdot}040486\times 931{\cdot}2=37{\cdot}7$ Mev (nearly).

Example : *Express energy, in ergs, equivalent to* 1 *mg of mass.*

[*H. S. Exam. 1980*]

Ans. We know $E=mc^2$; here $m=1\ mg=10^{-3}$ gm ; $c=3\times 10^{10}$ cm/s.

$\therefore\ E=10^{-3}\times(3\times 10^{10})^2$ ergs$=9\times 10^{17}$ ergs.

In joule, the energy, $E=\dfrac{9\times 10^{17}}{10^7}=9\times 10^{10}$ joules (approx.).

5.14. Radio-isotopes :

The French Physicist F. Joliot and his wife Irene Curie (the daughter of Madame Curie) found that when alpha-particles were allowed to bombard a piece of aluminium, *positrons* were given off. A positron is a particle identical with an electron, except that its charge is positive instead of negative. For this reason, it is sometimes known as *positive electron.* It is not permanent but soon unites with an ordinary negative electron, both disappearing in a flash of radiant energy in accordance with the mass-energy relation. In the experiment with aluminium, the Curie-Joliots found that the positron-activity did not stop at once when the stream of alpha particles was cut off, but continued for some time like natural beta-activity. It was found that α-particles due to collision had converted some of the aluminium atoms into isotopes of phosphorous which were radioactive. The equation representing the transformation is as follows :

$$_{13}Al^{27}+{}_2He^4\longrightarrow{}_{15}P^{*30}+{}_0n^1$$

Normal phosphorous ($_{15}P^{27}$) is not radioactive ; but the above isotope is. It is called **radio-isotope.** It disintegrates like natural radioactive substances but with the emission of *positron* instead of electron. The disintegreation takes place according to the following equation :

$$_{15}P^{*30}\longrightarrow{}_{14}Si^{30}+{}_1e^0 \text{ (positron)}$$

Radio-phosphorous disintegrates into silicon ($_{14}Si^{30}$) emitting a positron.

This phenomenon is known as **artificial radioactivity.** In later years, it was found that almost all non-radioactive elements could be rendered radioactive when bombarded with suitable energetic particles or when exposed to the radiations of a nuclear reactor. Illustrations of the formation of some other radio-isotopes are given below :

(i) When boron is bombarded with α-particle, radio-isotope of nitrogen is formed with the emission of a neutron :

$$_5B^{10}+{}_2He^4\longrightarrow{}_7N^{*13}+{}_0n^1$$

The mass number of normal nitrogen atom is 14 and it is not radioactive. But $_7N^{13}$ is radioactive and is not normally available. Its life is extremely short and it soon decays into carbon isotope emitting a positron :

$$_7N^{13*} \longrightarrow {}_6C^{13} + {}_1e^0 \text{ (positron)}$$

(ii) Normal sodium atom can be converted into radioactive aluminium isotope by the bombardment of α-particles.

$$_{11}Na^{23} + {}_2He^4 \longrightarrow {}_{13}Al^{26*} + {}_0n^1$$

Radio-aluminium disintegrates into magnesium and ejects positron.

$$_{13}Al^{*26} \longrightarrow {}_{12}Mg^{26} + {}_1e^0 \text{ (positron)}$$

Uses of radio-isotopes :

The contribution of radio-isotopes in the welfare of mankind is inestimable. Some of these isotopes find applications as *tracers* for checking the distribution of foods or fluids in plants and animals. Common salt in which some of the sodium atoms have been made radio-active can be fed to a patient and followed through his system by an electronic detector such as a Gieger counter held near the surface of the body. Some radio-isotopes can be used in place of radium or X-rays for treating tumors and other harmful growths. In this way, the physicians use radiobismuth-206 for the treatment of brain tumor, radio-cobalt for cancer theraphy, radio-iodine for thyrod trouble etc.

Radio-isotopes are also used widely in industry and technology for various purposes.

In industry, the radio-isotopes are being extensively used in testing and controlling thickness of metals, rubber, paper etc. without actual contact, to detect flaws in the interior of metal castings and to study many surface phenomenon like wetting, detergency, corrosion etc. In agriculture, radio-isotopes are being increasingly used to find effective processes for better yield of crops, assimilation, application and efficacy of fertilisers etc.

Radio-isotope dating :

An interesting application of radioactivity is the carbon dating technique for estimating the ages of archaeological and biological specimens. The neutrons in the cosmic ray, when slowed down, are captured by nitrogen nuclei in the air and the reaction produces a radio-isotope C^{14}. It is a β-emitter with a long half-life of 5568 years. This radio-carbon soon gets converted into carbon dioxide and will be consumed by plants to build up carbohydrates through photosynthesis. These carbohydrates, containing a certain amount of radiocarbon will be consumed by all living animals who return part of it to the atmosphere in respiration, where it decays to form N^{14}. Depending on the rate of production of radioactive C^{14} and its decay back into N^{14}, an equilibrium will be established with all living matter containing a certain percentage of C^{14}. It was suggested by Libby that once a living organism ceases to take part in the carbon cycle *i.e.* once it is dead, C^{14} in it will decay with its half life of 5568 years. Libby's suggestion has opened up the possibility of estimating the ages of carbonaceous matter which

has been dead for periods varying between 1000 years to probably 100,000 years, if it is assumed that the relative proportion of C^{14} in it at the time of its death is the same as that which exists at the present time in fresh samples of carbonaceous matter.

C^{14} dating method can be applied to determine the age of those samples which contain a detectable amount of this radio-isotopes *i.e.* samples which have been dead for not more than a few times of its half life. To estimate the age of geological specimens, scientists make use of the activity of the long-lived natural radioactive nuclides, like U^{238}, present in them.

5.15. Nuclear fission :

All the nuclear changes brought about by alpha particles, protons or neutrons involve the removal of fairly small pieces of the nucleus of an atom. These changes cannot be called major disruption because most of the nucleus in these changes remains in tact, the greatest fragment that we get as a result of disruptions being an alpha particle. In natural radioactivity, too, the same thing happens, with the result that the liberation of energy in all these cases is very small. In 1939, Hahn and Strassmann in Germany found that uranium nuclei can apparently be split into two parts of roughly the same mass, releasing huge amounts of energy in the process. The change was produced by bombarding uranium with neutrons, which are ideal nuclear bullets because—unlike the other heavy particles like alpha or deuteron,—they have no charge and so are not repelled by the target nucleus. The breaking down of a nucleus into two parts of comparable size was called **fission.** Fission produces at least, ten times energy than an ordinary disintegration. This energy is produced because the original mass of the nucleus is greater than the sum of the masses of the products produced after fission and this mass-excess appears as energy according to Einstein's equation $E=mc^2$.

Now, uranium has two isotopes of which U-238 is abundantly available but it does not undergo fission. The other isotope U-235 which is very rare in its occurrance, is a very fissile element. In natural uranium there are only about 7 U-235 atoms to 1000 of U-238. The entering neutron shakes up the structure of the nucleus, making it divide into two parts. An important feature of the action is that perhaps two or three nutrons are thrown off at the same time. The equation representing the fission may be written in the following way :

$$_{92}U^{235}+{}_0n^1 \longrightarrow {}_{56}Ba^{141}+{}_{36}Kr^{92}+3\,{}_0n^1$$

It expreses that a neutron (${}_0n^1$) of mass number 1 bombarding an uranium nucleus (${}_{92}U^{235}$) splits it into barium of mass number 141 and krypton of mass number 92 and three neutrons are released. It was recognised that this makes possible a *chain reaction* ; if more than one of the neutrons produced were able to cause fission of another nucleus, the process would go ahead faster and faster, until after a very short time, all the nuclei would be fissioned, with the release of an enormous amount of energy.

Chain reaction :

Fig. 5.6 shows a scheme of chain reaction. In the scheme, it has been assumed that out of the three neutrons produced in a fission one is absorbed by the material and the other two cause fission in the neighbouring nuclei and maintain the reaction. It is to be noted that in a small piece of *U*-235, many neutrons will escape and the chain reaction will not develop. In a large lump, the chance

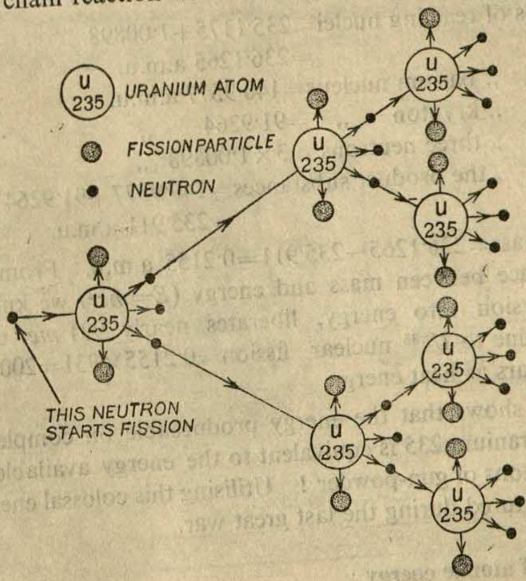


Fig. 5.6

of a neutron escaping without meeting fission nuclei is much less. So there must be a *critical size*—any small piece will not be able to maintain a chain reaction, a larger one will.

Calculations show that about 10,000 tons of gun-powder will be needed if the energy available from the fission of only a pound of *U*-235 is to be produced by the explosion of the gun-powder. The uranium fission reaction was first used successfully to produce heat energy on a large scale by the Italian physicist Enrico Fermi in 1942. *Atomic bombs* were, however, manufactured during the World War II, where this stupendous energy of atoms was used as an explosive.

5.16. Energy released in nuclear fission :

The energy released in nuclear fission is much more than the energy released in natural or artificial radioactive disintegration. The energy is derived from a loss of mass in the nucleus fissioned. Consider the case of fission of U^{235} nucleus by neutron. Suppose the two fragments obtained as a result of the fission of U^{235} are barium and krypton. The process can be represented by the following equation :

$$_{92}U^{235}+{}_0n^1 \rightarrow {}_{56}Be^{141}+{}_{36}Kr^{92}+3.{}_0n^1$$

It is to be noted that U^{235} nucleus may undergo fission in a number of ways, includ

ing the above which is very convenient for the calculation of energy release. Now, we shall find the total mass (actual) of the reacting nuclei (*i.e.* U^{235} and neutron) and also the total mass (actual) of the product substances (*i.e.* barium, krypton and three neutrons)

Mass of $U^{235} = 235{\cdot}1175$ a.m.u.

„ „ neutron $= 1{\cdot}00898$ „

Total mass of reacting nuclei $= 235{\cdot}1175 + 1{\cdot}00898$

$= 236{\cdot}1265$ a.m.u.

Also, „ „ barium nucleus $= 140{\cdot}9577$ a.m.u.

„ „ krypton „ $= 91{\cdot}9264$ „

„ „ three neutrons $= 3 \times 1{\cdot}00898$ „

Total „ „ the product substances $= 140{\cdot}9577 + 91{\cdot}9264 + 3 \times 1{\cdot}00898$

$= 235{\cdot}911$ a.m.u.

Loss of mass $= 236{\cdot}1265 - 235{\cdot}911 = 0{\cdot}2155$ a.m.u. From Einstein's equation of equivalence between mass and energy ($E = mc^2$) we know that 1 a.m.u. mass, on conversion into energy, liberates nearly 931 *mev* of energy. So, the energy liberated due to U^{235} nuclear fission $= 0{\cdot}2155 \times 931 = 200$ *m.e.v.* (nearly). This energy appears as heat energy.

Calculation shows that the energy produceable on complete disintegration of only 1 gm of uranium 235 is equivalent to the energy available from the combustion of 10,000 tons of gun-powder ! Utilising this colossal energy, atom bombs were first manufactured during the last great war.

5.17. Use of atomic energy :

The nuclear energy is now being put to peaceful use as a source of industrial power. Ordinarily, the chain reaction is a very rapid and devastating process. It generates huge amount of energy within a very short time, causing nothing but destruction. The reaction should be brought under control if it is to be put to the welfare of the people. **Nuclear reactor** is an arrangement for producing controlled nuclear energy. It is also called *a nuclear furnace.* In the reactor, some cadmium rods are used, known as control rods. Cadmium is a good absorber of neutrons and these rods by absorbing neutrons, control the release of energy. The tremendous heat generated in the reactor is used to produce steam which operates a steam turbo-generator and produces electric power. In western countries, a number of nuclear reactors is in operation, producing electrical energy for industrial and other purposes. In India, too, we have several nuclear reactors and atomic power stations. Besides supplying electrical power, these reactors and power stations are also providing us radio-isotopes which have manifold uses.

*5.18. Nuclear Reactor :

Atomic energy is now-a-day harnessed for human welfare. Chain reaction, in general, is very rapid and intense. It produces huge amount of energy in a very short time which can cause wanton destruction and nothing else.

If the release of energy is to be put to any welfare work, it should be controlled. Nuclear reactor is such an arrangement which gives controlled release of atomic energy. It may also be called an atomic furnace. The nuclear reactor shown in fig. 5.7 is designed to produce atomic energy which will generate electricity by producing steam.

A few rectangular graphite blocks are used as core in a hard steel vessel which can bear a very high pressure. In the graphite core there are several vertical narrow orifices which are filled up by rods of uranium. Uranium acts as a *fuel* in

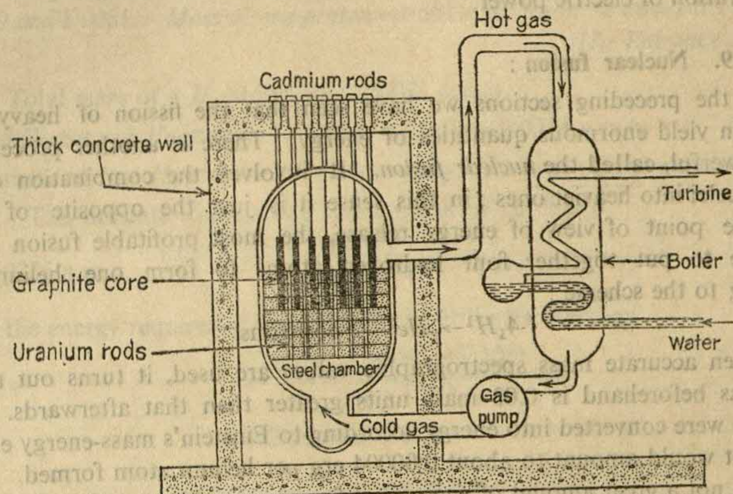


Fig. 5.7

the atomic furnace. A few cadmium rods are inserted at random within the rods of uranium. These cadmium rods can be raised or lowered through narrow orifices similar to those existing in the graphite core. Cadmium rods are called *control rods.*

The material—Uranium—used in a rector contains two varieties of atom. Of these two varieties, *U*-235 atoms are significant. If, somehow, one or two atoms of *U*-235 are fissioned, each produces light nuclei of barium and krypton and at the same time, releases three neutrons and some energy. These neutrons go to fission the neighbouring nuclei and maintain the chain reaction.

Now, the graphite core used here acts as a **moderator.** Its function is to slow down the fast neutrons to thermal ones. Thermal neutrons have been found to be more efficient than fast neutrons in respect of fission of *U*-235 nuclei. The cadmium rods are put in their proper positions before the rods of uranium are inserted into the graphite core. Cadmium absorbs the neutrons emitted by uranium fission. So, chain reaction will not be maintained and the reactor will be idle if the cadmimum rods are kept in their positions. Inserting sufficient uranium rods in the reactor, when the reactor assumes the *critical size* the steel chamber is sealed and the cadmium rods are raised up from the core. In this condition, the neutrons start nuclear fission process and the chain reaction

continues. The rate of fission can be easily controlled by raising or lowering a few cadmium rods. The process stops when all the cadmium rods are introduced fully ; only natural and spontaneous fission continues.

The heat energy that is produced as a result of fission, is carried away by a current of high-pressure carbon dioxide gas and is allowed to circulate around a steam boiler. Carbon dioxide gas, at high pressure, is injected continuously inside the steel chamber by means of pumps. The steam available from the boiler by this process is utilised in the conventional way to operate a turbo-generator for the generation of electric power.

5.19. Nuclear fusion :

In the preceding sections we have seen that the fission of heavy atomic nuclei can yield enormous quantities of energy. There is another process, even more powerful, called the *nuclear fusion*. It involves the combination of light-weight nuclei into heavier ones ; in this sense it is just the opposite of fission. From the point of view of energy release, the most profitable fusion reaction would be to put together four hydrogen atoms to form one helium atom according to the scheme :

$$4\,{}_1H^1 \rightarrow {}_2He^4 + 2 \text{ positrons.}$$

When accurate mass spectrographic values are used, it turns out that the total mass beforehand is 0.03 mass units greater than that afterwards. If the difference were converted into energy according to Einstein's mass-energy equation $E=mc^2$, it would amount to about 0·00004 erg per helium atom formed. This is seemingly not a great amount of energy but for any sizeable quantity of hydrogen, containing millions of atoms, it is enormous !

The process of fusion appears to be simple. But it is not easy to fuse the light nuclei into a single nucleus. The fusion process can be carried out only at extremely high temperatures. It is calculated that fusion is possible at a temperature of the order of 10^7 to 10^8 degree celsius. Therefore before fusion takes place the light nuclei must have their temperature raised by several millions of degrees celsius. These type of reactions are called *thermo-nuclear* reactions. This high temperature can be attained during the explosive type of fission process. For this reason, it is said that to get energy from nuclear fusion, nuclear fission must precede fusion.

In 1939, Bethe in the United States, concluded that fusion action can account for the tremendous heat of the sun and the stars. No other source ever suggested is at all large enough to furnish the huge quantities of energy that the sun has been radiating out into space for millions and millions of years. Einstein's mass energy equation tells us that as a result of this radiation of energy, the sun is losing mass at the rate of 4 million tons each second ! Yet the sun is so large that the present solar mass will almost remain in tact even after million of years of radiation.

It may be mentioned here that the *thermonuclear bomb* popularly known as *hydrogen bomb*, produces energy by a fusion reaction. For the same mass of reacting material, a fusion bomb produces about 30 times as much energy as a

fission bomb. In addition, a thermonuclear bomb does not require a critical size as in the case of a fission bomb and hence its size may be made as large as necessary. The possibility of a controlled fusion reaction is the subject of much current research. Limited success has been attained so far. Chief difficulty is the production of very high temperature of millions of degrees, which no known material, is capable of withstanding.

Example : *Find the energy, in electron-volt, necessary to transfer one helium atom into four hydrogen atoms. Masses of helium and hydrogen atoms are respectively* 4·0039 *and* 1·0081. *Mass of one proton*$=1{\cdot}661\times10^{-24}$ *gm* ; 1 *ev*$=1{\cdot}6\times10^{-12}$ *erg.* [*Jt. Entrance 1981*]

Ans. Total mass of 4 H_2-atoms$=4\times1{\cdot}0081$ a.m.u.

Mass difference in *He*-atom$=4\times1{\cdot}0081-4{\cdot}0039=0{\cdot}0285$ a.m.u.

Now, mass of one H_2-atom$=1{\cdot}0081$ a.m.u. and mass of one proton$=1{\cdot}661\times10^{-24}$ gm. This indicates that 1·0081 a.m.u.$=1{\cdot}661\times10^{-24}$ gm.

$$\therefore\ 0{\cdot}0285\text{ a.m.u.}=\frac{1{\cdot}661\times10^{-24}\times0{\cdot}0285}{1{\cdot}0081}\text{gm.}=0{\cdot}047\times10^{-24}\text{ gm}$$

Now, the energy required $E=mc^2=0{\cdot}047\times10^{-24}\times9\times10^{20}$ erg

$$=\frac{0{\cdot}047\times10^{-24}\times9\times10^{20}}{1{\cdot}6\times10^{-12}}\ e.v.$$

$$=2{\cdot}64\times10^{7}\ e.v.$$

Exercises

Eassy type :

1. What is radioactivity ? How was it discovered ? What are its characteristics ?
2. What radiations are given off by a radioactive body ? Mention the important properities of these radiations. [*H. S. Exam 1978, '81*]
3. State the difference between α and β particles in respect of charge, mass and ionising power. [*Jt. Entrance 1982*]
4. Describe an experiment to show that three different kinds of radiations are emitted by a radioactive body.
5. How do you explain the emission of γ-rays from a nucleus ? Does a nucleus always emit γ-ray of single energy ?
6. Electrons do not reside inside a nucleus. How then can you explain the emission of β-particles by radio active nuclei ?
7. What is artificial transmutation ? How was it first accomplished ? [*H. S. Exam. 1979*]
8. Complete the following nuclear reaction and explain fully the result obtained : [*Jt. Entrance 1983*]
$_7N^{16}+{}_2He^4\rightarrow$
9. What do you understand by nuclear fission ? Are nuclear fission and nuclear fusion reverse processes ? How do they give rise to energy ? [*cf. H. S. Exam. 1980*]
10. Explain, with two illustrations, the processes of nuclear fission and nuclear fusion. [*H. S. Exam. 1978, '82*]

11. State Einstein's law of equivalence of mass and energy. How do you get an explanation of binding energy from the law ?

12. Is the mass of an α-particle exactly equal to the sum of the masses of 2 free protons and 2 free neutrons ? If not, account for the difference.

13. Establish the relation between the half-life and disintegration constant of a radioactive substance.

14. What are radioactive isotopes ? Explain, with equation, the formation of a few radio-isotopes. What are their uses ?

15. How does a nuclear reactor function ?

16. What is nuclear fusion ? What is its difference with nuclear fission ? Are both of them sources of energies ? [*Jt. Entrance 1981*]

Short answer type :

17. What do you mean by half-life and decay constant of a radioactive substance ? What is their relation ? [*H. S. Exam. 1983*]

18. 'Half-life of radium is 1622 years'—Explain. [*H. S. Exam. 1978, '81*]

19. Does any change take place in the element when its radioactive nucleus disintegrates ? What is the law of such disintegration ?

20. Why are α-particles called the nuclei of helium atom ? What is their velocity ?

21. What are 'tracer' elements ? Why are they so called ?

22. By bombarding the nuclei of ${}_{13}Al^{27}$ by α-particles radioactive isotopes of ${}_{15}p^{30}$ are produced. It disintegrates into ${}_{14}Si^{30}$ by ejecting a positron. Write the transformation equations. [*H. S. Exam. 1979*]

23. ${}_{15}p^{32}$ radioisotope emits β-particle and is converted to ${}_{14}S^{32}$. Write the equation of transformation. [*H. S. Exam. 1981*]

24. If an aluminium atom (${}_{13}Al^{27}$) is bombarded with a proton (${}_{1}H^{1}$), a helium atom (${}_{2}He^{4}$) and another atom are produced. What is the structure of the second atom ? Can you identify it ? [*H. S. Exam. 1984*] [Ans. ${}_{12}Mg^{24}$]

25. To get energy from nuclear fusion, nuclear fission must precede fusion. Explain the meaning of the statement. [*Jt. Entrance 1981, '83*]

Objective type :

26. Fill in the gaps in the following cases ;

(*a*) Of the different radioactive radiations, in respect of penetrating power, first comes——, then——and last of all——.

(*b*) When a β-ray is emitted by a radioactive body, there is no change in the——of the product atom but——is raised by one unit.

(*c*) Electron and positron are——in their electrical nature but——in their masses.

(*d*) When an α-particle is emitted by a radioactive body, the————of the product atom is less by 4 units and——by 2 units.

(*e*) γ-rays are——like ordinary light and their——are of the order of 10^{-11} cm.

(*f*) Beta rays emitted by a radioactive material are (i) electromagnetic radiation (ii) electrons orbiting round the nucleus (iii) charged particles emitted by the nucleus (iv) neutral particles. Which is correct ?

Numerical problems :

27. An isotope of carbon has its mass number 14. How many protons and neutrons does it contain. This isotope of carbon has a half-life of about 6000 years. In how many years will an amount of this carbon be reduced to 1/8 th. of its original value ?

[*Jt. Entrance1982*] [Ans. 14 ; 18000 years]

28. The half-life of a radioactive substance is 2 hours. If the initial mass of the substance is 5 gm, how much of it will remain after 10 hours ? [Ans. 0·156 gm]

29. The radio-isotope $_{92}U^{239}$ of uranium undergoes two successive β-decays and transforms to an isotope of plutonium (Pu). Determine the atomic number and mass number of the new isotope. [*Jt. Entrance 1983*] [Ans. $Z=94$; $A=239$]

30. The radioactive isotope $Th^{232}(Z=90)$ emits successively six α-particles and four beta particles. What is the mass number and atomic number of the resulting isotope ? Can you identify it ? [*H. S. Exam. 1983*] [Ans. $Z=208$; $A=82$; lead]

31. If α-particles are used as projectiles to transmute a nucleus of mass number A and atomic number Z, what will be the values of A and Z of the product nucleus if (i) protons are ejected (ii) neutrons are ejected ?
[*H. S. Exam. 1979*] [Ans. (i) $A=A+3$; $Z=Z+1$ (ii) $A=A+3$; $Z=Z+2$]

32. The mass number and atomic number of radium atom are 226 and 88 respectively. It decays in radium-C atom through successive emission of 3 α-particles and 1 β-particle. Find the mass number and atomic number of radium-C. [Ans. 214 ; 83]

33. Find the decay constant of radium whose half-life is 1620 years.
[Ans. $4{\cdot}28\times10^{-4}$ per year]

34. Calculate the time in which the activity of a sample of thorium reduces to 90% of its original value. Half-life of thorium $=1{\cdot}4\times10^{10}$ years. [Ans. $2{\cdot}108\times10^{9}$ years]

[**Hints** : Apply the formula $N=N_0e^{-\lambda t}$]

35. $_6C^{11}$ nucleus disintegrates emitting positive β-particle. What is the atomic number and chemical name of the new nucleus ? [Ans. 5]

[**Hints** : $_8C^{14}\rightarrow{}_5B^{11}\rightarrow{}_1e^{\circ}$]

36. If an aluminium atom ($_{13}Al^{27}$) is bombarded with a proton ($_1H^1$) a helium atom ($_2He^4$) and another atom is produced. What is the structure of the nucleus of the second atom ?
[*H. S. Exam 1984*] [Ans. $_{12}Mg^{24}$]

37. From the following equations, pick out the possible nuclear fission reaction :

(*a*) $_6C^{13}+{}_1H^1\rightarrow{}_6C^{14}+4{\cdot}3$ *Mev.*

(*b*) $_6C^{12}+{}_1H^1\rightarrow{}_7N^{13}+2$*Mev.*

(*c*) $_7N^{14}+{}_1H^1\rightarrow{}_8O^{15}+7{\cdot}3$ *Mev.*

(*d*) $_{92}U^{235}+{}_0n^1\rightarrow{}_{64}Xe^{140}+{}_{38}Sr^{24}+{}_0n^1+{}_0n^1+\gamma+200$ *Mev.*

38. In the uranium radio active series, the initial nucleus is $_{92}U^{238}$ and the final nucleus is $_{82}Pb^{206}$. When uranium nucleus decays to lead, the number of α-particles emitted is .... and the number of β-particles emitted are..... [*I.I.T. 1985*] [Ans. 8 ; 6]

39. Find out X, Y and Z of the following :

(i) $_{13}Al^{27}+{}_1H^1\rightarrow{}_{14}Si^{28}+X$

(ii) $_{92}U^{238}+{}_2He^4\rightarrow{}_{94}Pu^{241}+Y$

(iii) $_{15}P^{30}\rightarrow{}_{14}Si^{30}+Z$

(iv) $_3Li^6+{}_1H^2\rightarrow{}_2He^4+X$

(v) $_{17}Cl^{35}+Y\rightarrow{}_{16}S^{32}+{}_2He^4$

(vi) $_4Be^9+{}_2He^4\rightarrow Z+{}_0n^1$.

40. If 1 kg. of a substance is fully converted into energy, how much energy is produced ?
[Ans. 9×10^{16} joules]

41. Calculate the binding energy of an alpha-particle in Mev, given mass of a proton $=1{\cdot}00758$ a.m.u. mass of a neutron $=1{\cdot}00897$ a.m.u. and mass of helium nucleus $=4{\cdot}0028$ a.m.u. [Ans. 28.21]

APPENDIX

Bohr's theory of hydrogen spectrum :

By applying Bohr's postulates on a hydrogen atom, we shall see how Bohr determined the wavelengths of different line spectra produced by hydrogen atom. Consider an atom whose atomic number is Z *i.e.* its nucleus contains Ze amount of positive charge and an electron of negative charge e revolves round the nucleus in a circular orbit of radius r_n with the nucleus at the centre. This atom is like a hydrogen atom because for hydrogen $Z=1$ and hydrogen atom has one revolving electron.

The centripetal force acting on the electron$=\dfrac{mv^2}{r_n}$. This force comes from the electrostatic attraction of the nucleus on the electron. This attractive

$$\text{force}=\frac{Ze.e}{r_n{}^2}=\frac{Ze^2}{r_n{}^2} \qquad \therefore \quad \frac{mv^2}{r_n}=\frac{Ze^2}{r_n{}^2} \quad \text{or} \quad mv^2=\frac{Ze^2}{r_n} \quad .. \text{ (i)}$$

According to Bohr's second postulate, $mvr_n=\dfrac{nh}{2\pi}$ or $v=\dfrac{nh}{2\pi mr_n}$

Putting this value in eqn. (i), we get

$$m\left(\frac{nh}{2\pi mr_n}\right)^2=\frac{Ze^2}{r_n}. \quad \text{or} \quad r_n=\frac{n^2h^2}{4\pi^2mZe^2} \quad .. \text{ (ii)}$$

Putting $n=1$, 2, 3, etc. eqn. (ii) we get the radii of different quantitised orbits. Further we can also calculate the velocity of the electron revolving in the nth quantised orbit from the above consideration. If v_n be the velocity of the electron,

than $v_n=\dfrac{nh}{2\pi mr_n}=\dfrac{nh}{2\pi m}\times\dfrac{4\pi^2mze^2}{n^2h^2}=\dfrac{2\pi ze^2}{nh}$.

Now we shall find the wavelength of spectial lines radiated by an atom by the application of Bohr's third postulate.

While revolving in the nth quantised orbit, the electron has kinetic energy as well as potential energy. From the definition of potential energy, we can write,

$$E_p=\int_0^{r_n}\frac{Ze^2}{x^2}\,dx=-\frac{Ze^2}{r_n}$$

Negative sign indicates that energy is to be supplied to the atom in order to take the 'electron' from the nucleus to the infinite orbit where the energy is zero.

The kinetic energy of the electron, $E_k=\frac{1}{2}mv_n{}^2=\dfrac{Ze^2}{2r_n}$ [From eqn. (i)]

So, the total energy of the electron in the nth orbit

$$E_n=E_p+E_k=-\frac{Ze^2}{r_n}+\frac{Ze^2}{2r_n}=-\frac{Ze^2}{2r_n}$$

Putting the value of r_n from eqn (ii), we get $E_n = -\dfrac{2\pi^2 me^4 Z^2}{n^2 h^2}$ .. (iii)

When transition of electron takes place from an orbit of quantum number n_2 to an orbit of quantum number n_1, the difference of energies in the two orbits is, according to Bohr's third postulate, radiated as a photon of energy $h\nu$. So,

$$E_{n_2} - E_{n_1} = h\nu \quad \text{or} \quad \frac{2\pi^2 me^4 Z^2}{h^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = h\nu$$

$$\text{or} \quad \frac{1}{\lambda} = \frac{2\pi^2 me^4 Z^2}{ch^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \left[\because \quad \nu = \frac{c}{\lambda}\right]$$

For hydrogen atom $Z=1$; So,

$$\frac{1}{\lambda} = \frac{2\pi^2 . me^4}{ch^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = R_H\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

Here $R_H = \dfrac{2\pi^2 me^4}{ch^3}$ = a constant, known as Rydberg constant.

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WEST BENGAL HIGHER SECONDARY EXAMINATION, 1985

PHYSICS—Second Paper

1. (*a*) Explain with a diagram what umbra and penumbra are.

(*b*) Why are the shadows of birds not seen when they fly at a height ?

(*c*) How is the rectilinear propagation of light proved by the pin hole camera ?

(*d*) A 6 cm high image of a tower is formed inside a pin hole camera when placed at some distance from the tower. From another point 10 metre farther from the tower in the same straight line the image is 4 cm in height. What is the height of the tower ? Camera box is 20 cm long. [Ans. 600 cm]

2. (*a*) Write down the two laws of refraction of light.

(*b*) What do you mean by the statement, 'Refractive index of light in water is 1·33'?

(*c*) Explain with diagrams what you mean by total internal reflection of light and critical angle.

(*d*) Refractive index of water is $\frac{4}{3}$. What is the critical angle for water ? [$\sin 48°30' = 0·75$]

A nail is fixed perpendicularly to a circular wooden disc at its centre. The disc is floated in water with the nail downwards. What must be the ratio of the longest possible length of the nail to the radius of the disc, so that the nail is completely invisible from air ? [Ans. 48°30′, $\sqrt{7}:3$]

3. (*a*) Define the principal focus of a *diverging* lens and show it in a diagram.

(*b*) With the symbols having their usual meaning deduce the following formula for a converging lens :

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

(*c*) Image of a film 50 times magnified in height is to be formed on a screen. If the focal length of the lens in 7·5 cm, at what distance from it should the screen be placed ? What will be the distance of the film from the lens under such condition ? [Ans. 382·5 cm. 7·65 cm]

4. (*a*) Discuss the defect of vision of the human eye called 'hypermetropia' or long-sight on the following points :

(i) nature of the defect ;
(ii) its cause ;
(iii) its remedy.

(*b*) A long-sighted person cannot read from points nearer than 40 cm. What should be the power of eyeglasses suitable for the person, if he wants to read from a distance of 25 cm ? [Ans. 1·5D]

5. (*a*) Define c.g.s. unit of magnetic pole.

(*b*) The pole strength of each pole of a bar magnet is 20 c.g.s. units. How much force will act on each pole of it when placed in a uniform magnetic field of intensity 100 Oersteds ?

If the axis of the magnet lies inclined to the magnetic field at an angle of 45 and its effective length is 7·5 cms, then what will be the moment of the couple acting on it ? [Ans. $7500\sqrt{2}$ dyne-cm.]

(*c*) Define the Dip of the terrestrial magnetism of a place and explain it with a simple diagram.

6. (*a*) What do you mean by the capacity of a capacitor ?

(*b*) Explain on what factors and how the capacity of a capacitor depends.

(*c*) On charging a capacitor with 10 e.s.u. of charge its potential rises from zero to 150 volts. What is its capacitance ? How much energy is stored in it ? [Ans. 5 e.s.u., 0·625 erg.]

7. (*a*) What is a secondary cell ? Why is it so called ?

(*b*) Explain the action of any type of secondary cell.

(*c*) What do you mean by the statement, 'The capacity of a secondary cell is 30 ampere-hours' ? How much electric charge can be drawn from it without damaging it ? [Ans. 108000 coulomb]

8. (*a*) State Ohm's law.

(*b*) What do you mean by the resistance of a conductor ?

(*c*) What is resistivity ? On what factors does it depend ?

(*d*) A lump of copper weighing 10 gms is given. What should be the length and the cross-section of a wire made from it so that its resistance may be 2 ohms ? [Density and resistivity of copper are 9 gm/cc and $1{\cdot}8\times10^{-6}$ ohm-cm respectively.] [Ans. 1111·1 cm., 10^{-3} sq.cm.]

9. (*a*) Through a coil of wire of resistance 50 ohms a current of 2 amperes is passed for 5 minutes. Find :

(i) How much electric charge has flown through the coil ?
(ii) How much work has been done by the source of e.m.f. ?
(iii) How much heat has been produced ?

[Ans. (i) 600 coulomb, (ii) 60000 joule, (iii) 14400 cal.]

(*b*) A tangent galvanometer and a silver voltameter are connected in series and a current is passed through them for some time. On the cathode of the voltameter 0·112 gm of silver is deposited in 5 minutes and the deflection of the galvanometer remains 30° all through. What is the value of the reduction factor of the galvanometer ? [E.C.E. of silver = ·00112 gm/coulomb]

$$\left[\text{Ans. } \frac{1}{10\sqrt{3}}\right]$$

10. (*a*) A current is passed through a wire for some time. How does the heat developed in the wire depend on each of the following factors ?—

(i) Length of the wire, (ii) its cross-section, (iii) resistivity, and (iv) the p.d. across the two terminals of the wire.

[In the first three cases, assume the p.d. to be constant.]

(*b*) Define the B.O.T. unit of the electrical energy consumption.

(*c*) The main-meter of a house is marked 10 Amp-220 volts. How many 60 watt lamps may be used in the house with safety ? [Ans. 36]

11. (*a*) Draw a diagram of a simple A. C. Dynamo and explain its action.

(*b*) State how the e.m.f. of the dynamo stated above will be affected in the following cases :

(i) Intensity of the magnetic field is doubled.
(ii) Number of turns of the coil is increased 5 times.
(iii) Rate of rotation of the coil is decreased.
(iv) Area of the coil is increased.

In each of the above cases, assume that the other factors remain unchanged.

(*c*) What is the source of electrical energy generated in a dynamo ?

12. (*a*) What are cathode rays ? How are they produced ?

(*b*) How can you experimentally show that cathode ray particles travel in straight lines and carry negative charge ?

13. (*a*) Describe a thermionic diode and explain its action.

(*b*) Explain the principle of rectification of an A.C. voltage by a diode.

14. (*a*) What is radioactivity ?

(*b*) What is the ratio of the masses of alpha and beta particles emitted from radioactive atoms ? What is the ratio of their charges ?

(*c*) The nucleus of a Polonium atom is represented as $_{84}Po^{218}$. What do you mean by this ? Due to radioactivity an alpha particle and two beta particles are ejected from it in succession. State the nuclear structure of the atoms formed at each stage.

15. Write short notes on any *two* :

(*a*) Semiconductor diode.
(*b*) Phototube and its use.
(*c*) Artificial transmutation.

JOINT ENTRANCE EXAMINATION, 1985

1. (*a*) A swimmer sees only hazy contours of objects when he opens his eyes under water, while they are distinct when using a mask. Why ?

(*b*) The ratio between the wavelengths of the incident and refracted waves of light is its refractive index. Give reasons.

(*c*) A block of ice floats on water in a beaker filled to the brim. What would happen to the level of water when the ice melts into water (*i*) at 0°C (*ii*) at 4°C ?

(*d*) Explain why felt rather than air is used for thermal insulation even though the thermal conductivity of air is less than that of felt.

(*e*) Explain with scientific reasoning, 'Water placed in a vacuum boils at a lower temperature and gets cooler during boiling.'

2. (*a*) 6 cu ft of water is heated in a bath tub on a gas burner to raise it from 55°F to 100°F. The heat of combustion of the gas is 600 B.T.U. per cu ft and it costs one rupee per thousand cubic foot. What is the cost of heating the water in the bath tub if 70 per cent of the fuel heat is utilised in heating the same ? [Ans. 4 paisa]

(*b*) If 70,000 cal of heat are abstracted from 100 gm of steam at 100°C, what will be the result ?

[Latent heat of fusion of ice=80 cal/gm and latent heat of steam=539 cal/gm]

[Ans. 76·25 gm ice and 23·75 gm water at 0°]

3. (*a*) What effect will be produced on (*i*) the dew point and (*ii*) the relative humidity, when the temperature of a room is increased ?

(*b*) Two rods *A* and *B* are of equal length. The temperatures at the two ends of each rod are T_1 and T_2. Under what condition will the rate of flow of heat in the rods be equal ? [Ans. $K_1\alpha_1=K_2\alpha_2$]

(*c*) An iron rod connected the opposite sides of a circular iron ring. If the system is equally heated, will the ring remain circular ? Explain.

(*d*) A concave lens made of a material of refractive index μ is immersed in a medium whose refractive index is greater than μ. Trace the path of the emergent rays when a parallel beam of light is incident on the lens.

(*e*) Define candela as the unit of luminous intensity. How is it related to lumen ?

4. (*a*) In which case will the tension in a rope be greater : when two men pull the ends of the rope with equal force *F* in opposite directions or when one end of the rope is fastened to a support and the other is pulled by a man with a force 2*F* ? Why ?

(*b*) A car is moving with a uniform velocity. Is the engine of the car doing any work under this condition ?

(*c*) When a gas filled baloon rises up, it gains both kinetic and potential energy. How does the principle of conservation of energy apply in this case ?

(*d*) How would you test whether the space above mercury column in a barometer tube contains air or not ?

(*e*) Why don't we observe interference effects between the light beams emitted by two torch lights ?

5. (*a*) (*i*) Two 60 cm long identical sonometer wires are stretched by the same tension to give a note of frequency 300. By how much should the length of one of them be altered to give 5 beats per second ?

[Ans. 0·99 cm decrease or 0·01 cm increase]

(*ii*) A tuning fork of frequency 256 vibrations per second is to be mounted on a wooden box with one end open to reinforce its sound. How long should be the sound box ? [Ans. 32·42 cm]

(*b*) A tuning fork having a frequency of 340 vibration/sec is vibrated just above a cylindrical tube. The height of the tube is 120 cm. Water is slowly

poured into it. What is the minimum height of water required for the resonance. [Ans. 46·76 cm]

6. (*a*) A person travels half the distance to his destination at an average speed of 24 miles/hr. At what speed must he travel so that the average speed for the entire trip is 32 miles/hr.? [Ans. 48 miles/hr.]

(*b*) A cricket ball is caught by a player 5 seconds after being hit by the batsman. How high did the ball rise in its flight ? [$g=980$ cm/sec^2] [Ans. 30·62 metre]

(*c*) A bullet is fired from an aeroplane travelling horizontally at a speed of 300 miles/hr. The pilot hears the echo of the sound reflected from the ground after 4 secs. Find the height of the plane from the ground. [Vel. of sound in air=1120 ft/sec] [Ans. 2060 ft]

(*d*) A glass ball of density 2·6 gm/c.c. is coated with a thick layer of wax of density 0·8 gm/c.c. If the combination floats in water completely submerged, compare the volume of wax with that of glass ball. [Ans. 8]

(*e*) How are the harmonics produced related with the fundamentals in the case of a closed organ pipe and an open organ pipe ?

7. (*a*) Explain why the spectrum of hydrogen has many lines although a hydrogen atom has only one electron.

(*b*) In what way does the passage of electric current through electrolyte differs from conduction of current through metal ?

(*c*) Two conductors carry like charges of the same magnitude. Can there be a potential difference between the conductors ?

(*d*) A permanent bar magnet falls through a metal ring. Will the magnet fall with the acceleration of a freely falling body ?

(*e*) An alternating voltage can be amplified by a triode. Where does the added energy in an amplifier come from ?

8. (*a*) Two identical magnetised needles each of mass 4 gm and length 20 cm are suspended with their south poles together. In equilibrium the two north poles move apart until the distance between them is 4 cm. Find the pole strength of each needle. Assume that the poles are concentrated at the ends of the needles. [Ans. 56 unit (nearly)]

Explain what is meant by saying that the pole strength of a magnetic pole is 30 unit.

(*b*) Two negative charges of unit magnitude each and a positive charge q are placed along a straight line. At what position and for what value of q the system will be in equilibrium ? Check whether it is in stable, unstable or neutral equilibrium. [Ans. centre, unstable]

A positive charge Q e.s.u. is located at a point. What is the work done if a unit positive charge is carried from one point to another along a circle of radius r about this charge Q ? Explain the answer. [Ans. 0]

9. (*a*) A parallel plate condenser of area 1 sq metre and dielectric constant 7 is charged to a potential of 300 volts. It the distance between the plates is 0·01 cm, find the energy stored in the condenser. [Ans. $2{\cdot}8\times10^5$ erg (nearly)]

(*b*) The temperature of 0·3 kg of paraffin oil in a vacuum flask rises 1° per min with an immersion heater of 12·3 watts input.

On repeating with 0·4 kg of oil the temperature rises by 1·2°C per minute for an input of 19·2 watts.

Find the specific heat capacity of the oil and the thermal capacity of the flask (assumed constant). [Ans. s=0·53, Th. cap=17·3 cal.]

10. (*a*) 12 cells each having the same e.m.f. are connected in series and are kept in a closed box. Some of the cells are wrongly connected. This battery is connected in series with an ammeter and two cells and the battery aid each other. The current is 3 amp when the cells and the battery aid each other and is 2 amp when the cells and battery oppose each other. How many cells in the battery are wrongly connected ? [Ans. 1]

(*b*) A single electron orbits around a stationary nucleus of charge Ze, where 'Z' is constant and 'e' is the magnitude of the electronic charge. It requires 47·2 e.v. to excite the electron from the second Bohr orbit to the third Bohr orbit. Find the value of Z

$e=4{\cdot}8\times10^{-10}$ e.s.u. ; $h=6{\cdot}55\times10^{-27}$ ergs/sec. ; $m=9{\cdot}06\times10^{-28}$ gm
1 ev$=1{\cdot}6\times10^{-12}$ ergs. [Ans. $z=5$]

WEST BENGAL HIGHER SECONDARY EXAMINATION, 1986

PHYSICS—Second Paper

1. (*a*) Explain the annular eclipse of the sun with the help of a figure.

(*b*) Prove that the reflected ray rotates through twice the angle of rotation of a plane mirror.

(*c*) Show how an image is formed by a plane mirror and prove that the image distance is equal to the object distance.

(*d*) What is meant by virtual image and real image ? Explain with examples.

2. (*a*) What do you mean by total internal reflection of light and critical angle ? Find the relation between them.

(*b*) Determine the velocity of light in glass, if the velocity of light in vacuum is 3×10^{10} cm/sec, and refractive index of glass is 1·5.
[Ans. 2×10^{10} cm/sec]

(*c*) Prove that the minimum distance between the screen and the object should be four times the focal length of a convex lens for the formation of a real image by such a lens.

(*d*) The power of a lens is $+4$ *D*. What is the focal length and nature of the lens ?
[Ans. -25 cm ; convex]

3. (*a*) Deduce the relation between the object distance, image distance and focal length of a concave mirror.

(*b*) What type of mirrors are used (i) as shaving mirror and (ii) as a driving mirror by the side of the driver of a motor car and why ?

(*c*) Using a spherical mirror, an image of a candle flame is formed four times enlarged on a screen 9 cm away from the flame. Find the nature, position and focal length of the mirror. [Ans. 2·4 cm.; concave, 3 cm. from candle]

4. (*a*) Explain the angle of minimum deviation of a prism with the help of a graph.

(*b*) Explain the principle of action of a total reflecting prism.

(*c*) Define emission spectrum and absorption spectrum. How do Fraunhofer lines appear in the solar spectrum ?

(*d*) What do you mean by short-sight of the human eye ?

5. (*a*) Define intensity of a magnetic field at a point.

(*b*) What are magnetic lines of force and what are their characteristics ?

(*c*) Define neutral point. Explain why neutral point for a bar magnet whose north pole is pointing north, lies on the perpendicular bisector of the magnet and not at any other point.

(*d*) The distance between two magnetic poles is 10 cm. The pole strength of one is five times that of the other. The force acting between them in air is equal to 80 dyne. Find out their pole strengths. [Ans. 40; 200 unit]

Or (*a*) Describe briefly the molecular theory of magnetism.

(*b*) The horizontal component of earth's magnetic field at Calcutta is 0·35 oersted and dip angle is 30° N. Find the total intensity of the earth's magnetic field at Calcutta. [Ans. $\cdot 7/\sqrt{3}$ oe.]

(*c*) What do you understand by Curie point ?

(*d*) What will you choose—iron or steel—in making a permanent magnet and why ?

6. (*a*) What is the working principle of a lightning arrester ?

(*b*) Describe with the help of diagrams the charging of a gold-leaf electroscope positively by induction.

(*c*) A positive charge of 20 e.s.u. is situated at a distance of 30 cm from a negative charge of 30 e.s.u. What is the potential at a point lying between the two charges on the straight line joining them at a distance of 10 cm from the first charge ? At what point will the potential be zero ?

[Ans. 0·5 e.s.u.; 12 cm.]

(*d*) What do you mean by an electron-volt ?

Or Describe a Van de Graaff generator and explain how it works.

7. (*a*) State and explain Faraday's laws of electrolysis.

(*b*) What are the defects of a simple voltaic cell ? State the method of rectifying any one of them.

(*c*) A metal plate of surface area 300 sq. cm has to be coated with nickel. What will be the thickness of nickel coating on the plate if a current of 1·5 ampere flows for three hours ? Density of nickel is 8·8 gm/cc and E.C.E. of nickel is 0·000304 gm/coulomb. [Ans. $27 \cdot 3 \times 10^{-4}$ cm]

8. (*a*) *ABC* is a triangle made up of wire. The resistances of the sides *AB*, *BC* and *CA* are respectively 40, 60 and 100 ohms. What is the equivalent resistance between the points *A* and *B* ? [Ans. 32 ohm]

(*b*) Explain with diagram how an unknown resistance can be determined using Wheatstone bridge principle.

(*c*) A moving coil galvanometer has a resistance of 10 ohms and the maximum current that can flow through it is 1 milliampere. What will you do to convert it into an ammeter reading upto 10 amperes ? [Ans. 0·001 ohm shunt]

9. (*a*) State and explain Faraday's laws of electro-magnetic induction.

(*b*) What do you mean by a non-inductive coil ?

(*c*) What is Seebeck effect ? Explain neutral temperature with the help of a diagram.

10. Write short notes on (*a*) alternating current and (*b*) d.c. motor.

11. (*a*) What is photo-electric effect ?

(*b*) Explain Einstein's photoelectric equation.

(*c*) When a radiation of frequency $7{\cdot}5\times10^{14}$ Herz is incident on a metal surface, electrons are emitted whose maximum energy is $1{\cdot}6\times10^{-19}$ joule. What is the lowest frequency of radiation required for emission of electrons from the said metal surface ? $h=6{\cdot}62\times10^{-27}$ erg$-$second. [Ans. $5{\cdot}08\times10^{14}$ Hz]

12. (*a*) Describe an apparatus for production of X-rays and explain its working principle.

(*b*) What are the differences between X-rays and cathode rays ?

(*c*) Write down the postulates of Niels Bohr relating to atomic structure.

13. (*a*) What are N-type and P-type semiconductors ?

(*b*) Explain how a P-N junction acts as a rectifier.

(*c*) What is the nature of β-rays ?

14. Write short notes on (any *two*) :

(*a*) Nuclear fission.

(*b*) Radio-isotope and its use.

(*c*) Basic principle of radio broadcast.

(*d*) Nuclear fusion.

JOINT ENTRANCE EXAMINATION, 1986

1. What will happen to the value of the acceleration due to gravity, g if (*i*) the earth stops rotating, (*ii*) the speed of the earth increases ?

2. A vessel of water filled up to the rim is placed on one of the pans of a balance. Then on the other pan is placed another vessel, also filled up to the rim, but with a piece of wood floating in it. Which of the two is heavier ?

3. A hollow iron ball just floats in water at 10°C. What will happen if both water and the ball are heated to 50°C ?

4. Light from an object falls on a concave mirror forming a real image of the object. If both the object and the mirror are immersed in water, will there be any change in the position of the image ?

5. Two hollow conductors are charged positively. The smaller one is at 50 V and the bigger one is at 100 V. How should they be arranged such that positive charges flow from the smaller to the bigger conductor when connected by wire ?

6. A metallic wire has a certain resistance. If the wire is stretched so that its length is doubled, what happens to the value of its resistance ? It may be assumed that the volume and resistivity of the wire remain unchanged.

7. The liquid inside an electric kettle begins to boil 15 min after being switched on. The heating element consists of a coil of wire 6 m long. How should the heating element be modified so that the liquid inside the kettle begins to boil 10 min after being switched on ? Neglect the loss of heat to the surrounding atmosphere. [Ans. 9 cm]

8. A boy sitting on a vehicle moving at a constant acceleration throws a ball straight up into the air. Where will the ball fall ? Justify your answer.

9. Why is it necessary to note the barometric height when determining the upper or lower fixed point of a thermometer ?

10. Distinguish between a gas and a vapour. The critical temperature of CO_2 is 31·4°C, state if it is a gas at 25°C.

11. What will be the work done on a unit positive charge to move it from one point to another on an equipotential surface ? Explain.

12. Why a potentiometer and not a voltmeter is used for accurate measurement of the e.m.f. of a cell ?

13. Calculate the energy in electron volt corresponding to the compton wavelength 0·0242 A. [Ans. $1{\cdot}7 \times 10^5$ ev]

14. What repulsive coulomb force exists between two protons in a nucleus of iron ? Assume a separation of $4{\cdot}0 \times 10^{-15}$ metre. [Ans. $1{\cdot}44 \times 10^6$ erg]

15. Give interpretation of temperature of a gas from the stand-point of kinetic theory of gas.

16. A simple pendulum with a hollow spherical bob is taken.

(*a*) How will the time-period change if the bob is completely filled with a liquid ?

(*b*) How will the time-period change if the bob is partly filled with a liquid ? Give reasons of your answer.

17. The equation of a progressive wave is given by $y=15 \sin (660\pi t-0{\cdot}02\pi x)$ cm. Calculate the frequency and velocity of the wave. [Ans. 330 ; 3,300 m/sec]

18. What do you mean by elasticity ? Which is more elastic between steel and diamond ? Give reason of your answer.

19. A piece of ice of 100 gms of sp gr 0·9 is floating on a liquid of sp gr 1·06, kept in a vessel, with a portion within liquid. Will there be change of level of liquid when the entire piece of ice melts ? Give reason of your answer.
[Density of water is 1 gm/cc] [Ans. No change]

20. Why do the rising and the setting sun appear red ?

21. Under what circumstance a person, standing on a lift, feel himself weightless ?

22. Why do you adjust deflection near about 45° while performing experiments with tangent galvanometer ?

23. Define atomic weight and atomic number of an element. Explain which one of these two determines the chemical properties of the element.

24. What is meant by half-life period of a radio-active substance ? If half-life period of a radio-active substance be 2 days then after how many days will $\frac{1}{64}$th part of the substance be left behind ? [Ans. 12 days]

25. Show that the equivalent resistance of parallel combination of resistances is less than the smallest resistance of the combination.

26. Steam containing some water at 100°C is passed into an empty 10 kg vessel originally at 15°C. When the temperature has risen to 60°C, the water in the vessel is found to weigh 150 gm. What is the percentage of water in the mixture ? [Latent heat of steam=540 cal/gm ; Specific heat of the material of the vessel=0·12] [Ans. 3·7%]

27. (*a*) Find the angle of incidence and also the angle of deviation of a ray of light that passes symmetrically through a glass prism having refracting angle of 80°. [μ of glass=1·5 ; sin 40°=0·6428 ; sin 74°37′=0·9642]
[Ans. 74°37′; 69°14′]

(*b*) What is the greatest value of the refractive index for which light can pass in this way through an 80° prism ? What is the corresponding angle of deviation ? [Ans. 1·56, 100°]

28. A magnetic needle of mass 7·5 gm has a magnetic moment of 98 units. If the needle is to be maintained horizontal in the northern hemisphere, where should the point of support lie relative to its centre of gravity ? Assume that the vertical component of the earth's magnetic field is 0·25 oersted.
[Ans. $3{\cdot}33\times10^{-3}$ cm. from C.G.]

29. A factory requires a power of 90 kw. It has to be transmitted to the factory through lines of total resistance 2·5 Ω. If 10% of the power generated is lost in transmission, calculate (*i*) the transmission line current, (*ii*) the potential difference at the generating station and (*iii*) the drop of potential due to line resistance. [Ans. 63·26 amp. 1581 V; 158·13 V]

30. A glass plate of length 0·10 metre, breadth 0·0154 metre, and thickness 2×10^{-3} metre, weighs $8{\cdot}2 \times 10^{-3}$ kgm in air. With its long side horizontal, if it is held vertically and its lower half is immersed in water, what will be its apparent weight ? Surface tension of water=0·073 N/m. [Ans. 8·18 gm-wt.]

31. A body initially at 353°K cools down to 337°K in 5 minutes, and to 325°K in 10 minutes. What will be its temperature after 15 minutes and what is the temperature of the surroundings ? [Ans. 316°K, 289°K]

32. Determine the amount of ${}_{84}Po^{210}$ necessary to provide a source of α-particle of 5 millicurie strength. Half-life (T) of ${}_{84}Po^{210}$=138 days.

[Ans. $1{\cdot}1 \times 10^{-6}$ gm]

33. (*a*) A train is whistling, while approaching a platform with a speed of 90 km/hr. The frequency of sound of the whistle is 600 cycles/sec. If the velocity of sound be 325 km/sec, calculate the value of apparent frequency of sound of the whistle to an observer standing on the platform. [Ans. 650]

(*b*) A person, while running towards a cliff with a speed of 4 metre/sec, fires a gun when he is 2 km away from the cliff. Where and when will he hear the echo if the velocity of sound be 330 metres/sec ? [Ans. 11·9 sec., 1952 km]

34. (*a*) A scale made of steel gives correct length at 0°C. A copper rod shows 1·00007 metre when measured by this scale at 10°C. Calculate the actual length of the copper rod at 0°C.

$\alpha_{cu} = 19 \times 10^{-6}/°C$, $\alpha_{steel} = 12 \times 10^{-6}/°C$ [Ans. 1 metre]

(*b*) A metal rod is placed between two supports such that there cannot be any expansion in either direction. Calculate the thermal stress when the temperature of the rod is raised by 30°C.

Coeff. of linear expansion of the metal$=12 \times 10^{-6}/°C$
Young modulus of the metal$=20 \times 10^{11}$ dynes/cm^2

[Ans. $7{\cdot}2 \times 10^{8}$ dynes/sq. cm.]

35. (*a*) An object 5 cm long is placed 12 cm from a convex lens of focal length 8 cm. Find out the position of the image. What would be the size of the image ? [Ans. 24 cm opposite to the lens ; 10 cm.]

(*b*) Explain with diagram the half-wave rectification of alternating current by using vacuum tube diode.

WEST BENGAL HIGHER SECONDARY EXAMINATION, 1987

SECOND PAPER

Group—A

1. (a) State the two laws of reflection of light at a plane mirror. Explain with the help of a diagram the lateral inversion of an image formed by such reflection.

(b) Two plane mirrors are inclined such that for any angle of incidence on a mirror, the reflected ray, obtained after reflections from the two mirrors will be parallel to the incident ray. Find the angle of inclination between the mirrors and give a neat ray diagram.

(c) Why is a cinema screen white and rough? What would be the harm if the screen be polished?

2. (a) Explain with neat diagram the formation of image of an extended object by reflection from a convex mirror. Obtain the relation between object distance, image distance and the focal length of the mirror.

(b) An object is placed at right angle to the principal axis of a concave mirror of focal length 25 cm and the image is formed in front of the mirror at a distance of 100 cm from the mirror. Find the position of the object and the magnification of the image. [Ans. 33·3 cm; 3]

(c) Explain how you would determine whether a mirror is concave, convex or plane.

3. (a) Write down the laws of refraction on plane surface. What is refractive index? Deduce the relation, $\mu = \dfrac{\sin[(A+\delta m)/2]}{\sin[A/2]}$ in case of refraction of light through a prism, assuming that in case of minimum deviation of emergent ray, the angle of incidence is equal to the angle of emergence. μ, A and δm have their usual significance.

(b) Show by diagram how an incident ray can be deviated by 180° with the help of a prism.

4. (a) If an object be placed at a distance of 60 cm from a convex lens of focal length 20 cm, a real image is formed behind the lens. Obtain the position of the image and its magnification. [Ans. —30 cm ; $\frac{1}{2}$]

(b) What do you mean by pure and impure spectrum? Describe in brief how a pure spectrum of white light can be obtained on a screen.

(c) Why does the colour of a green body appear green when it is illuminated by white light? What will its colour look like when it is illuminated by yellow light? Explain your answer.

Group—B

5. (a) Define the magnetic axis and magnetic length of a magnet.

(b) Describe in brief the production of magnetism by induction. Explain the statement "Induction precedes attraction". How can you explain induction in the light of molecular theory of magnetism?

(c) What are self-demagnetisation and magnetic keeper?

Or

5. (a) State Coulomb's laws of force between two magnetic poles and hence define unit magnetic pole. Define intensity at a point in a magnetic field. Two similar poles of strengths 9 and 16 C. G. S. units are separated by a distance of 14 cms. Obtain the position on the line joining them where the magnetic intensity will be zero. [Ans. 6 cm ; 8 cm.]

(b) Define magnetic dip at a place. What is understood by the statement that the dip at Calcutta is 31°N?

6. (a) With the help of the electron theory, discuss the origin of electricity. Explain electric induction on the basis of this theory.

Write down the uses of gold-leaf electroscope. How can you determine the ature of electric charge on a body, which is negatively charged with its help?

(b) Prove by a simple experiment that charge resides on the outer surface of a charged conductor.

Or

6. (a) Define capacitance of a capacitor. Two capacitors are combined (i) in series and (ii) in parallel. Obtain the relation between the equivalent capacitance and the capacitances of the capacitors for the two cases.

(b) What is an equipotential surface?

(c) What are the factors on which the capacitance of a parallel plate capacitor depends?

Group—C

7. (a) State Ohm's law and verify the law experimentally. A galvanometer of resistance 200 Ω, a coil of resistance 20 Ω and a cell of e.m.f. 2V and negligible internal resistance are connected in series and a conductor of resistance 2 Ω is connected in parallel with the galvanometer. Calculate the current through the galvanometer. [Ans. 0·0009 amp.]

(b) Electric current is maintained through a metallic conductor and an electrolytic solution in two different circuits. What will be the effect on currents in the two circuits if the temperature of the conductor and the solution be increased?

8. (a) State Joule's laws of heating by current. Describe in brief the electrical method for the determination of mechanical equivalent of heat and deduce the formula you use.

(b) What do you understand by 220V, 66W bulb? The bulb is run by connecting it to the 220V main supply. Calculate the resistance of its filament when it is incandescent. [Ans. 733·3 ohm]

9. (a) Explain Fleming's Left hand rule and apply the same to explain with diagram the action of Barlow's wheel. Mention the factors on which the rate of rotation of the wheel depends.

(b) What is Seebeck effect?

10. (a) What do you mean by induced e.m.f. and induced current? State and explain Lenz's law of electro-magnetic induction. Justify the law from the principle of conservation of energy. What would be the direction of induced current in a closed coil when a bar magnet is moved towards the coil with its N-pole towards the coil?

(b) What do you mean by self-induction and mutual induction?

Group—D

11. (a) Describe in brief the production of cathode rays. Mention the principal properties of cathode rays.

(b) What do you mean by thermionic emission?

(c) Express an electron-volt in ergs.

12. (a) Explain with diagram the full wave rectification of an alternating current by a diode valve.

(b) Write down the characteristics of photoelectric effect.

(c) What is threshold frequency?

13. (a) How does a semiconductor differ from a conductor and an insulator? Give two examples of each of these three classes of materials. Draw the current voltage characteristic of a p-n junction diode and explain the same.

(b) Define amplification factor of a triode valve.

14. (a) What is radioactivity? What do you mean by artificial transmutation of elements? Explain how nuclear fission is utilised for the benefit of mankind.

(b) When ${}_4Be^9$ is bombarded by α-particles, neutron is ejected and another atom is produced. Write down the equation of reaction.

[Ans. ${}_4Be^9 + {}_2He^4 \rightarrow {}_6C^{12} + {}_0n^1$]

JOINT ENTRANCE EXAMINATION, 1987

1. Explain whether the magnitude of resultant of two vectors of equal magnitude can be equal to the magnitude of either of the two vectors.

2. The earth is moving round the sun in a circular orbit, say and is acted on by a force due to the sun. Is sun doing any work on the earth during this movement? Explain.

3. Assuming that the earth moves round the sun in a circular orbit, show that the area swept out by the earth in unit time is constant.

4. A block of wood floats in water contained in a closed vessel provided with a small hole at the top. The hole is connected to a compression pump from which air is introduced into the vessel half filled with water. Explain whether the block will rise or sink further.

5. What would be the pressure at a point inside a liquid, kept in a vessel on an artificial satellite, moving along a circular path round the earth? Give reason of your answer.

6. What fraction of the whole volume of a glass vessel should be filled with mercury in order that the volume of the flask not occupied by mercury may remain the same at all temperatures? (Coefficient of linear expansion of glass is $9 \times 10^{-6}/°C$ and coefficient of volume expansion of mercury is $1.8 \times 10^{-4}/°C$.) [Ans. 3/20]

7. There are two thermometers—one with spherical bulb and the other with cylindrical bulb of equal volume. Which of them will respond more quickly to the change of temperature? Explain your answer.

8. Two identical pieces of ice flying towards each other with equal velocities collide and converted into vapour due to impact. Find the minimum possible velocities of the pieces just before impact if their temperature is —12°C and sp. heat is 0.5. [Ans. 229×10^3 cm/s (nearly)]

9. An iron and a copper strip of identical dimensions are rivetted together. Explain what will happen if the combination be heated.

10. The lower and upper fixed points of a thermometer are 0·2° and 101·7° respectively. What would be the reading of this thermometer corresponding to 60°C? [Ans. 61·1°]

11. A clock having marks instead of numbers on its dial appears to indicate 7-25 when viewed in a plane mirror. What is the correct time? [Ans. 5·35]

12. If a green body be illuminated by white light and red light in turn what would be the apparent colour of the body? Explain your answer.

13. An object 5 cm long is placed at right angles to the principal axis of a concave mirror at distance of 75 cm from the mirror. If the radius of curvature of the mirror be 60 cm, obtain the position and size of the image.

[Ans. 50 cm; 3·33 cm]

14. The near-point of a person is 50 cm. He wants to bring it at the least distance of normal distinct vision. What type of lens should he use and what is its power? What is the name of this type of defect of the eye? [Ans. concave ; —2D]

15. What should be the maximum angle of a glass prism of refractive index μ so that light cannot emerge from it if incident on the other refracting face? If this prism is immersed in water, emergent light is again obtained. Why?

16. Can electric intensity at a point exist even if the electric potential at the point be zero ? Justity your answer by giving one example.

17. In a thermocouple with one junction at 0°C and the other at t°C, the e.m.f. is $16{\cdot}7t—0{\cdot}019t^2$ microvolt. Determine the neutral temperature for this couple. [Ans. 440°C]

18. One 220V-60W carbon filament bulb is connected in series with one 220V-60W metal filament bulb and the combination is placed across 220V mains. Which bulb will glow more? Explain you answer. [Ans. carbon filament bulb.]

19. A conducting wire is bent in the form of a circle and a straight conductor AB outside but near the circle. If the current flowing from A to B gradually increases in magnitude, will there be any current in the circle? If so, in what direction?

20. Equal number of identical cells are joined in series and also in parallel. Under what condition, the currents in both the cases will be the same?

21. The velocity of sound in solids is generally greater than that in gases. Can you give any explanation for this discrepancy?

22. Determine the percentage change in velocity of sound when the temperature changes from 10°C to 20°C. [Ans. 3·4%]

23. A vertical pillar of soft iron, introduced partly in the earth, is found to be magnetised after a long time. What would be the nature of the polarity at the top of the pillar if it is in the northern hemisphere? Explain your answer.

24. How do N-type of semiconductors differ from P-type of semiconductors?

If suitable amount of arsenic be introduced into a crystal of germanium, what type of semiconductor be obtained?

25. Positively charged protons and neutral neutrons are packed inside atomic nucleus, but protons, though similarly charged, do not repel each other. How do you account for the phenomenon?

26. A loaded lorry weighing 5000 kilogram freewheels down an incline of 1 in 40 at a constant speed of 18 km/h. How much horse power would the engine of the lorry need to develop to drive it up the same incline at the same speed, assuming the frictional resistance to be the same in each case? [Ans. 42 H.P.]

27. Mercury is poured into two cylindrical communicating vessels with different cross sections of 15 and 5 sq cms. A cubical iron block of 2 cm thickness is dropped into the broader vessel and as a result the level of mercury in it rises. Then water is poured into the vessel until mercury reaches the previous level. Find the height of the water column. (Sp. gravity of iron=7.8) [Ans. 13 cm]

28. (*a*) Water is boiling in a kettle, placed on a stove. The area and thickness of the bottom of the kettle are respectively 300 sq cm and 3 mm. If the rate of formation of steam per minute be 3 gms, calculate the difference of temperature between outer and inner surface of the bottom of the kettle.

Latent heat of vaporisation of water=540 cals/gm conductivity of the material of the kettle=0·5 C.G.S. unit. [Ans. 0·054°C]

(*b*) The height of a waterfalls is 200 metres. What will be the difference of temperature of water at the top and the bottom of the falls if 90% of the potential energy of water at the top be converted to heat and the heat be within water?

$J=4{\cdot}2$ Joules/cal $g=980$ cm/sec^2 [Ans. 0·42°C]

29. When 100 gm of a substance, initially in the solid state, was supplied with heat at a steady rate of 200 calories per minute, its temperature was observed

(*i*) to increase from −5°C to 0°C in a period of 1·25 mins;

(*ii*) to remain constant at 0°C for a period of 40 mins;

(*iii*) to increase thereafter at a steady rate of 2°C per minute until it reaches 100°C;

(*iv*) to remain constant at 100°C while the amount of the substance appears to decrease to approximately one half of its original volume in a time of 135 mins.

What information regarding the thermal constants of the material can be derived from the above observations? Neglect any heat loss.

[Ans. Sp-heat=0·5 ; Latent heat=80 cal/gm; Sp. ht in liquid state=1; Latent heat of vapourisation=540 cal/gm.]

30. Two thin equiconvex lenses of focal lengths 10 cm and 20 cm are placed inside a thin-walled glass box with curved sides, side by side such that these are tightly filled inside. The glass box is then filled with water and used as a lens. Determine the position of the object so that an image twice the size of the object is formed due to this lens combination. R.I. of glass=$\frac{3}{2}$, R.I. of water=$\frac{4}{3}$.

[Ans. virtual→0·42 cm; real→1·25 cm.]

31. Show that light ray will not emerge after refraction through a prism if its refracting angle be greater than twice the critical angle between the material of the prism and the surrounding medium.

32. (*a*) A coil of resistance 100 ohms is placed in a magnetic field of 1 milliweber. The coil has 100 turns and a galvanometer of 400 ohms, resistance is connected in series with it. Find the average e.m.f. and current if the coil is moved in $\frac{1}{0 1}$the second from the given field to a field of strength 0·2 milliweber.

[Ans. 0·8V ; $1{\cdot}6\times10^{-3}$ amp.]

(*b*) 8 spherical liquid drops each of 2 mm diameter charged with 5 micro-statcoulomb coalesce to form a single drop. What is the potential in volts at the surface of the drop so formed? [Ans. 10^{-6} stat volt.]

33. What is fundamental difference between a voltmeter and an ammeter? Explain :

(*i*) How a voltmeter reading up to 150 volts can be converted to an ammeter of 8 amperes range. The resistance of the voltmeter is 300 ohms.

[Ans. 20 ohm shunt]

(*ii*) How an ammeter of 0.05 amp. range can be made to read up to 5 amperes. The resistance of the ammeter is 5 ohms. [Ans. ·05 ohm shunt]

34. A wire is stretched between two bridges 25 cm apart and is subjected to an elongation of 0·04 cm by the tension. If the density and Young's modulus of the material of the wire be respectively 10 gm/cc and 9×10^{11} dynes/cm^2, calculate the frequency of the fundamental of this stretched wire. [Ans. 120]

35. In what way is full wave rectification better than half wave rectification? What is the necessity of a rectifier? Draw the circuit diagram of a full wave junction diode rectifier.

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SECOND PAPER

Group—A

1. (a) Show that when a plane mirror is rotated through an angle then the reflected ray of a given ray of light will be rotated twice the angle.

(b) Establish the relation between object distance, image-distance and the focal length of a concave mirror in case of formation of image by reflection on it.

(c) Two plane mirrors are mutually inclined at 60°. If a point object is placed on the bisector of the angle, explain with diagram how many images in all will be formed. [*Ans.* 5]

(d) Why are the shadows of birds not seen when they fly at a height ?

2. (a) Write the conditions for total internal reflection and derive the relation between critical angle and refractive index.

(b) Show that when a ray of light is incident on a thin prism, the deviation of the ray is given by $\delta=(\mu-1)A$, where A=the refracting angle of the prism, and μ=the refractive index of the material of the prism.

(c) A ray of light is incident at an angle of 60° on the face of a prism which has an angle of 30°. The ray emerges out of the prism through the opposite face. If the imerging ray makes an angle of 30° with the incident ray, show that the imergent ray is perpendicular to the face of the prism through which it emerges.

(d) Explain how the mirages are formed in a desert.

3. (a) What do you understand by the terms principal axis and principal focus of a lense ?

(b) A convex lens is placed between an object and a screen. For two positions of the lens, real images are produced on the screen. If the lengths of the two images be I_1 and I_2 and that of the object be O, prove that $O=\sqrt{I_1 I_2}$.

(c) Show that the image formed by a concave lens will always be virtual and diminished.

(d) If a convex lens be placed at a distance of 15 cm from an object, a real image, magnified four times is formed. For which position of the lens will the emage be virtual and magnified three times ? [*Ans.* 8 cm]

4. (a) What do you mean by accommodation of human eye? What is least distance of distinct vision ? Mention and explain the various defects of vision of the human eye. What is the advantage of having two eyes ?

(b) A short-sighted person can see clearly object at a distance of 20 cm. What type of lens will be used to see clearly an object at a distance of 100 cm ? Calculate the power of the lens. [Concave ; 4*D*]

Group—B

5. (a) "Repulsion is a sure test of magnetisation."—Explain the statement. For what reason can a magnet have similar poles at this two ends ?

(b) There are three similar bars, of which one is a magnet, one a magnetic substance and the other one a non-magnetic substance. Without suspending the bars, how would you identify them ?

(c) Explain magnetic saturation in the light of the molecular theory of magnetism.

(d) If a vertical iron pillar is kept embeded partly into earth for a long time, it is found to be magnetised. What type of pole would be formed at the top of such a pillar in the northern hemisphere ? Answer with justification.

Or, 5. (a) Define unit pole and pole-strength of a magnet. Prove experimentally that the pole-strength at the two ends of a magnet are equal and opposite.

(b) Distinguish between para and diamagnetic substances. How can you transform a ferro-magnetic substance to a para-magnetic one ? What is magnetic permeability and magnetic susceptibility ?

6. (a) State coulomb's law of force between two point charges and hence define unit charge and dielectric constant.

(b) Define electric intensity at a point. Can electric intensity at a point exist if the electric potential at the point be zero ? Explain your answer with example.

(c) Two capacitors of capacitance 20 μF and 60 μF are connected in series. If the potential difference between the two ends of the combination is made 40 volts, calculate the potential difference between the ends of each of the capacitors. [*Ans.* 30 v ; 10 v]

Or, 6. (a) What do you mean by electric lines of force ? What are their properties ? Show that electric lines of force are perpendicular to equipotential surface. How would you show experimentally that the surface of a charged conductor is equipotential ?

(b) An insulated conductor is positively charged. Another insulated uncharged conductor is brought near the first conductor. Will there be any change of potential of the charged conductor ? Explain your answer and justify the same experimentally. What would happen if the second conductor is connected to the earth ?

Group—C

7. (a) What is a storage cell ? Why it is so called ?

(b) Two identical cells of e.m.f. 1.5 volts are connected in series. When the combination is connected in series with a resistance and a galvanometre, the current through the circuit is 1 amp. If two cells be connected in parallel, then the current through circuit becomes 0.6 amp. Calculate the internal resistance of the cell. [*Ans.* $\frac{1}{3}$ ohm]

(c) The e.m.f. of two cells is E volts and their internal resistances are r_1 and r_2 respectively. They are connected in series and the combination is connected to a resistance R such that the terminal potential difference across the plates of the first cell is zero. Calculate the value of R. [*Ans.* $R = r_1 - r_2$]

(d) Define Volt and Ampere.

8. (a) The liquid inside an electric kettle begins to boil 15 min after being switched on. The heating elements consists of a coil of wire 6 metre long. How should the heating element be modified so that the liquid inside the kettle begins to boil 10 min after being switched on. Neglect the loss of heat to the surrounding atmosphere. [*Ans.* To be shorten 2 metre]

(b) A 25 watt and a 100 watt bulb are joined in series and connected to the mains. Explain with reason which bulb will glow brighter.

(c) What do you mean by Peltier effect and Seebeck effect ? What is the difference between Joule effect and Peltier effect ? Mention one practical application of Peltier effect.

9. (a) State Faraday's laws of electrolysis. Define electro-chemical equivalent of an element.

(b) Explain with diagram how currect can be measured by the principle of electrolysis.

(c) A tangent galvanometer and a silver voltameter are connected in series and current is passed through them for sometime. On the cathode of voltameter 0.112 gm of silver is deposited in 5 minutes and the deflection of the galvanometer remains 30° all through. What is the value of the reduction factor of the galvanometer ? [E.C.E. of silver=0.00112 gm/coulomb.] [*Ans.* 0.07]

10. (a) State and explain Faraday's laws relating to electromagnetic induction.

(b) Explain with the help of diagram the principle of action of a simple a.c. dinamo. State how the e.m.f. generated by the dynamo stated above will be affected in the following cases :—

(i) Intensity of the magnetic field is doubled.

(ii) Rate of rotation of the coil is decreased.

Group—D

11. (a) Describe in brief the production of X-rays. State three properties of X-rays which are similar to those of visible light.

(b) Mention two major uses of X-rays.

12. (a) How was photo-electric-emission explained by Einstein ?

(b) The maximum K.E. of photo-electrons, emitted from a metal due to incidence of light of wavelength 5000 Å is 0.3 eV. Find out the work function of the metal. [$h=6.640\times10^{-27}$ erg. sec. ; 1 eV$=1.6\times10^{-12}$ erg.] [*Ans.* 2·19 eV]

(c) Draw the anode and mutual characteristic curves of a triode valve.

13. Write down the postulates of Bohr model of an atom. How can the origin of hydrogen spectra be explained by Bohr's theory ? What is meant by 'heavy water' ?

14. Write short notes on the following :

(i) n and p type of semiconductor.

(ii) Semiconductor diode and its uses.

JOINT ENTRANCE EXAMINATION, 1988

1. Two stones are let fall from a roof, the first from rest and the second with an initial horizontal velocity. Which one will hit the ground earlier ? Explain your answer.

2. Between escape velocities in case of earth and moon—which is greater ? Justify your answer.

3. Is Archemedes' principle applicable to freely falling bodies ? Explain.

4. A body of mass 25 gms is under water at a depth of 50 cms. If the sp.gr. of the material of the body be 5.0 and g=980 cms/sec^2 ; find the amount of work required to lift it very slowly to the surface. [*Ans.* 9.8×10^5 ergs]

5. A parachutist having weight 75 kg descends with a constant velocity. What is the force of air resistance acting on him ? [*Ans.* 75 kgwt]

6. A rectangular parallelopiped of mass M and sides l, $2l$, $3l$ is placed on a horizontal surface. What position will be the most stable ? What is the reason behind it ? [*Ans.* $3l$-base ; l-height]

7. Two identical glass spheres filled with air are connected by a glass tube whose middle portion is horizontal containing a pellet of mercury. Air in one vessel is at 0°C and another at 20°C. Will the column of mercury be displaced if both the vessels are heated through 10°C and if so, in which direction ?
[*Ans.* Towards higher temp. side.]

8. Heat is supplied at a uniform rate to a piece of ice. The melting begins after 2 seconds and is complete in the next 20 seconds. Calculate the initial temperature of the ice. (Sp. heat of ice=0.5, latent heat of fusion of ice=80 cal/gm).
[*Ans.* -16°C]

9. Bodies of different colour in the same enclosure come to the same final temperature. Explain how this shows that a body which is a good radiator of heat must also be a good absorber.

10. When a hot body warms a cold body, do their temperatures change in equal magnitudes ?—Explain.

11. Sawdust is found to be thermally less conducting than the piece of wood from which the dust is derived.—Why ?

12. What will happen to the image if one half of the converging lens is closed by an opaque screen ? Hence explain what happens when the aperture opening of camera lens is reduced by irising.

13. Explain how you can identify a telescope and a microscope from their appearance.

14. A gas is compressed to half of its original volume in two ways, first rapidly and then slowly. In which case the work done is greater and why ?

15. To reduce the response of a galvanometer by 25 times a shunt is joined with it. What is the resistance of shunt if the internal resistance of the galvanometer is 1000 ohm ? [*Ans.* 41.67 ohm]

16. Two insulated conductors A and B are connected by a metal wire. A positively charged rod is placed near A (on the further side from B). What will be difference of potential between A and B ? [*Ans.* Zero]

17. A closed circuit consists of n cells connected in series. Each element has an e.m.f. ϵ and internal resistance r. The resistance of the connecting wires is assumed to be zero. What will be the reading of a voltmeter connected to the terminals of one of the cells ? It is assumed the voltmeter has an infinitely high resistance, as usual. Can it be realised in practice ? [*Ans.* Zero]

18. A voltmeter with resistance 100 ohms connected to the terminals of a cell shows a difference of 2 volts. When this cell is connected across a resistance of 15 ohms, an ammeter of resistance 1 ohm connected to the curcuit shows a current of 0.1 amp. Find the e.m.f. of the cell. [*Ans.* 2.1 Volt]

19. If the length of the closed pipe producing a tone of 400 Hz is 20 cm, find the length of the open pipe which produces a tone of 600 Hz at the same temperature. [*Ans.* 26.67 cm]

20. Give physical interpretation of magnetic suspectibility and permeability.

21. A steel spiral spring has an unstretched length of 8 cm and when a weight is hung on it, its length becomes 14 cm. Calculate the periodic time of oscillation of the weight, if displaced vertically. [*Ans.* 0·49 sec]

22. Indicate the zero potential line of a bar-magnet.

23. How will a flexible free conductor placed near a strong long bar-magnet arrange itself if a current is passed through it from the top to the bottom ? The north pole of the magnet is arranged vertically upward.

24. A small metal ball is suspended by means of a weightless string between the plates of a large plane capacitor. If the ball is charged to $+q$ and the upper plate is charged positively, what will be the period of oscillation of such pendulum ?

$$\left[\textit{Ans.} \quad 2\pi\sqrt{\frac{l}{g+\dfrac{QE}{m}}} \right]$$

25. Why does the sea water seem much darker directly below an aircraft flying over the sea than that of when flying at the horizon ?

26. (a) The length of a second pendulum is taken as unity, find the value of g. [*Ans.* π^2 Sec^{-2}]

(b) A thin steel ring is heated to a temperature of 95°C. At this temperature it just fits over a steel cylinder which has a diameter of 10 cm at 20°C. If the system is allowed to cool until the temperature of both the ring and the cylinder is 20°C, what will be the stress in the ring ? Assume that the cylinder does not change its diameter. Young's modulus $=21\times10^5$ kg/cm^2 and coefficient of thermal expansion, $\alpha=12\times10^{-6}$ per degree C. [*Ans.* 1890 kg/cm^2]

27. A tube contains a column of mercury isolating a certain volume of air form the medium. The tube can be turned in a vertical plane. In the horizontal position of the tube, the column of air in the tube has the length l_1 and in the vertical position with air column at the top, l_2. Find the length l_3 of the column of air when the tube is inclined at an angel of α to the vertical.

$$\left[\textit{Ans.} \quad \frac{l_1 l_2}{l_2-(l_2-l_1)\cos\alpha} \right]$$

28. Three resistances A, B and C are connected in such a way that their combined equivalent resistance is equal to that of B. If A and B are 10 ohms and 30 ohms respectively and $C \neq O$; find the three possible values of C and the ways in which they are connected in these three cases. [*Ans.* 120, 60 and 22.5 ohm]

29. A man having mass m stands on a rope-ladder which is tied to a free ballon. The ballon is at rest. Find the velocity of the ballon when the man starts to climb the ladder with a constant velocity v relative to the ladder.

$$\left[Ans.\ \frac{Mv}{M+m} \right]$$

30. A solid spherical ball having densities d and volume v floats on the interface of two immiscible liquids. The density of the upper liquid is d_1 and that of the lower one is d_2. What fraction of the volume of the ball will be in the upper and lower liquids, if $d_1 < d < d_2$?

[*Ans.* $(d_2 - d)/(d_2 - d_1)$; $(d - d_1)/(d_2 - d_1)$]

31. A metal block having density 8 gm/cc is suspended by means of a weightless string from a hook of a wooden frame. The tension in the string is 56 gm wt. If the whole system is immersed in a liquid at 40°C, what will be the new tension in the string ? The ambient temperature during the experiment is 20°C. The specific gravity of liquid at 20°C is 1.24. The cubical expansion of the liquid and the material of the metal block are $4 \times 10^{-5}/°C$ and $8 \times 10^{-4}/°C$ respectively. [*Ans.* 47.19 gm-wt]

32. A common hydrometer completely immerses and floats in water of specific gravity 1. It floats with its stem fully above a liquid of specific gravity 1.6. What is the density of a liquid in which that hydrometer floats with 2/3 of its stem above the liquid ? [*Ans.* 1.33 gm/cc]

33. A thin flat glass is placed in between a convex mirror of focal length 20 cm and a point source. The distance between the mirror and the glass plate is 5 cm. What is the distance between the plate and the source if its image produced by the rays reflected from the front surface of the glass plate coincides with the image formed by the rays reflected by the mirror ? How can you establish the coincidence of the images by direct observation ? Trace the path of rays.

[*Ans.* 15 cm]

34. An object is placed at a distance of 36 cm in front of a converging lens of focal length 30 cm. A plane mirror is placed at a distance of 100 cm behind the lens. The angle between the optical axis of the lens and the plane of the mirror is 45°. A tray with water of depth 20 cm is placed beneath the mirror in such a way that a sharp image of the object is formed at the bottom of the ray. What is the distance between the optical axis and the bottom of the tray ? The refractive index of water is 4/3. Draw the ray diagram. [*Ans.* 85 cm]

35. Two strings of length 70 cm and 60 cm are sounded with a tuning fork and produced 5 beats per second. What is the frequency of fork ? [*Ans.* 65 Hz]

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